**KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS**
**PHYSICS DEPARTMENT**

**PHYS-430: Thermal Physics**
**Term 061**
**EXAM # 1**

**Name:** ______________________________ **ID#** ______________

**Please Solve all the Problems.**

**Problem 1.**

A bit in computer sciences is some physical object that can be in two different states 0 or 1. A word is a sequence of bits. Lets assume that the word here consist of N bits some of them ($N_0$) have values of 0 and some ($N_1$) have values of 1.

a. What is the probability that half of the bits in the word are 1's and half are 0's?
b. What is the probability that the word has a value of 1?
c. What is the probability that the word has a value of 0?
d. What is the probability that the number of 0's is larger than the number of 1's?

Solution

$$a.\ P = \frac{1}{2^N}\frac{N!}{(N/2)!^2}$$

$$b.\ P = \frac{1}{2^N}$$

$$c.\ P = \frac{1}{2^N}$$

$$d.\ P > \frac{1}{2^N}\frac{N!}{\left(\frac{N}{2}+1\right)!\left(\frac{N}{2}-1\right)!}$$

**Problem 2.**

A gas expands isothermally from volume $V_1$ to $V_2$ in an isothermal process. Find the work if the equation of state for this gas is

$$P = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

Solution:

$$W = -\int PdV = -\int_{V_1}^{V_2} nRT \frac{dV}{V - nb} + \int_{V_1}^{V_2} an^2 \frac{dv}{V^2}$$

$$= -nRT \ln\left(\frac{V_2 - nb}{V_1 - nb}\right) + an^2\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

**Problem 3.**

The multiplicity of an Einstein solid containing N oscillators and q energy units is given as

$$\Omega = \left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N$$

Show that in the limit of $T \to \infty,$ the heat capacity $C = Nk$.

$$S = k \ln \Omega = k \left[ q \ln\left(\frac{q+N}{q}\right) + N \left(\frac{q+N}{N}\right) \right]$$

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \frac{\partial q}{\partial U} \frac{\partial S}{\partial q} = \frac{1}{\varepsilon} \frac{\partial S}{\partial q}$$

$$= \frac{k}{\varepsilon} \ln\left(1 + \frac{N}{q}\right)$$

$$T = \frac{\varepsilon}{k \ln\left(1 + N\,\varepsilon/U\right)}$$

$$U = \frac{N\,\varepsilon}{e^{\varepsilon/kT} - 1}$$

$$C = \frac{\partial U}{\partial S} = \frac{N\,\varepsilon^2}{kT^2}\left(\frac{e^{\varepsilon/kT}}{(e^{\varepsilon/kT} - 1)^2}\right)$$

$$\text{when } T \to \infty,\ (kT > \varepsilon) \Rightarrow e^{\varepsilon/kT} \approx 1 + \frac{\varepsilon}{kT}$$

$$C \approx \frac{N\,\varepsilon^2}{kT^2} \frac{1 + \varepsilon/kT}{\left(\varepsilon/kT\right)^2} \approx Nk\left(1 + \frac{\varepsilon}{kT}\right) \approx kT$$

**Problem 4.**

A 10 g ice cube whose temperature is -10 $^oC$ is placed in a lake whose temperature is 15 $^oC$. Calculate the change in entropy of the cube-lake system as the ice cube comes to thermal equilibrium with the lake.

The specific heat of ice is 2220 J/kg K
The specific heat of water is 4190 J/kg K

Solution

For the ice cube:

$$\Delta S_{ice} = \Delta S\,(T = -10^oC \to T = 0^oC) + \Delta S\,(T = 0) + \Delta S\,(T = 0 \to T = 15^oC)$$
$$= mc_{ice}\ln\left(T_f / T_i\right) + \frac{mL}{T} + mc_{water}\ln\left(T_f / T_i\right)$$
$$= 0.01\times 2220\times\ln\left(273/263\right) + \frac{0.01\times 333\times 10^3}{273} + 0.01\times 4190\ln\left(288/273\right)$$
$$= 15.27 \text{ J/K}$$

the change of the entropy of the lake is $\Delta S_{lake} = \dfrac{Q_{lake}}{T}$,

where this Q is the energy lost by the lake. This is equal to the negative of the heat gained by the ice $Q_{\mathrm{ice}}$.

$$\mathrm{Q_{ice}} = mc_{ice}\Delta T + mL + mc_{water}\Delta T$$
$$= 0.01\times 2220\times(273-263) + 0.01\times 333\times 10^3 + 0.01\times 4190\times(288-273)$$
$$= 4.18\times 10^3 \text{ J}$$

$$\Delta \mathrm{S_{lake}} = -\frac{4.18\times 10^3}{288} = -14.51 \text{ J/K}$$

The change of the entropy of the ice cube-lake system is

$$\Delta S = \Delta S_{ice} + \Delta S_{lake} = 0.76 \text{ J/K}$$

**Problem 5.**

The fundamental relation for what may be regarded as a gas of photons in thermal equilibrium is

$$S = \left[ AU^{3/4} V^{1/4} \right],$$

where A is a constant.

a. Calculate the pressure of the photon gas and its equation of state
b. Calculate the energy of the photon gas
c. What is the chemical potential of the photon gas

Solution

$$a.\ P = \left( \frac{\partial S}{\partial V} \right) = \frac{A}{4} T U^{3/4} V^{-3/4}$$

$$\frac{1}{T} = \left( \frac{\partial S}{\partial U} \right) = \frac{3A}{4} U^{-1/4} V^{1/4}$$

$$P = \frac{27}{64} A^4 T^4$$

$$b.\ U = \frac{3}{4} A T V^{1/4}$$

$$c.\ \mu = -T \frac{\partial S}{\partial N} = 0$$