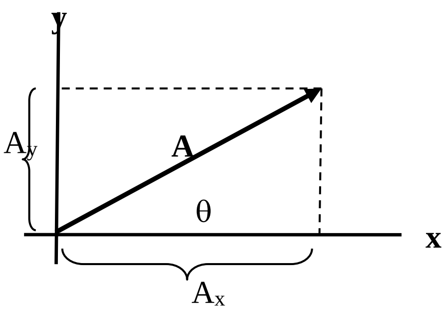
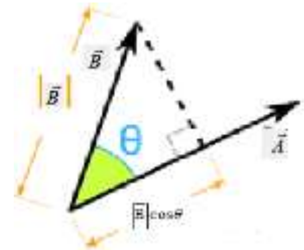
Chapter 3
Summary

Consider vector $\mathbf{A}$, with a direction of θ above the horizontal.



$A_x = |\mathbf{A}|\cos\theta \equiv$ The projection of **A** along x-axis ,
$\equiv$ The component of **A** along x-axis,
$A_y = |\mathbf{A}|\sin\theta \equiv$ The projection of **A** along y-axis
$\equiv$ The component of **A** along y-axis.



Define the two vectors $\vec{A} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)$,
$\vec{B} = \left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$ and the angle between them is θ. Then we can construct the following table:

	Dot-product	Cross-product
Definition	$\vec{A}\cdot\vec{B} = \lvert\vec{A}\rvert\lvert\vec{B}\rvert\cos\theta$	$\vec{A}\times\vec{B} = \lvert\vec{A}\rvert\times\lvert\vec{B}\rvert\sin\theta\ \hat{n}$
Unit vector	$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \cdots = 1,$ $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = \cdots = 0$	$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \hat{0},$ $\hat{i}\times\hat{j} = \hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{i} = \hat{j}$
Expansion	$\vec{A}\cdot\vec{B} = \left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\cdot\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$ $= A_xB_x + A_yB_y + A_zB_z$	$\vec{A}\times\vec{B} = \left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\times\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$ $= \left(A_yB_z - A_zB_y\right)\hat{i} - \left(A_xB_z - A_zB_x\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$ $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

Q1: A vector $\vec{A}$ is added to the sum of two vectors $\vec{B} = 3.0\hat{i} - 2.0\hat{j} - 2.0\hat{k}$ and $\vec{C} = 2.0\,\hat{i} - \hat{j} + 3.0\hat{k}$ such that $\vec{A} + \vec{B} + \vec{C} = \hat{k}$. The vector $\vec{A}$ is:

Answer:

$$\vec{A} + \vec{B} + \vec{C} = \hat{k} \quad \Rightarrow \quad \vec{A} + 3.0\hat{i} - 2.0\hat{j} - 2.0\hat{k} + 2.0\,\hat{i} - \hat{j} + 3.0\hat{k} = \hat{k}$$
$$\Rightarrow \quad \vec{A} + 5.0\hat{i} - 3\hat{j} = 0$$

A) $-5.0\,\hat{i} + 3.0\,\hat{j}$
B) $5.0\,\hat{i} - 3.0\,\hat{j}$
C) $-3.0\,\hat{i} - 1.0\,\hat{j}$
D) $-1.0\,\hat{i} + 3.0\,\hat{j}$
E) $3.0\,\hat{j}$

Q2. Consider the vector $\vec{A} = 3.0\hat{i} + 4.0\hat{j}$. Which of the following vectors is perpendicular to vector $\vec{A}$:

Answer:

$$\vec{A} \cdot \vec{B} = 0 \quad \Rightarrow \quad A_x B_x + A_y B_y = 0 \quad \Rightarrow \quad \frac{B_y}{B_x} = -\frac{A_x}{A_y} = \frac{-3}{4} \text{ or } \frac{3}{-4}.$$

A) $4.0\hat{i} - 3.0\,\hat{j}$
B) $6.0\,\hat{i} - 4.0\,\hat{j}$
C) $4.0\hat{i} + 3.0\,\hat{j}$
D) $-3.0\hat{i} - 4.0\,\hat{j}$
E) $3.0\hat{i} + 4.0\,\hat{j}$

Q3. Find the sum of the following two vectors: $\vec{A}$: 8.66 in +*x*-direction, $\vec{B}$: 10.0 , at 60° from +*y*-axis measured counterclockwise.

Answer:

$$\vec{A} + \vec{B} = 8.66\,\hat{i} + 10.0\,\mathrm{Cos}\left(60^o\right)\hat{j} - 10.0\,\mathrm{Sin}\left(60^o\right)\hat{i} = 5.00\,\hat{j}.$$

A) $5.00\,\hat{j}$
B) $3.00\hat{i} + 4.00\,\hat{j}$
C) $6.00\hat{i} + 8.00\,\hat{j}$
D) $8.66\hat{i} + 10.0\,\hat{j}$
E) $\hat{i} + 16.7\,\hat{j}$

Q5.

A vector $\vec{B}$, when added to the vector $\vec{C} = 3.0\,\hat{i} + 4.0\,\hat{j}$, yields a resultant vector that is in the positive *y* direction and has a magnitude equal to that of vector $\vec{C}$. What is the magnitude of vector $\vec{B}$?

A) 3.2
B) 1.9
C) 2.4
D) 0.2
E) 0.6
Answer:
Define B = *Bx*i + *By*j then the resultant of B and C is
B + C = (B_x + 3.0)i + (B_y + 4.0)j =(*0*)i + ($\sqrt{3^2+4^2}=5$)j
Then:
B_x + 3.0 = 0 => $B_x = -3.0$, B_y = 5-4=1
$$|B| = \sqrt{1^2+3^2} = \sqrt{10} = 3.16$$
The magnitude of vector B is 3.2.

```
CC = {3, 4, 0}; BB = {a, b, 0};

Solve[CC + BB - {0, Norm[CC], 0} == 0, {a, b}]

{{a → -3, b → 1}}

Norm[BB] /. {a → -3, b → 1} // N

3.16228
```

Q6.
A man walks 3.00 km due East, then 7.00 km 25° South of East, and then 12.0 km due South. What is the final location, in km, of the man from the starting point?

A) $9.34\,\hat{i} - 15.0\,\hat{j}$
B) $6.21\,\hat{i} - 13.2\,\hat{j}$
C) $-8.04\,\hat{i} + 11.1\,\hat{j}$
D) $-15.0\,\hat{i} - 8.45\,\hat{j}$
E) None of the other answers

Ans:
$$A_x = 3 + 7\cos\left(25^\circ\right) \approx 9.34,$$
$$A_y = -12 - 7\sin\left(25^\circ\right) \approx -15$$

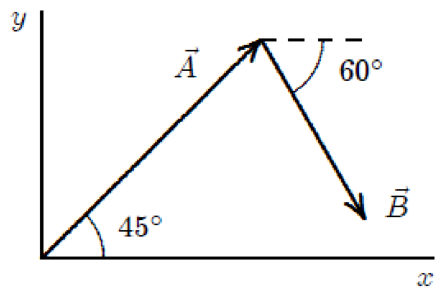
Q7.

Consider two vectors, $\vec{A}$ and $\vec{B}$, each has magnitude L and having an angle 60° between them. The magnitude of the product $\left(\vec{A}\times\vec{B}\right)\cdot\vec{A}$ is:

A) 0
B) $3L^2/2$
C) $L^2/2$
D) $3L^2$
E) $3L^2/4$

$$\left(\vec{A}\times\vec{B}\right)\cdot\vec{A}=\vec{C}\cdot\vec{A}=\left|\vec{C}\right|\left|\vec{A}\right|\cos(90^o)=0$$

Q8:
In the diagram, vector ($\vec{A}$) has magnitude 12 m and vector ($\vec{B}$) has magnitude 8.0 m. The y-component of vector ($\vec{A}-\vec{B}$) is about:



A. 4.48 m
B. 7.61 m
C. 15.4 m
D. 12.5 m
E. None of the other answers

Q9:
Let $\vec{A}=\hat{\imath}+2.00\,\hat{\jmath}+2.00\,\hat{k}$ and $\vec{B}=3.00\,\hat{\imath}+4.00\,\hat{k}$. The angle between these two vectors is:

A.21.0°
B. 87.2°
C. 62.5°
D. 42.8°
E. None of the other answers

Q10:
A certain vector ($\vec{A}$) in the xy plane has an x component of A_x = 4.0 m and a y component of A_y = 10 m. It is then rotated in the xy plane so its x component is doubled. Its new y component is about:

A. 20 m
B. 7.2 m
C. 5.0 m
D. 4.5 m
E. None of the other answers

Answer:

The rotation will not change the magnitude of $\vec{A}$. If we have the vector $\vec{A'}$ after rotation, then

$$\left|\vec{A}\right|^2 = \left|\vec{A'}\right|^2 \Rightarrow 4^2 + 100^2 = 8^2 + y'^2$$

$$y' = \underline{7.2\ \text{m}}$$

1. Vector **A** = (5.0***i*** + 3.0**j**) m, and vector **B** is 6 m in length and making 120 degrees angle with +ve x–axis. Find A–B.

a. (8.0*i* – 2.2***j***) m b. (8.0***i*** + 8.2***j***) m c. (–2.0***i*** + 8.2***j***) m d. (2.0***i*** – 5.6***j***) m

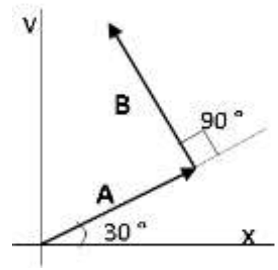
2. If **a** = (3.0***i*** + 4.0***j***) m and **b** = (5.0***i*** –2.0***j***) m, find the angle between the two vectors.

a. 75 ° b. 31 ° c. 82 ° d. 55 ° e. 93 °

3. For the following three vectors; **A** = 2***i***+3***j***+4***k***, **B** = 4***i***+4***j*** and **C** = 2***i***+2***k***, find **A.(B×A)**.

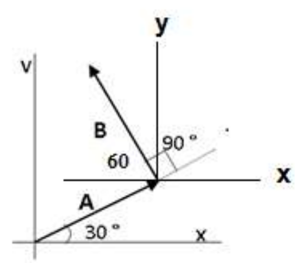
a. 0 b. –16***i*** +16***j*** –8***k*** c. 16***i*** –16***j*** +8***k***
d. 8***i*** – 8***j*** –8***k*** e. –8***i*** + 8***j*** +8***k***

4. The two vectors **A** and **B** shown in the Figure have equal magnitudes of 10.0 m. Find the magnitude of the resultant of these vectors and the angle it makes with the positive x-axis.



a. 14.1 m, 75 ° b. 10.0 m, 90 °
c. 12.0 m, 60 ° d. 16.0 m, 30 ° e. 20.0 m, 45 °

Answer:
For vector B, take the axis as shown in the figure



$$\vec{A} = 10.0\,\mathrm{Cos}\left(30^o\right)\hat{i} + 10.0\,\mathrm{Sin}\left(30^o\right)\hat{j}$$

$$\vec{B} = -10.0\,\mathrm{Cos}\left(60^o\right)\hat{i} + 10.0\,\mathrm{Sin}\left(60^o\right)\hat{j}$$

$$\vec{A} + \vec{B} =$$

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In[24]:= A = 10 Cos[30 Degree] i + 10 Sin[30 Degree] j
Out[24]= 5 √3 i + 5 j

In[25]:= B = -10 Cos[60 Degree] i + 10 Sin[60 Degree] j
Out[25]= -5 i + 5 √3 j

In[26]:= A + B // Simplify
Out[26]= 5 ((-1 + √3) i + (1 + √3) j)

In[28]:= Mresultant = Sqrt[(5 (-1 + √3))^2 + (5 (1 + √3))^2] // N
Out[28]= 14.1421

In[32]:= θ = ArcTan[(1 + √3)/(-1 + √3)] * 180 / π // N
Out[32]= 75.
```

5. A vector in the xy-plane has a magnitude of 25.0 and an x-component of 12.0. The angle that it makes with the positive x-axis is:

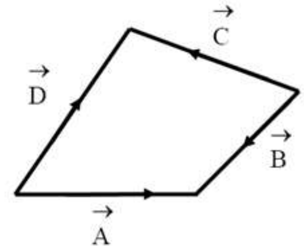
a. 61.3 ° b. 25.6 ° c. 28.7 ° d. 64.3 °

6. The unit vectors in the positive directions of the x, y, and z axes are labeled ***i***, ***j***, and ***k***. The value of **$\boldsymbol{i}.(\boldsymbol{j} \times \boldsymbol{k})$** is:

a. +1 b. –1 c. 0 d. –i e. +j

7. Unit vectors ***i***, ***j***, ***k*** have magnitudes of unity and are directed in the +ve directions of the x, y, z axes. The value of $\boldsymbol{k}.(\boldsymbol{k} \times \boldsymbol{i})$ is:

a. 0 b. –1 c. +1 d. i e. j



8. The Figure shows four vectors **A**, **B**, **C**, **D**. Which one of the following statements is correct?

a. C = D + B – A
b. **C** = **A** + **B** + **D**
c. **C** = –**D** – **B** + **A**
d. **C** = **A** – **B** + **D**
e. **C** = –**A** – **B** – **D**

9. If we have two vectors **A** = (a ***i*** – 2 ***j***) and **B** = (2 ***i*** + 3 ***j***) such that **A.B** = 4, find the value of a.

a. 5 b. 4 c. 0 d. –5 e. –4

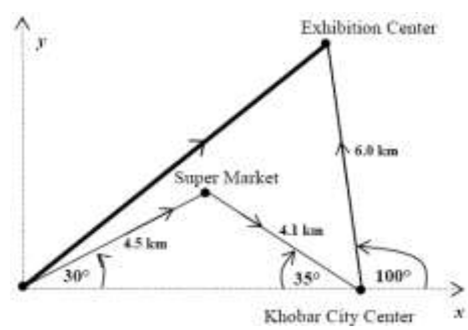
10. Which of the following is NOT a unit vector?

a. ½ (***i*** + ***j***) b. vector **a** / |a| c. ***j*** × ***i***
d. (***i*** + ***j*** + ***k***) / √3 e. 0.6 ***j*** + 0.8 ***k***

11. What is the angle between the two vectors **A** = (***i*** –2***j*** +2***k***) and **B** = (–2***i*** +***j*** +2***k***) ?

a. 90 ° b. 30 ° c. 45 ° d. 60 ° e. 0 °

12. A student makes the journey from KFUPM to a Super Market and then to Khobar City Center and finally to Exhibition Center. The magnitude and the direction of each of these displacements are indicated in the Figure. Give the resultant displacement from KFUPM to the Exhibition Center in unit vector notation.

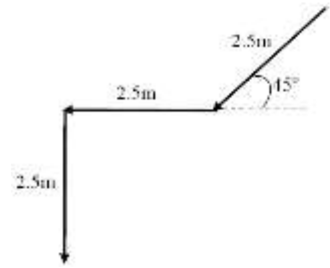


a. (6.2 ***i*** + 5.8 ***j***) km b. (–0.5***i*** +12.1 ***j***) km
c. (5.2***i*** + 5.8 ***j***) km d. (13.2***i*** +12.1 ***j***) km
e. (9.1***i*** + 8.7 ***j***) km

13. The angle between the two vectors **A** = 2 ***i*** + 4 ***j*** and **B** = 4 ***i*** – 2 ***j*** is:

a. 90 ° b. 27 ° c. 39 ° d. 180 °

14. As shown in the Figure, a block moves down on a 45 ° inclined plane of 2.5 m length, then horizontally for another 2.5 m, and then falls down vertically a height of 2.5 m. Find the magnitude and direction of the resultant displacement vector of the block.



a. 6.0 m and 45 ° below horizontal axis
b. 3.5 m and 30 ° degrees below horizontal axis
c. 6.0 m and 30 ° below horizontal axis
d. 3.5 m and 45 ° below horizontal axis
e. 5.5 m and 60 ° below horizontal axis

15. Given the vectors **A** = 3 ***j*** + 6 ***k*** , **B** = 15 ***i*** + 21 ***k***. Find the magnitude of vector **C** that satisfies equation 2**A** + 3**C** - **B** = 0.

a. 6.16 b. 5.48 c. 18.5 d. 6.71

16. Two vectors are given as: **A** = –3.0 ***i*** + 5.0 ***j*** + 4.0 ***k*** and **B** = 4.0 ***i*** + 5.0 ***j*** + 3.0 ***k***, where ***i***, ***j*** and ***k*** are the unit vectors in the positive x, y and z directions. Find the angle between the vectors **A** and **B**.

a. 60 ° b. 45 ° c. 150 ° d. 30 ° e. 90 °

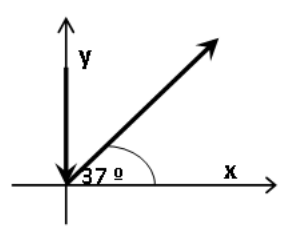
17. In the cross product **F** = **v** × **B**, take **v** = 2.0 ***i***, F = 6.0 ***j*** and the x-component of vector **B** equals zero. What then is **B** in unit-vector notation?

a. –3.0 ***k*** b. 3.0 ***k*** c. 2.0 ***j*** + 6.0 ***k***
d. 2.0 ***j*** – 6.0 ***k*** e. –2.0 ***j*** + 6.0 ***k***

18. Two displacement vectors **A** and **B** have equal magnitudes of 10 m. Vector **A** is along the +y axis and vector **B** makes 45 ° counterclockwise with +x axis. Find the vector **C** such that **B** + **C** = 2**A** .

a. **C** = –7 ***i*** + 13 ***j*** b. **C** = –7 ***i*** + 3 ***j***
c. **C** = 7 ***i*** + 13 ***j*** d. **C** = 7 ***i*** + 3 ***j*** e. **C** = 7 ***i*** + 27 ***j***

19. An object is displaced initially by -30 ***j*** m then by 50 m in a direction making an angle of 37 ° with +x axis (see the Figure). What is the resultant displacement?



a. (40***i***) m b. (40***i*** +30***j***) m
c. (–40***i*** –60***j***) m d. (– 40***i***) m e. (–40***i*** –30***j***) m

20. Vector **A** has a magnitude of 40.0 cm and is directed 60.0 degrees above the negative x-axis. Vector **B** has magnitude of 20.0 cm and is directed along the positive x-axis. Find the resultant vector (***i*** and ***j*** are unit vectors along positive x and y axes, respectively).

a. 34.6 ***j*** cm b. 34.6 ***i*** cm c. 20.0 ***i*** cm
d. 20.0 ***j*** cm e. (20.0 ***i*** + 34.6 ***j***) cm

21. Consider two vectors **A** = (3 ***i*** + 4 ***j***) cm and **B** = (- 4 ***i*** + 3 **j**) cm. Find the angle between these two vectors.

a. 90 ° b. 45 ° c. 120 ° d. 0 ° e. 25 °

22. If vector **A** is added to vector **B**, the result is (6 ***i*** + 1***j***) m. If **A** is subtracted from **B**, the result is (- 4 ***i*** + 7 ***j***) m. Find the magnitude of **B**.

a. 4 m. b. 8 m. c. 2 m. d. 1 m. e. 9 m.

Q: Two vectors are given by $\vec{A} = 2.00\hat{i} + 2.00\hat{j}$ and $\vec{B} = -2.00\hat{i} + 4.00\hat{j}$, find the angle between $\vec{A}$ and $\vec{B}$.

A) 71.6°
B) 45.0°
C) 56.1°
D) 18.4°
E) 24.5°

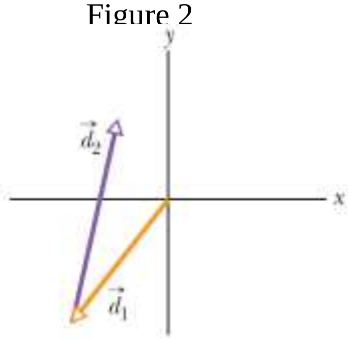
Ans:

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\phi$$
$$(2)(-2) + (2)(4) = \sqrt{4+4}\,\sqrt{4+16}\,\cos\phi$$
$$\cos\phi = \frac{4}{\sqrt{8}\sqrt{20}} \Rightarrow \phi = \mathbf{71.6°}$$

Q8.

The two vectors shown in **Figure 2** lie in an *xy* plane. What are the signs of the *x* and *y* components, respectively, of the vector $(\vec{d_2} - \vec{d_1})$?

Figure 2



A) +,+
B) +, -
C) -, +
D) -, -
E) None of the other answers is correct.

Ans:

Drag $\vec{d}_2$ to orgin then reverse. $\vec{d}_1$ and drag it to tip of $\vec{d}_2$

Q9.

For the following three vectors, find $\vec{C}\cdot(2\vec{A}\times\vec{B})$

$\vec{A} = 2.00\hat{i} + 3.00\hat{j}$

$\vec{B} = -3.00\hat{i} + 4.00\hat{j}$

$\vec{C} = 7.00\hat{i} + 3.00\hat{k}$

A) 102
B) -14.0
C) 0
D) 56.0
E) 78.0

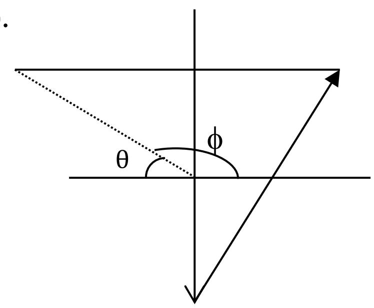
Ans:

$$2\vec{A} \times \vec{B} = (2)[8\,\hat{k} + 9\,\hat{k}] = 34\,\hat{k}$$

$$\vec{C} \cdot (2\,\vec{A} \times \vec{B}) = (3)(34) = 102$$

Q10.

A man makes three successive displacements; 3.50 m south, 8.20 m northeast, and 15.0 m west, respectively. Find the resultant displacement (both the magnitude and direction relative to the east and measured counter-clock wise).



A) 9.48 m, 166°
B) 9.48 m, 45.0°
C) 9.48 m, 225°
D) 5.80 m, 45.0°
E) 5.80 m, 225°

Ans:

$$R = \sqrt{(9.2)^2 + (2.3)^2} = \mathbf{9.48\ m}$$

$$\tan\theta = \frac{2.3}{9.2} \Rightarrow \theta = 14.0°$$

$$\Rightarrow \Phi = 180° - 14° = \mathbf{166°}$$

Q10.

A vector in the *xy* plane has a magnitude of 25 and the magnitude of its *x*-component is 12. The angle this vector makes with the positive *y*-axis is:

A) 29°
B) 64°
C) 61°
D) 24°
E) 41°

Ans:

$$A_x = 12 = x\cos\psi = 25\cos\psi$$

$$\cos\psi = \frac{12}{25} \Rightarrow \psi = 61.3^\circ$$

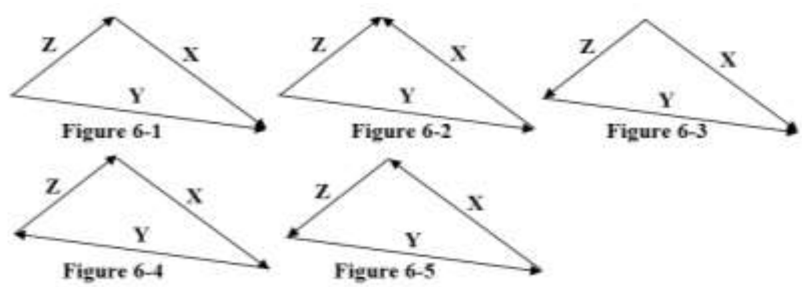
Then $90 - \psi \approx 29^\circ$

Q11.

The vectors **X**, **Y**, and **Z** are related by **Z - Y + X = 0**. Which diagram in **Figure 6** illustrates this relationship?

Figure 6

A) Figure 6-1
B) Figure 6-2
C) Figure 6-3
D) Figure 6-4
E) Figure 6-5



Q12.

The result of $(\hat{j}\times\hat{k})\times(\hat{k}\times\hat{i})$ is:

[$\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in the x, y and z-direction, respectively]

A) $\hat{k}$
B) 0
C) $\hat{i}$
D) $\hat{j}$
E) $-\hat{k}$

Ans:

$$\left(\hat{j}\times\hat{k}\right)\times\left(\hat{k}\times\hat{i}\right) = \hat{i}\times\hat{j} = \hat{k}$$

Q13.

A particle undergoes a displacement, $\Delta\vec{r} = 2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k}$, ending with the position vector, $\vec{r} = 3.0\hat{j} - 4.0\hat{k}$ in meters. What was the particle's initial position vector? [$\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in the x, y and z-direction, respectively]

A) $-2.0\hat{i} + 6.0\hat{j} - 10\hat{k}$
B) $6.0\hat{j} + 10\hat{k}$
C) $2.0\hat{i} + 3.0\hat{k}$

D) $2.0\hat{k}$

E) $-2.0\hat{i}+3.0\hat{j}-9.0\hat{k}$

Ans:

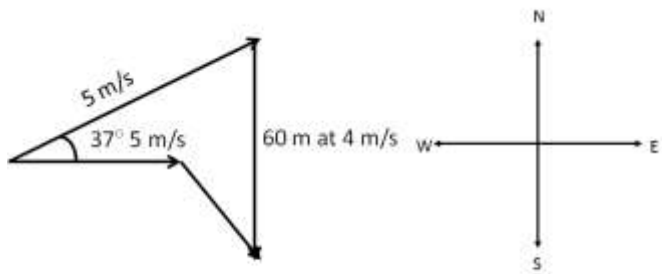
$$\Delta r = \widehat{r_f} - \widehat{r_i} = \Delta\vec{r} = 2.0\hat{i} - 3.0\hat{j} + 6.0\widehat{k} = \left(3.0\hat{j} - 4.0\widehat{k}\right) - \widehat{r_i}$$

$$\widehat{r_i} = 2.0\hat{i} + 6.0\hat{j} - 10\widehat{k}$$

Q7.

A man walks 50 m in a direction 37^o north of east at 5.0 m/s, then 60 m south at 4.0 m/s. How long would it take him to get back to his starting point at 5.0 m/s by the shortest path?

A) 10 s
B) 15 s
C) 20 s
D) 5.0 s
E) 3.5 s



Ans:

$$\Delta x = 50\cos 37 = 39.9 \text{ m}$$
$$\Delta y = 50\sin 37 - 60 = 30.09 - 60 = -29.9$$
$$r = \sqrt{(\Delta x)^2 + (\Delta y)^2} = 49.9 \text{ m}, \qquad t = \frac{r}{v} = \frac{49.9}{5} = 9.98 \text{ sec}$$

Q8.

Vector $\vec{A}$ has a magnitude of 35.0 m and makes an angle of 37.0^o with the positive x axis. Find a vector $\vec{B}$ that is in the direction opposite to vector $\vec{A}$ and is one fifth the magnitude of $\vec{A}$.

A) $-(5.59\text{ m})\ \hat{i}\ -(4.21\text{m})\ \hat{j}$

B) $(5.59\text{ m})\ \hat{i}\ +(4.21\text{ m})\ \hat{j}$

C) $(0.798\text{ m})\ \hat{i}\ -(0.602\text{ m})\ \hat{j}$

D) $-(1.56\text{ m})\ \hat{i}\ -(5.06\text{ m})\ \hat{j}$

E) $-(0.798\text{ m})\ \hat{i} + (0.602\text{ m})\ \hat{j}$

Ans:

$$\vec{B} = -\frac{\vec{A}}{5}$$
$$\vec{A} = 35\cos 37\vec{\imath} + 35\sin 37\vec{\jmath}$$
$$\vec{A} = 27.95\vec{\imath} + 21.06\vec{\jmath}$$
$$\vec{B} = -\frac{1}{5}(27.95\vec{\imath} + 21.06\vec{\jmath})$$
$$= -5.59\vec{\imath} - 4.21\vec{\jmath}$$

Q9. If $\vec{A} = 2\hat{i} + 3\hat{j}$, $\vec{B} = \hat{i} - \hat{j}$ and $\vec{C} = \hat{i} + \hat{j}$, find $(\vec{A} \times \vec{B}) \cdot \vec{C}$.

A) 0
B) –6
C) +6
D) $-3\hat{k}$
E) $+2\hat{i}$

Ans:

$C = \vec{\imath} + \vec{\jmath}\,;\ \vec{D} = \vec{A} \times \vec{B} = \vec{k}$

$\vec{D}.\vec{C} = 0$

Q10.

The scalar product of vectors $\vec{A}$ and $\vec{B}$ is 6.00 and the magnitude of their vector product is 9.00. Find the angle between these two vectors.

A) 56.3°
B) 43.0°
C) 23.4°
D) 37.5°
E) 90.0°

Ans:

$AB\cos\theta = A.B = 6\,, |A \times B| = 9.0 = AB\sin\theta$

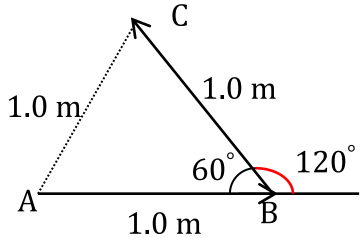
$\tan\theta = \dfrac{A.B}{|A \times B|} = \dfrac{9}{6} = 1.5$

$\theta = \tan^{-1}(1.5) = 56.3°$

Q7.

Initially an object moves 1.00 m in a straight-line from point A to point B. Then, it changes direction and moves another 1.00 m in a straight-line until it reaches point C. Point C is at a distance of 1.00 m from point A. Through what angle did the object changes its direction with respect to its initial direction of motion?

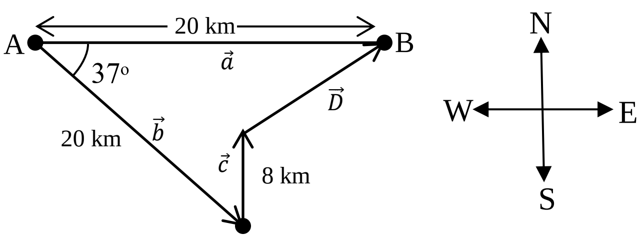
A) 120°
B) 70.0°
C) 100°
D) 135°
E) 140°



Q8.

Oasis B is 20 km due east of oasis A. Starting from Oasis A, a camel walks 20 km in a direction 37° south of east and then walks 8.0 km due north. How far is the camel then from oasis B?

A) 5.7 km
B) 4.0 km
C) 6.6 km
D) 2.7 km
E) 1.4 km



Ans:

$$\vec{D} = \vec{a} - \vec{b} - \vec{c}$$

$$= 20\vec{\imath} - bcos37°\vec{\imath} + bsin37°\vec{\jmath} - c\vec{\jmath}$$

$$= 20\vec{\imath} - 15.97\vec{\imath} + 12.04\vec{\jmath} - 8\vec{\jmath}$$

$$\vec{D} = 4.03\vec{\imath} + 4.04\vec{\jmath} \Rightarrow |D| = \sqrt{(4.03)^2 + (4.04)^2} = 5.7\ km$$

Q9.

Vector $\vec{A}$ has a magnitude of 5.0 units and vector $\vec{B}$ has a magnitude of 10 units. Which of the following values is not possible for the scalar product of vectors $\vec{A}$ and $\vec{B}$?

A) 55
B) 45
C) 35
D) Zero
E) 25

Ans:

$$|A.B|_{max} = 50; |A.B|_{min} = 0$$

Q10.

Vector $\vec{A}$ = 1.00 $\hat{\imath}$ +3.00 $\hat{\jmath}$, vector $\vec{B}$ = 4.00 $\hat{\imath}$ –1.00 $\hat{\jmath}$ and the vector $\vec{C}$ = 2.00 $\hat{k}$. Find the angle (in degrees) between vector $\vec{A}$ and vector $\vec{B}\times\vec{C}$.

A) 176
B) 103
C) 76.0
D) 1.1
E) 24.0

Ans:

$$\vec{D} = \vec{B} \times \vec{C}\ then\ angle\ between\ \vec{A}\ and\ \vec{D} = \theta = \cos^{-1}\left(\frac{A_xD_x + A_yD_y}{|A||B|}\right)$$

$$\vec{D} = \vec{B} \times \vec{C} = (4\vec{\imath} - 1.0\vec{\jmath}) \times 2\vec{k} = -8\vec{\jmath} - 2\vec{\imath}$$

$$|D| = \sqrt{68} = 8.25;\ |A| = \sqrt{10} = 3.16$$

$$\theta = \cos^{-1}\left(\frac{-2 - 24}{8.25 \times 3.16}\right) = 175.8° \cong 176°$$

Q7.

A car travels 20.0 km due north and then 35.0 km due west. Find the car's resultant displacement relative to the starting point?

A) 40.3 km, 60.3° west of north
B) 45.3 km, 30.3° north of west
C) 65.0 km, 65° north
D) 30.5 km, 45.0° west of south
E) 65.8 km, 25.0° east

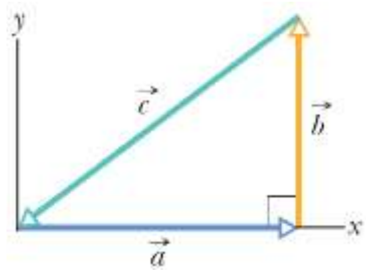
Q8.

If $\vec{A} = 2.0\hat{i} + 3.0\hat{j}$, $\vec{B} = -3.0\hat{i} + 4.0\hat{j}$ and $\vec{C} = 7.0\hat{i} + 3.0\hat{j}$, find $\vec{C} \times (2\vec{A} - \vec{B})$?

A) $-7.0\hat{k}$
B) $7.0\hat{k}$
C) $2.0\hat{i} + 1.0\hat{j}$
D) 0
E) $-6.0\hat{j}$

Q9.

In **Figure 1**, the magnitudes of vector $\vec{a} = 4.0$ m, $\vec{b} = 3.0$ m, and $\vec{c} = 5.0$ m. If the + z axis is out of the page, find the magnitude and direction of $\vec{c} \times \vec{b}$?



A) 12 m, along the – z axis
B) 9.0 m, along the + y axis
C) 12 m, along the –y axis
D) 12 m, along the + z axis
E) 9.0 m, along the + z axis

Answer: From the figure, it is clear that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.
$|\vec{b} \times \vec{c}| = |\vec{b} \times (-\vec{a} - \vec{b})| = |-(\vec{b} \times \vec{a})| = |(\vec{a} \times \vec{b})| = 12$.
along the – z axis

Q8:

A vector **B,** when added to the vector **C** = 3.0 **i** + 4.0 **j,** yields a resultant vector that is in the positive *y* direction and has a magnitude equal to that of **C.** What is the magnitude of **B?**

Answer:

Define **B** = B_x**i** + B_y**j** then the resultant of **B** and **C** is
B + **C** = (B_x + 3.0)**i** + (B_y + 4.0)**j** .
We are told that the resultant points in the positive *y* direction, so its *x* component must be zero. Then:
$B_x + 3.0 = 0 \Rightarrow B_x = -3.0$.
Now, the magnitude of **C** is

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(3.0)^2 + (4.0)^2} = 5.0$$

so that if the magnitude of **B** + **C** is also 5.0 then we get

$$|\mathbf{B} + \mathbf{C}| = \sqrt{(0)^2 + (B_y + 4.0)^2} = 5.0 \qquad \Longrightarrow \qquad (B_y + 4.0)^2 = 25.0 \ .$$

The last equation gives $(B_y + 4.0) = \pm 5.0$ and apparently there are two possible answers $B_y = +1.0$ and $B_y = -9.0$, but the second case gives a resultant vector **B** + **C** which points in the negative *y* direction so we omit it. Then with $B_y = 1.0$ we find the magnitude of **B**:

$$B = \sqrt{(B_x)^2 + (B_y)^2} = \sqrt{(-3.0)^2 + (1.0)^2} = 3.2$$

The magnitude of vector **B** is 3.2.

A. 3.2
B. 1.9
C. 2.4
D. 0.2
E. 0.6

Q9: If vector **B** is added to vector **A**, the result is 6.0 **i** +1.0 **j** . If **B** is subtracted from **A**, the result is −4.0 **i** + 7.0 **j**. What is the magnitude of **A**?

a. 4.1
b. 5.1
c. 5.4
d. 5.8
e. 8.2

Ans: **B+ A =** 6 **i** + **j**
-B + A = - 4 **i** + 7 **j**
----

2 A = 2 **i** + 8 **j** => **A =** 1 **i** + 4 **j** => **| A |= Sqrt(1^2 + 4^2) = 4.12**

Q10:

Vectors **A** and **B** each have magnitude L and having an angle of 60° between them. The magnitude of the product **(A × B) . A** is:

A. 0
B. $3L^2/2$
C. $L^2/2$
D. $3L^2$
E. $3L^2/4$

Answer: **(A × B) . A = C.A=CA cos 90º=0**

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Q11:

A particle is at the origin of coordinates at time t = 0. For the time interval from 0 to 15 s, the particle's average velocity:

$$\mathbf{V}_{average} = -3.8\,20\ \mathbf{i} + 4.4\ \mathbf{j}\ \text{(m/s)}.$$

How far is the particle from the origin at t = 15 s?

a. [illegible] m 87
b. 57 m
c. 69 m
d. 15 m
e. 72 m

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \frac{\vec{r}_f}{t_f}$$

$$\Rightarrow \vec{r}_f = t_f \cdot \vec{v}_{avg} = 15\times(-3.8\hat{i} + 4.4\hat{j}) = -57\hat{i} + 66\hat{j}\ \text{m}$$

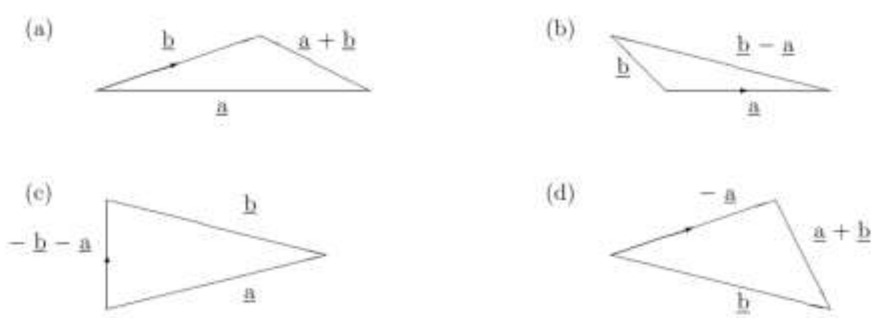
87.2

$$r_f = (57^2 + 66^2)^{1/2} = \cancel{87.4986}\ \text{m} \rightarrow 87\ \text{m}$$

EXERCISES

1. Which of the following are vectors and which are scalars ?
(a) Kinetic Energy; (b) Volume; (c) Force;
(d) Temperature; (e) Electric Field; (f) Thrust.

2. Fill in the missing arrows for the following vector diagrams:



3. ABCDE is a regular pentagon with centre O. Use the Triangle Law of Addition to show that AB + BC + CD + DE + EA = O.
4. Draw to scale a diagram which illustrates the identity
4a + 3(b − a) = a + 3b.
5. a, b and c are any three vectors and
p = b + c − 2a, q = c + a − 2b, r = 3c − 3b.
Show that the vector 3p − 2q is parallel to the vector 5p − 6q + r.

3. ABCDE is a regular pentagon with centre O. Use the Triangle Law of Addition to show that

$$\underline{AB} + \underline{BC} + \underline{CD} + \underline{DE} + \underline{EA} = \mathbf{O}.$$

4. Draw to scale a diagram which illustrates the identity

$$4\underline{a} + 3(\underline{b} - \underline{a}) = \underline{a} + 3\underline{b}.$$

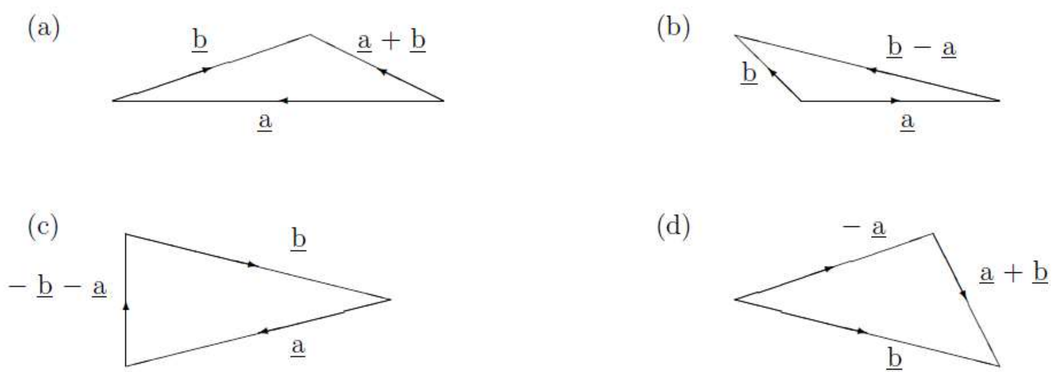
5. $\underline{a}$, $\underline{b}$ and $\underline{c}$ are any three vectors and

$$\underline{p} = \underline{b} + \underline{c} - 2\underline{a}, \quad \underline{q} = \underline{c} + \underline{a} - 2\underline{b}, \quad \underline{r} = 3\underline{c} - 3\underline{b}.$$

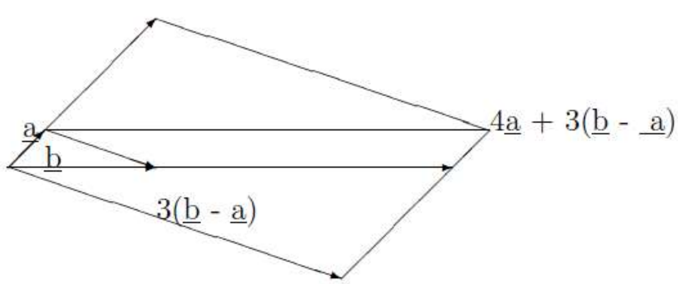
Show that the vector $3\underline{p} - 2\underline{q}$ is parallel to the vector $5\underline{p} - 6\underline{q} + \underline{r}$.

ANSWERS TO EXERCISES

1. (a) Scalar; (b) Scalar; (c) Vector:
(d) Scalar; (e) Vector; (f) Vector.
2. The completed diagrams are as follows:



3. Join A,B,C,D and E up to the centre, O.
4. The diagram is



5. One vector is a scalar multiple of the other.