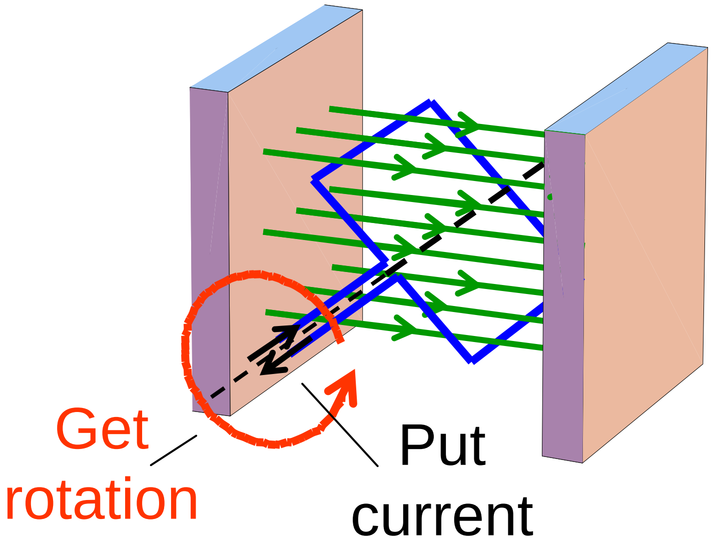
Chapter 31

Induction and Inductance

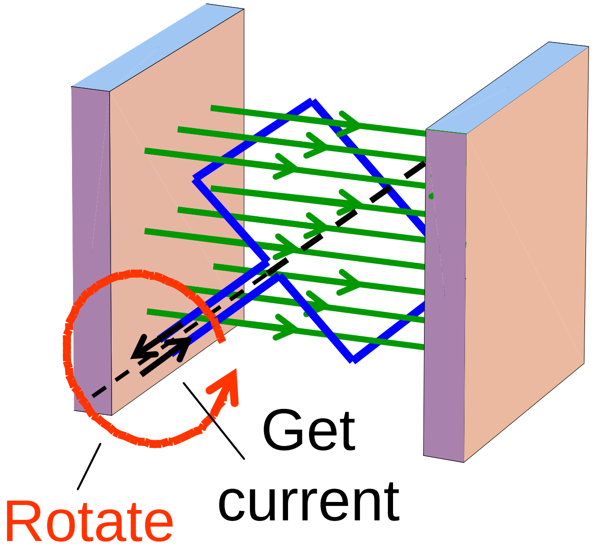
31-1 Two Symmetric Situation

Electric motor



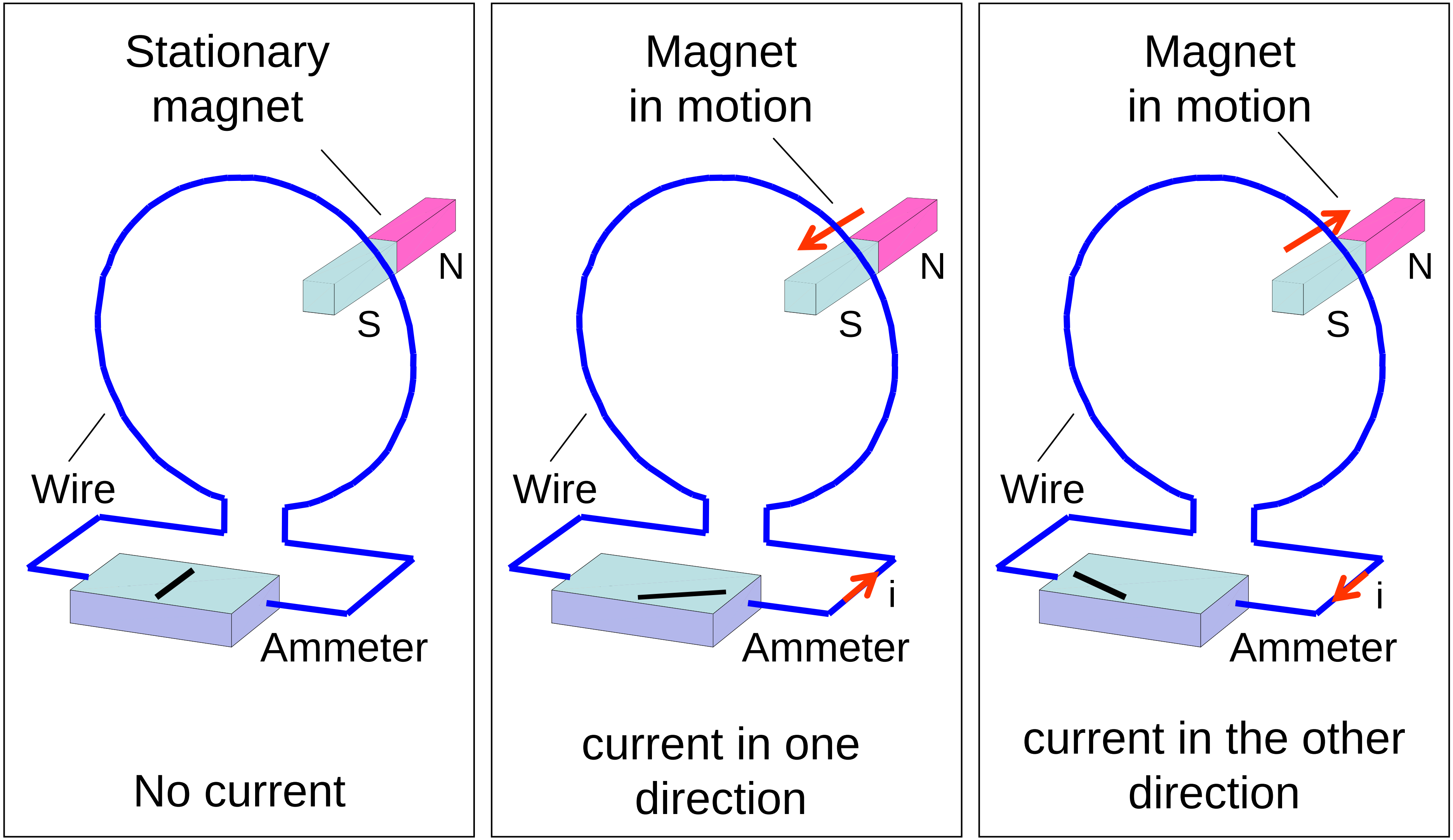
current loop + magnetic field
=
torque

Electric generator



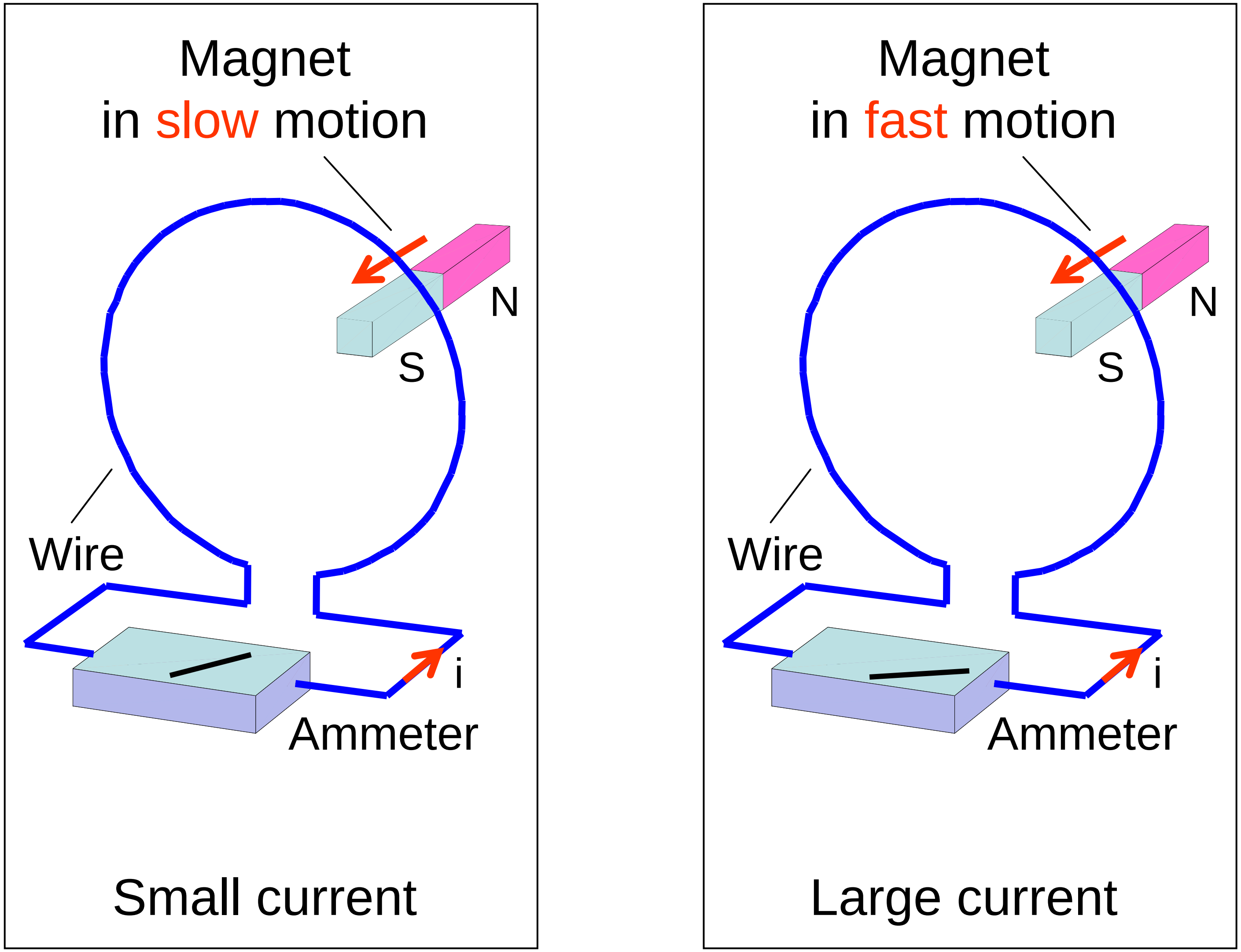
torque + magnetic field
=
current

31-2 Two Experiments



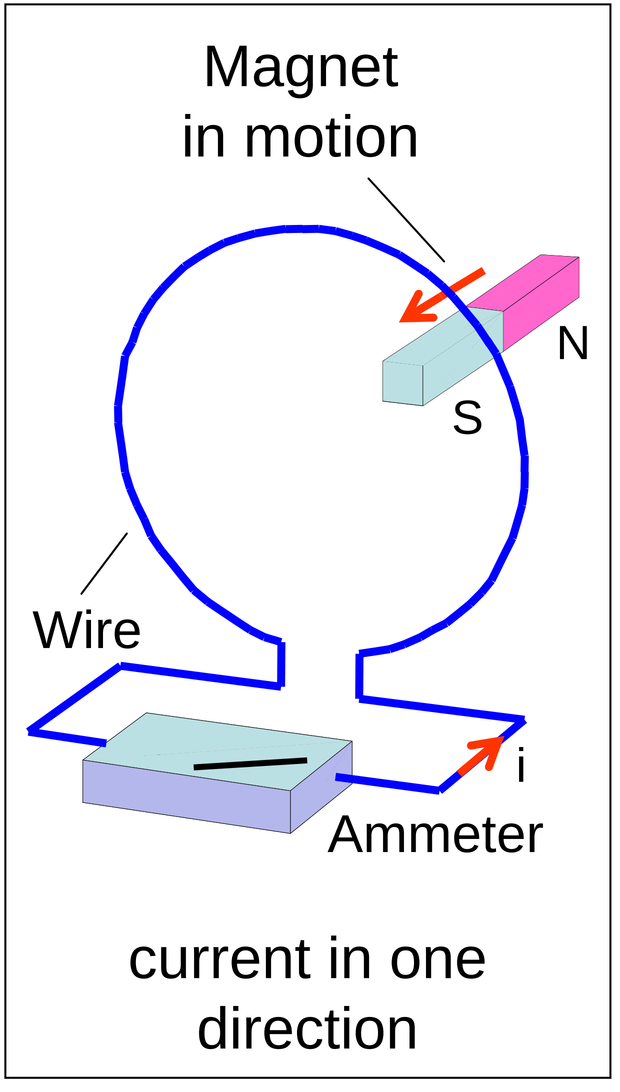
A current appears only if there is a relative motion between the loop and the magnet

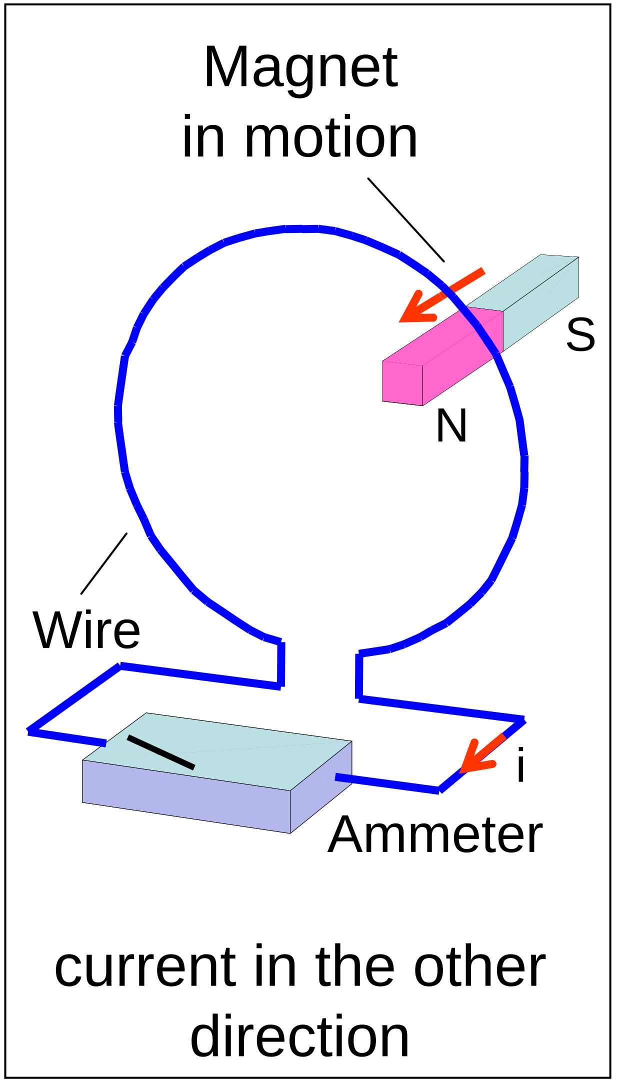
31-2Two Experiments



Faster motion produces a greater current

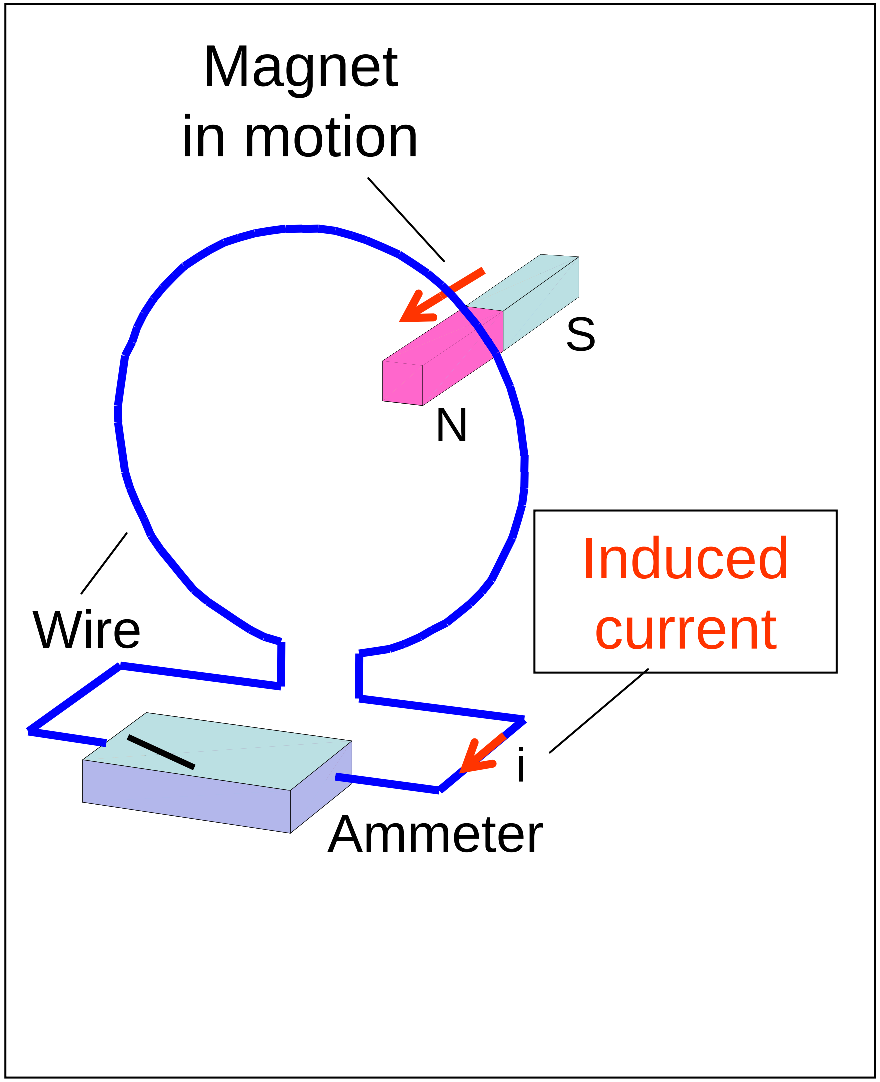
31-2 Two Experiments





If magnet is reversed, current is also reversed

31-2 Two Experiments



Induced emf

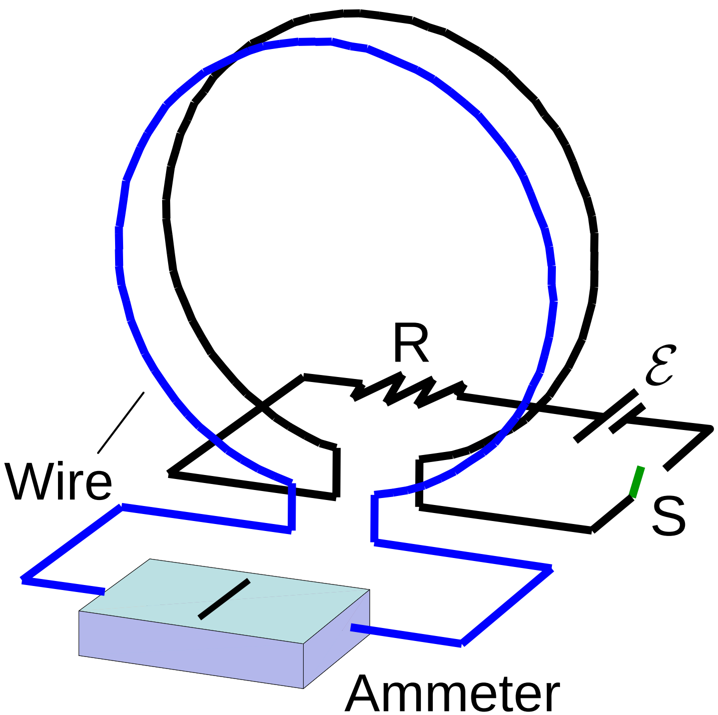
work done per unit charge to produce an induced current

Induction

process of producing an induced current and emf

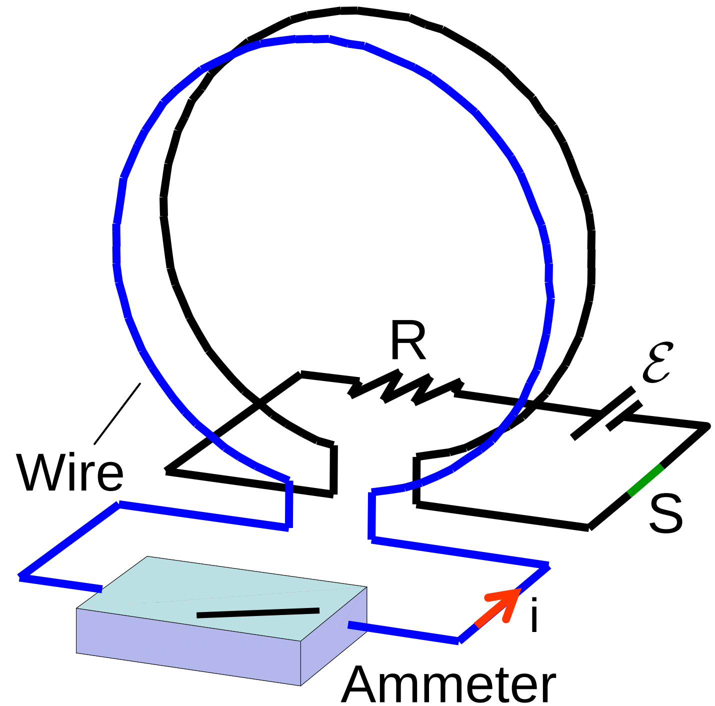
31-2 Two Experiments

Switch is open long time ago



No current
in the blue coil

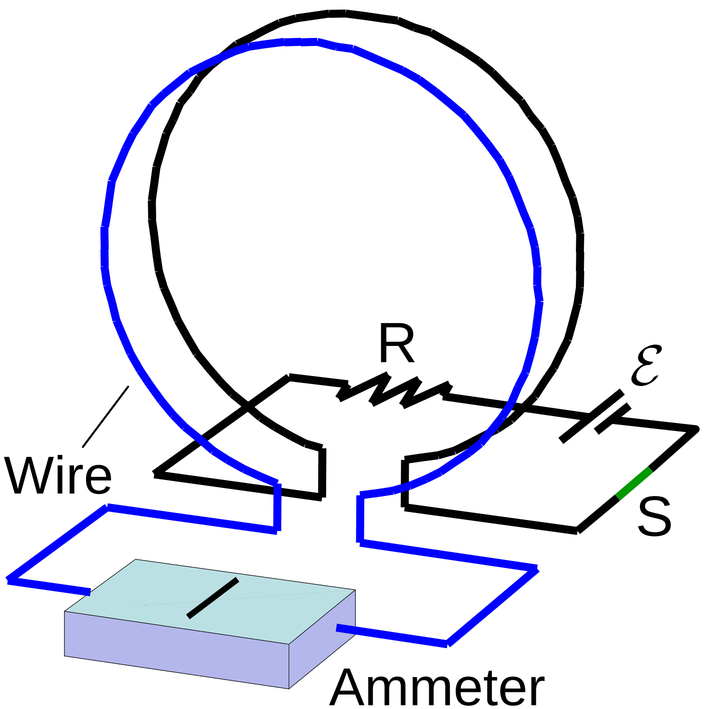
Just after the switch is closed



current briefly appears
in the blue coil

31-1 Two Symmetric Situation

Switch is closed long time ago



No current
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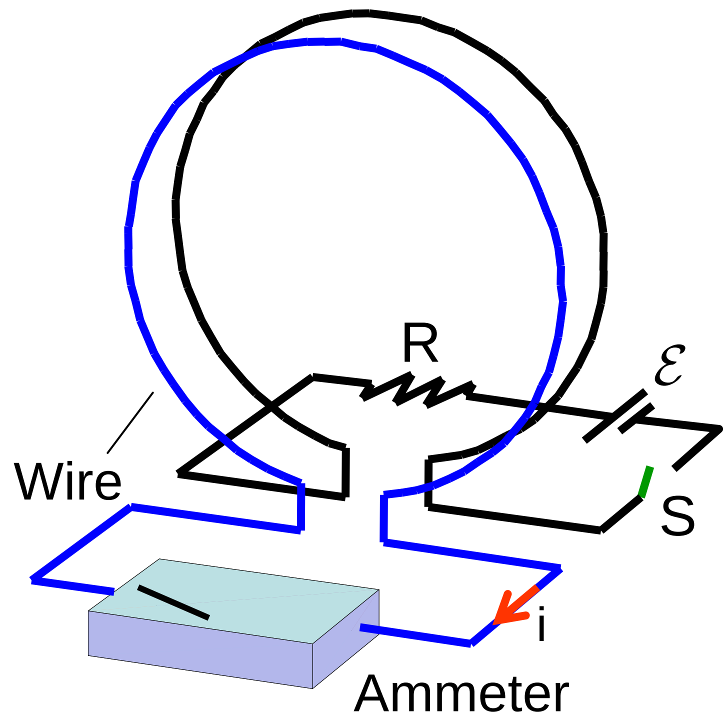
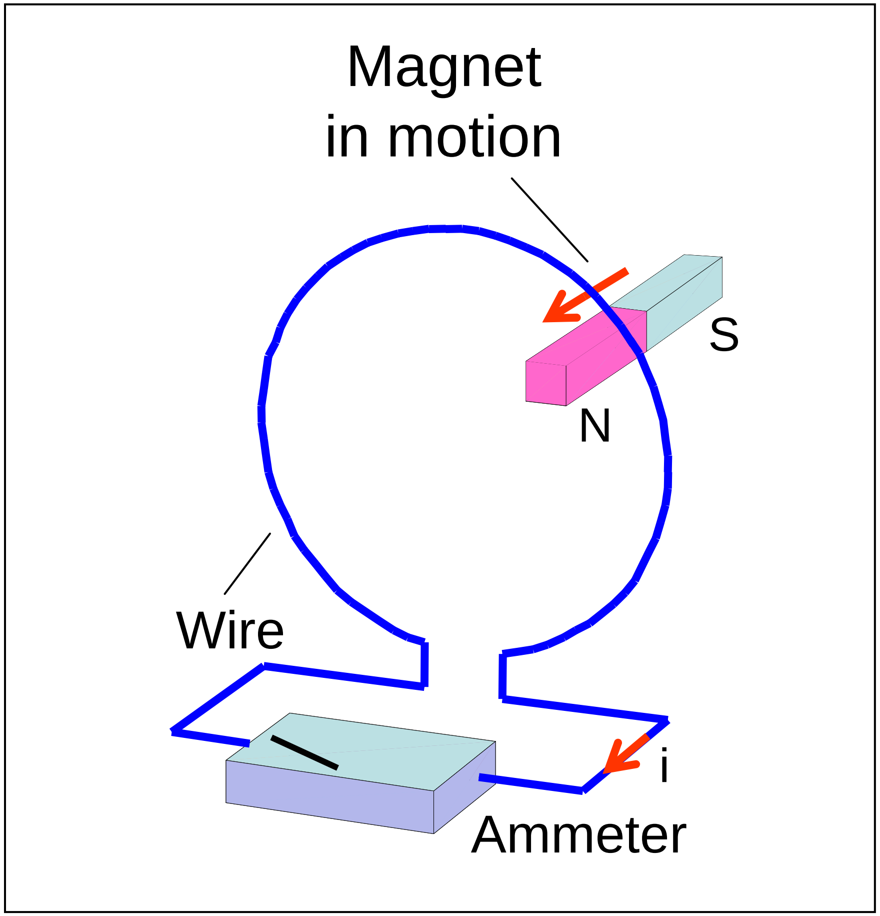
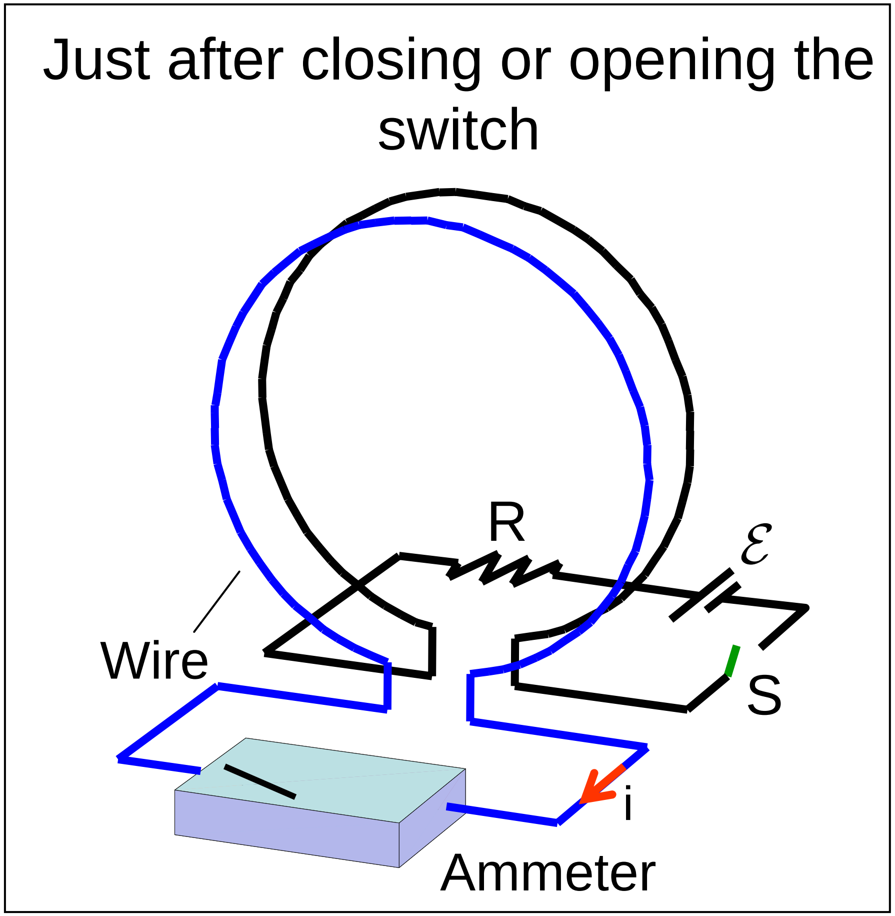
Just after the switch is open



current briefly appears
in the blue coil

31-3 Faraday's Law of Induction

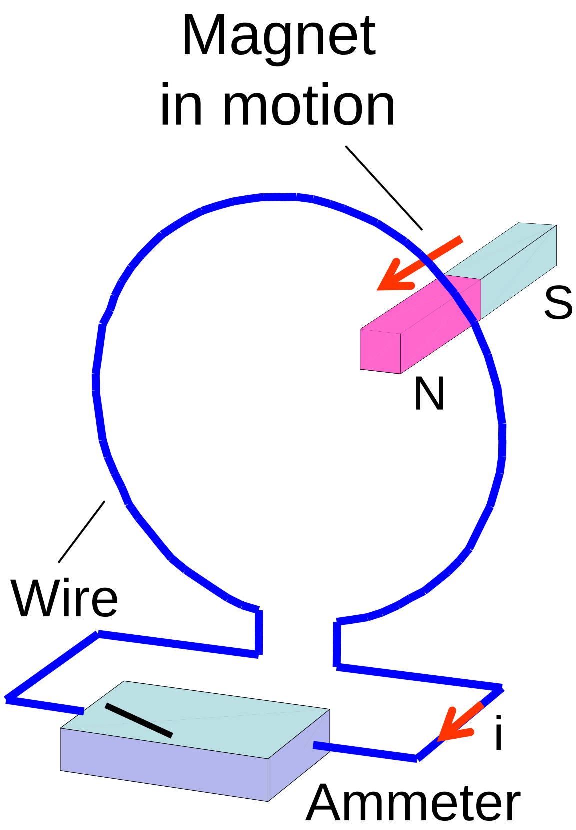




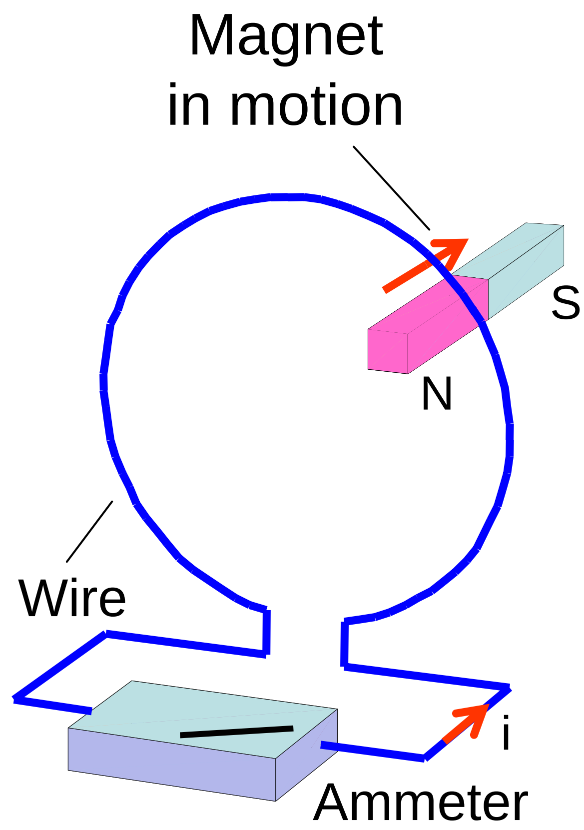
An emf induced in a loop when the number of magnetic field lines that pass through the loop is changing

The value of the induced emf is determined by the rate at which the number of magnetic field lines changes

31-3 Faraday's Law of Induction



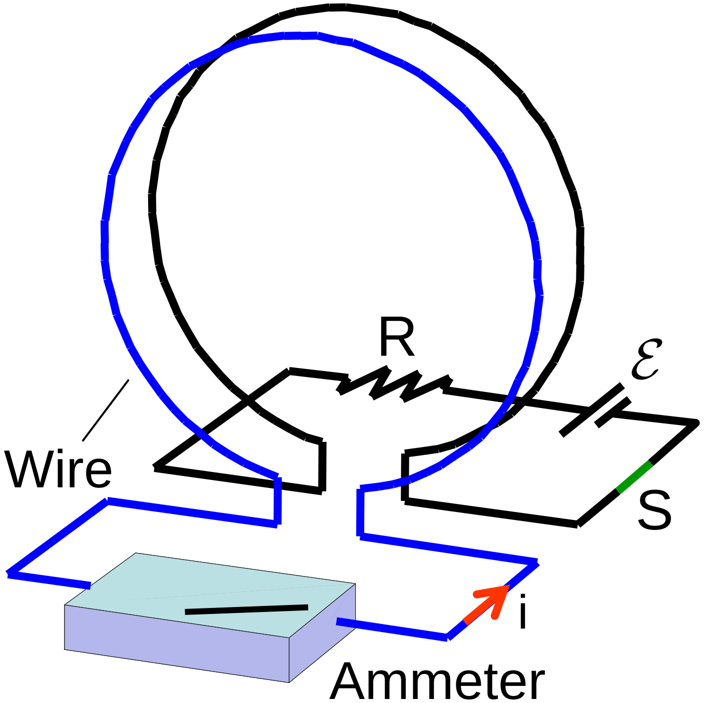
the number of magnetic field lines are increasing



the number of magnetic field lines are decreasing

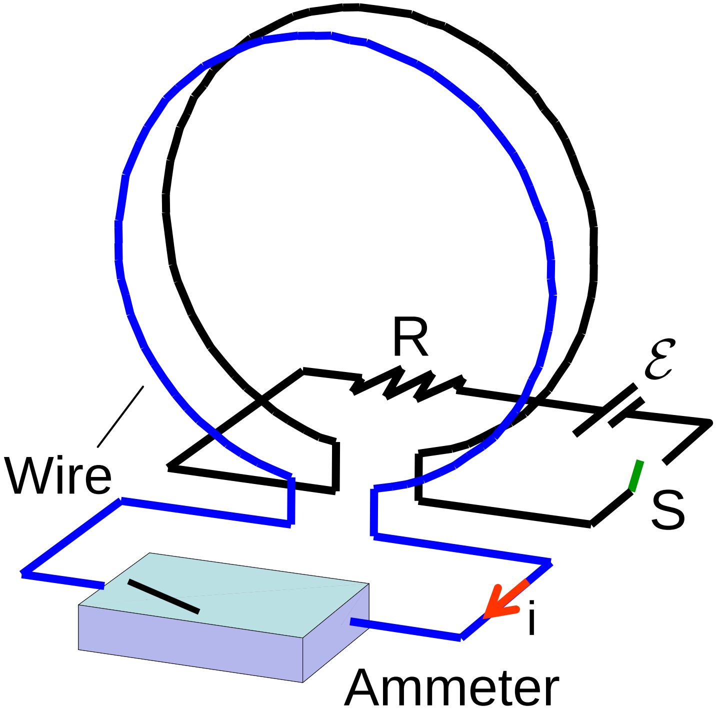
31-3 Faraday’s Law of Induction

Just after the switch is closed



the number of magnetic field lines are increasing

Just after the switch is open



the number of magnetic field lines are decreasing

31-3 Faraday's Law of Induction

Magnetic Flux trough area A

$$\Phi_B = \int_A \vec{B} \bullet d\vec{A}$$

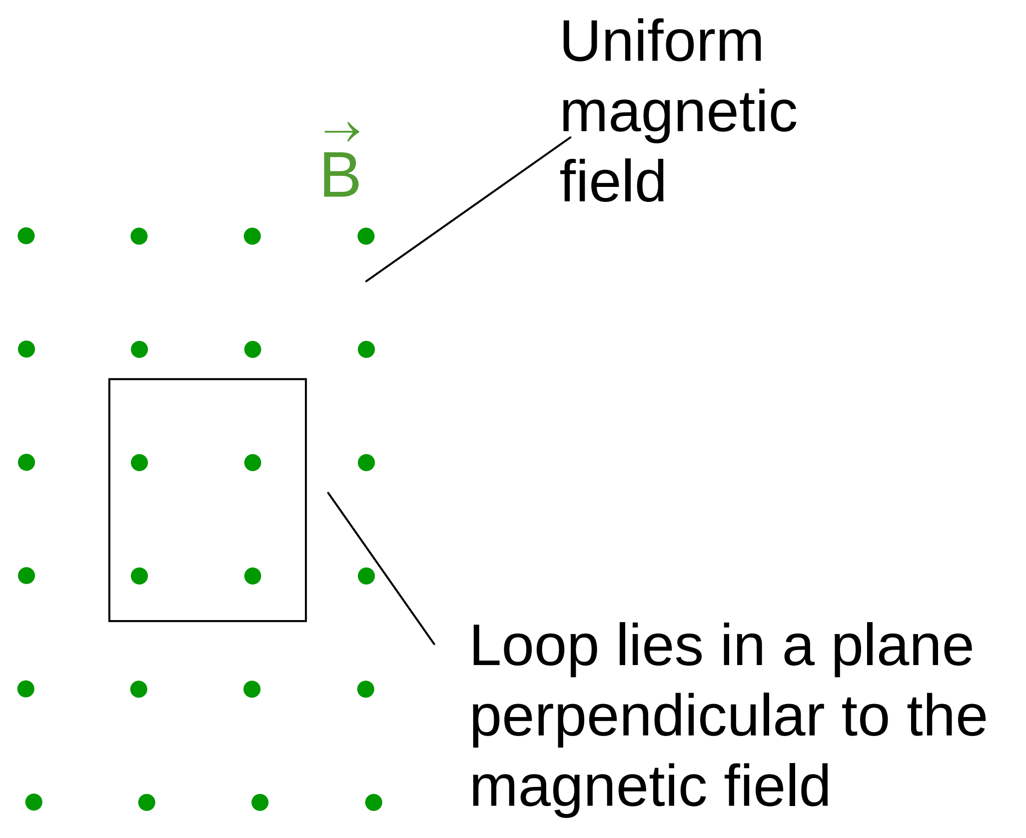
Area vector

Magnitude: area of dA

Direction: perpendicular to area dA

31-3 Faraday's Law of Induction

Special case



$$\Phi_B = \int \vec{B} \bullet d\vec{A} = \int B \, dA = B \int dA = B A$$

$$\boxed{\Phi_B = B A}$$

31-3 Faraday's Law of Induction

SI unit for magnetic flux Φ_B

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

Weber

Wb

1 Weber =1 Tesla (meter)^2

31-3 Faraday’s Law of Induction

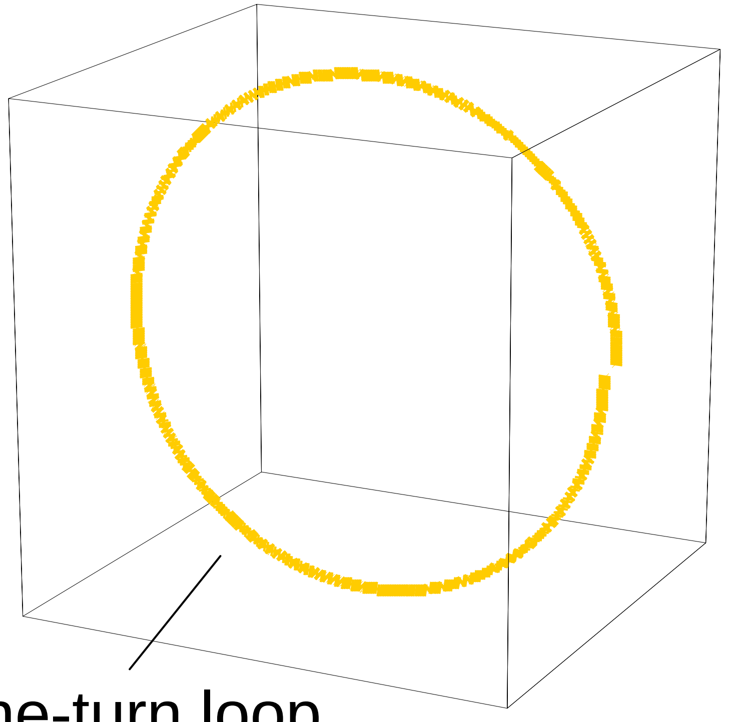
Faraday's Law

The magnitude of the emf $\mathcal{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time

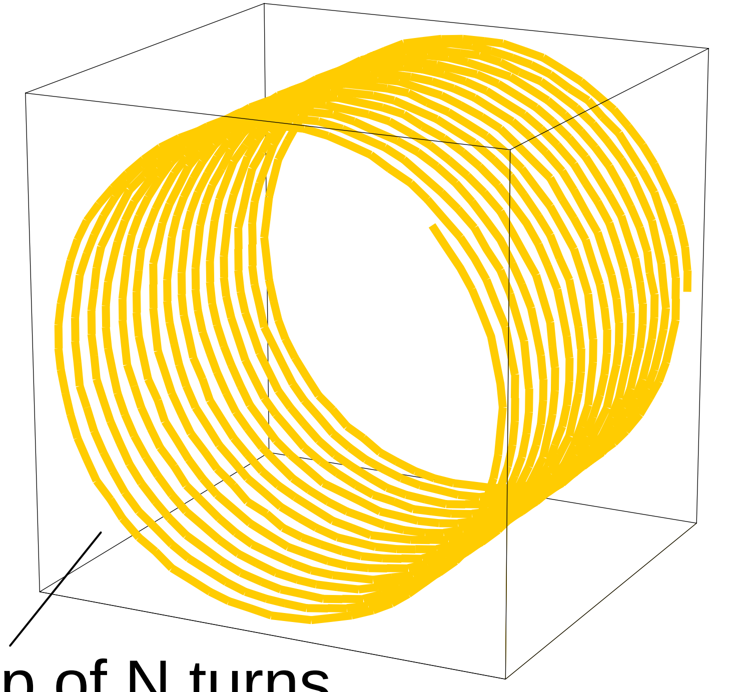
$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

Induced emf tends to oppose the flux change

31-3 Faraday's Law of Induction



$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$



$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

An emf of $\frac{d\Phi}{dt}$ is induced in every turn.

Total emf is the sum of individual induced emf

31-3 Faraday's Law of Induction

$$\Phi_B = \int_A \vec{B} \bullet d\vec{A}$$

Magnetic flux through a loop can be changed
by
changing the magnitude of the magnetic field B
or
changing the size of the loop in the magnetic field
or
changing the angle between B and the loop.

31-3 Faraday's Law of Induction

Checkpoint 1

Rank the time intervals according to the magnitude of the emf induced in a conducting loop perpendicular to B, greatest first.

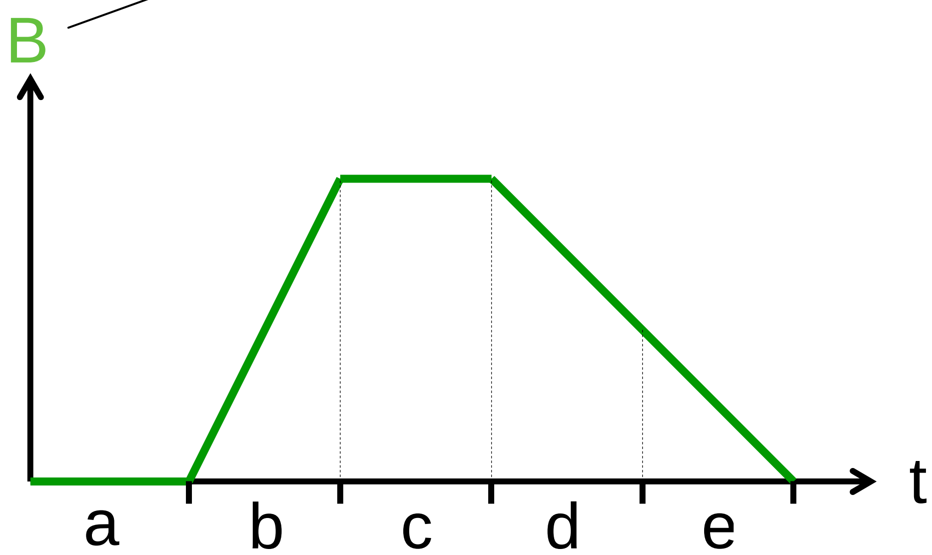
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = B\,A$$

$$\mathcal{E} = -A\,\frac{dB}{dt}$$

Slope of B-t curve

uniform magnetic field



b,
then d and e tie,
then a and c tie

31-3 Faraday's Law of Induction

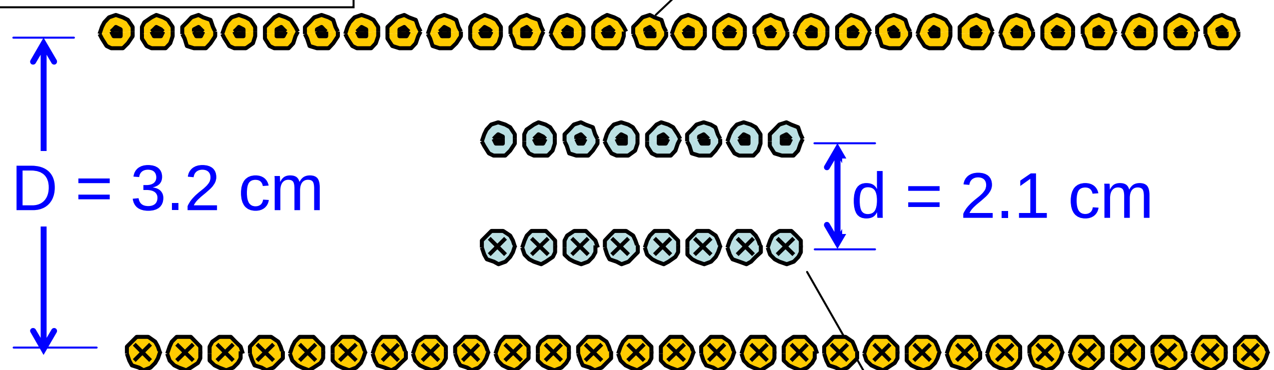
Sample Problem 31-1

Current in the solenoid is reduced at steady rate from 10 A to zero in 25 ms. What is the emf induced in coil C?

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

Flux through coil C

Number of turns of coil C



$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

$$= B\,A$$

Produced by Solenoid S

$$B = \mu_0\, n\, i$$

Number of turns per unit length of the solenoid S

Cross sectional area of coil C

$$A = \pi \left(\frac{d}{2}\right)^2$$

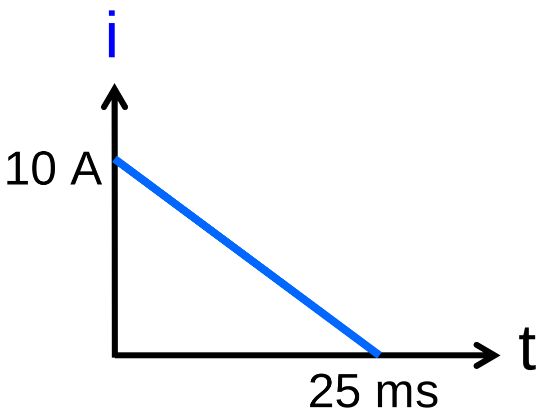
$$\Phi_B = (\mu_0\, n\, i)\, \pi \left(\frac{d}{2}\right)^2$$

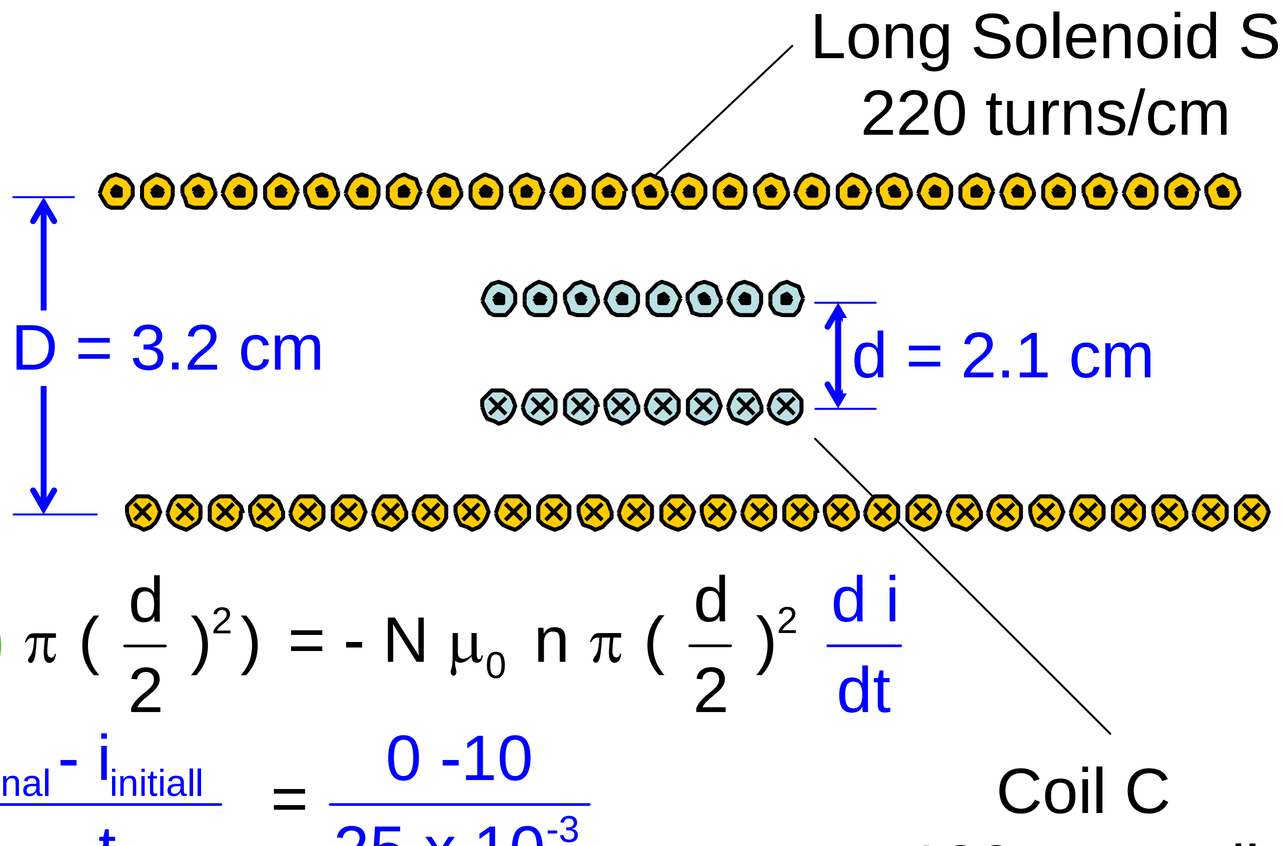
$$\mathcal{E} = -N \frac{d}{dt}\left((\mu_0\, n\, i)\, \pi \left(\frac{d}{2}\right)^2\right)$$

31-3 Faraday's Law of Induction

Sample Problem 31-1

Current in the solenoid is reduced at steady rate from 10 A to zero in 25 ms. What is the emf induced in coil C?





$$\mathcal{E} = -N \frac{d}{dt}\left((\mu_0\ n\ i)\ \pi \left(\frac{d}{2}\right)^2\right) = -N\mu_0\ n\ \pi\left(\frac{d}{2}\right)^2 \frac{d\,i}{dt}$$

$$\frac{d\,i}{dt} = \frac{i_{final} - i_{initiall}}{t_{final} - t_{initiall}} = \frac{0 - 10}{25 \times 10^{-3}}$$

$$\mathcal{E} = -(130)(4\pi x 10^{-7})(220)\pi\left(\frac{.021}{2}\right)^2\left(\frac{0 - 10}{25 \times 10^{-3}}\right) = 75\ mV$$

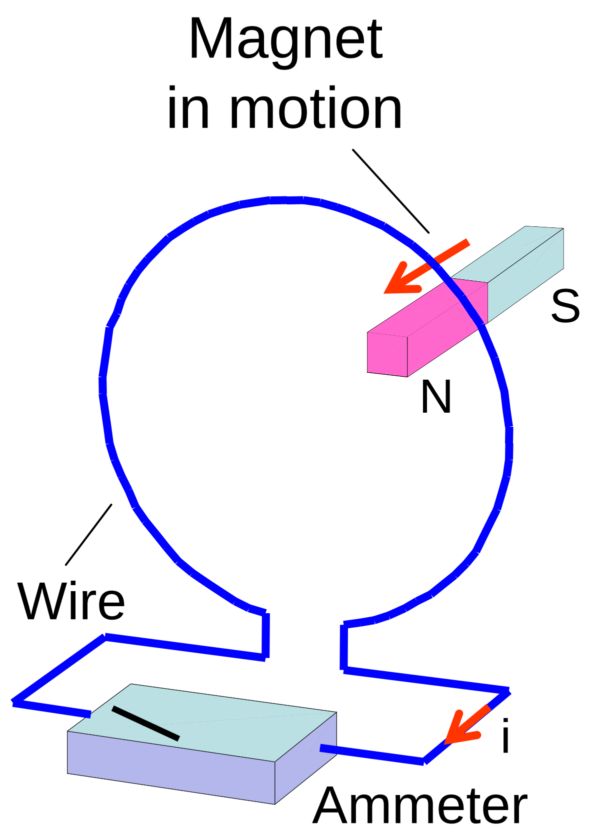
31-4 Lenz's Law

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current

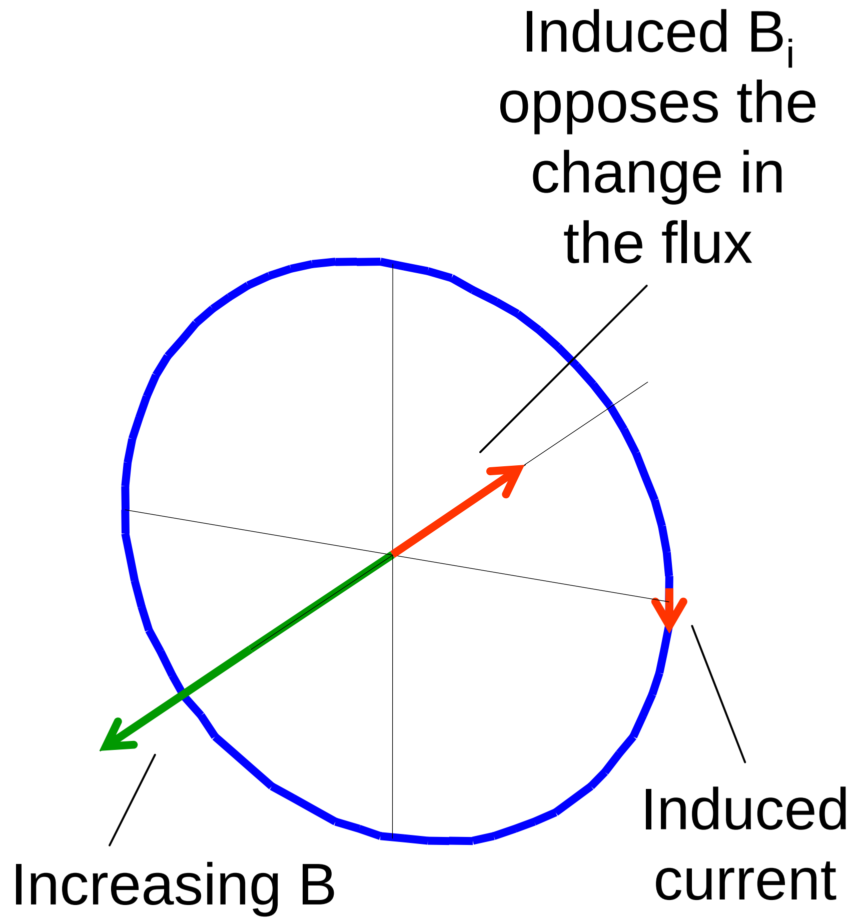
$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

Induced emf tends to oppose the flux change

31-4 Lenz's Law

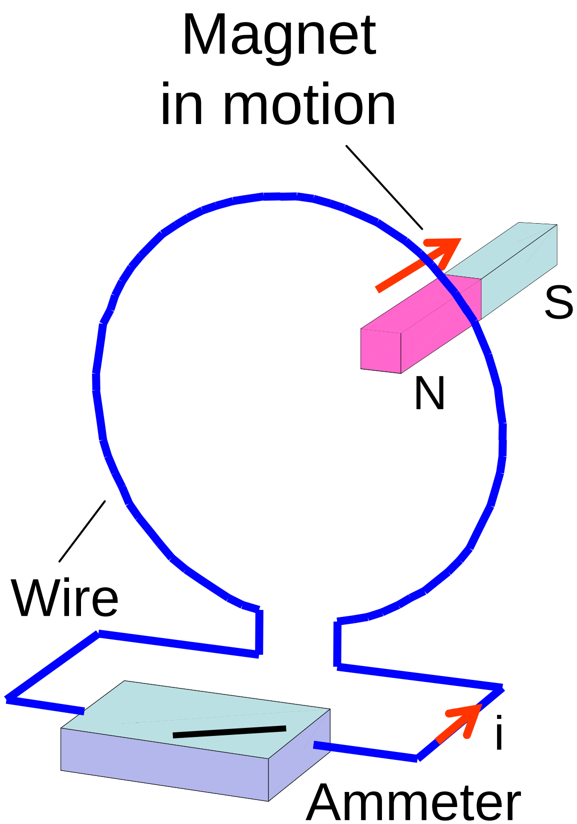


the number of magnetic field lines are increasing

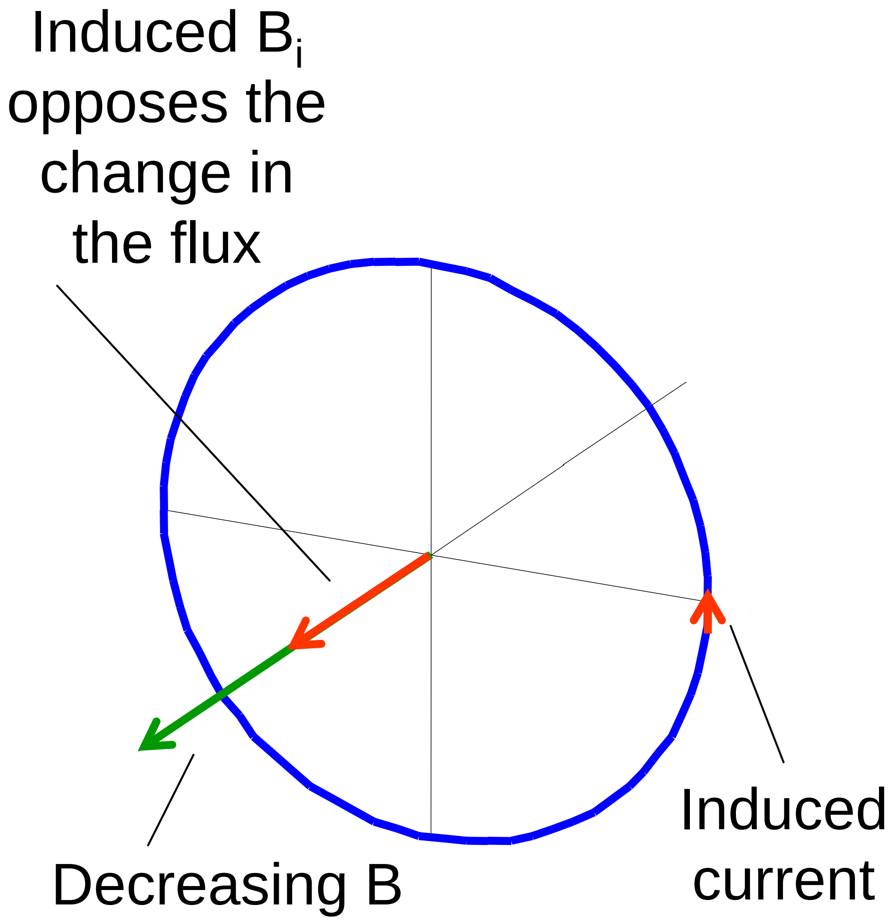


Use the right-hand rule to find the direction of the current

31-4 Lenz's Law



the number of magnetic field lines are decreasing



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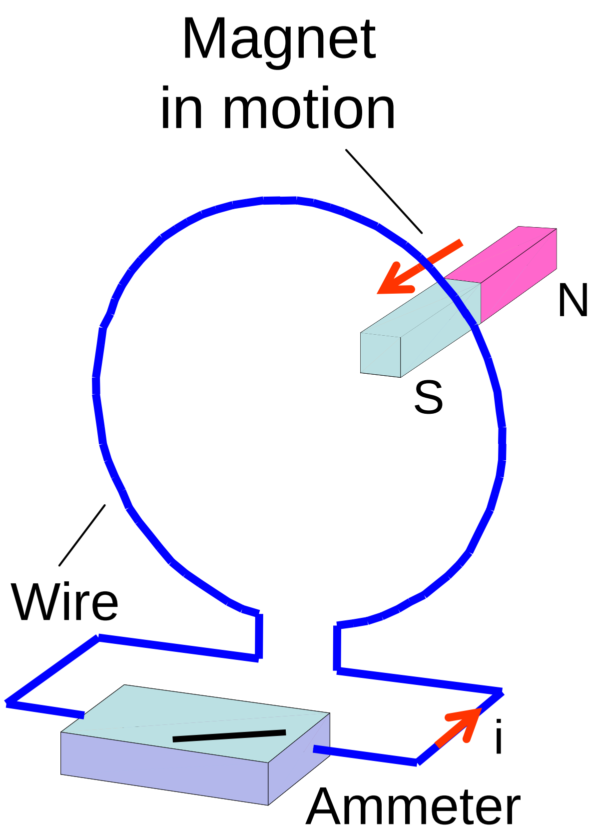

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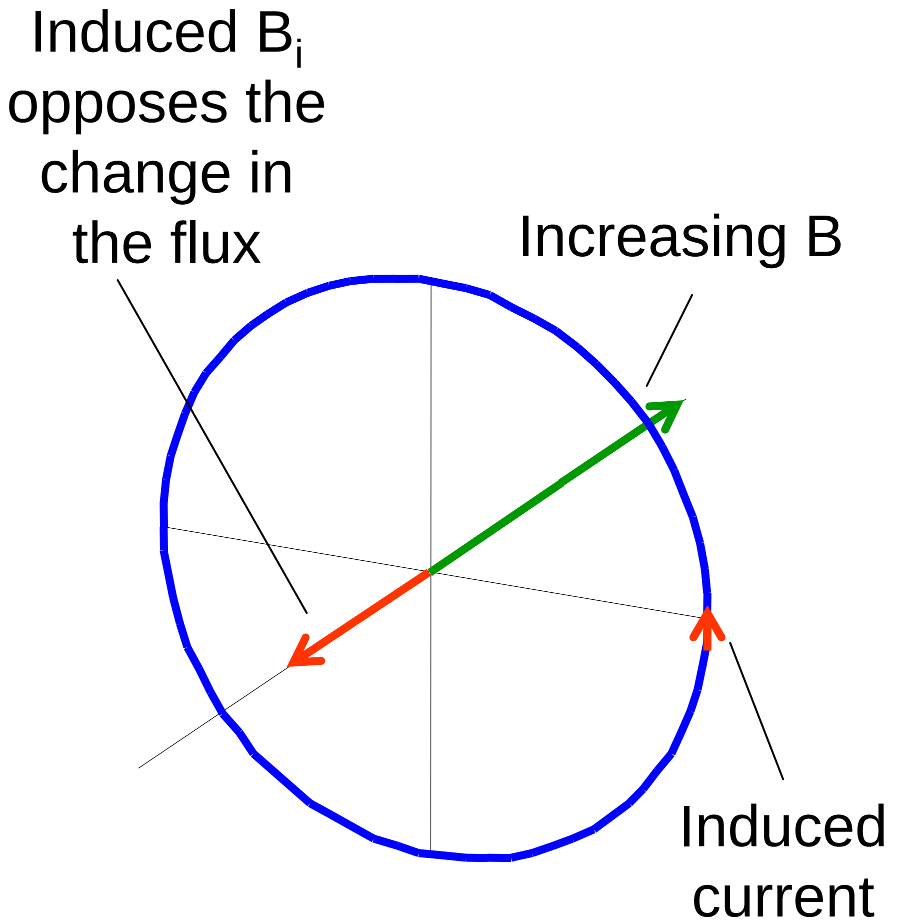

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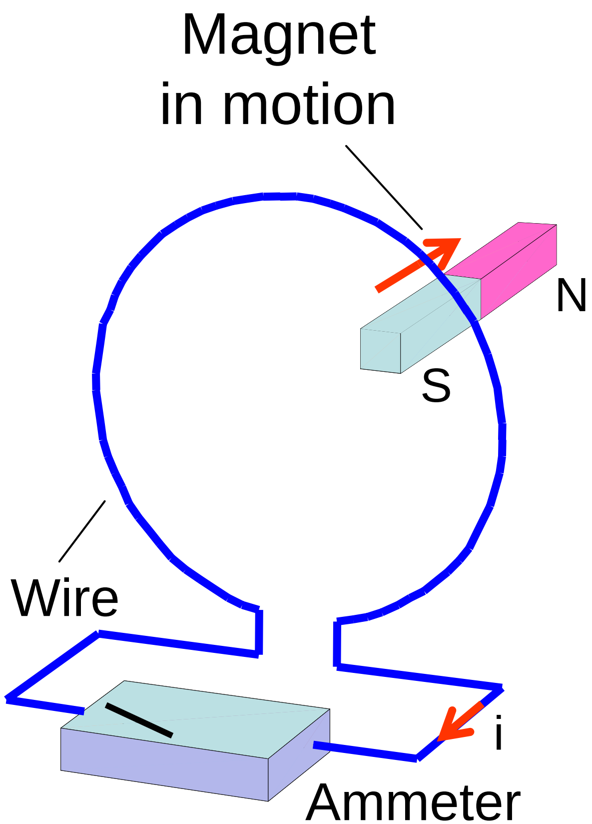
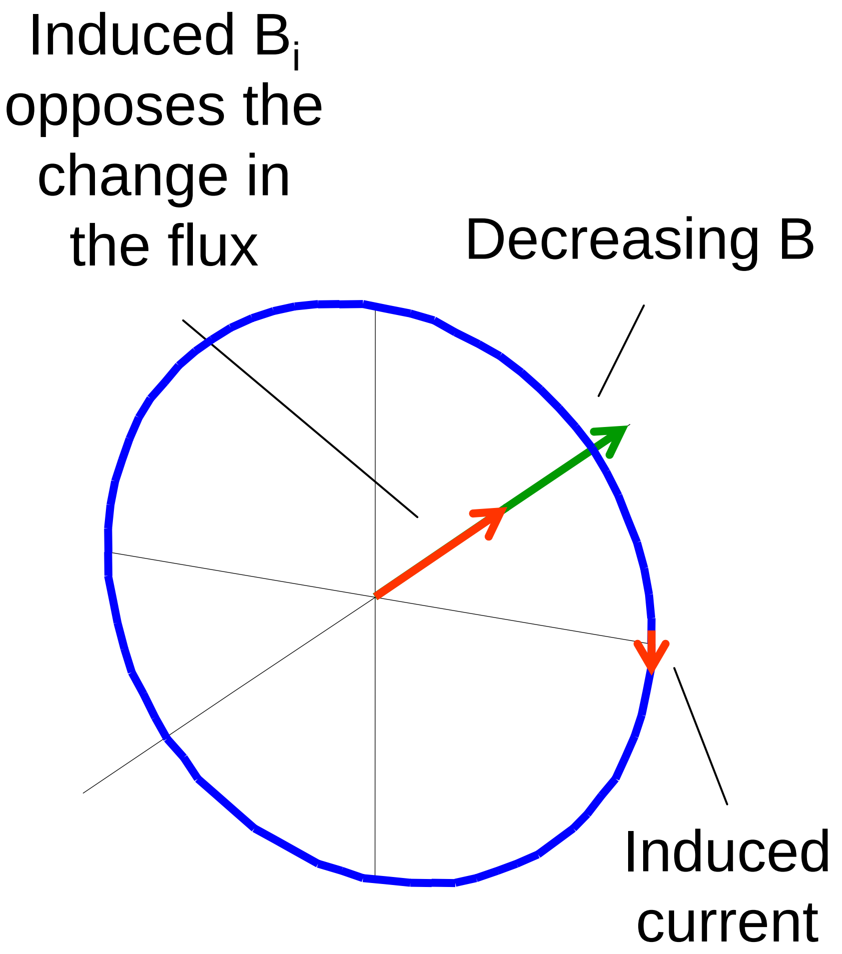


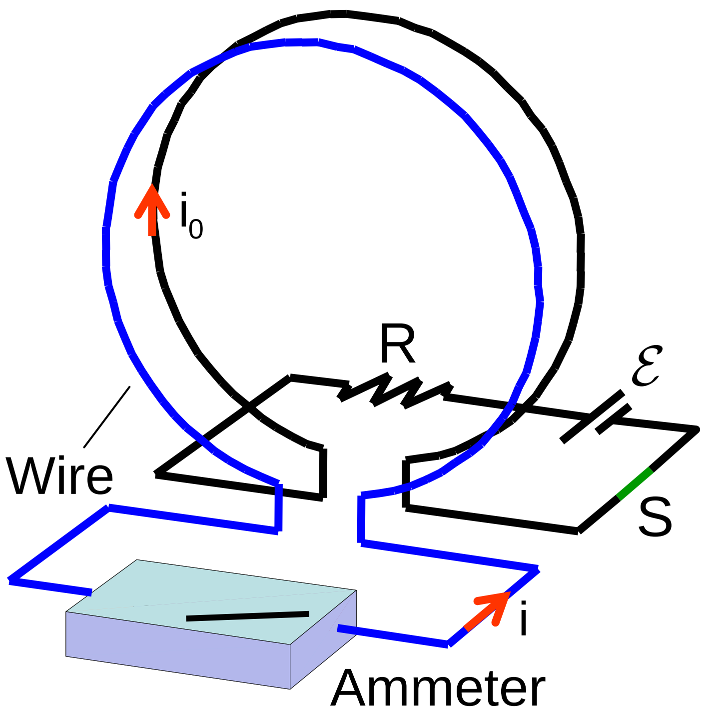
the number of magnetic field lines are decreasing



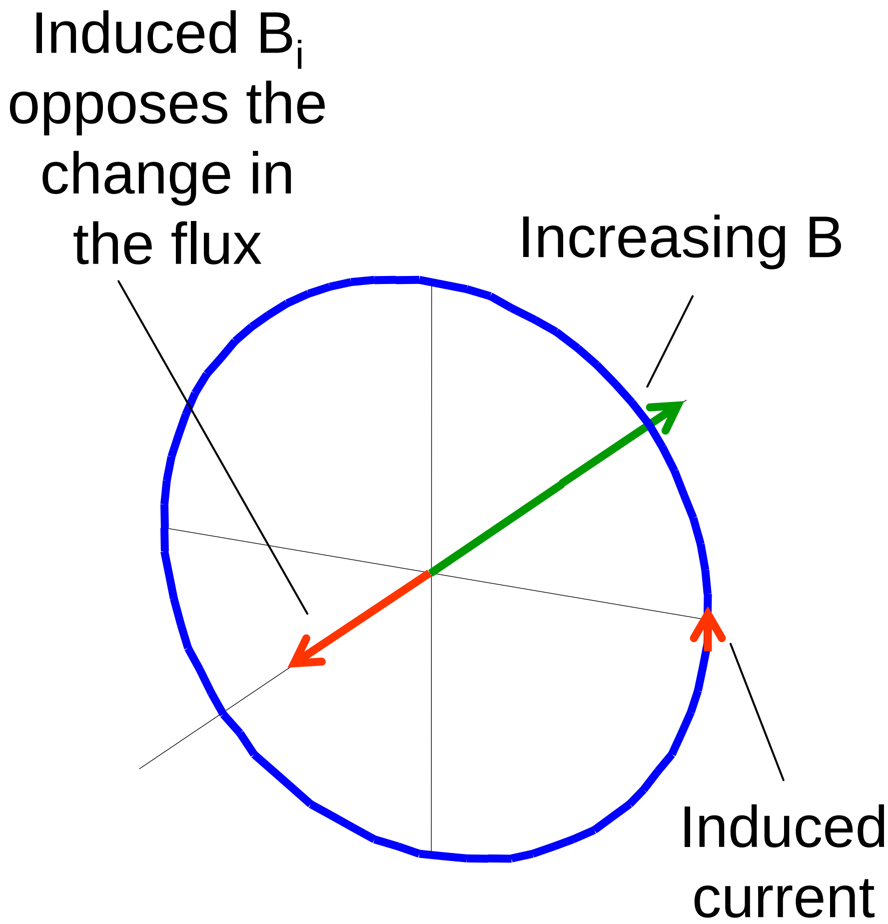
Use the right-hand rule to find the direction of the current

31-4 Lenz's Law

Just after the switch is closed



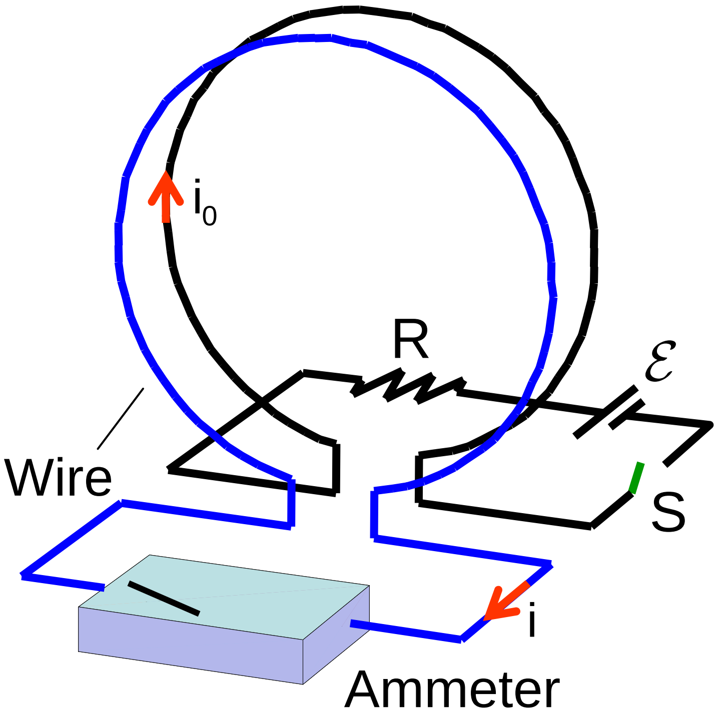
the number of magnetic field lines are increasing



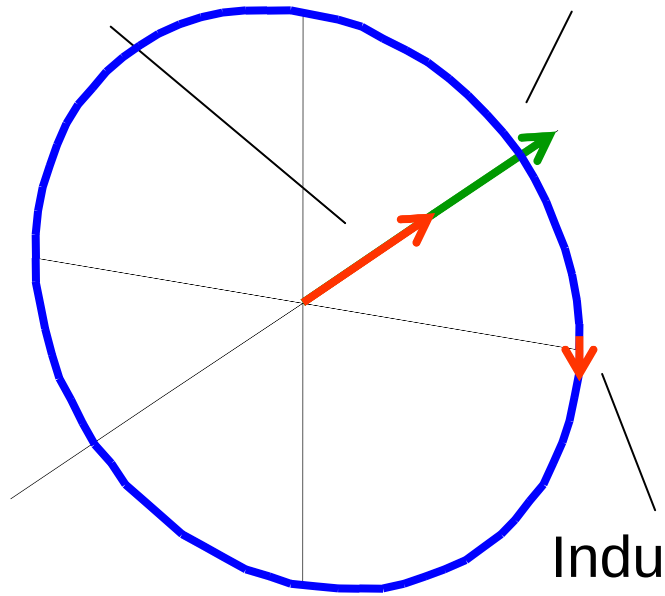
Use the right-hand rule to find the direction of the current

31-4 Lenz's Law

Just after the switch is open



the number of magnetic field lines are decreasing



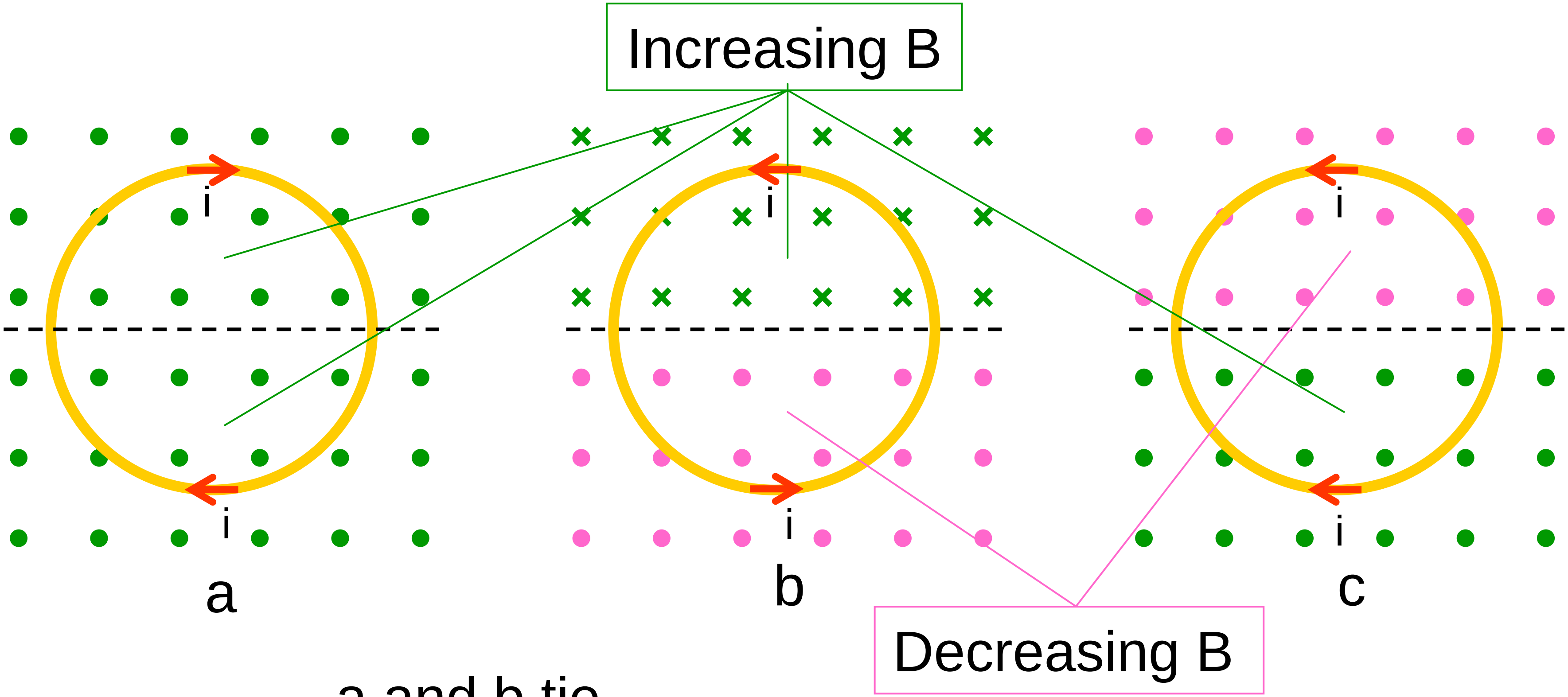
Use the right-hand rule to find the direction of the current

31-4 Lenz's Law

Checkpoint 2

Rank according to the magnitude of the current induced in the loop, greatest first.

Magnetic fields change at identical rates



a and b tie,
then c (zero)

31-4 Lenz's Law

Sample Problem 31-2

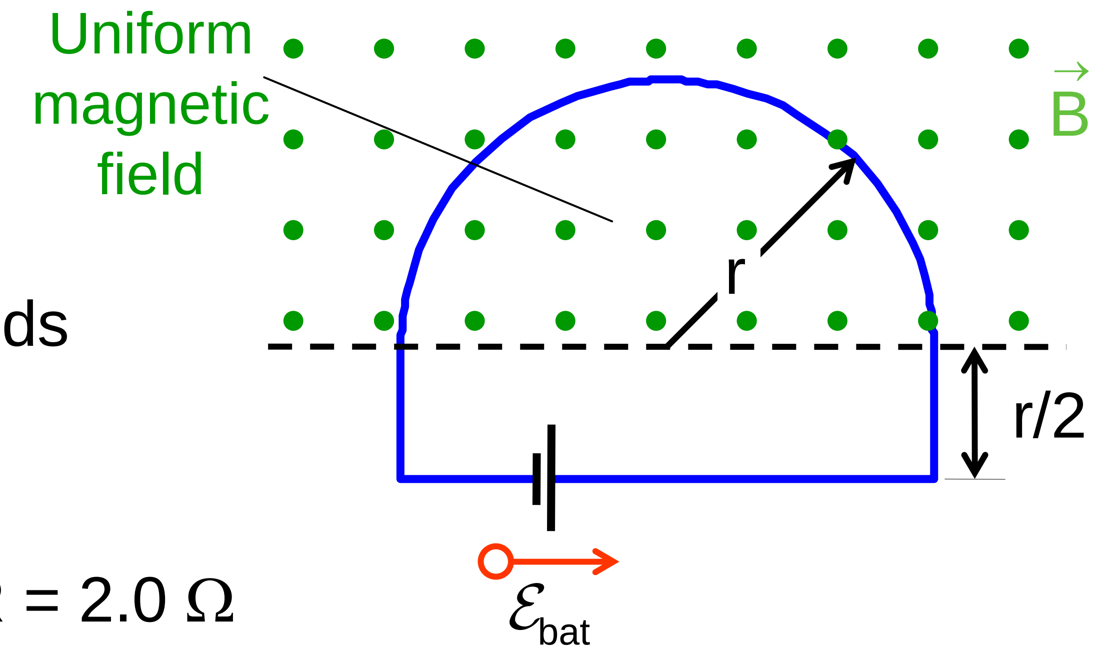
$B = 4.0\,t^2 + 2.0\,t + 3.0$

B in Tesla and t in seconds

$\mathcal{E}_{bat} = 2.0$ V

r = 0.2 m

Resistance of the loop R = 2.0 Ω



What is the magnitude and direction of the emf $\mathcal{E}_{ind}$ induced around the loop by the magnetic field at t = 10 s?

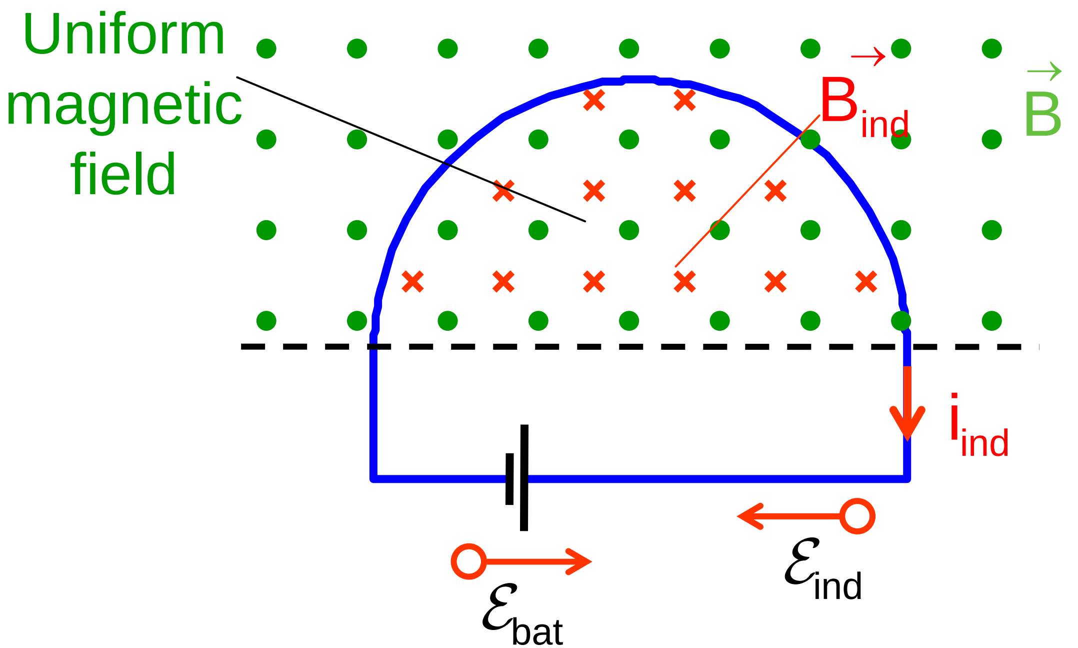
$$\mathcal{E}_{ind} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = \frac{\pi r^2}{2}\frac{dB}{dt}$$

$$= \frac{\pi r^2}{2}\frac{d}{dt}(4.0\,t^2 + 2.0\,t + 3.0) = \frac{\pi r^2}{2}(8.0\,t + 2.0)$$

$$= \frac{\pi (0.2)^2}{2}(8.0\,(10) + 2.0) = 5.15 \text{ V}$$

31-4 Lenz's Law

Sample Problem 31-2



At 10 s, $\dfrac{d\Phi_B}{dt} = 5.15\ \text{V} > 0.$ Flux is increasing.

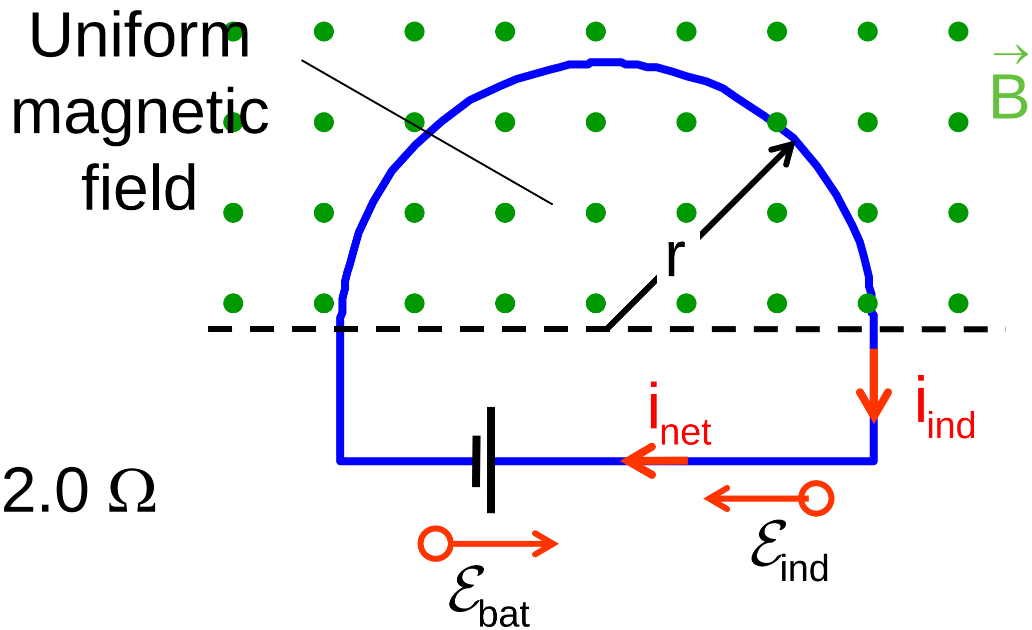
The induced B_{ind} must be into the page to oppose this increase.

From the right-hand rule, the induced current must be clockwise.

The induced emf $\mathcal{E}_{ind}$ must be clockwise.

31-4 Lenz's Law

Sample Problem 31-2



$\mathcal{E}_{bat} = 2.0$ V

Resistance of the loop R= 2.0 Ω

At t = 10 s, $\mathcal{E}_{ind} = 5.15$ V

What is the current in the loop at t = 10 s?

$$i_{net} = \frac{\mathcal{E}_{ind} - \mathcal{E}_{bat}}{R} = \frac{5.15 - 2.0}{2.0} = 1.6 \text{ A}$$

Since $\mathcal{E}_{ind} > \mathcal{E}_{bat}$, the net current is clockwise

31-4 Lenz's Law

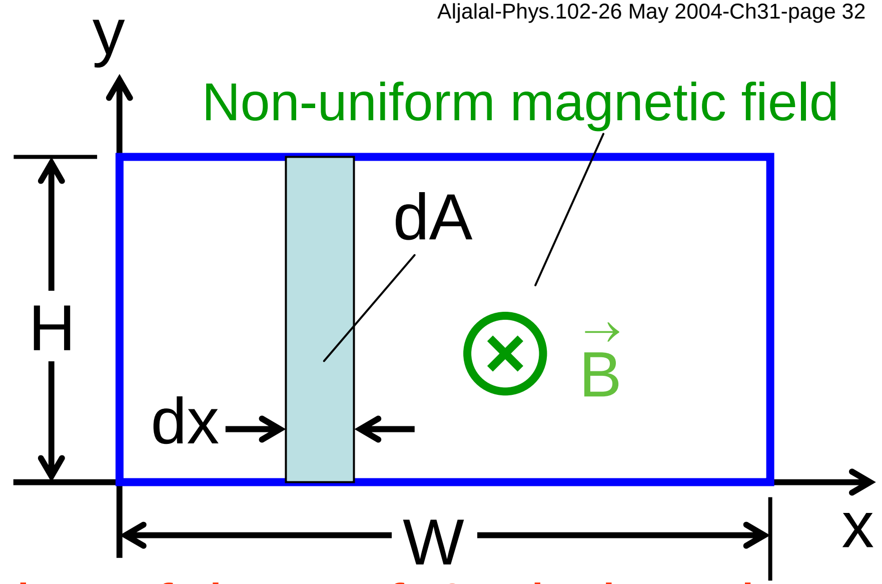
Sample Problem 31-3

$B = 4\, t^2\, x^2$

B in Tesla and t in seconds

W = 3.0 m

H = 2.0 m



What is the magnitude and direction of the emf $\mathcal{E}_{ind}$ induced around the loop by the magnetic field at t = 0.10 s?

$$\mathcal{E}_{ind} = \frac{d\Phi_B}{dt}$$

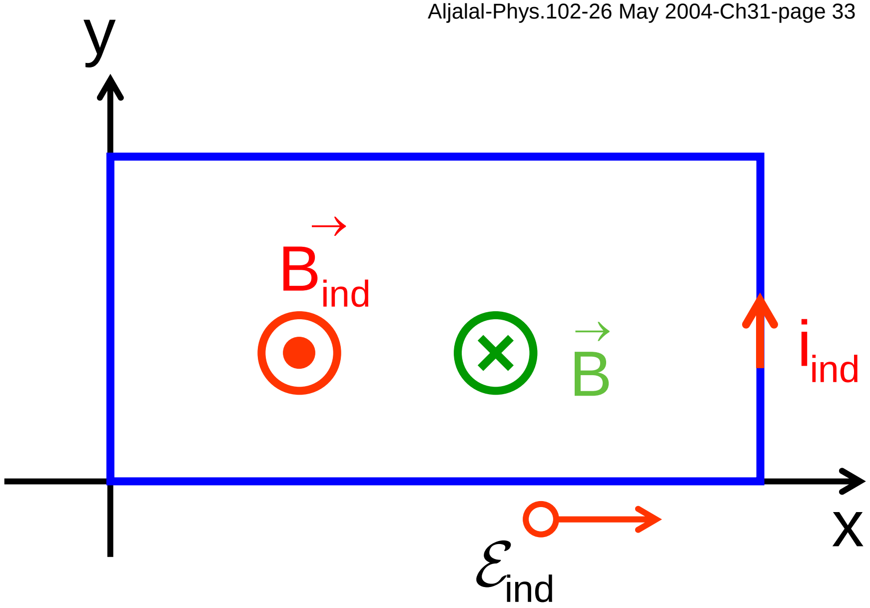
$$\Phi_B = \int \vec{B} \bullet d\vec{A} = \int B\, dA = \int_0^W B\, H\, dx = \int_0^W 4\, t^2\, x^2\, H\, dx$$

$$= 4\, t^2 H \int_0^W x^2\, dx = 4\, t^2\, H\, \frac{W^3}{3} = 4\, t^2\, 2\, \frac{3^3}{3} = 72\, t^2$$

$$\mathcal{E}_{ind} = \frac{d\Phi_B}{dt} = \frac{d}{dt}(72\, t^2) = 144\, t = 144\,(0.1) = 14.4\text{ V}$$

31-4 Lenz's Law

Sample Problem 31-3



At 0.10 s, $\frac{d\Phi_B}{dt} = 14.4 \text{ V} > 0.$ Flux is increasing

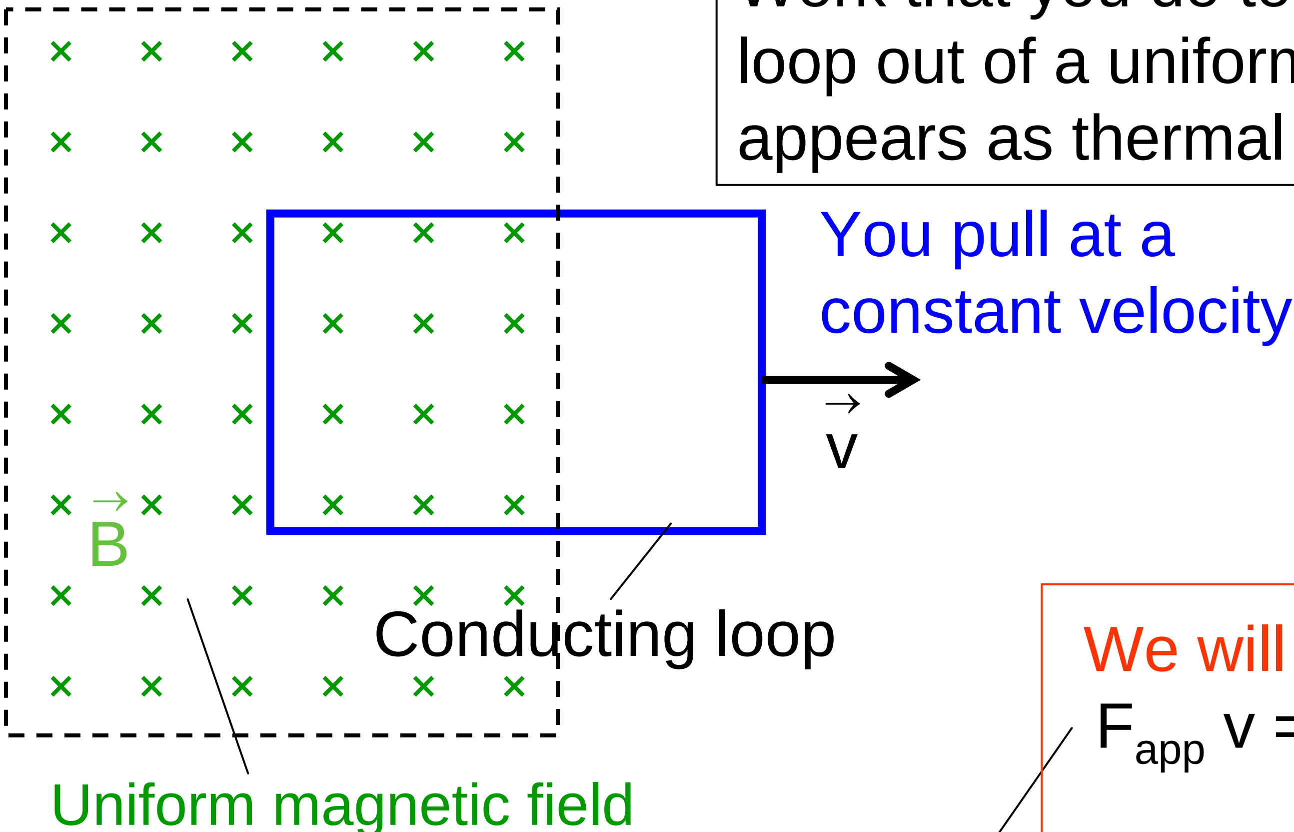
The induced B_{ind} must be out of the page to oppose this increase.

From the right-hand rule, the induced current must be counterclockwise

The induced emf $\mathcal{E}_{ind}$ must be also counterclockwise

31-5 Induction and Energy Transfer

Work that you do to pull a conducting loop out of a uniform magnetic field appears as thermal energy In the loop



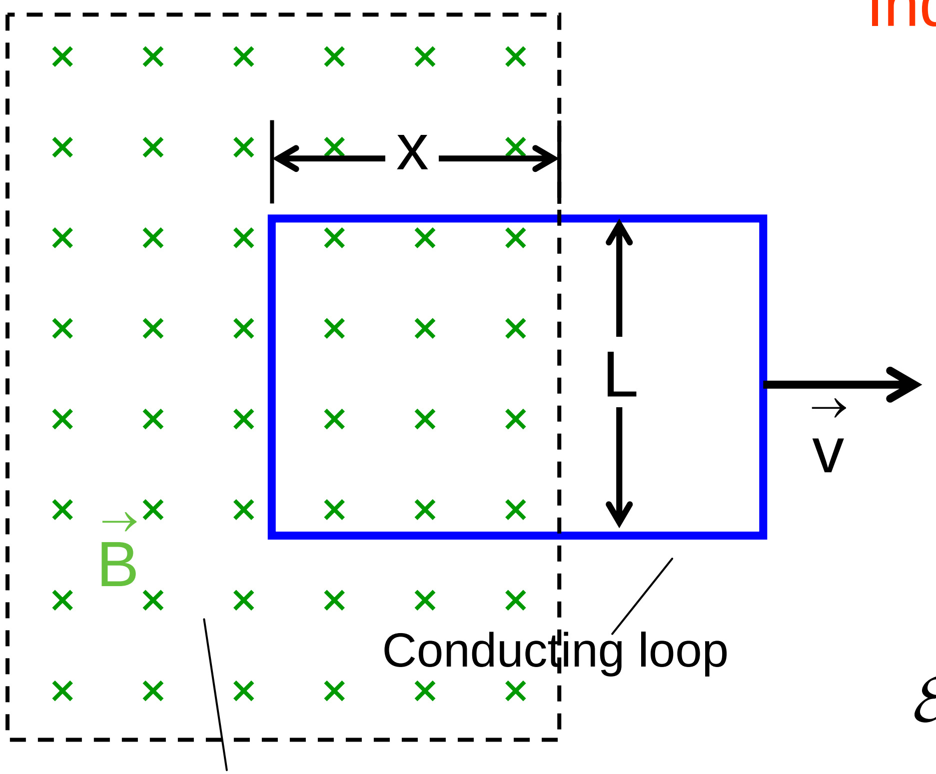
We will show

$$F_{app}\, v = i^2 R$$

Rate at which work is done to pull a loop out of a uniform magnetic field

Rate at which thermal energy is dissipated in the loop

31-5 Induction and Energy Transfer



Induced emf

$$\mathcal{E}_{ind} = \frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \, dA$$

$$= B \int dA = B A = B L x$$

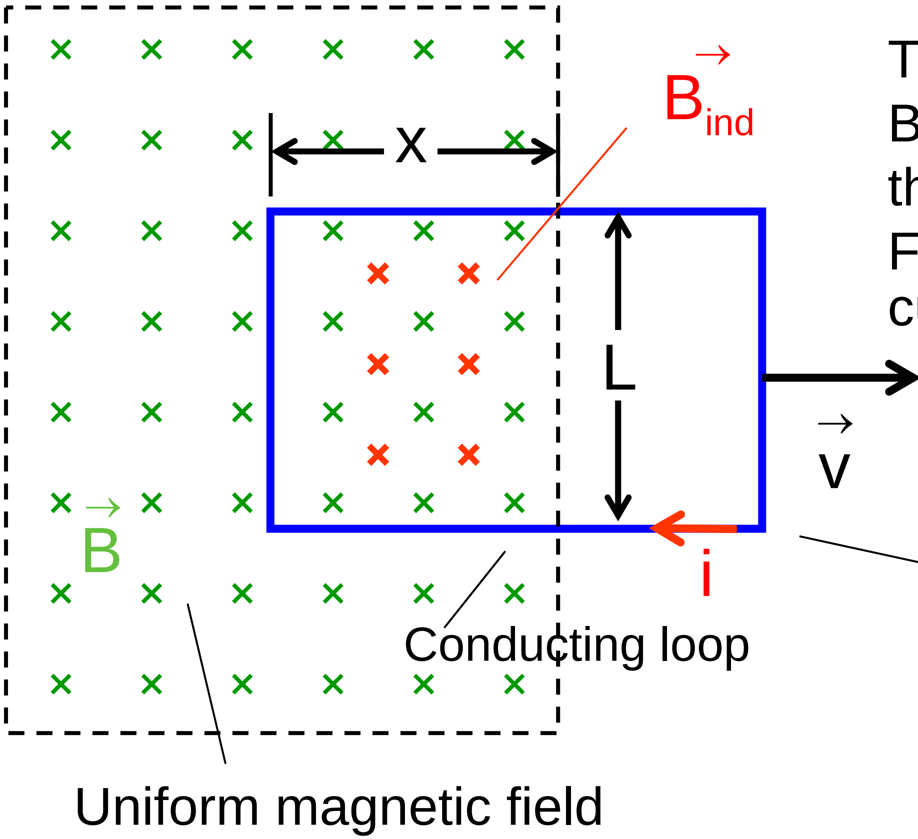
$$\mathcal{E}_{ind} = \frac{d\Phi_B}{dt} = \frac{d}{dt}(B L x)$$

$$= B L \frac{dx}{dt} = B L v$$

$$\boxed{\mathcal{E}_{ind} = B L v}$$

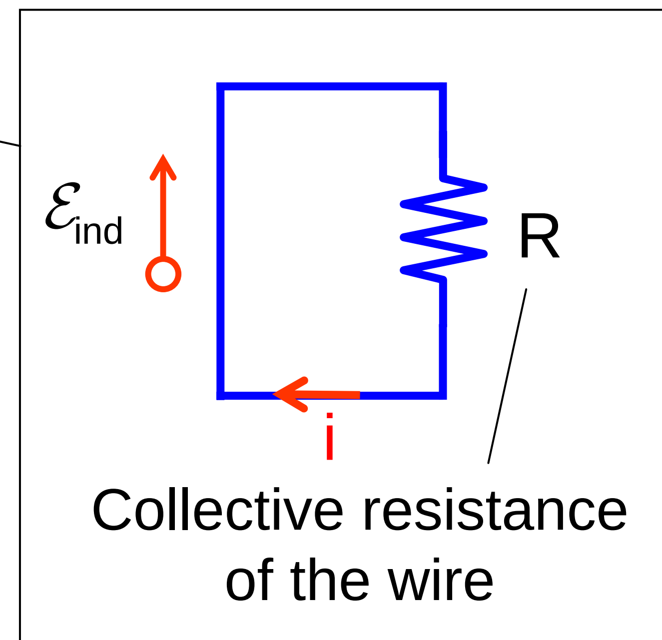
31-5 Induction and Energy Transfer

$$\mathcal{E}_{ind} = B\,L\,v$$



The flux is decreasing, the induced B_{ind} must be into the page to oppose this decrease.

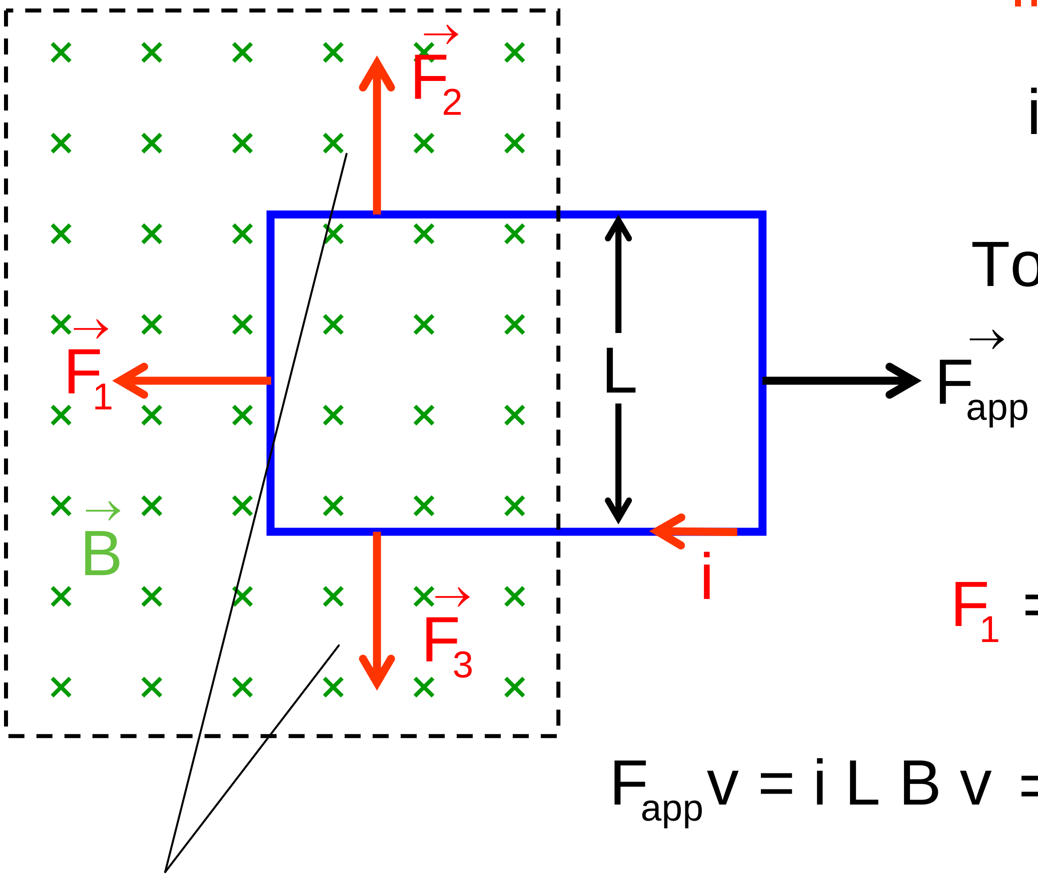
From the right-hand rule, the induced current and emf $\mathcal{E}_{ind}$ must be clockwise



Induced current

$$i = \frac{\mathcal{E}_{ind}}{R} = \frac{B\,L\,v}{R}$$

31-5 Induction and Energy Transfer



Induced current

$$i = \frac{\mathcal{E}_{ind}}{R} = \frac{B\,L\,v}{R}$$

To pull at constant velocity,

$$\vec{F}_{app} + \vec{F}_1 = m\,a = 0$$

$$F_{app} = F_1$$

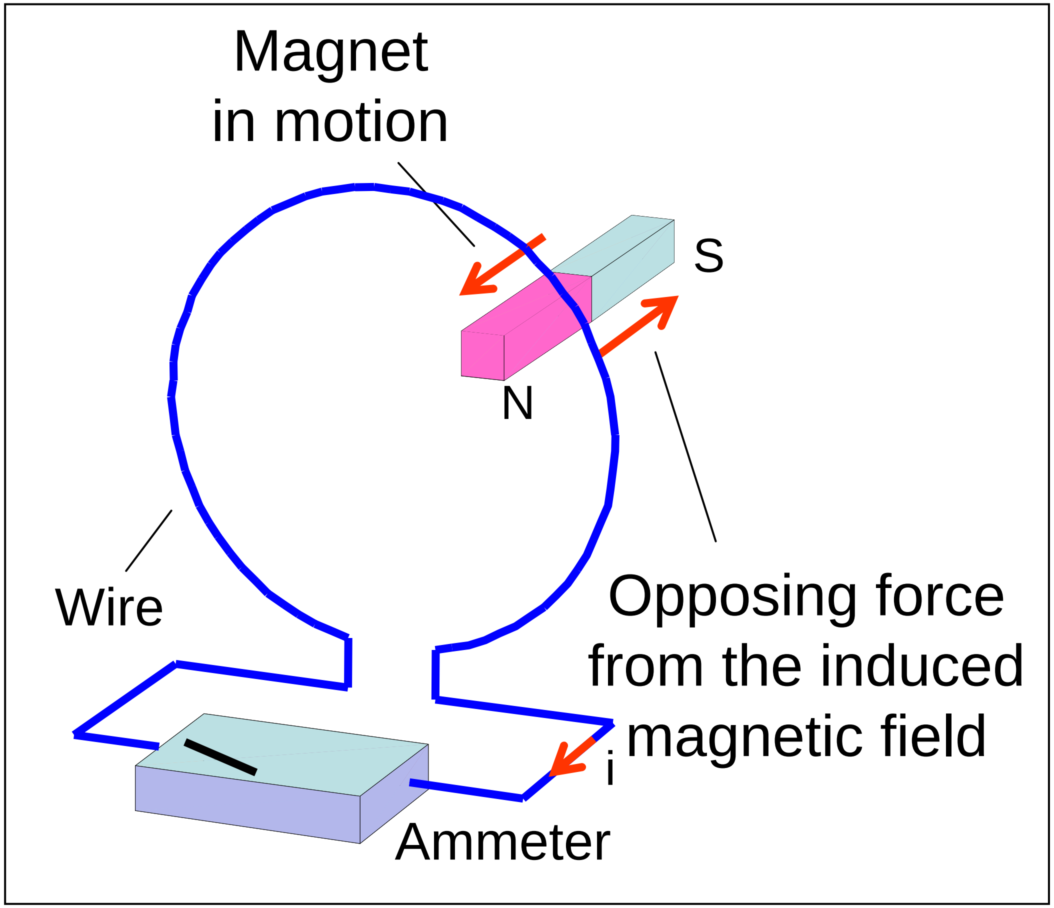
$$F_1 = i\,L\,B\,\sin 90^0 = i\,L\,B$$

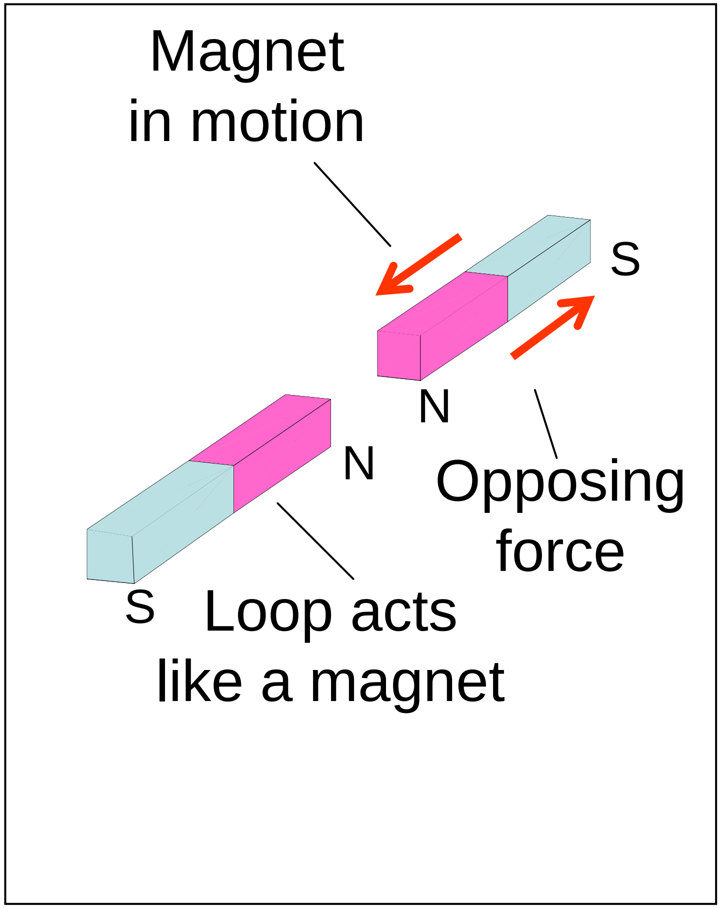
$$F_{app} v = i\,L\,B\,v = \frac{B\,L\,v}{R}\,L\,B\,v = \frac{B^2\,L^2\,v^2}{R}$$

$$i^2\,R = \left(\frac{B\,L\,v}{R}\right)^2 R = \frac{B^2\,L^2\,v^2}{R}$$

Work that you do to pull a conducting loop out of a uniform magnetic field appears as thermal energy In the loop

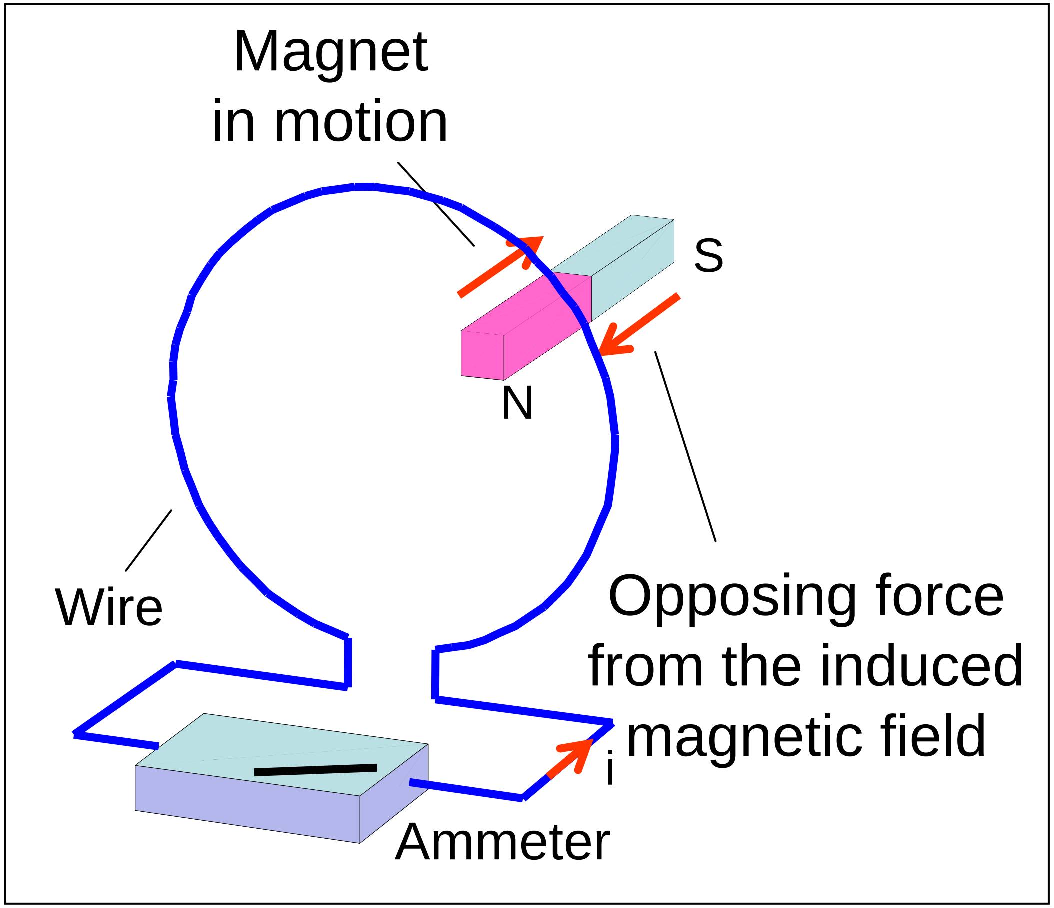
31-5 Induction and Energy Transfer

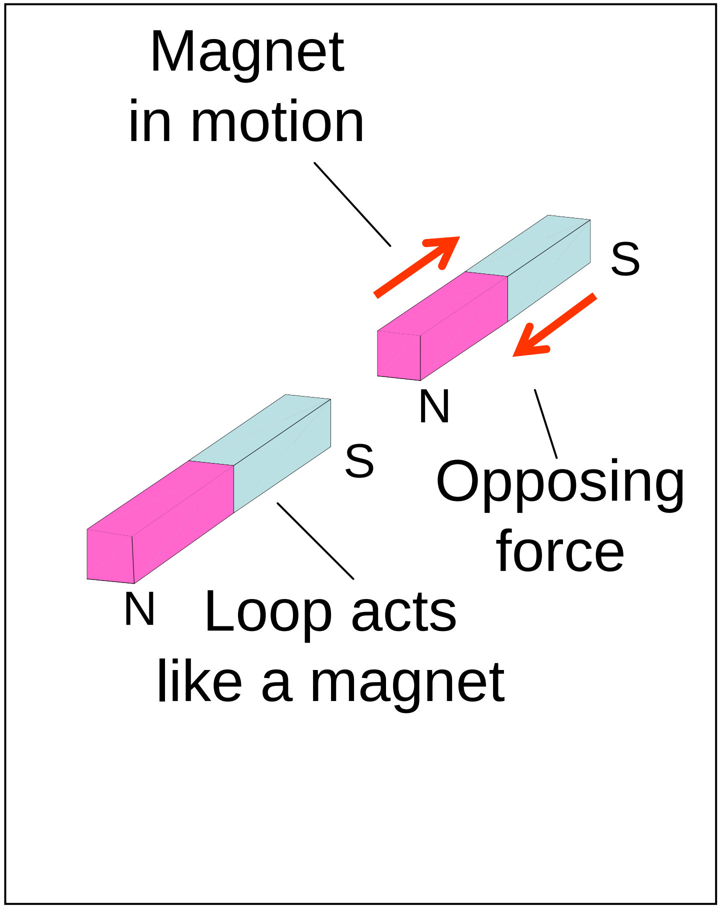




Whenever you move a magnet toward a conducting closed loop, a magnetic force resists the motion, requiring you to do positive work The energy you transfer to the system (magnet + loop), appears as thermal energy in the loop

31-5 Induction and Energy Transfer





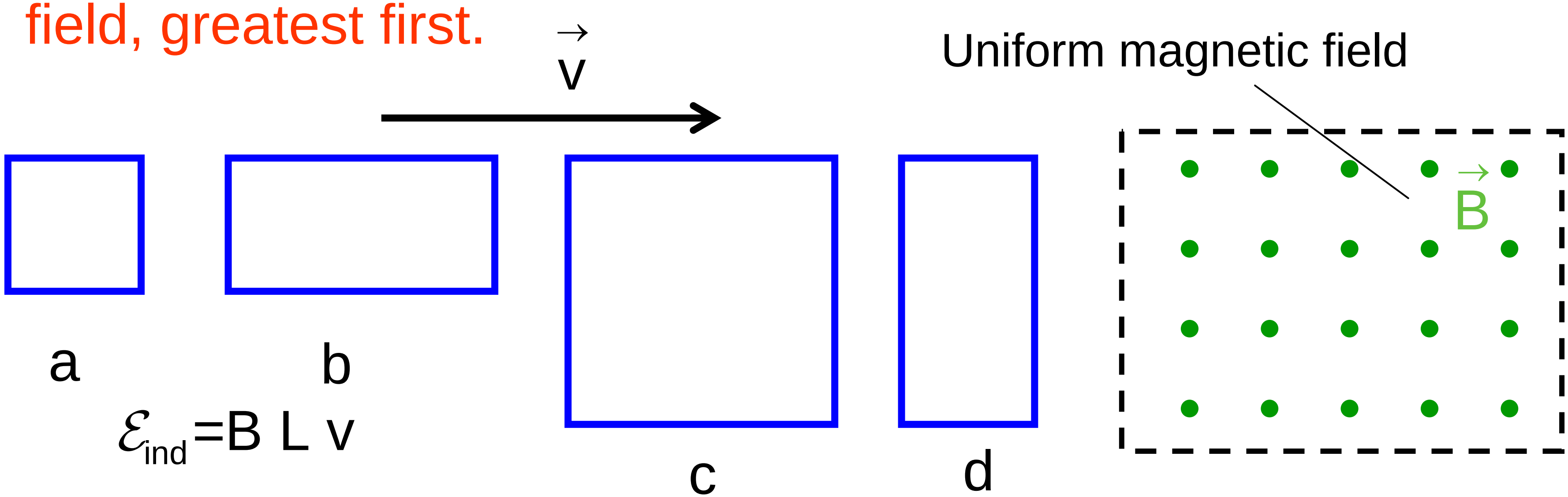
Whenever you move a magnet away from a conducting closed loop, a magnetic force resists the motion, requiring you to do positive work
The energy you transfer to the system (magnet + loop), appears as thermal energy in the loop

31-5 Induction and Energy Transfer

Checkpoint 3

Loops with edge lengths either L or 2L, move at the same constant speed

Rank according to the maximum magnitude of the emf induced in the loops as the loops move into the magnetic field, greatest first.



$\mathcal{E}_{ind} = B\,L\,v$

$\mathcal{E}_{ind} = B\,(2L)\,v$

c and d tie,
then a and b tie.