Exactly solvable model for the specific-heat amplitude ratio with uniaxial dipolar interaction

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The specific-heat amplitude ratio C_+/C_- with strong uniaxial dipolar interaction, is calculated in the Gaussian model for two different cases. The crossover behavior of the calculated ratio, with and without uniaxial dipolar behavior, is also discussed. [S0163-1829(99)01429-0]

I. INTRODUCTION

The Gaussian region is defined as being sufficiently far from the critical point so that only noninteracting order-parameter fluctuations need to be considered. In such a region, where one can use the so-called Gaussian model,[1] the fluctuations are kept only to the second order. The specific-heat amplitude ratio C_+/C_-, of the divergence of the specific heat above and below the critical point, is an important quantity and has been calculated in the Gaussian model for superconductors,[2] and suggested for magnetic systems.[3] Because of the nonasymptotic behavior of this region, one does not expect the ratio to be universal but it may depend on the nonuniversal parameters of the Gaussian model.[2,4]

Close to the critical point, it is unusual to find a pure critical behavior because many interactions could be competing within a specific region. An example of such an interaction is the long-range dipolar interaction. Much of the current research has been devoted to studying the crossover behavior in ferromagnets with dipolar interactions.[5] On the other hand, very little research has been done on the Lifshitz point that includes the dipolar term.[6,7] This has motivated us to study the crossover behavior of C_+/C_- for the cases of uniaxial ferromagnetic and a system exhibiting multicritical uniaxial Lifshitz point using the Gaussian model. The results of these calculations may be helpful in understanding the experimental results in systems whose phase diagrams show a Lifshitz point.[8] It is worth mentioning that a better solution to the given problem can only be obtained within the framework of the renormalization-group theory to the second-loop order, if it is feasible.[9,10]

To calculate the specific heat C above and below T_c, we consider the partition function[1]

$$Z=\int_{\varphi} e^{-H[\varphi]}, \tag{1}$$

where $H[\varphi]$ is the free-energy functional and is related to the order parameter (φ) and the Gaussian propagator $[G(r)]$ by the relation:

$$H[\varphi]=\frac{1}{2}\int_k G^{-1}(r)\varphi(\mathbf{k})\varphi(-\mathbf{k}), \tag{2}$$

where $r=(T-T_c)/T_c$ is the reduced temperature. The other arguments in G have been suppressed for simplicity and will be defined in the following sections. In the d dimension, the wave vector $\mathbf{k}$ is decomposed into $\mathbf{q}$ and $\mathbf{p}$ components of dimension m and $d-m$, respectively, and we used the notation $\int_k=[1/(2\pi)^d]\int d^{d-m}p\,d^m q$. Then, the specific heat is computed according to the definition

$$C=-T\frac{\partial^2 F}{\partial T^2}, \tag{3}$$

where F is the free energy per unit volume, $F=-(1/V)\ln Z$. Above T_c is trivial and C_+ is given by

$$C_+=\frac{1}{(2\pi)^d}\int G^2(r)d^{d-m}p\,d^m q. \tag{4}$$

Below T_c one expands around the broken symmetry state which leads to a shift of $|r|$ to 2 $|r|$, and

$$C_-=\frac{2^2}{(2\pi)^d}\int G^2(2|r|)d^{d-m}p\,d^m q, \tag{5}$$

where the factor 2^2 comes from the shift of $|r|$ to 2 $|r|$ at $T<T_c$.

II. UNIAXIAL DIPOLAR FERROMAGNETS

Consider a simple model in which the Gaussian propagator[11] can be written as

$$G(r,g)=\left(r+p^2+q^2+g^2\frac{q^2}{p^2}\right)^{-1}, \tag{6}$$

where g is the parameter of the dispersion. g contains the limits of a usual behavior $(g=0)$ and the strong uniaxial dipolar behavior $(g>0)$.

Above T_c, the singular part of the specific heat reads

$$C_+\propto\int dq\int G^2(r,g)d^{d-1}p=\text{const}\,\frac{\partial I_1(r,g)}{\partial r}, \tag{7}$$

where $I_1(r,g)$ has the following form:[12]

$$I_1(r,g)=\int dq\int G(r,g)d^{d-1}p=\pi S_{d-1,1}I_c(r,g), \tag{8}$$

$S_{d-1,1}=4\pi^{[(d-1)/2]}/\Gamma[(d-1)/2]$, $(d>1)$ is the geometrical factor of integration, $\Gamma(\alpha)$ is the usual gamma function and

$$I_c(r,g)=\int\frac{d^d p}{\sqrt{r+p^2}\sqrt{g^2+p^2}} \tag{9}$$

