Applications of the Schrödinger's Wave Equation

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(1926)

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. … E V T

-1

:

."ψ ”   :

(   )

"r" $|\psi(r,t)|^2$.

: ψ :   "t"

: (a,b) ψ

(Single valued)   -

(Finite and bounded)   -

.(Continous everywhere)   (   ) -

.   (   )

:

$$\int\limits_{\text{all space}} \psi_m^*(r)\,\psi_n(r)\,d\tau = \delta_{mn}, \quad \delta_{mn} = \begin{cases} 1 & \text{if } m=n \\ 0 & \text{if } m \neq n \end{cases} \qquad \textbf{(1)}$$

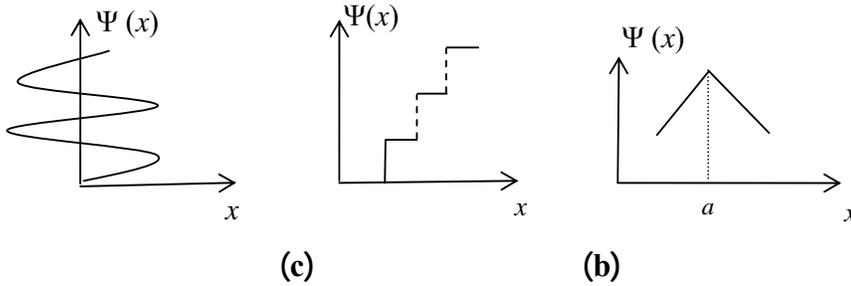$\delta_{ij}=1$. δ_{ij} ψ ψ^*

. $\delta_{ij}=0$

:

$$\text{Prob. } \{a \leq x \leq b\} = \int\limits_a^b \psi_n^*(x)\,\psi_n(x)\,dx$$

$$= \int\limits_a^b |\psi_n(x)|^2 dx$$

$$\{a \le x \le b\} \equiv (a,b).$$

:

(c)   **(b)**   **(a)**

:   **شروط ميكانيكا الكم**   :

(a)   (   ) $\Psi(x)$ $x = a$.

(b) $\Psi(x)$.

(c) $\Psi(x)$ x .

:   _

$(a)\quad \psi_1 = e^{-x} \quad (-\infty, 0),$

$(b)\quad \psi_2 = e^{-|x|} \quad (-\infty, \infty),$

$(c)\quad \psi_3 = \dfrac{1}{x-4} \quad (0,5).$

:

(a) $x \to -\infty$ $\psi_1 \to \infty$.

(b) ψ_2 $x = 0$.

(c) $x \to 4$ $\psi_3 \to \infty$.

:   (   ) $(0,L)$:

$$\psi(x) = c \sin(bx)$$

$b = \pi / L$:

-                        c

-                        $0 \to 0.5L$

-                        $0.25L \to 0.75L$.

:

:   -

$$I = \int_0^L \psi_n^*(x)\,\psi_n(x)\,dx = c^2 \int_0^L \sin(bx)\sin(bx)\,dx$$

$$= c^2 \int_0^L \sin^2(bx)\,dx = \frac{c^2}{2}\underbrace{\left[x - \frac{\sin(2bx)}{(2b)} \right]_0^L}_{=L}$$

$$= \frac{c^2}{2}(L)$$

:   " $I = 1$ "

$$c^2\left(\frac{L}{2}\right) = 1 \quad \Rightarrow \quad \boxed{c = \sqrt{\frac{2}{L}}}$$

:

$$\text{Prob. } \{a \le x \le b\} = \int_a^b \psi^*_n(x)\,\psi_n(x)\,dx = \int_a^b \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)dx$$

$$= \frac{2}{L}\int_a^b \sin^2\left(\frac{n\pi}{L}x\right)dx = \left| \frac{x}{L} - \frac{1}{2\pi n}\sin(\frac{2n\pi x}{L}) \right|_a^b$$

: $0 \to 0.5L$ -

$$\text{Prob.}\{0 \le x \le L/2\} = \left| \frac{x}{L} - \frac{1}{2\pi n}\sin(\frac{2n\pi x}{L}) \right|_0^{L/2}$$

$$= \frac{1}{2} \qquad\qquad\qquad \text{(for all } n\text{)}$$

: $0.25L \to 0.75L$ -

$$\text{Prob. } \{0.25L \le x \le 0.75L\} = \left| \frac{x}{L} - \frac{1}{2\pi}\sin(\frac{2\pi x}{L}) \right|_{0.25L}^{0.75L}$$

$$= 0.818$$

:

. $X = \{2,4\}$
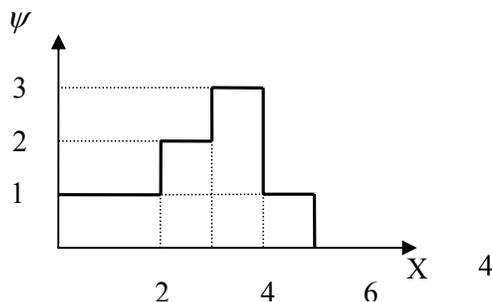
$$: X = \{2, 4\}$$:

$$I = \sum_{i=2}^{4} \psi_i^2 = 9 + 4 = 13$$

$$: X = \{0, 5\}$$

$$II = \sum_{i=1}^{5} \psi_i^2 = 1 + 1 + 4 + 9 + 1 = 16,$$

:

$$\text{Prob. } \{2 \leq X \leq 4\} = \frac{I}{II} = \frac{\sum_{i=2}^{4} \psi_i^2}{\sum_{i=0}^{5} \psi_i^2} = \frac{13}{16}$$

."   (Operator)   "   :

....   (−)   (+)

:   ."^"

$$\underbrace{\hat{A}}_{\text{operator}} \underbrace{\varphi}_{\text{eigenfunction}} = \underbrace{a}_{\text{eigenvalue}} \underbrace{\varphi}_{\text{eigenfunction}}$$ (2)

a (   ) (Eigenfunction) φ

.(Eigenvalue)

.

:   (   )

$$\hat{A}[f(x) + g(x)] = \hat{A}f(x) + \hat{A}g(x),$$

$$\hat{A}[kf(x)] = k\hat{A}f(x)$$ (3)

. $f(x)$ $g(x)$ k

.

$\hat{x} = x$	x	

Schrödinger equation and its applications

$\hat{p}_x = -i\,\hbar\dfrac{\partial}{\partial x}$	p_x	
$\hat{p} = -i\,\hbar\nabla$	p	
$\hat{K} = -\dfrac{\hbar^2}{2m}\nabla^2$	$K = p^2/2m$	
$\hat{L} = \hat{r}\times\hat{p}$	L	
$\hat{V} = V$	V	
$\hat{E} = i\,\hbar\dfrac{\partial}{\partial t}$	E	

$$\sin(ax)\qquad \frac{d^2}{dx^2}\qquad :$$

$$:\qquad \sin(ax)\qquad \frac{d^2}{dx^2}\qquad :$$

$$\frac{d^2}{dx^2}\big[\sin(ax)\big] = -a^2\big[\sin(ax)\big]$$

$$\sin(ax)\qquad \frac{d^2}{dx^2}$$

$$.-a^2$$

$$:$$

$(a)\ \sqrt{(\ )}\qquad (b)\ \sin(\ )\qquad (c)\ e^{(\ )}\qquad (d)\ \dfrac{d}{dx}(\ )$$

$$:$$

$(a)\ \sqrt{(\psi_1+\psi_2)} \neq \sqrt{(\psi_1)} + \sqrt{(\psi_2)}$

$(b)\ \sin(\psi_1+\psi_2) \neq \sin(\psi_1) + \sin(\psi_2)$

$(c)\ e^{(\psi_1+\psi_2)} \neq e^{(\psi_1)} + e^{(\psi_2)}$

$(d)\ \dfrac{d}{dx}(\psi_1+\psi_2) = \dfrac{d}{dx}(\psi_1) + \dfrac{d}{dx}(\psi_2)$

$$.\qquad\qquad (d)$$

$$\hat{A} = \frac{d}{dx} - 2x \qquad :$$

: $\hat{A}\psi = a\psi$:

$$\left(\frac{d}{dx} - 2x\right)\psi = a\psi$$

$$\frac{d}{dx}\psi = (a + 2x)\psi$$

:

$$\int \frac{d\psi}{\psi} = \int (a + 2x)\,dx$$

$$\Rightarrow \qquad \boxed{\psi = c\,e^{ax + x^2}}$$

. c

$$\hat{D}\,x \equiv \frac{d}{dx}x$$:

: $\dfrac{d}{dx}x$ ψ :

$$\left(\frac{d}{dx}x\right)\psi = \frac{d}{dx}(x\psi) = x\frac{d\psi}{dx} + \psi\frac{dx}{dx} = \left(x\frac{d}{dx} + 1\right)\psi$$

$$\boxed{\left(\hat{D}x\right) \equiv \left(x\hat{D} + 1\right)}$$

$.\hat{D} = \dfrac{d}{dx}$ $(\hat{D} + \hat{x})^2$:

:

$$(\hat{D} + \hat{x})^2 = (\hat{D} + \hat{x})(\hat{D} + \hat{x})$$

$$= \hat{D}^2 + \hat{x}\,\hat{D} + \underbrace{\hat{D}\,\hat{x}}_{\hat{x}\,\hat{D}+1} + \hat{x}^2 = \hat{D}^2 + \hat{x}\,\hat{D} + \hat{x}\,\hat{D} + 1 + \hat{x}^2$$

$$= \underline{\hat{D}^2 + 2\hat{x}\,\hat{D} + \hat{x}^2 + 1}$$

.

:   (   )   " :

$$\overline{\hat{A}} \equiv \left\langle \hat{A} \right\rangle = \frac{\int \psi^* \hat{A}\,\psi\,d\tau}{\int \psi^* \psi\,d\tau}$$ **(4)**

.

:   :

$$|\psi> = \underbrace{\sqrt{\frac{1}{19}}}_{a_1}|\varphi_1> + \underbrace{\sqrt{\frac{4}{19}}}_{a_2}|\varphi_2> + \underbrace{\sqrt{\frac{2}{19}}}_{a_3}|\varphi_3> + \underbrace{\sqrt{\frac{3}{19}}}_{a_4}|\varphi_4> + \underbrace{\sqrt{\frac{5}{19}}}_{a_5}|\varphi_5>$$

:

$$H|\varphi_n\rangle = E_n|\varphi_n\rangle, \qquad E_n = n\varepsilon_o$$
$$\langle\varphi_m|\varphi_n\rangle = \delta_{mn}$$

: a_n φ_n :

$$<\psi|\psi> = \sum a_n^2 <\varphi_n|\varphi_n> = \sum a_n^2 = \frac{1}{19} + \frac{4}{19} + \frac{2}{19} + \frac{3}{19} + \frac{5}{19} = \frac{15}{19}$$

.

:   :

$$P_i = \frac{<\varphi_i|\psi>}{<\psi|\psi>}$$

:

$$P_1 = \frac{|a_1|^2}{15/19} = \frac{1/19}{15/19} = \frac{1}{15}, \quad P_2 = \frac{4}{15}, \quad P_3 = \frac{2}{15}, \quad P_4 = \frac{3}{15}, \quad P_5 = \frac{5}{15}.$$

:   :

$$<E> = \sum_n P_n E_n = \frac{1}{15}(\varepsilon_o) + \frac{4}{15}(2\varepsilon_o) + \frac{2}{15}(3\varepsilon_o) + \frac{3}{15}(4\varepsilon_o) + \frac{5}{15}(5\varepsilon_o) = \frac{52}{15}\varepsilon_o.$$

:

$$\bar{E} = \frac{<\psi|H|\psi>}{<\psi|\psi>} = \frac{\sum a_n^2 <\varphi_n|H|\varphi_n>}{15/19} = \frac{19}{15}\left(\frac{1}{19} + \frac{8}{19} + \frac{6}{19} + \frac{12}{19} + \frac{25}{19}\right)\varepsilon_o = \frac{52}{15}\varepsilon_o$$

2.2

T

. E

:(x)

Schrödinger equation and its applications

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:   **-1**

$$\psi = e^{i(kx-wt)} = e^{i(p_x x - Et)/\hbar} = e^{ip_x x/\hbar} e^{-iEt/\hbar} = f(x)f(t) \tag{1}$$

. $p_x = \hbar k \quad E = \hbar\omega$

:   **(1)** $\hat{E}$ **-2**

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}E\psi \quad \Rightarrow \quad i\hbar\frac{\partial}{\partial t}\psi = E\psi \tag{2}$$

: $\hat{E}\psi = E\psi$

$$\boxed{\hat{E} = i\hbar\frac{\partial}{\partial t}} \tag{3}$$

: x **(1)** $\hat{p}_x$ **-3**

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar}p\, e^{i(p_x x - Et)/\hbar}$$

$$= \frac{i}{\hbar}p_x \psi \tag{4}$$

$$\Rightarrow \quad \boxed{\hat{p}_x \psi = -i\hbar\frac{\partial}{\partial x}\psi}$$

$$\boxed{\hat{p}_x = -i\hbar\frac{\partial}{\partial x}} \qquad\qquad \hat{p}_x \psi = p_x \psi$$

$$. \hat{p}_x^2 = \hat{p}_x \hat{p}_x = -\hbar^2 \frac{\partial^2}{\partial x^2}$$

:   **-4**

$$\hat{K} = \frac{\hat{p}_x^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{5}$$

) $\hat{H}$ $\hat{H}\psi = \hat{E}\psi$

(

$$\boxed{\hat{H} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \hat{V}\right)}, \qquad \boxed{\hat{E} = i\hbar\frac{\partial}{\partial t}} \tag{6}$$

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(   )   **-3**

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Schrödinger equation and its applications

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$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\hat{V}\right)\Psi(x,t)=i\hbar\frac{\partial}{\partial t}\Psi(x,t) \tag{1}$$

)

.(

:

$\Psi(x,t)$:   **-1**

$$\Psi(x,t)=\psi(x)\psi(t) \tag{2}$$

$\psi(x)$ $\psi(t)$.

:   **(2)**   **-2**

$$\frac{\partial}{\partial t}\Psi(x,t)=\psi(x)\frac{\partial\psi(t)}{\partial t} \tag{3a}$$

$$\frac{\partial^2}{\partial x^2}\Psi(x,t)=\psi(t)\frac{\partial^2\psi(x)}{\partial x^2} \tag{3b}$$

$\psi(x)\,\psi(t)$ **(1)**   **(3b)**   **(3a)**   **-3**   :

$$\frac{1}{\psi(x)}\left\{-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+\hat{V}\right\}=\frac{1}{\psi(t)}\left\{i\hbar\frac{\partial\psi(t)}{\partial t}\right\} \tag{4}$$

"x"   **(4)**   **-4**

"t"   .

"c"

$$i\hbar\frac{\partial\psi(t)}{\partial t}=c\psi(t) \tag{5a}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+\hat{V}\psi(x)=c\psi(x) \tag{5b}$$

(5a)   **-5**

$$\psi(t)=e^{-ict/\hbar} \tag{16}$$

.

E c **(5)**   **(6)** c **-6**

.( )

:   **-7**

Schrödinger equation and its applications

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$$-\frac{\hbar^2}{2m}\nabla_x^2\psi(x)+V(x)\psi(x)=E\psi(x)$$

$$\Rightarrow \quad \frac{d^2\psi(x)}{dx^2}+\frac{2m}{\hbar^2}\left[E-V(x)\right]\psi(x)=0$$

(2.17)

V (   ) E

.

4–

: ψ^* ψ

$$-\frac{\hbar^2}{2m}\nabla^2\psi=i\hbar\frac{\partial\psi}{\partial t}$$

(1)

$$-\frac{\hbar^2}{2m}\nabla^2\psi^*=-i\hbar\frac{\partial\psi^*}{\partial t}$$

(2)

: ψ (2) ψ^* (1)

$$-\frac{\hbar^2}{2m}\left(\psi^*\nabla^2\psi-\psi\nabla^2\psi^*\right)=i\hbar\left(\psi^*\frac{\partial\psi}{\partial t}+\psi\frac{\partial\psi^*}{\partial t}\right)$$

(3)

:

$$\left(\psi^*\nabla^2\psi-\psi\nabla^2\psi^*\right)=\nabla.\left(\psi^*\nabla\psi-\psi\nabla\psi^*\right)$$

(4)

$$\psi^*\frac{\partial\psi}{\partial t}+\psi\frac{\partial\psi^*}{\partial t}=\frac{\partial}{\partial t}\left(\psi^*\psi\right)$$

(5)

:

:

$$\boxed{\mathbf{J}=-\frac{i\hbar}{2m}\left(\psi^*\nabla\psi-\psi\nabla\psi^*\right)}$$

:

$$\boxed{\rho=\psi^*\psi}$$

(3)

$$\nabla\cdot\mathbf{J}+\frac{\partial\rho}{\partial t}=0$$

(6)

(   )

.

: $\psi(x)=A\,e^{ikx}$ A .

Schrödinger equation and its applications

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:

$$J = -\frac{i\hbar}{2m}\left(\psi*\nabla\psi - \psi\nabla\psi*\right) = \frac{\hbar}{m}I_m\left(\psi*\frac{\partial}{\partial x}\psi\right)$$

$$= \frac{\hbar}{m}I_m\left(A*e^{-ikx}\frac{\partial}{\partial x}Ae^{ikx}\right) = \frac{\hbar k}{m}|A|^2 = v|A|^2$$

$$A, B \qquad \psi(x) = A\,e^{ikx} + B\,e^{-ikx} \qquad :$$

$$.J = v\left(|A|^2 - |B|^2\right)$$

-5

5.I   ($V = 0$)

:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\,\psi(x)$$

$$\Rightarrow \quad \frac{d^2\psi}{dx^2} = -k^2\psi,$$

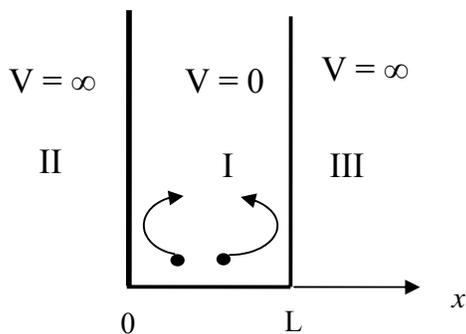k $k^2 = \frac{2mE}{\hbar^2}$ A $\psi(x) = A\,e^{\pm ikx/\hbar}$

.

-5.II

:

$$V(x) = \begin{cases} 0, & 0 \le x \le L \\ \infty, & \text{otherwise} \end{cases} \tag{1}$$

(6)   .

. $V = \infty$ $0 \le x \le L$

V = ∞ | V = 0 | V = ∞

II | I | III

(6)

0   L x

:

.   :

:   (I)   **(6)**

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Schrödinger equation and its applications

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$$-\frac{\hbar^2}{2m}\frac{d^2\psi_I}{dx^2}=E\psi_I \atop \Rightarrow \frac{d^2\psi_I}{dx^2}=-k^2\psi_I,\}$$
(2)

$(II),(III)$. E m $k^2=\frac{2mE}{\hbar^2}$

$x=0$ $V=\infty$

: $x=L$

$\psi_{II}(x=0)=\psi_{III}(x=L)=0$.

(2)   .   ( )   :

:

$$\psi_I(x)=A\sin(kx)+B\cos(kx)$$
(3)

$$\psi_I(x)=a\,e^{ikx}+b\,e^{-ikx}$$
(4)

. a,b,A,B $i=\sqrt{-1}$

.**(3)**   :

:   .   :

$$\psi_I(0)=0$$
(5)

:   **(3)** B

$$\psi_I(x)=A\sin(kx)$$
(6)

A,k :

: $\psi_I(x=L)=0$ k -

$$\left.{\because \quad \psi_I(x=L)=0 \atop \therefore \quad A\sin(kL)=0}\right\} \Rightarrow k_n=\frac{n\pi}{L}, \qquad n=1,2,3,\cdots$$
(7)

. n

: A -

$$\left.{\because \qquad \int_0^L |\psi_I|^2\,dx=1 \atop \therefore A^2\underbrace{\int_0^L \sin^2(k_n x)dx}_{\frac{L}{2}}=1}\right\} \Rightarrow A=\sqrt{2/L}$$
(8)

:

$$\psi_I(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi}{L}x), \qquad n = 1,2,3,\cdots \qquad \textbf{(9)}$$

:

: E_n k_n **-1**

$$E_n = \frac{p^2}{2m} = \frac{(\hbar k_n)^2}{2m} = n^2\underbrace{\left(\frac{\hbar^2\pi^2}{2mL^2}\right)}_{E_1} = n^2 E_1, \qquad n = 1,2,3,\cdots \qquad \textbf{(10)}$$

. $$E_1 = \frac{\hbar^2\pi^2}{2mL^2}$$.(   )

. $n = 0$ **-2**

$$\left|\psi_I(0 < x < L)\right|^2 = 0$$

.

.   **-3**

.n^2 **-4**

: n "ΔE"   **-5**

$$\Delta E = E_{n+1} - E_n = (n+1)^2 E_1 - n^2 E_1 = (2n+1)E_1$$

$(7-a$ $)$

: $< x >$ **-6**

$$\langle x \rangle = \int_0^L x\,|\psi_I|^2\,dx = \frac{2}{L}\int_0^L x\,\sin^2(k_n x)\,dx = \frac{2}{L}\left(\frac{L^2}{4}\right) = \frac{L}{2} \qquad \textbf{(11)}$$
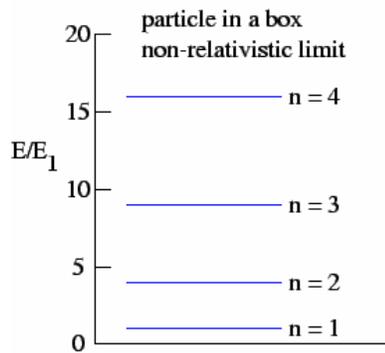
.$\frac{L}{2}$ (   )

$(7-b$ $)$.

(a)

(b)

-b   -a **(6)**   **(7)**
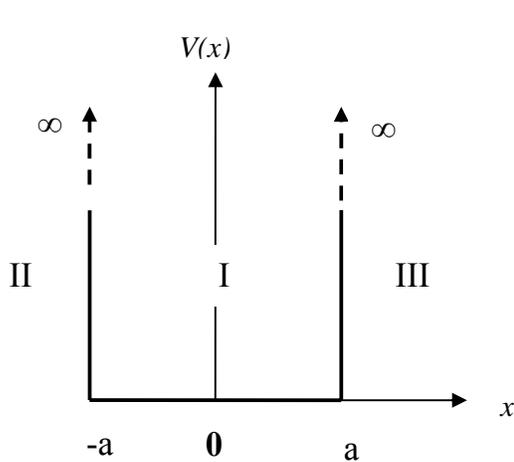
: $\langle \hat{p} \rangle$ **–7**

$$\langle \hat{p} \rangle = \int_0^L \psi_I^* \hat{p} \, \psi_I \, dx = \frac{2}{L}\int_0^L \sin(k_n x)\left(-i\,\hbar\,\frac{\partial}{\partial x}\sin(k_n x)\right) dx = 0 \qquad \textbf{(12)}$$

(12)

.

.

:   :

$$V(x) = \begin{cases} 0, & |x| < a \\ \infty, & |x| > a \end{cases}$$

$$\psi_I(x) = \begin{cases} A\,\sin(\frac{n\pi}{2a}x) & n \text{ is even} \\[2mm] B\,\cos(\frac{n\pi}{2a}x) & n \text{ is odd} \end{cases}$$

$$E = \frac{\pi^2\hbar^2}{2ma^2}n^2, \quad A = B = \sqrt{\frac{1}{a}}$$

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:

.(III)   (II)   (I)

:   (I)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_I}{dx^2}=E\,\psi_I$$

$$\Rightarrow \quad \frac{d^2\psi_I}{dx^2}=-k^2\psi_I,$$ **(1)**

:   (1) $.k^2=\dfrac{2mE}{\hbar^2}$

$$\psi_I(x)=A\sin(kx)+B\cos(kx) \qquad (2)$$

$(-a)$ (a) . B A

:

$$\psi_I(a)=0 \quad \Rightarrow \quad A\cos(ka)+B\sin(ka)=0, \qquad (3)$$

$$\psi_I(-a)=0 \quad \Rightarrow \quad A\cos(ka)-B\sin(ka)=0 \qquad (4)$$

:   **(4)**   **(3)**

$$2A\cos(ka)=0$$
$$\Rightarrow \quad A=0 \ \text{ or } \ \cos(ka)=0 \qquad (5)$$

:   **(4)**   **(3)**

$$2B\sin(ka)=0$$
$$\Rightarrow \quad B=0 \ \text{ or } \ \sin(ka)=0 \qquad (6)$$

$\psi_I(x)$ B A

k $\cos(ka)$ $\sin(ka)$.

:   .E

$$A=0 \ \ B\neq 0 \ \Rightarrow \ \sin(ka)=0, \qquad (i)$$

$$B=0 \ \ A\neq 0 \ \Rightarrow \ \cos(ka)=0 \qquad (ii)$$

:

$$\sin(ka)=0 \quad \Rightarrow \quad ka=\pi,2\pi,3\pi,\cdots=n\frac{\pi}{2},$$

:   . n

$$\cos(ka) = 0 \quad \Rightarrow \quad ka = \frac{\pi}{2}, 3\frac{\pi}{2}, 5\frac{\pi}{2}, \cdots = n\frac{\pi}{2},$$

. n

:

$$\psi_I(x) = \begin{cases} A \sin(\frac{n\pi}{2a}x) & n \text{ is even} \\\\ B \cos(\frac{n\pi}{2a}x) & n \text{ is odd} \end{cases}$$

$$k_n^2 = \frac{2mE_n}{\hbar^2}, \qquad \Rightarrow \qquad E_n = \frac{k_n^2\hbar^2}{2m} = n^2 \underbrace{\frac{\pi^2\hbar^2}{2ma^2}}_{E_1} = n^2 E_1$$

$$A = B = \sqrt{\frac{1}{a}}$$

_____

III.5-

:

$$V(x) = \begin{cases} 0, & x < 0 \\ V_o, & x \geq 0 \end{cases}$$

V_o V_o V_o E

-                      -   .(2.III.1   )

.   -   -   -

(b)   (a)

$E < V_o$ (b) $E > V_o$ (a)   2.*III*.1

:

.

. $E > V_o$ E V_o

. $E < V_o$

.

$E > V_o$

: $x = 0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_I}{dx^2} = E\psi_I$$

$$\Rightarrow \quad \frac{d^2\psi_I}{dx^2} = -k^2\psi_I, \quad k^2 = \frac{2mE}{\hbar^2}$$

(1)

:

$$\psi_I(x) = A\ \underbrace{e^{ikx}}_{\text{Incident wave}} + B\ \underbrace{e^{-ikx}}_{\text{Reflected wave}}$$

(2)

e^{ikx}

e^{-ikx} (Incident wave)

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B A . (Reflected wave)

.

: $(x = 0)$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} + V_o\psi_{II} = E\psi_{II}$$

$$\Rightarrow \frac{d^2\psi_{II}}{dx^2} = -\alpha^2\psi_{II}, \quad \alpha^2 = \frac{2m}{\hbar^2}(E - V_o)$$

(3)

:

$$\psi_{II}(x) = C\,e^{i\alpha x} + D\,e^{-i\alpha x}$$

(4)

$$e^{i\alpha x}$$

(Transmitted wave)

: $x = 0$ $. D = 0$

$$\because \psi_I(x = 0) = \psi_{II}(x = 0)$$

$$\therefore \quad A + B = C$$

(5)

:

$$\because \psi'_I(x = 0) = \psi'_{II}(x = 0)$$

$$\therefore \quad ik(A - B) = i\alpha C$$

(6)

:   (6) (5)

$$B = \left(\frac{k - \alpha}{k + \alpha}\right)A, \qquad C = \left(\frac{2k}{k + \alpha}\right)A$$

(7)

:

$$v_1|A|^2 = \frac{q}{m}|A|^2 =$$

$$v_1|B|^2 = \frac{q}{m}|B|^2 =$$

$$v_2|C|^2 = \frac{\alpha}{m}|C|^2 =$$

: $. v_i = \hbar k_i / m$

$\left(\dfrac{k-\alpha}{k+\alpha}\right)^2 =$		
		$= (R)$

20

$$\frac{4k\,\alpha}{\left(k+\alpha\right)^2} = \boxed{\quad} = (T)$$

$$T + R = 1 \qquad\qquad \textbf{-1}$$

$$E > V_o \qquad\qquad \textbf{-2}$$

$$E < V_o$$

$$x = 0$$

$$x = 0 \qquad\qquad\qquad\qquad \textbf{.(2) (1)}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} + V_o\psi_{II} = E\,\psi_{II}$$

$$\Rightarrow \frac{d^2\psi_{II}}{dx^2} = \beta^2\psi_{II}, \quad \beta^2 = \frac{2m}{\hbar^2}(V_o - E) \qquad\qquad \textbf{(8)}$$

$$\psi_{II}(x) = C\,e^{-\beta x} + D\,e^{\beta x} \qquad\qquad \textbf{(9)}$$

$$\lim_{x\to\infty} e^{\beta x} = \infty \qquad \{0,\infty\} \qquad\qquad e^{\beta x}$$

$$x = 0 \qquad\qquad .\,D = 0$$

$$\because \psi_I(x=0) = \psi_{II}(x=0)$$
$$\therefore A + B = C \qquad\qquad \textbf{(10)}$$

$$\because \psi'_I(x=0) = \psi'_{II}(x=0)$$
$$\therefore k(A - B) = -\beta C \qquad\qquad \textbf{(11)}$$

$$\textbf{(11) (10)}$$

$$B = \left(\frac{ik+\beta}{ik-\beta}\right)A = \left(\frac{k-i\beta}{k+i\beta}\right)A,$$

$$C = \left(\frac{2ik}{ik-\beta}\right)A = \left(\frac{2k}{k+i\beta}\right)A \qquad\qquad \textbf{(12)}$$

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$$k = r\cos\delta, \qquad \beta = r\sin\delta,$$

$$r = \sqrt{k^2 + \beta^2}, \qquad \delta = \tan^{-1}\left(\frac{\beta}{k}\right) = \sqrt{\frac{V_o - E}{E}} \qquad \textbf{(13)}$$

$$\textbf{(12)}$$

:   B   :

$$B = \left(\frac{ik + \beta}{ik - \beta}\right)A = \left(\frac{k - i\beta}{k + i\beta}\right)A = \left(\frac{\cos\delta - i\sin\delta}{\cos\delta + i\sin\delta}\right)A$$

$$= \frac{re^{-i\delta}}{re^{i\delta}}A \qquad\qquad \textbf{(14)}$$

$$= e^{-2i\delta}A$$

:   C

$$C = \left(\frac{2ik}{ik - \beta}\right)A = \left(\frac{2k}{k + i\beta}\right)A = \left(\frac{2k}{k + i\beta} - 1 + 1\right)A$$

$$= \left(\frac{k - i\beta}{k + i\beta} + 1\right)A \qquad\qquad \textbf{(15)}$$

$$= \left(e^{-2i\delta} + 1\right)A$$

:

:   **-1**

$$|B|^2 = B^{*}B = \left(\frac{k - i\beta}{k + i\beta}\right) \times \left(\frac{k + i\beta}{k - i\beta}\right)|A|^2$$

$$= |A|^2$$

$$E < V_o$$

$$.(R = \frac{|B|^2}{|A|^2} = 1 \qquad)$$

$$."T = 0"$$

:   **(14) (13)**   **-2**

$$\psi_I(x) = 2Ae^{-i\delta}\cos(kx + \delta), \qquad\qquad \textbf{(16)}$$

$$\psi_{II}(x) = \left(2A\cos\delta e^{-i\delta}\right)e^{-\beta x} \qquad\qquad \textbf{(17)}$$

$$(II) \qquad\qquad \textbf{-3}$$

$$.E < V_o \qquad\qquad (T = E - V_o)$$

:   **(17)**   **-4**

.   (II)   -

:   (II)   -

$$P_{II}(x) = |\psi_{II}|^2 = (2A \cos \delta)^2 e^{-2\beta x} \qquad (15)$$

.b 2.III.1   (   ) $x = 0$

: $\psi_I(x)$ $V_o \to \infty$ $\psi_{II}(x)$ -5

$$\psi_I(x) = A \sin kx$$

$$.x = 0$$

.(7) $i\alpha \to -\beta$ (12)   -6

___

(Rectangular potential barrier)   -5. IV

:

$$V(x) = \begin{cases} V_o, & 0 < x < a \\ 0, & a \le x \le 0 \end{cases}$$

V_o V_o V_o E

-              -   .(2.III.1   )

.   -   -   -

(b)   (a)

$E < V_o$ (b) $E > V_o$ (a)   2.III.1

$E < V_o$

: $-\infty \leq x \leq a$ (I)

$$\frac{d^2\psi_I}{dx^2} = -k^2\psi_I , \tag{1}$$

:   **(1)** $. k^2 = \dfrac{2mE}{\hbar^2}$

$$\psi_I(x) = A \underbrace{e^{ikx}}_{\text{Incident wave}} + B \underbrace{e^{-ikx}}_{\text{Reflected wave}} \tag{2}$$

e^{-ikx} e^{ikx}

A .

. B

: $0 \leq x \leq a$ (II)

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} + V_o\psi_{II} = E\psi_{II}$$

$$\Rightarrow \frac{d^2\psi_{II}}{dx^2} = \alpha^2\psi_{II} \tag{3}$$

:   **(3)** $. \alpha^2 = \dfrac{2m}{\hbar^2}(V_o - E)$

$$\psi_{II}(x) = C\, e^{\alpha x} + D\, e^{-\alpha x} \tag{4}$$

$$C \; e^{\alpha x}$$

$$De^{-\alpha x} \qquad . x$$

$$. \qquad\qquad a \quad 0 \qquad\qquad x \qquad\qquad . x$$

$$: \qquad\qquad a \le x \le \infty \quad \text{(III)}$$

$$\frac{d^2 \psi_{III}}{dx^2} = -k^2 \psi_{III}, \qquad\qquad \textbf{(5)}$$

$$: \qquad \textbf{(5)}$$

$$\psi_{III}(x) = G \quad \underbrace{e^{ikx}}_{\text{Transmitted wave}} \quad + H \quad \underbrace{e^{-ikx}}_{\text{Reflected wave}} \qquad \textbf{(6)}$$

$$e^{ikx}$$

$$e^{-ikx} \;\; . \,(\text{Transmitted wave})$$

$$. \infty \qquad\qquad\qquad H \qquad\qquad\qquad\qquad G \; .$$

$$. H = 0 \qquad\qquad\qquad \infty$$

$$:$$

$$\psi_I(x) = A \, e^{ikx} + B \, e^{-ikx}$$

$$\psi_{II}(x) = C \, e^{\alpha x} + D \, e^{-\alpha x} \qquad\qquad \textbf{(4)}$$

$$\psi_{III}(x) = G \, e^{ikx}$$

$$: \qquad\qquad\qquad\qquad\qquad x = 0 \qquad\qquad :$$

$$\because \psi_I(x = 0) = \psi_{II}(x = 0)$$

$$\therefore \quad A + B = C \qquad\qquad\qquad \textbf{(5)}$$

$$:$$

$$\because \psi'_I(x = 0) = \psi'_{II}(x = 0)$$

$$\therefore \quad ik(A - B) = \alpha C - \alpha D \qquad\qquad \textbf{(6)}$$

$$: \qquad\qquad\qquad\qquad x = a$$

$$\because \psi_{II}(x = a) = \psi_{III}(x = a)$$

$$\therefore \quad Ce^{\alpha a} + De^{-\alpha a} = Ge^{ika} \qquad\qquad \textbf{(7)}$$

$$:$$

$$\because \quad \psi'_{II}(x=a)=\psi'_{III}(x=a)$$

$$\therefore \quad \alpha C e^{\alpha a}-\alpha D e^{-\alpha a}=ikGe^{ika} \tag{8}$$

B A (8 7 6 5)   .

: G

$$A=\left[\cosh(\alpha a)+\frac{i}{2}\left(\frac{\alpha}{k}-\frac{k}{\alpha}\right)\sinh(\alpha a)\right]Ge^{ika}$$

$$B=-\frac{i}{2}\left(\frac{\alpha}{k}+\frac{k}{\alpha}\right)\sinh(\alpha a)Ge^{ika} \tag{9}$$

$v_1|B|^2 =$ $v_1|A|^2 =$

: (R) $.\, v_i=\hbar k_i/m$ $v_2|G|^2 =$

$$R=\frac{BB^*}{AA^*}=\left(\frac{B}{A}\right)\left(\frac{B^*}{A^*}\right)=\frac{\dfrac{1}{4}\left(\dfrac{\alpha}{k}+\dfrac{k}{\alpha}\right)^2\sinh^2(\alpha a)}{\cosh^2(\alpha a)+\dfrac{1}{4}\left(\dfrac{\alpha}{k}-\dfrac{k}{\alpha}\right)^2\sinh^2(\alpha a)}$$

$$=\frac{\dfrac{V_o^{\,2}}{4E(V_o-E)}\sinh^2(\alpha a)}{1+\dfrac{V_o^{\,2}}{4E(V_o-E)}\sinh^2(\alpha a)}\qquad =\left[1+\frac{4E(V_o-E)}{V_o^{\,2}\sinh^2(\alpha a)}\right]^{-1} \tag{10}$$

$.k$ α

: (T)

$$T=\frac{GG^*}{AA^*}=\left(\frac{G}{A}\right)\left(\frac{G^*}{A^*}\right)=\left(1+\frac{V_o^{\,2}}{4E(V_o-E)}\sinh^2(\alpha a)\right)^{-1} \tag{11}$$

:

. $T+R=1$ (11)   (10)   **-1**

$.\infty$ k α $\hbar=0$ **-2**

. $T\to 0$ $R\to 1$

$.E=0$ $T=0$ **-3**

. T $0<E<V_o$

V_o

$.V_o$

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.(Tunneling effect)   .

:

.   ✸

.   (Tunnel diodes)   ( )   ✸

.   ✸

. α ✸

.   ✸

.   ✸

a .α $(V_o - E)$ E **-4**

.E T .$(V_o - E)$ $\sinh^2(\alpha a)$

. T $\sinh^2(\alpha a)$ a **-5**

.$\alpha a \gg 1$:

: $\alpha a \gg 1$:

$$e^{-a\alpha} \to 0, \quad \sinh^2(a\alpha) = \left(\frac{e^{a\alpha} - e^{-a\alpha}}{2} \right)^2 \approx \frac{1}{4} e^{2a\alpha}$$

:

$$T = \left(1 + \frac{V_o^2}{4E(V_o - E)} \sinh^2(\alpha a) \right)^{-1}$$

$$\approx \frac{4E(V_o - E)}{V_o^2} \sinh^{-2}(\alpha a) \approx \frac{16E(V_o - E)}{V_o^2} e^{-2\alpha a}$$

$$= \frac{16}{V_o} (1 - \frac{E}{V_o}) e^{-2a\alpha}$$

$$E > V_o$$

.**(6)**   **(2)** (III) (I)

: $0 \le x \le a$ (II)

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$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2}+V_o\psi_{II}=E\psi_{II}$$

(12)

$$\Rightarrow \frac{d^2\psi_{II}}{dx^2}=-\beta^2\psi_{II}$$

: **(12)** $\beta^2=\frac{2m}{\hbar^2}(E-V_o)$

$$\psi_{II}(x)=C\,e^{i\beta x}+D\,e^{-i\beta x}$$

(13)

:

$$\psi_I(x)=A\,e^{ikx}+B\,e^{-ikx}$$

$$\psi_{II}(x)=C\,e^{i\beta x}+D\,e^{-i\beta x}$$

(14)

$$\psi_{III}(x)=G\,e^{ikx}$$

:

$$\psi_I(x=0)=\psi_{II}(x=0)$$

$$\psi'_I(x=0)=\psi'_{II}(x=0)$$

$$\psi_{II}(x=a)=\psi_{III}(x=a)$$

(15)

$$\psi'_{II}(x=a)=\psi'_{III}(x=a)$$

:   **(11)**   **(10)** $\alpha=i\beta$

$$R=\left[1+\frac{4E(E-V_o)}{V_o^{\,2}\sin^2(\beta a)}\right]^{-1}$$

(16)

$$T=\left(1+\frac{V_o^{\,2}}{4E(E-V_o)}\sin^2(\beta a)\right)^{-1}$$

(17)

:

. $T+R=1$ **(17)**   **(16)**   **-1**

: $\sin(\beta a)\approx\beta a$ $E\to V_o$ **-2**

$$T=\left(1+\frac{mV_o a^2}{2\hbar^2}\right)^{-1}$$

$$E\,(E>V_o)$$ **-3**

.**(17)**

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-4   **(17)** $T = 1$:

$$\beta a = n\pi, \qquad\qquad n = 1, 2, 3, \cdots$$

:

$$\frac{2m(E - V_o)}{\hbar^2}\, a = n\pi$$

:

$$\frac{2\pi}{\lambda} a = n\pi \qquad \Rightarrow \qquad a = n\frac{\lambda}{2}$$

.   :

$$E = V_o\left[1 + n^2\,\frac{\pi^2\hbar^2}{8mV_o a}\right]$$

-5   **(17)** T :

$$\beta a = (2n + 1)\frac{\pi}{2}, \qquad\qquad n = 0, 1, 2, \cdots$$

:

$$E = V_o\left[1 + (2n + 1)^2\,\frac{\pi^2\hbar^2}{8mV_o a}\right]$$

:

$$T = T_{\min} = \left(1 + \frac{V_o^2}{4E(E - V_o)}\right)^{-1}$$

-6 $(E > V_o)$

.

$(\frac{E}{V_o})$

.

$.(\frac{E}{V_o} < 1)$

- 5. V

    :

$$V(x,y,z) = \begin{cases} 0, & 0 \langle x \langle a, 0 \langle y \langle b, 0 \langle z \langle c \\ \\ \infty, & \text{otherwise} \end{cases}$$

(   )   **2.4**

(V)

      $(0,0,0) \ \langle \ (x,y,z) \ \langle \ (a,b,c)$:

      :

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)\psi(x,y,z) = E\psi(x,y,z) \qquad \textbf{(1)}$$

(1) $.(x,y,z)$ ψ

      :

      -1

$$\psi(x,y,z) = X(x)Y(y)Z(z) \qquad \textbf{(2)}$$

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$$E = E_x + E_y + E_z \qquad \textbf{(1)} \qquad \textbf{(2)} \qquad \textbf{-2}$$

$$\psi(0, y, z) = \psi(a, y, z) \qquad \text{for all } y, \text{ and } z$$

$$\psi(x, 0, z) = \psi(x, b, z) \qquad \text{for all } x, \text{ and } z \qquad \textbf{(3)}$$

$$\psi(x, y, 0) = \psi(x, y, c) \qquad \text{for all } x, \text{ and } y$$

:

$$\frac{d^2 X(x)}{dx^2} + k_x^2 X(x) = 0, \quad k_x^2 = \frac{2mE_x}{\hbar^2},$$

$$\frac{d^2 Y(y)}{dy^2} + k_y^2 Y(y) = 0, \quad k_y^2 = \frac{2mE_y}{\hbar^2}, \qquad \textbf{(4)}$$

$$\frac{d^2 Z(z)}{dz^2} + k_z^2 Z(z) = 0, \quad k_z^2 = \frac{2mE_z}{\hbar^2}.$$

$(2.II.9)$.

:

$$X(x) = \sqrt{\frac{2}{a}} \sin k_x x, \quad k_x = \frac{n_x \pi}{a}, \quad n_x = 1, 2, \cdots$$

$$Y(y) = \sqrt{\frac{2}{b}} \sin k_y y, \quad k_y = \frac{n_y \pi}{b}, \quad n_y = 1, 2, \cdots \qquad \textbf{(5)}$$

$$Z(z) = \sqrt{\frac{2}{c}} \sin k_z Z \quad k_z = \frac{n_z \pi}{c}, \quad n_z = 1, 2, \cdots$$

$$E = E_x + E_y + E_z = \frac{\hbar^2 \pi^2}{2m} \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right) \qquad \textbf{(6)}$$

:

$$\psi(x, y, z) = \sqrt{\frac{8}{abc}} \sin\left(\frac{n_x \pi}{a} x \right) \sin\left(\frac{n_y \pi}{a} y \right) \sin\left(\frac{n_z \pi}{a} z \right), \quad \begin{array}{l} n_x = 1, 2, \cdots \\ n_y = 1, 2, \cdots \\ n_z = 1, 2, \cdots \end{array} \qquad \textbf{(7)}$$

:

(   )   (6)   -1

. (n_x, n_y, n_z) -2

.

:   -3

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$$\int_{-\infty}^{\infty} |\psi|^2 dx dy dz = \int_0^a |X(x)|^2 dx \int_0^b |Y(y)|^2 dy \int_0^c |Z(z)|^2 dz = 1 \qquad \textbf{(8)}$$

$$: \quad (a = b = c = L) \qquad \textbf{-4}$$

$$E = \frac{\hbar^2 \pi^2}{2mL^2} \underbrace{\left(n_x^{\,2} + n_y^{\,2} + n_z^{\,2} \right)}_{n^2} = n^2 E_1, \qquad E_1 = \frac{\hbar^2 \pi^2}{2mL^2} \qquad \textbf{(9)}$$

$$(\qquad) \qquad\qquad\qquad (\qquad) \qquad\qquad \textbf{(9)} \qquad \textbf{-5}$$

.   .

.

	n^2	n_x	n_y	n_z	ψ_{n_x,n_y,n_z}
1	3	1	1	1	$\psi_{1,1,1}$
3	6	1	1	2	$\psi_{1,1,2}$
	6	1	2	1	$\psi_{1,2,1}$
	6	2	1	1	$\psi_{2,1,1}$
3	9	1	2	2	$\psi_{1,2,2}$
	9	2	1	2	$\psi_{2,1,2}$
	9	2	2	1	$\psi_{2,2,1}$
3	11	1	1	3	$\psi_{1,1,3}$
	11	1	3	1	$\psi_{1,3,1}$
	11	3	1	1	$\psi_{3,1,1}$
1	12	2	2	2	$\psi_{2,2,2}$

:   **-1**

(a) $\dfrac{\sin(x)}{x}$ $(0,\infty)$ (b) $a\,x$ $(-5,5)$ (c) e^{-x} $(0,\infty)$

:   **-2**

(a) $\left(\dfrac{d}{dx}+x\right)^2 = \left(\dfrac{d^2}{dx^2}+2x\dfrac{d}{dx}+x^2+1\right)$ b) $\left(\dfrac{d}{dx}+\dfrac{1}{x}\right)^3 = \dfrac{d^3}{dx^3}+\dfrac{3}{x}\dfrac{d^2}{dx^2}$

(c) $\left(x\dfrac{d}{dx}\right)^2 = x^2\dfrac{d^2}{dx^2}+x\dfrac{d}{dx}$ (d) $\left(\dfrac{d}{dx}x\right)^2 = x^2\dfrac{d^2}{dx^2}+3x\dfrac{d}{dx}+1$

(e) $\left(\dfrac{d}{dx}\pm x\right)\left(\dfrac{d}{dx}\pm x\right) = \left(\dfrac{d^2}{dx^2}-x^2\mp1\right)$

                                                                                        -3

(a) $\dfrac{d}{dx}\sin\left(\dfrac{\pi x}{2}\right)$ (b) $\dfrac{d^2}{dx^2}\sin\left(\dfrac{\pi x}{2}\right)$ (c) $\dfrac{d^n}{dx^n}e^{\alpha x}$ (d) $-i\hbar\dfrac{\partial}{\partial x}e^{ikx}$

: $\psi = 45xy^2$ $\hat{C} = \dfrac{1}{(\ )}$ $\hat{A} = \dfrac{d}{dx}(\ )$ **-4**

(a) $\hat{A}(\hat{C}\psi) = \dfrac{d}{dx}\left(\dfrac{1}{\left(45xy^2\right)}\right) = -\dfrac{1}{45x^2y^2}$

(b) $\hat{C}(\hat{A}(\psi)) = \hat{C}(\dfrac{d}{dx}\left(\dfrac{1}{\left(45xy^2\right)}\right) = \hat{C}(-\dfrac{1}{45x^2y^2}) = -45x^2y^2$

:   **-5**

$$V(x) = \begin{cases} 0, & -a \le x \le a \\ \infty, & \text{otherwise} \end{cases}$$

$$\psi_I(x) = \begin{cases} A \sin(\frac{n\pi}{2a}x) & n \text{ is even} \\ \\ B \cos(\frac{n\pi}{2a}x) & n \text{ is odd} \end{cases}$$

$$E = \frac{\pi^2 \hbar^2}{2ma^2} n^2, \quad A = B = \sqrt{\frac{1}{a}}$$

-6

$$f(x) = \begin{cases} A, & a \le x \le b \\ 0, & otherwise \end{cases}$$

:   .   A

$$A = \frac{1}{\sqrt{b-a}}, \; <x> = \frac{b+a}{2}, \; <x^2> = \frac{b^2+ab+a^2}{3},$$

$$\Delta x = \sqrt{<x^2> - <x>^2} = \frac{\sqrt{b-a}}{\sqrt{12}}$$

: $\psi(x) = Ne^{-x^2/2a}, \quad -\infty \le x \le \infty$ **-7**

$$N = \left(\frac{1}{\pi a}\right)^{1/4}, \; <x> = 0, \; <x^2> = \frac{a}{2}, \; <p> = 0, <p^2> = \frac{\hbar^2}{2a},$$

$$\Delta x = \sqrt{<x^2> - <x>^2} = \sqrt{\frac{a}{2}}, \; \Delta p = \sqrt{<p^2> - <p>^2} = \sqrt{\frac{\hbar^2}{2a}}, \text{ and}$$

$$<E = \frac{p^2}{2m}> = \frac{1}{2m}\sqrt{\frac{\hbar^2}{2a}}, \quad \Delta x \, \Delta p = \frac{\hbar}{2}$$

: $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}, \quad \int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}},$

-8

$$f(x) = \begin{cases} cx(a-x), & 0 \le x \le a \\ 0, & \text{otherwise} \end{cases}$$

:

$$c = \sqrt{\frac{30}{a^5}}, \; <x> = \frac{a}{2}, \; <x^2> = \frac{2}{6}a^2, \; <p> = 0, <p^2> = 10\frac{\hbar^2}{a^2},$$

$$\Delta x = \sqrt{<x^2> - <x>^2} = \frac{\sqrt{7}}{14}a, \; \Delta p = \sqrt{<p^2> - <p>^2} = \frac{\sqrt{10}}{a}\hbar, \text{ and}$$

$$<E = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}> = 5\frac{\hbar^2}{ma^2}$$

$$\Delta x \, \Delta p = 0.6\hbar > \frac{\hbar}{2}$$

$$: \qquad\qquad 0 \leq x \leq L \qquad \psi_1(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{L}x\right) \qquad \text{-9}$$

$$\langle x \rangle = \frac{L}{2}; \quad \langle p \rangle = 0; \quad \langle x^2 \rangle = \frac{L^2}{6n^2}\left(\frac{2\pi^2 n^2 - 3}{\pi^2}\right); \quad \langle \hat{p}^2 \rangle = \frac{n^2\hbar^2\pi^2}{L^2}$$

$$\Delta x = \sqrt{<x^2> - <x>^2} = \sqrt{\frac{L^2}{6n^2}\left(\frac{2n^2\pi^2 - 3}{\pi^2}\right) - \frac{L^2}{4}}$$

$$\Delta p = \sqrt{<p^2> - <p>^2} = \sqrt{\frac{n^2\hbar^2\pi^2}{L^2}}$$

$$\Delta x \, \Delta p = \frac{\hbar}{2}\sqrt{\frac{\pi^2 n^2 - 2}{3}} = .57\hbar \rangle \frac{\hbar}{2}$$

1.A

Simple Differential Equation

:

$$\frac{d^2\Psi}{dx^2} + k^2\Psi = 0 \tag{1}$$

:   (1)   . k^2

$$\Psi = e^{mx} \tag{2}$$

$.i = \sqrt{-1}$ $m = \pm i\ k$ $m^2 + k^2 = 0$

: e^{-imx} e^{imx} (1)

$$\Psi = A\ e^{imx} + B\ e^{-imx} \tag{3}$$

: $e^{\pm ix} = \cos x \pm i\ \sin x$

$$\Psi = (A+B)\cos(mx) + (A-B)i\ \sin(mx)$$
$$= C\cos(mx) + D\sin(mx) \tag{4}$$

. A, B, C, D

:

$$\frac{d^2\Psi}{dx^2} - m^2\Psi = 0 \tag{5}$$

:

$$\Psi = A\ e^{mx} + B\ e^{-mx}$$
$$= C\cosh(mx) + D\sinh(mx) \tag{6}$$