

Math 101

1. Find the derivative of $y = x^2 + 3x - 5$ with respect to x .

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 3x - 5) = 2x + 3$$

2. Find the derivative of $y = \sin(x)$ with respect to x .

$$\frac{dy}{dx} = \frac{d}{dx}(\sin(x)) = \cos(x)$$

3. Find the derivative of $y = e^x$ with respect to x .

$$\frac{dy}{dx} = \frac{d}{dx}(e^x) = e^x$$

4. Find the derivative of $y = \ln(x)$ with respect to x .

$$\frac{dy}{dx} = \frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

5. Find the derivative of $y = x^3 \sin(x)$ with respect to x .

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 \sin(x)) = 3x^2 \sin(x) + x^3 \cos(x)$$

6. Find the derivative of $y = \cos(x)$ with respect to x .

$$\frac{dy}{dx} = \frac{d}{dx}(\cos(x)) = -\sin(x)$$

7. Find the derivative of $y = x^2 \cos(x)$ with respect to x .

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 \cos(x)) = 2x \cos(x) - x^2 \sin(x)$$

8. Find the derivative of $y = \tan(x)$ with respect to x .

$$\frac{dy}{dx} = \frac{d}{dx}(\tan(x)) = \sec^2(x)$$

Problems on Entropy

Q1 The change in entropy for melting 6.0 kg of a solid which melts at 27 °C is: [The latent heat of fusion of the solid is 2.5×10^4 J/kg] (Ans: $+5.0 \times 10^2$ J/K)

$$\Delta S = \frac{mL}{T} = \frac{6 \times 2.5 \times 10^4}{27 + 273} = 500 \text{ J/K}$$

Q2. An ice cube of mass 400 g at temperature of 0 °C melts to water at 0 °C. The process takes place very slowly, so it is reversible. What is the change in entropy of the ice when it has all melted. (Ans: 488 J/K)

$$\Delta S = \frac{mL}{T} = \frac{0.4 \times 333 \times 10^3}{273} = 488 \text{ J/K}$$

Q3. Liquid water having a mass of 50 grams was initially at 0 °C. Heat was added to the water so that its entropy increases by 94.0 J/K, what is the final temperature of the water? (Ans: 428 K)

$$\Delta S = mc \ln(T_f/T_i) \Rightarrow T_f = T_i e^{\frac{\Delta S}{mc}} = 273 e^{\frac{94}{0.05 \times 4190}} = 428 \text{ K}$$

Q#4 A 5.00-kg block of copper is at 296 K. If it is heated that its entropy increases by 1.07 kJ/K, what is the final temperature? [The specific heat of copper is 386 J/(kg·K)] [A: 515 K.]

$$T_f = T_i e^{\frac{\Delta S}{mc}} = 515 \text{ K}$$

Q5. Calculate the change in entropy of 1.0 kg of ice at 0.0 °C when its temperature is increased to 20.0 °C [$L_{\text{fusion-ice}} = 333$ kJ/kg; $c_{\text{water}} = 4190$ J/kg·K. (Ans: 1.5×10^3 J/K)

$$\Delta S = \Delta S_1 + \Delta S_2 = \frac{mL}{T} + mc_w \ln \frac{T_f}{T_i} = \frac{1 \times 333 \times 10^3}{273} + 1 \times 4190 \times \ln \left(\frac{273 + 20}{273} \right)$$

Q#6. One mole of an ideal gas expands reversibly and isothermally at temperature $T = 27^\circ \text{C}$ until its volume is doubled. The change of entropy of this gas for this process is: (Ans: 5.8 J/K) $V_f = 2V_i$

$$\Delta S = nR \ln \frac{V_f}{V_i} = 1 \times 8.31 \ln 2 = 5.76 \text{ J/K}$$

Q7: An ideal monatomic gas is confined to a cylinder by a piston. The piston is slowly pushed in so that the gas temperature remains at 27 degree C. During the compression, 750 J of work is done on the gas. The change in the entropy of the gas is: (Ans: -3.0 J/K) .

$$\left. \begin{aligned} W &= nRT \ln \left(\frac{V_f}{V_i} \right) \\ \Delta S &= nR \ln \left(\frac{V_f}{V_i} \right) \end{aligned} \right\} \text{ note that } W \text{ is -ve (done on gas)} \\ \text{so } S \text{ is -ve} \\ \Delta S = \frac{W}{T}$$