Chapter 30

Magnetic Fields Due to Currents

30-1 Calculating the Magnetic Field Due to a Current

Permeability constant

$\mu_0 = 4\pi \times 10^{-7}$ T·m/A

Magnetic field db produced at point P by length ds of the wire

Length vector

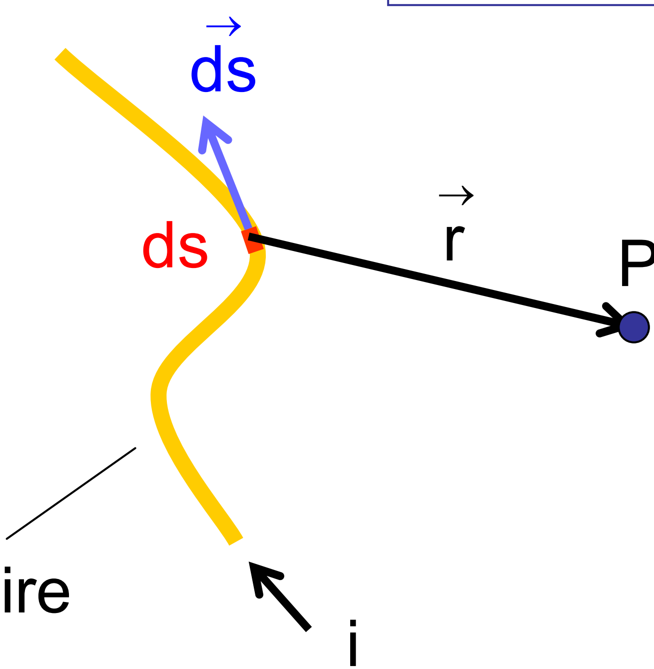
Magnitude : length of segment ds

Direction : along the wire segment in the direction of conventional current

Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \vec{r}}{r^3}$$

Distance between point P and segment ds

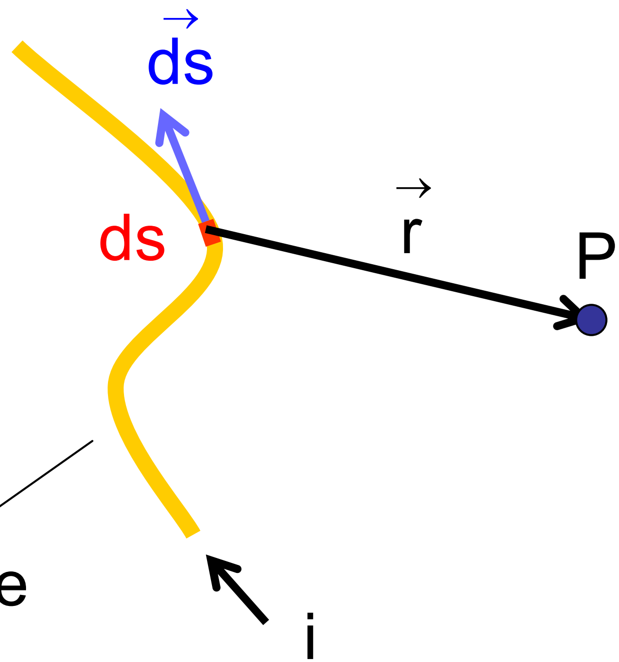


30-1 Calculating the Magnetic Field Due to a Current

Magnetic field dB
produced at point P by
length ds of the wire

Biot-Savart law

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i\,\vec{ds}\times\vec{r}}{r^3}$$



Magnetic field B
produced at point P by
the hole wire

$$\vec{B} = \int_{wire} \vec{dB} = \int_{wire} \frac{\mu_0}{4\pi}\frac{i\,\vec{ds}\times\vec{r}}{r^3}$$

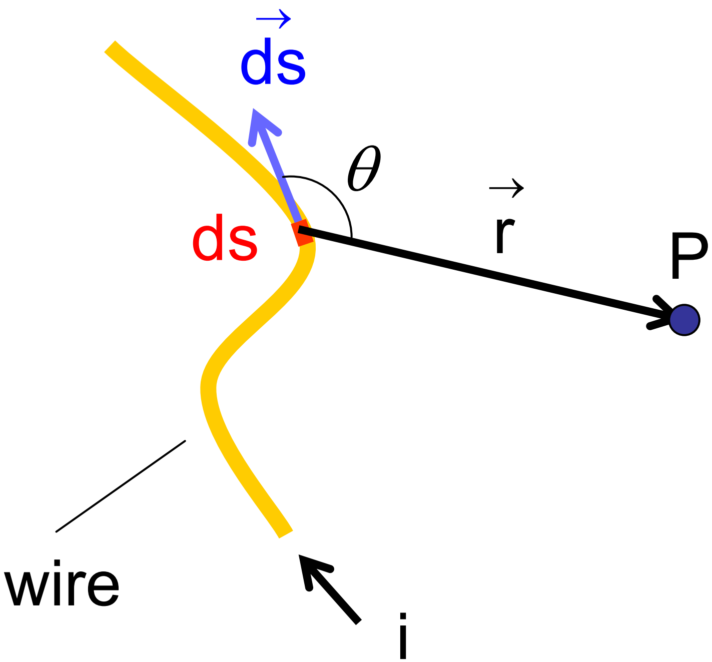
Vector sum

30-1 Calculating the Magnetic Field Due to a Current

Magnetic field dB produced at point P by length ds of the wire

Biot-Savart law

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{i\, \vec{ds} \times \vec{r}}{r^3}$$



Magnitude of dB vector

$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds\, r \sin\theta}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2}$$

Inverse-square law

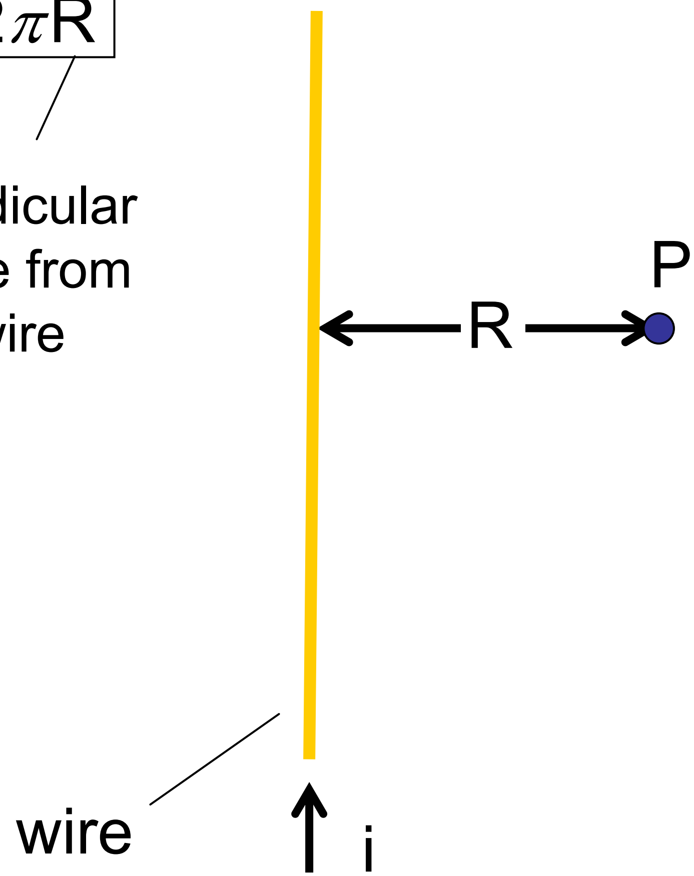
30-1 Calculating the Magnetic Field Due to a Current

Magnetic field due to a current in a long straight wire

Magnitude

$$B = \frac{\mu_0 i}{2\pi R}$$

Perpendicular distance from the wire

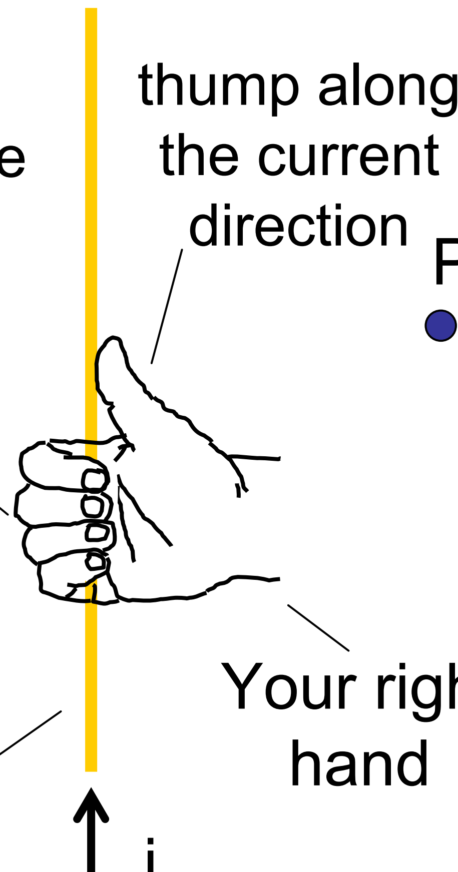


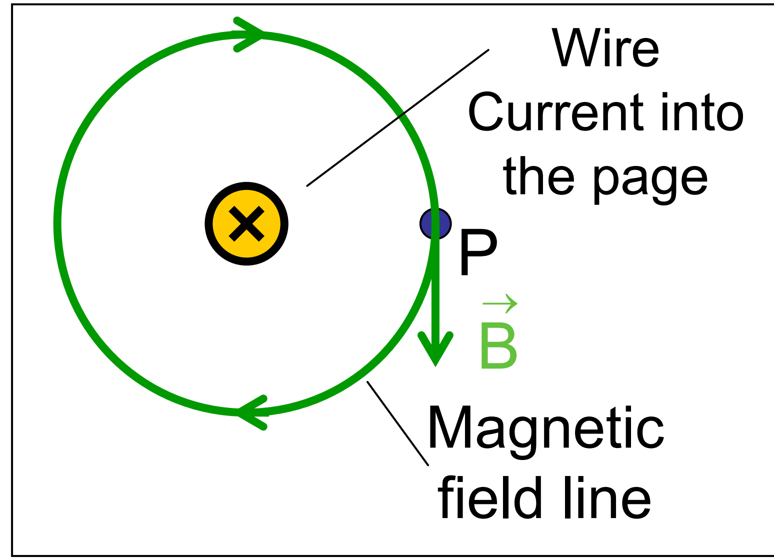
30-1 Calculating the Magnetic Field Due to a Current

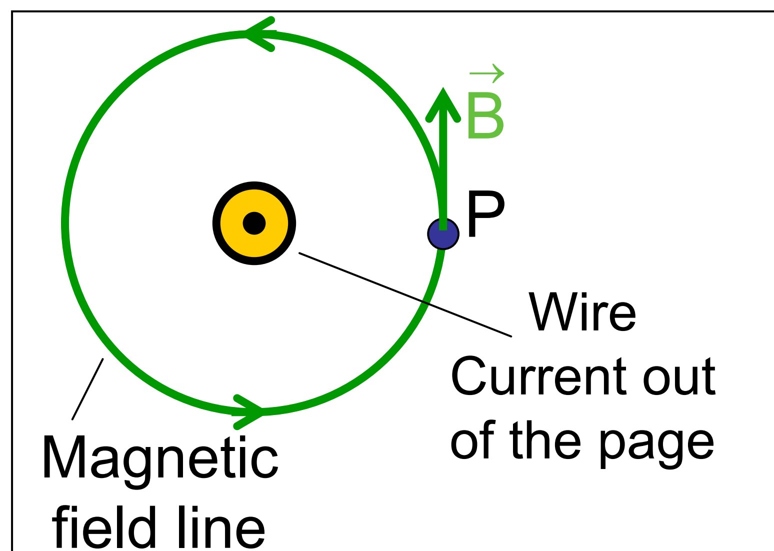
Magnetic field due to a current in a long straight wire

Direction

Right-hand rule



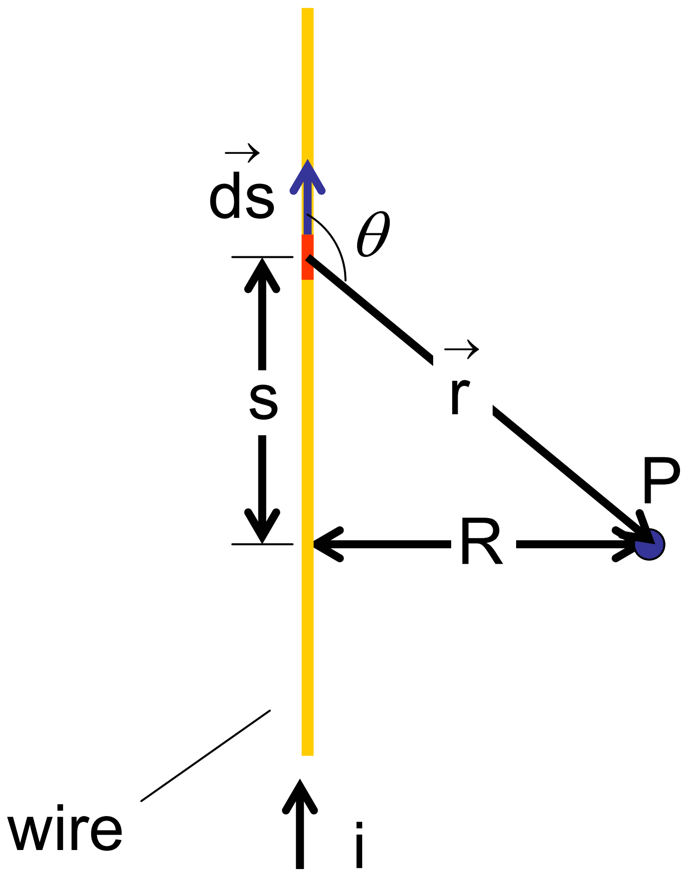




30-1 Calculating the Magnetic Field Due to a Current

Magnetic field due to a current in a long straight wire

Derivation of $B = \dfrac{\mu_0 i}{2\pi R}$



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\, d\vec{s} \times \vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i\, ds \sin\theta}{r^2}$$ Into the page

$$\sin\theta = \frac{R}{r} \qquad r = \sqrt{s^2 + R^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i\, ds\, R}{(s^2+R^2)^{3/2}}$$

$$B = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi}\frac{i\, R}{(s^2+R^2)^{3/2}}\, ds$$

$$B = \frac{\mu_0 i}{4\pi R}\left[\frac{s}{(s^2+R^2)^{1/2}}\right]_{s=-\infty}^{s=\infty}$$

$$B = \frac{\mu_0 i}{4\pi R}\, 2 = \frac{\mu_0 i}{2\pi R}$$

30-1 Calculating the Magnetic Field Due to a Current

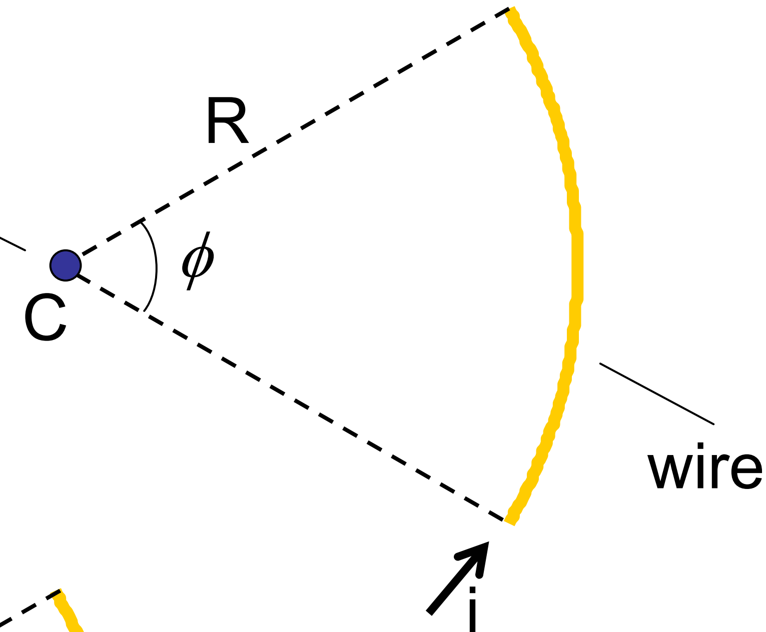
Magnetic field due to a current in a Circular Arc of Wire

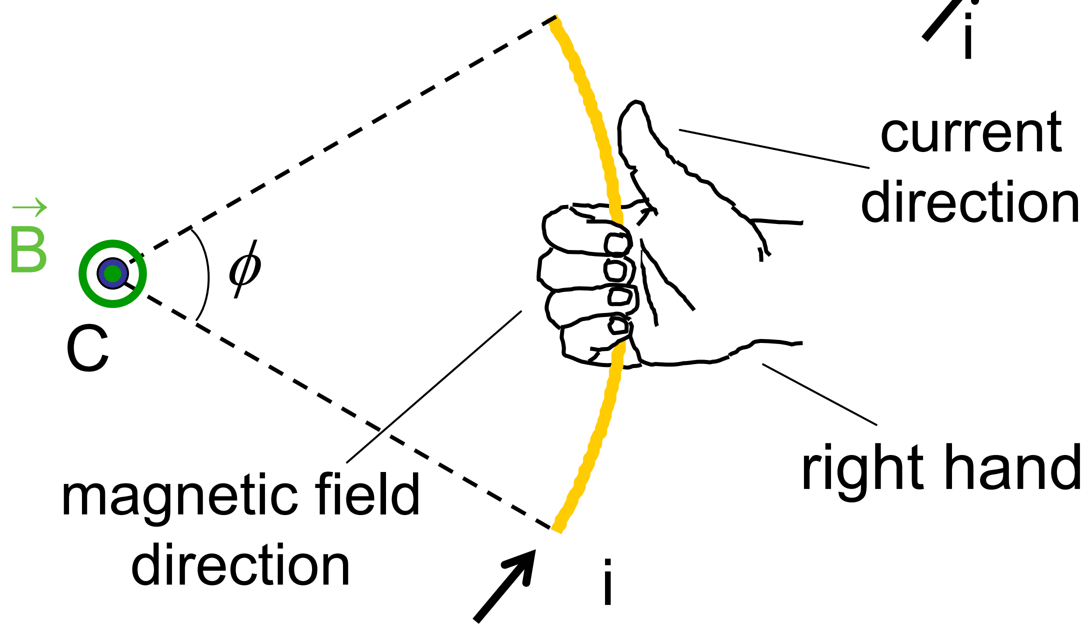
At center of circular arc

$$B = \frac{\mu_0 \, i \, \phi}{4\pi R}$$

In radians

Radius of the arc



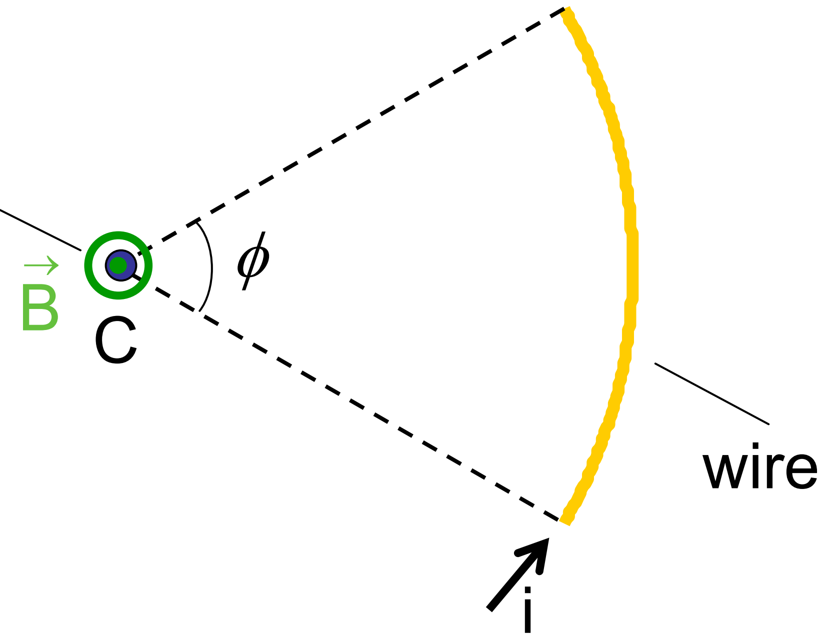


30-1 Calculating the Magnetic Field Due to a Current

Magnetic field due to a current in a Circular Arc of Wire

At center of circular arc

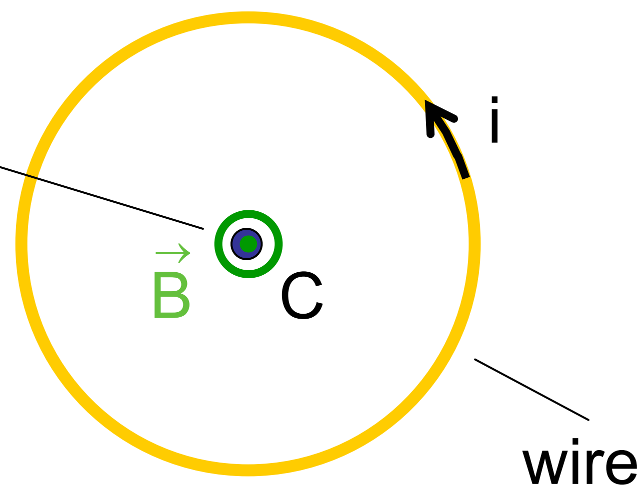
$$B = \frac{\mu_0 \, i \, \phi}{4\pi R}$$



At center of full circle

$$\phi = 2\pi$$

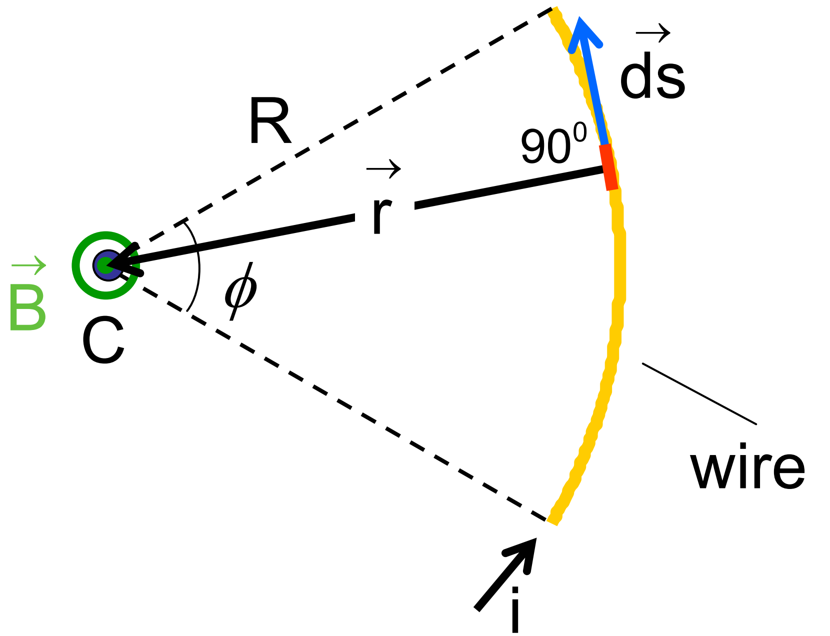
$$B = \frac{\mu_0 \, i}{2R}$$



30-1 Calculating the Magnetic Field Due to a Current

Magnetic field due to a current in a Circular Arc of Wire

Derivation of $$B = \frac{\mu_0 i \phi}{4\pi R}$$



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\, d\vec{s} \times \vec{r}}{r^3}$$

At center of circular arc

$$dB = \frac{\mu_0}{4\pi}\frac{i\, ds \sin 90^0}{R^2}$$

Out of the page

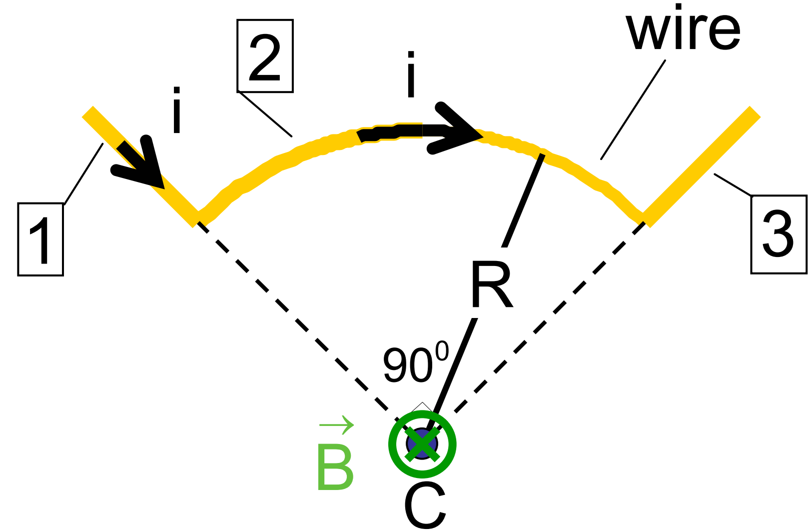
$$dB = \frac{\mu_0}{4\pi}\frac{i\, ds}{R^2} = \frac{\mu_0}{4\pi}\frac{i\, R d\phi}{R^2} = \frac{\mu_0\, i}{4\pi\, R}\, d\phi$$

$$B = \int_0^{\phi} \frac{\mu_0\, i}{4\pi\, R}\, d\phi = \frac{\mu_0\, i}{4\pi\, R}\int_0^{\phi} d\phi = \frac{\mu_0 i\, \phi}{4\pi\, R}$$

30-1 Calculating the Magnetic Field Due to a Current

Sample Problem 30-1

What magnetic field does the current produce at the center?



$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$B_1 = 0$$

$$B_3 = 0$$

$$B_2 = \frac{\mu_0 \, i \, \phi}{4\pi R} = \frac{\mu_0 \, i \, \frac{\pi}{2}}{4\pi R}$$

$$B_2 = \frac{\mu_0 \, i}{8R}$$

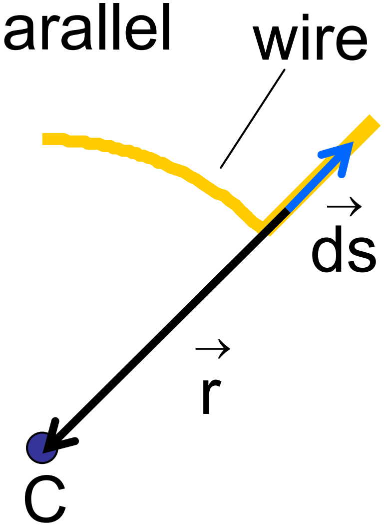
$$B = 0 + \frac{\mu_0 \, i}{8R} + 0 = \frac{\mu_0 \, i}{8R}$$

Direction: into the page

For segments 1 and 3

$\vec{ds}$ and $\vec{r}$ are parallel

$\vec{ds} \times \vec{r} = 0$



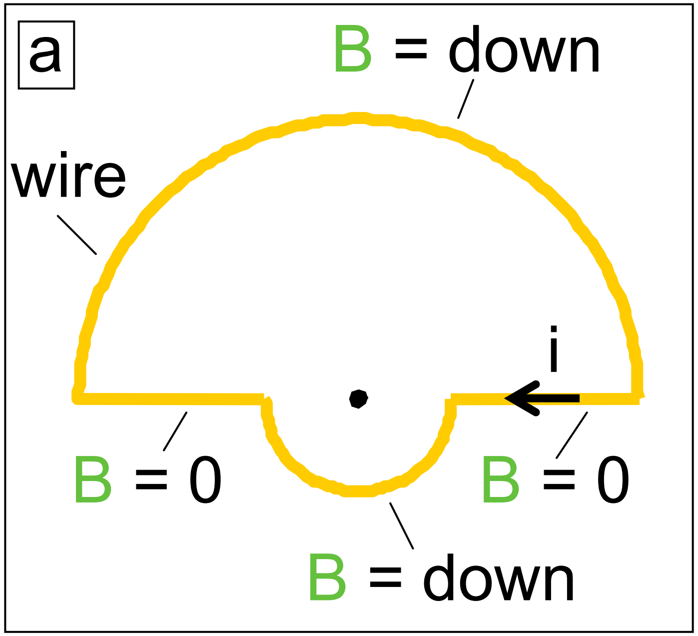
30-1 Calculating the Magnetic Field Due to a Current

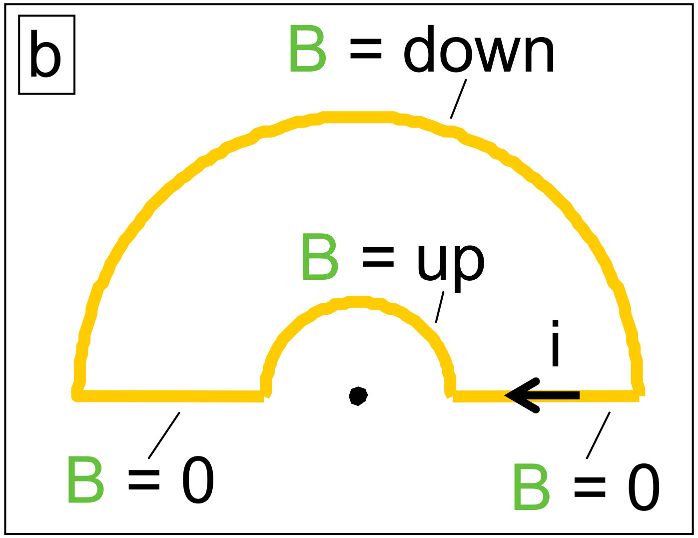
Checkpoint 1

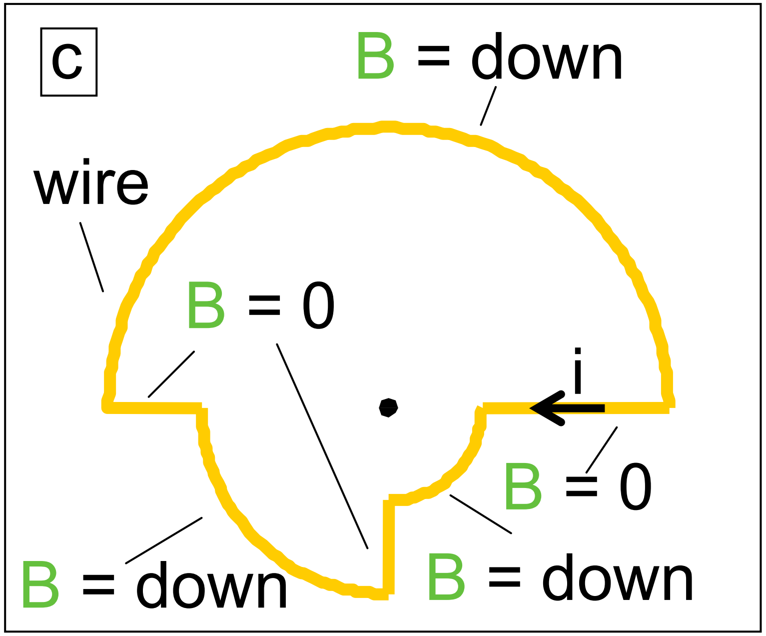
Rank the circuits according to the magnitude of the magnetic field at the center, greatest first

a,
then c,
then b.

$$B = \frac{\mu_0 \, i \, \phi}{4\pi R}$$







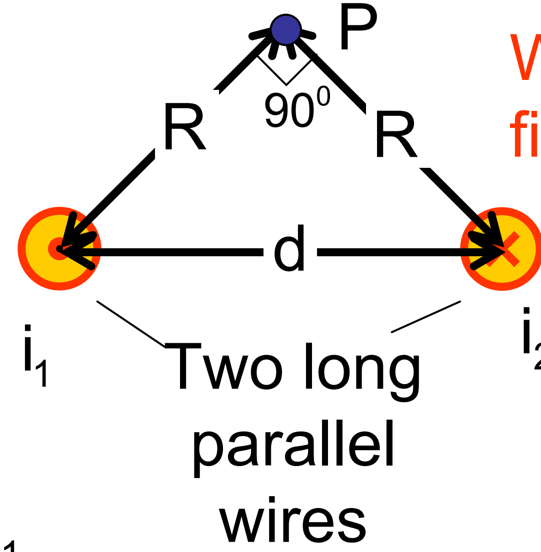
30-1 Calculating the Magnetic Field Due to a Current

Sample Problem 30-1

$i_1 = 15$ A

$i_2 = 32$ A

$d = 5.3$ cm



What is the magnetic field at P?

$$R = d \cos 45^0$$

$$B_1 = \frac{\mu_0 \, i_1}{2\pi R} = \frac{\mu_0 \, i_1}{2\pi d \cos 45^0}$$

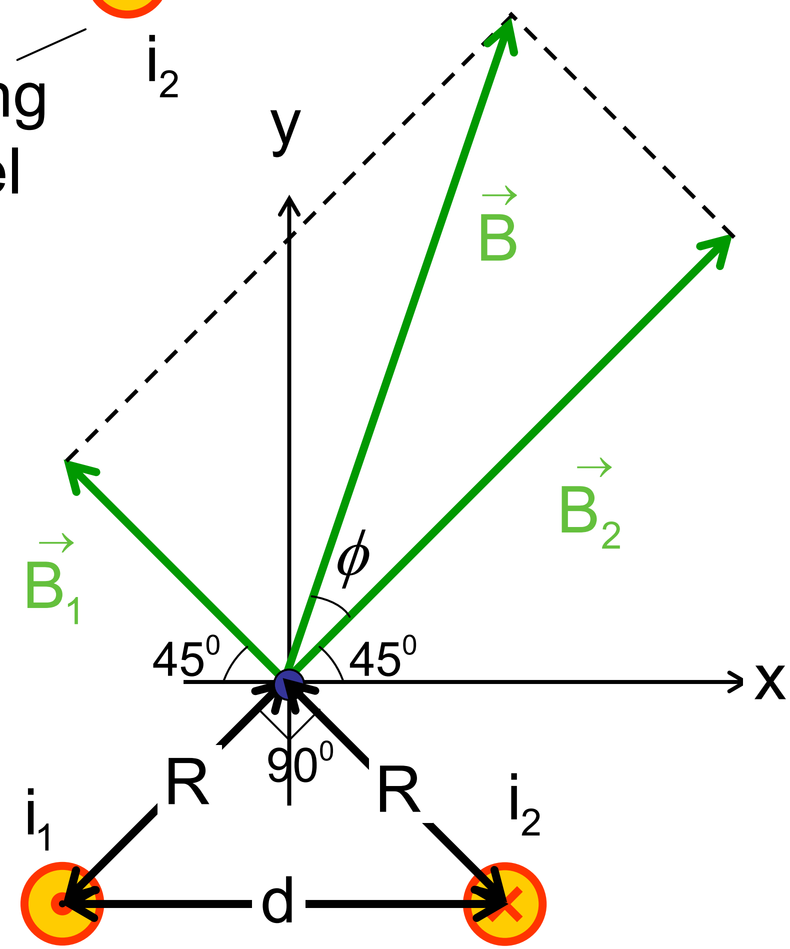
$$B_2 = \frac{\mu_0 \, i_2}{2\pi R} = \frac{\mu_0 \, i_2}{2\pi d \cos 45^0}$$

$$\vec{B}_1 \perp \vec{B}_2$$

$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{2\pi d \cos 45^0} \sqrt{i_1^2 + i_2^2}$$

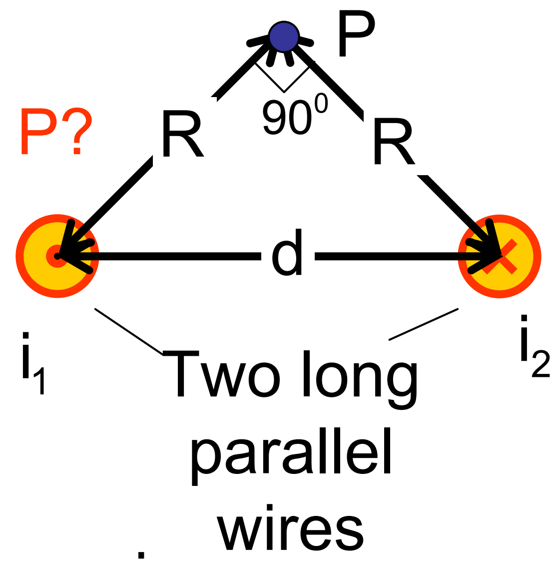
$$= 1.89 \times 10^{-4} \text{ T}$$



30-1 Calculating the Magnetic Field Due to a Current

Sample Problem 30-1

What magnetic field at P?

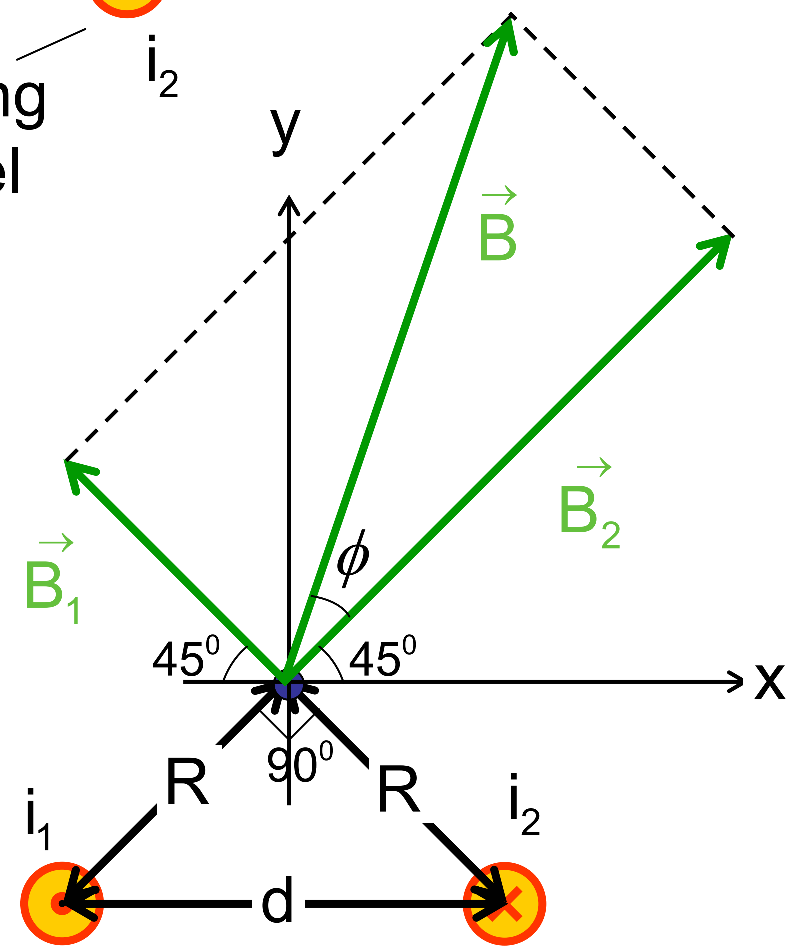


$$\phi = \tan^{-1}\left(\frac{B_1}{B_2}\right) = \tan^{-1}\left(\frac{i_1}{i_2}\right)$$

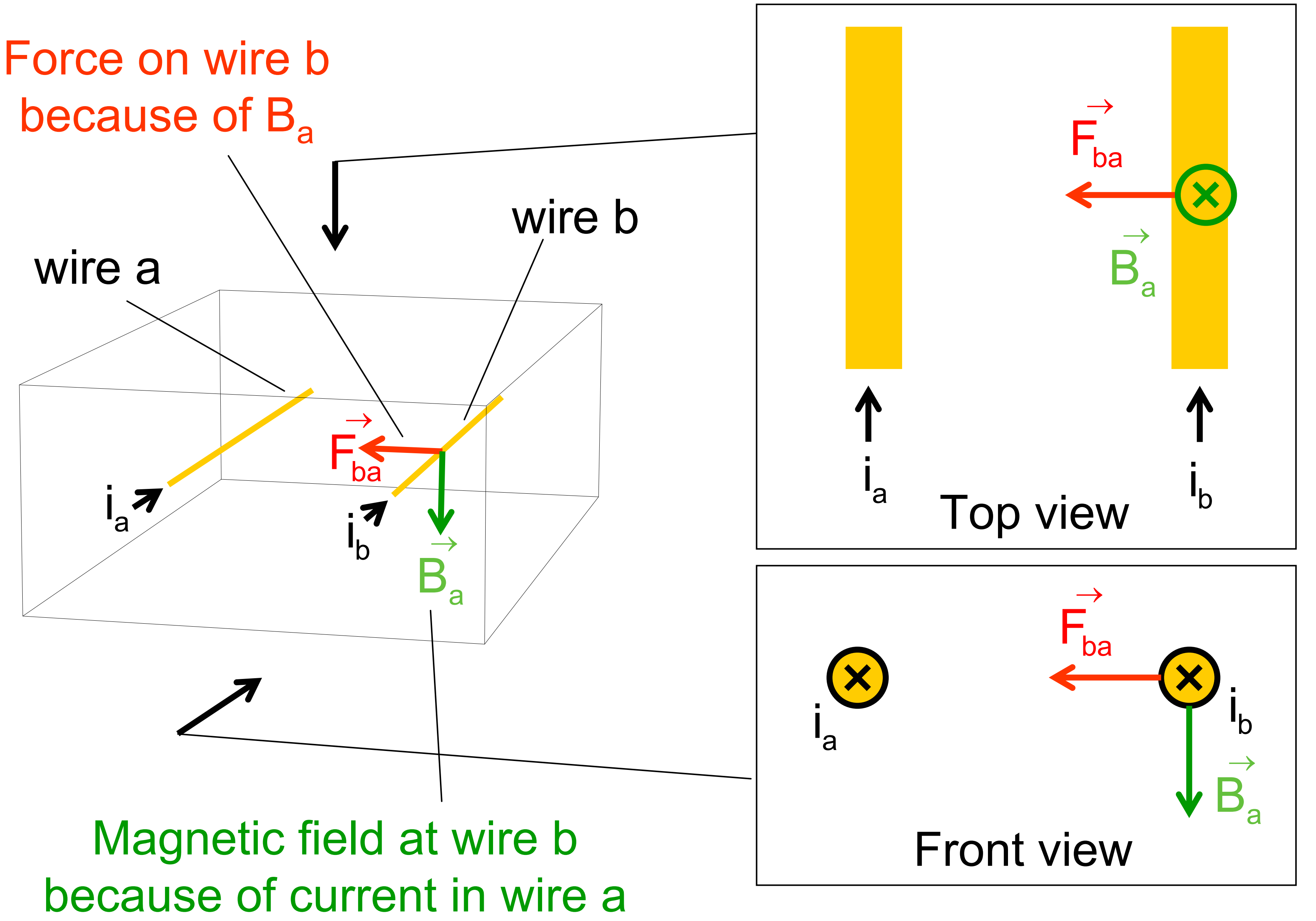
$$= 25^0$$

The angel between the total magnetic field and the x axis

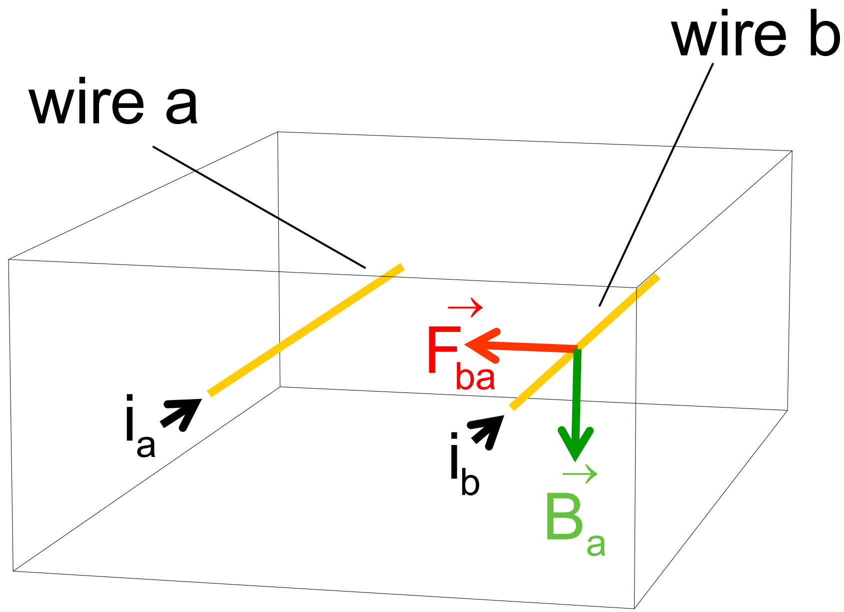
$$\phi + 45^0 = 70^0$$

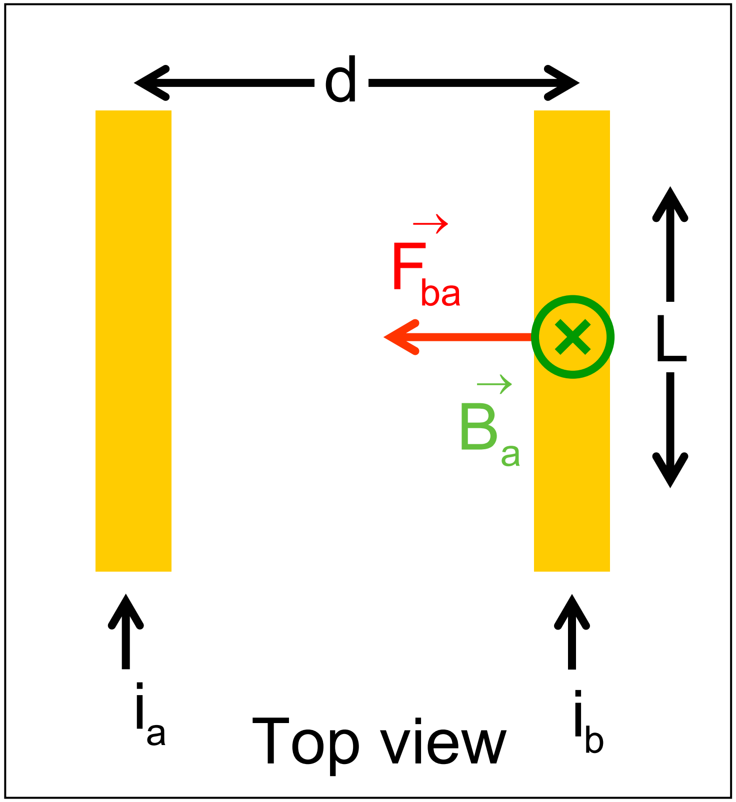


30-2 Force Between Two Parallel Currents



30-2 Force Between Two Parallel Currents





Magnetic field at wire b because of current in wire a

$$B_a = \frac{\mu_0\, i_a}{2\pi d}$$

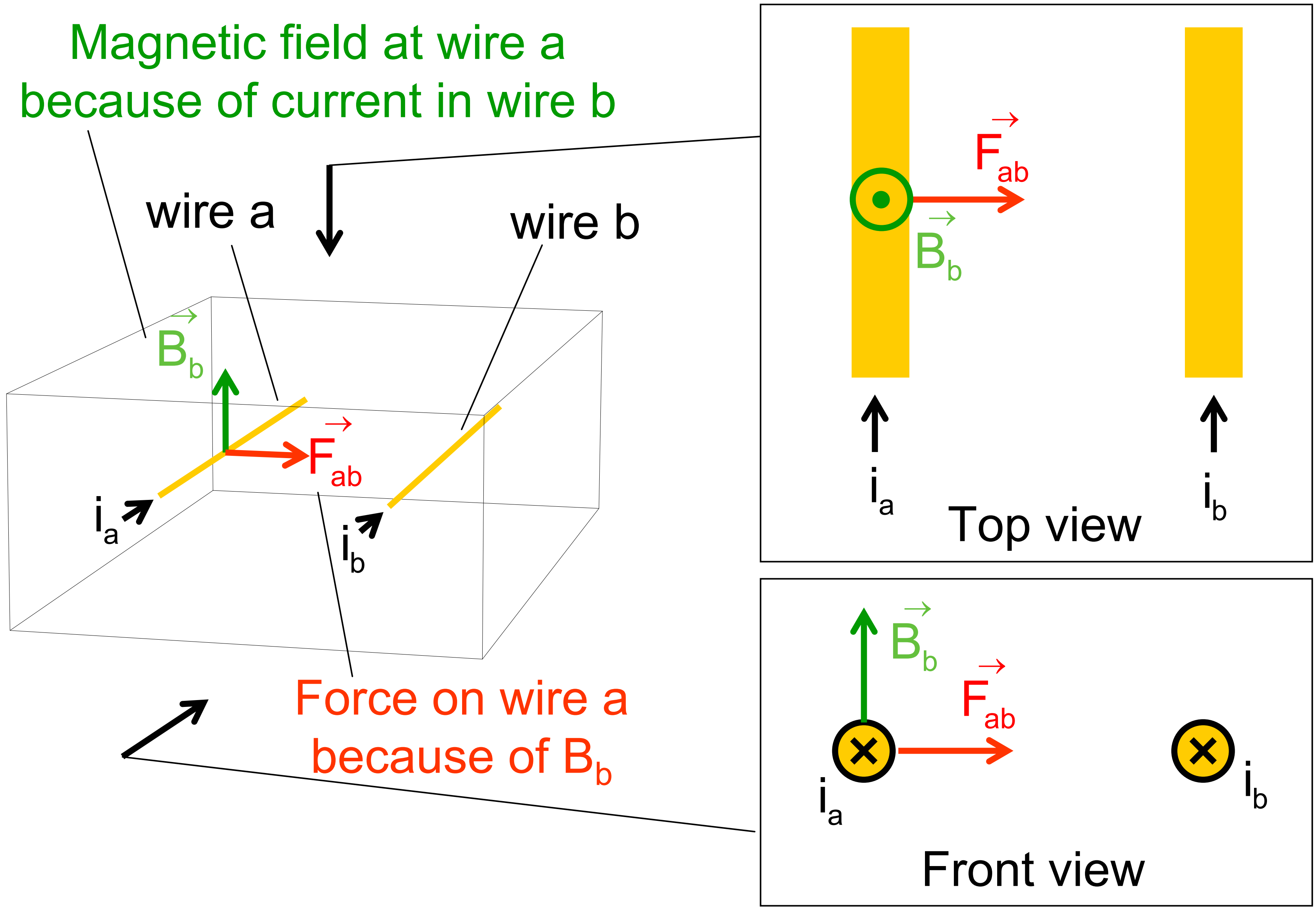
Force on length L of wire b due to B_a

$$\vec{F}_{ba} = i_a\, \vec{L} \times \vec{B}_a$$

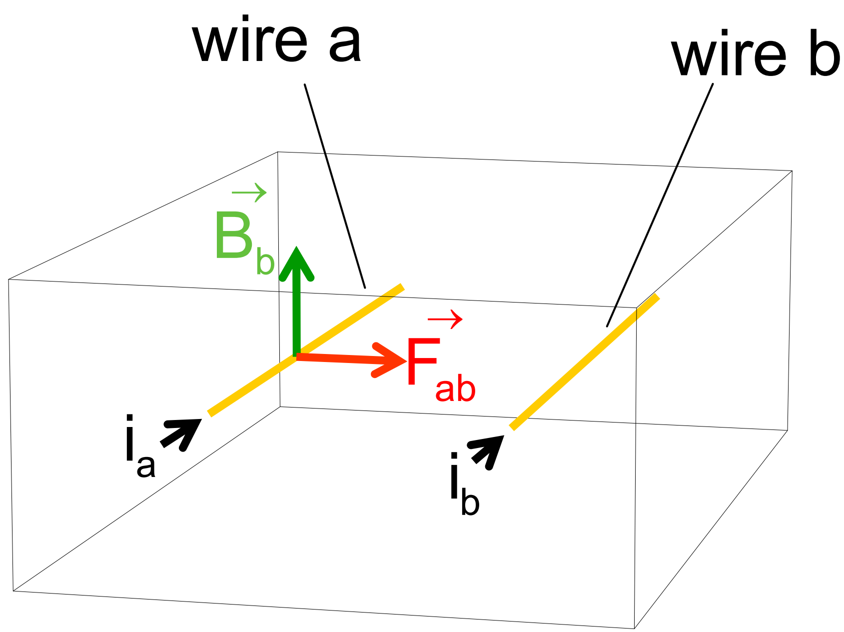
$$F_{ba} = i_b\, L\, B_a\, \sin 90^0 = i_b\, L\, B_a$$

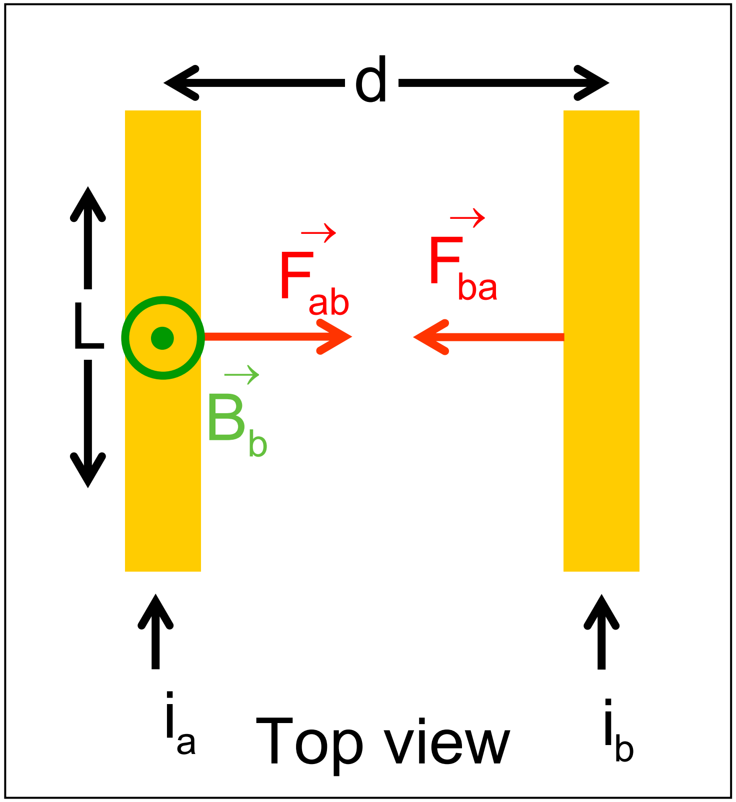
$$F_{ba} = \frac{\mu_0\, L\, i_a\, i_b}{2\pi d}$$

30-2 Force Between Two Parallel Currents



30-2 Force Between Two Parallel Currents





Magnetic field at wire a because of current in wire b

$$B_b = \frac{\mu_0 \, i_b}{2\pi d}$$

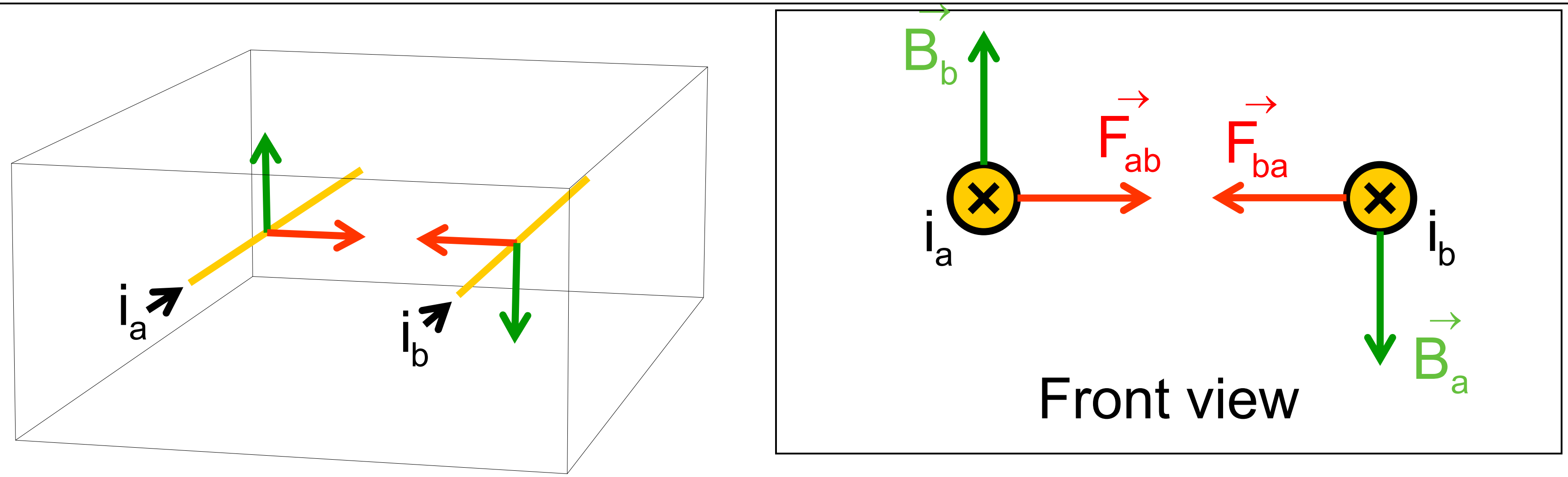
Force on length L of wire a due to B_b

$$\vec{F}_{ab} = i_a \, \vec{L} \times \vec{B}_b$$

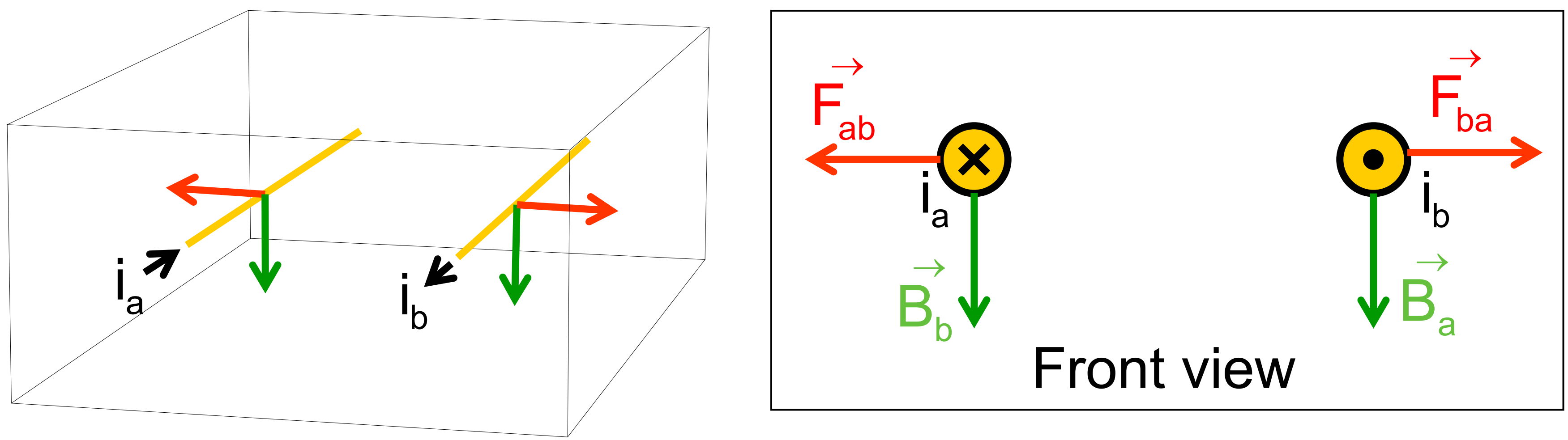
$$F_{ba} = i_a \, L \, B_b \, \sin 90^0 = i_a \, L \, B_{\backslash b}$$

$$F_{ab} = \frac{\mu_0 \, L \, i_a \, i_b}{2\pi d} = F_{ba}$$

30-2 Force Between Two Parallel Currents



Parallel current attract



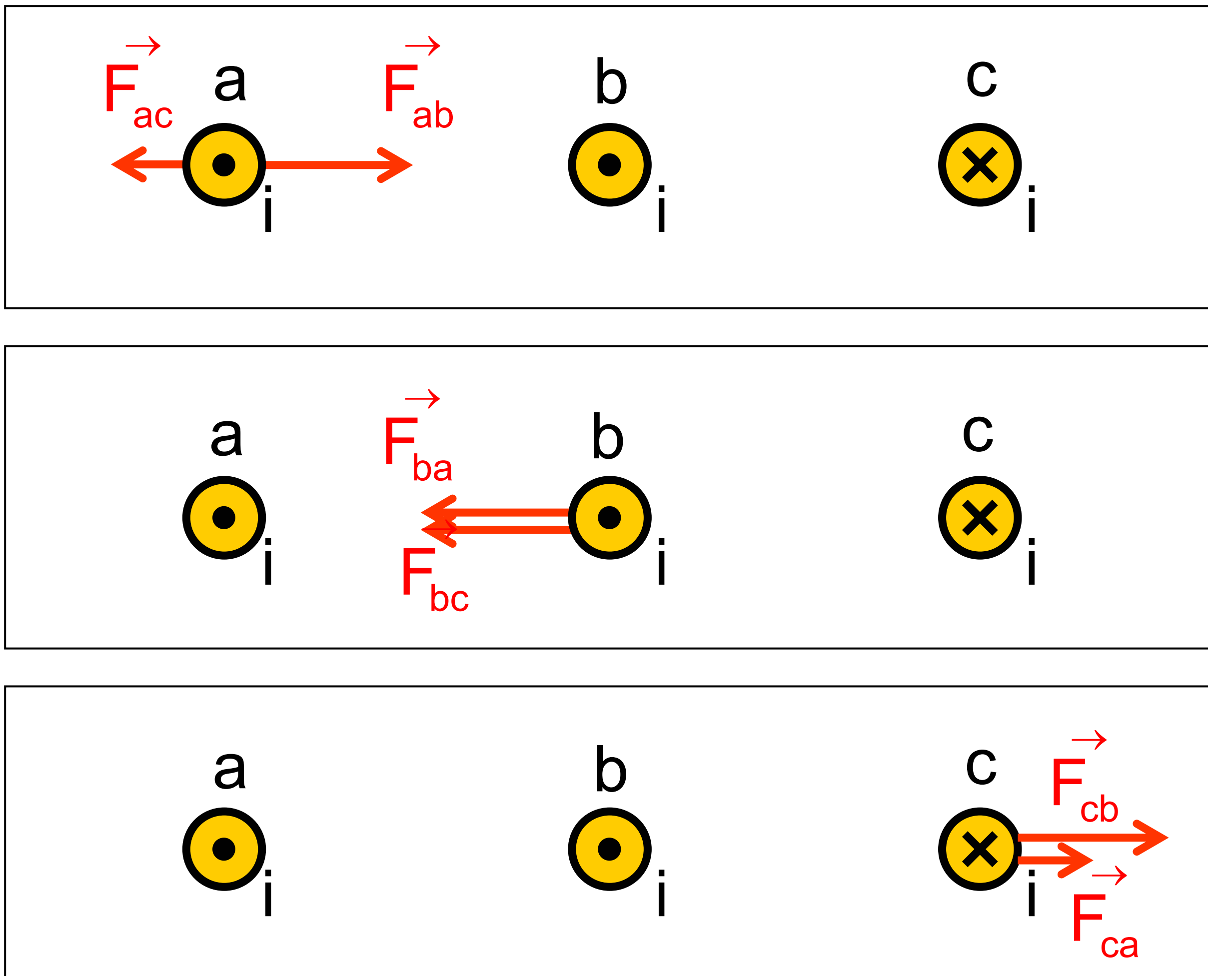
Antiparallel current repel

30-2 Force Between Two Parallel Currents

Checkpoint 2

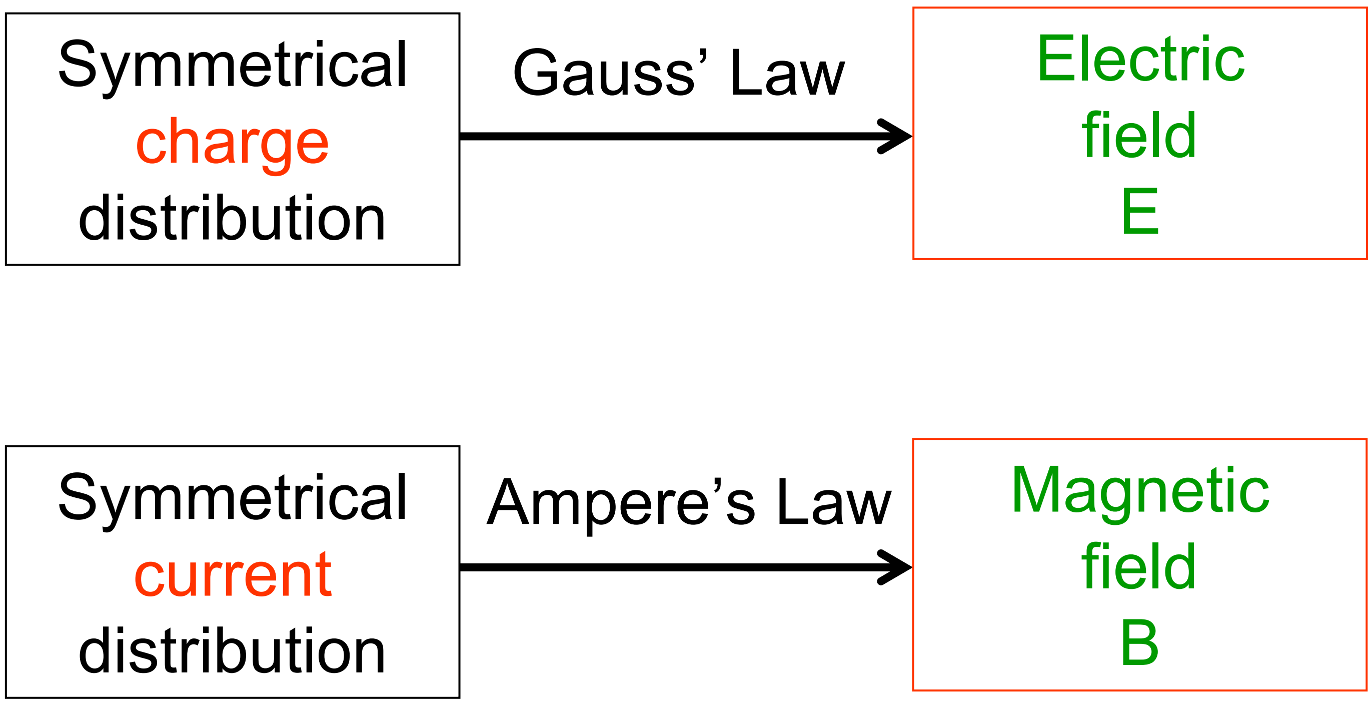
Rank according to the magnitude of the force on each wire, greatest first.

b,
then c,
then a



Wires are parallel and equally spaced

30-3 Ampere's Law



30-3 Ampere's Law

Ampere's Law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \, i_{enc}$$

Over imaginary closed loop (Apmerian loop)

Current enclosed by the loop

Length vector

Magnitude: length of a small segment of the loop

Direction: tangent to the loop

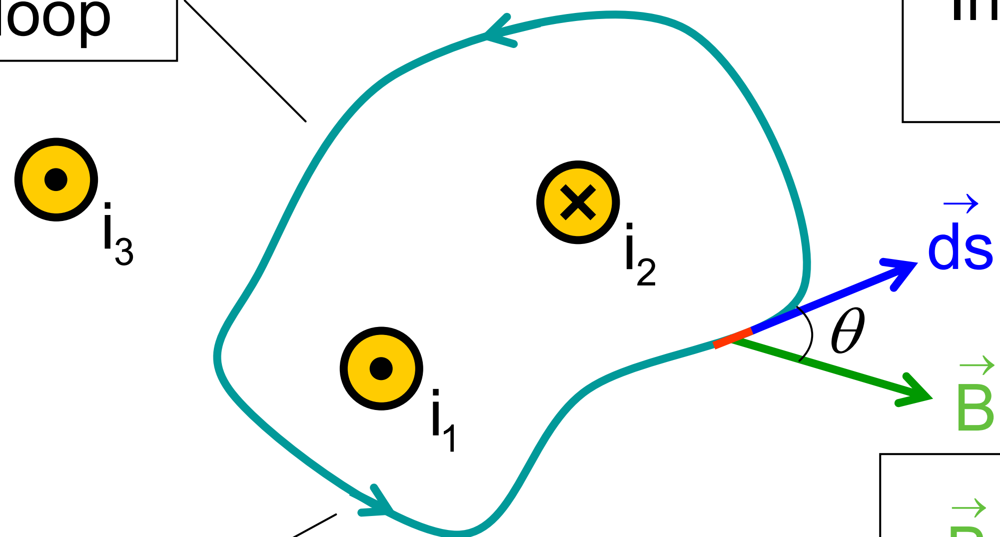
30-3 Ampere's Law

Ampere's Law

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \, i_{enc}$$

Imaginary closed loop

In the direction of the loop



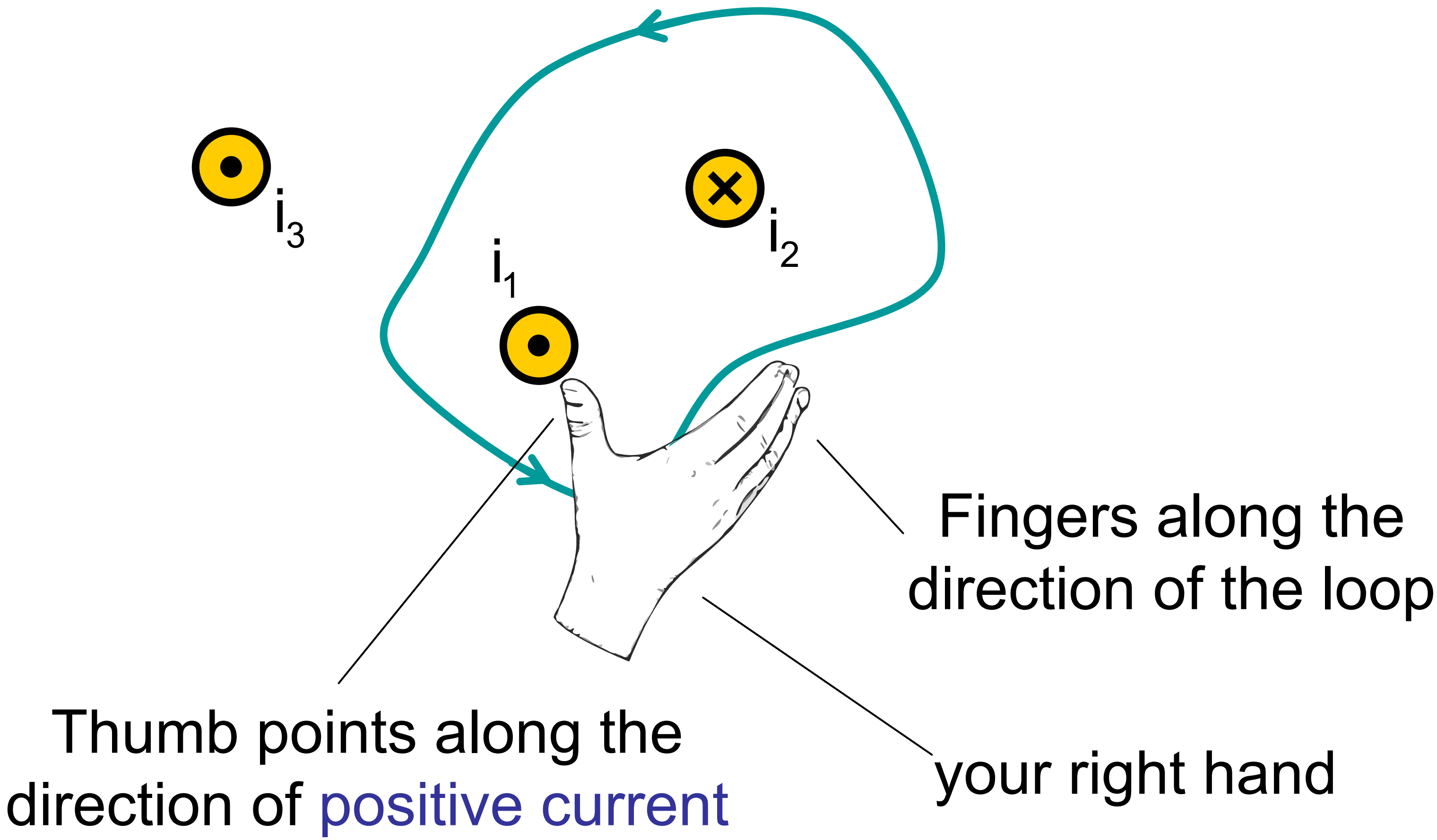
$\vec{B} \bullet \vec{ds} = B \, ds \cos\theta$

Scalar quantity

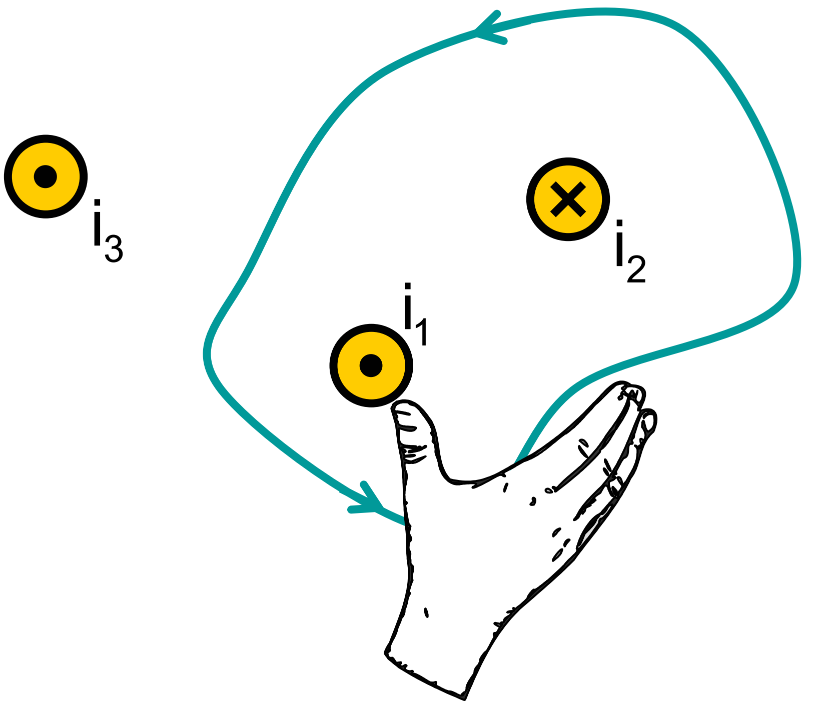
Choose any direction, the positive and negative directions of currents are determined by the right-hand rule

30-3 Ampere's Law

Right-hand rule



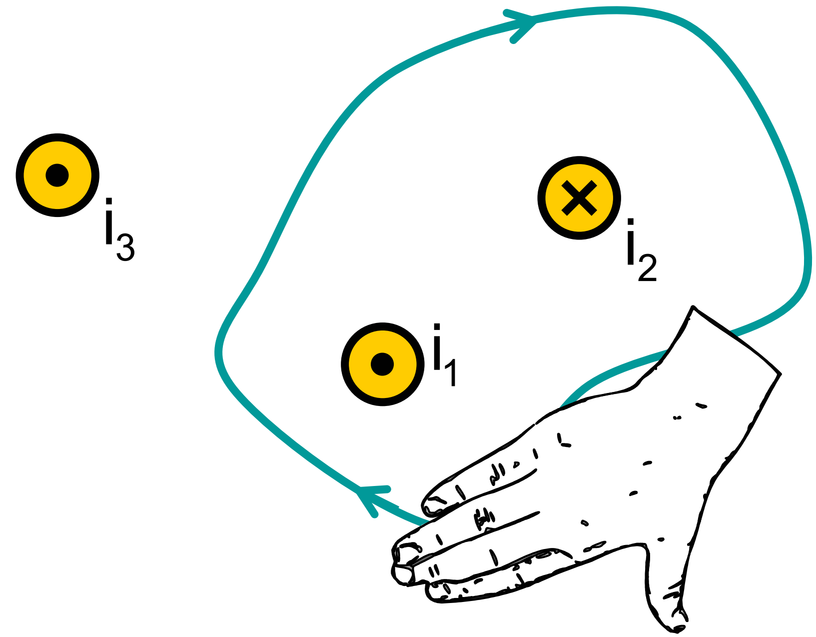
30-3 Ampere's Law



Current pointing out of the page is positive

$$i_{enc} = +i_1 - i_2$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \, (+i_1 - i_2)$$



Current pointing into the page is positive

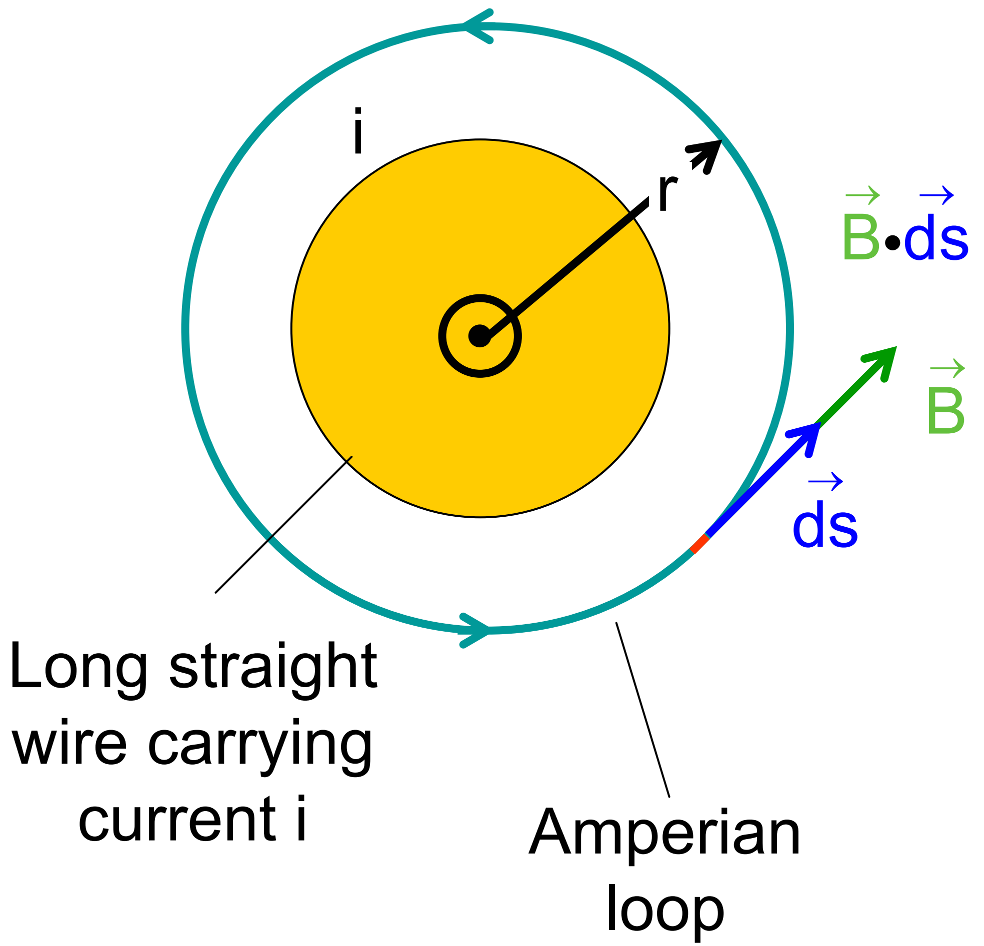
$$i_{enc} = -i_1 + i_2$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \, (-i_1 + i_2)$$

30-3 Ampere's Law

The magnetic field outside a long straight wire

From symmetry, B has cylindrical symmetry about the wire i



$\vec{B} \bullet \vec{ds} = B\, ds \cos 0^0 = B\, ds$

Ampere's law

$$\oint \vec{B} \bullet \vec{ds} = \mu_0\, i$$

$$\oint B\, ds = \mu_0\, i$$

$$B \oint ds = \mu_0\, i$$

$$B\, 2\pi r = \mu_0\, i$$

$$B = \frac{\mu_0\, i}{2\pi r}$$

30-3 Ampere's Law

The magnetic field inside a long straight wire

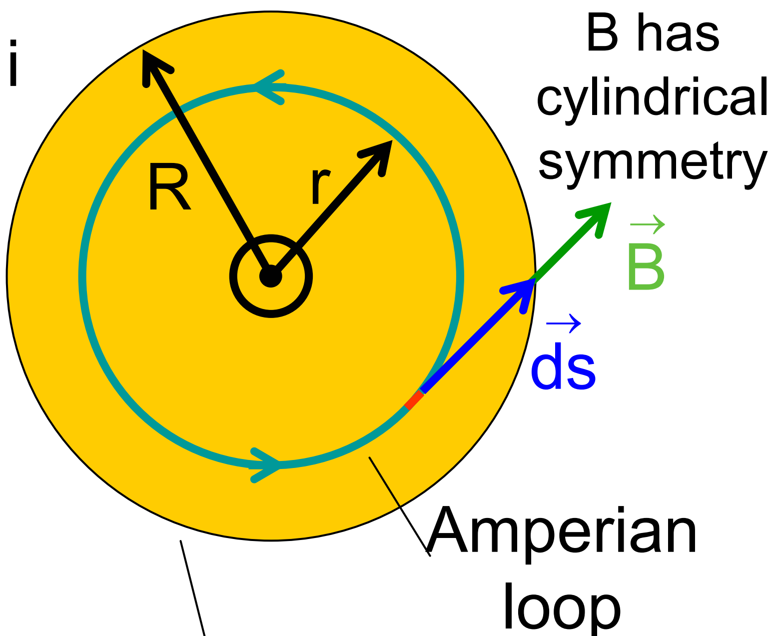
$$\oint \vec{B}\bullet\vec{ds} = \mu_0 \; i_{enc}$$

$$\oint B\,ds = B \oint ds = B\,2\pi r$$

current density

Area enclosed

$$i_{enc} = J\,(\pi r^2) = \frac{i}{\pi R^2}(\pi r^2) = i\,\frac{r^2}{R^2}$$



Long straight wire of radius R carrying uniformly distributed current i

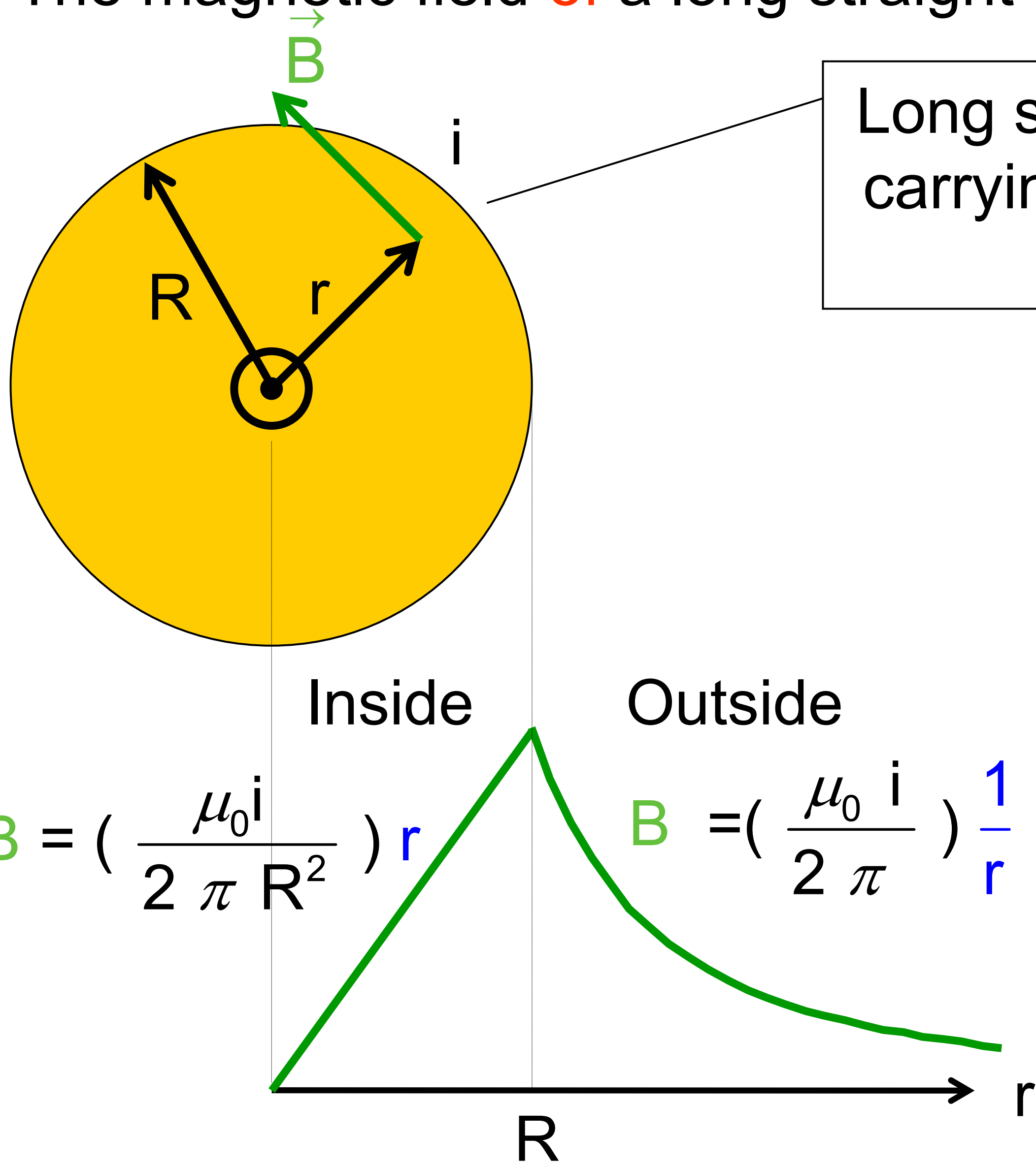
$$B\,2\pi r = \mu_0 i\,\frac{r^2}{R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$

30-3 Ampere's Law

The magnetic field of a long straight wire

Long straight wire of radius R carrying uniformly distributed current i



Inside: $B = \left(\dfrac{\mu_0 i}{2 \pi R^2} \right) r$

Outside: $B = \left(\dfrac{\mu_0 i}{2 \pi} \right) \dfrac{1}{r}$

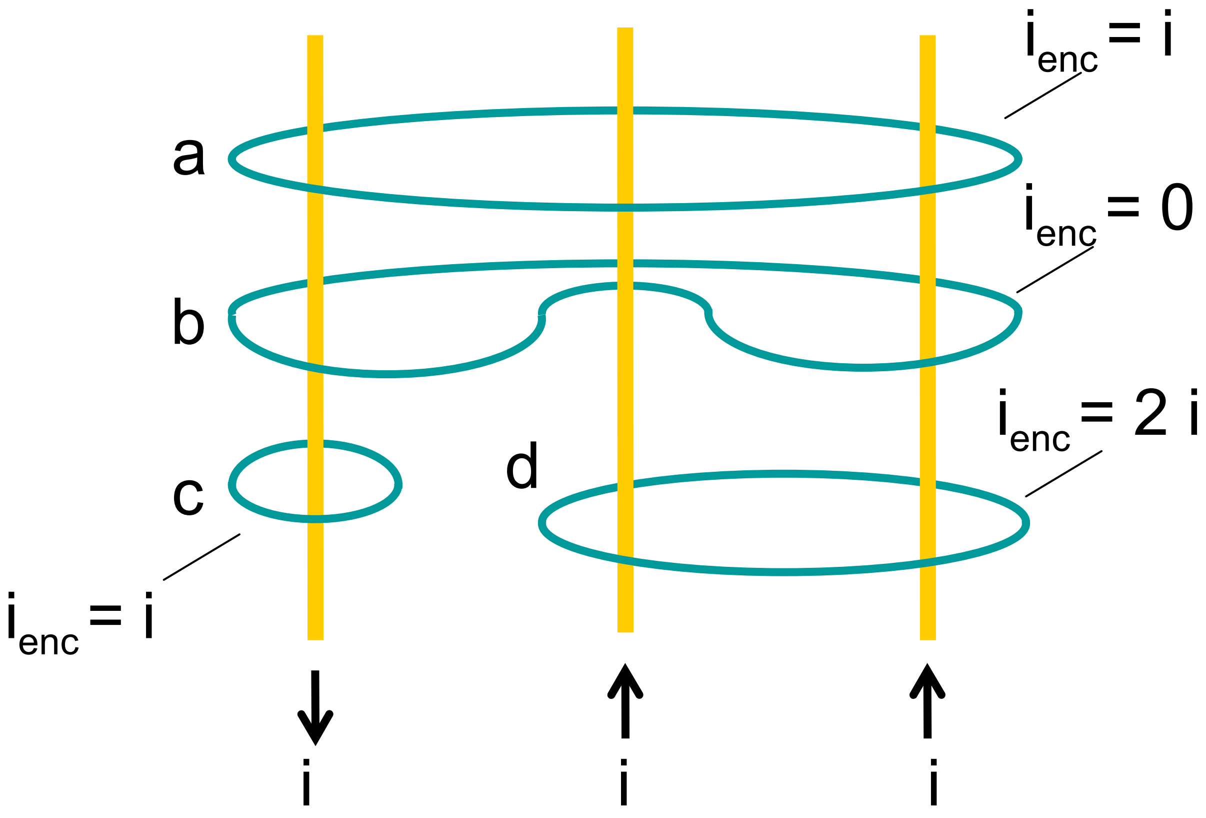
30-3 Ampere's Law

Checkpoint 3

Rank the loop according to the magnitude of $\oint \vec{B} \bullet d\vec{s}$ along each, greatest first.

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 \, i_{enc}$$

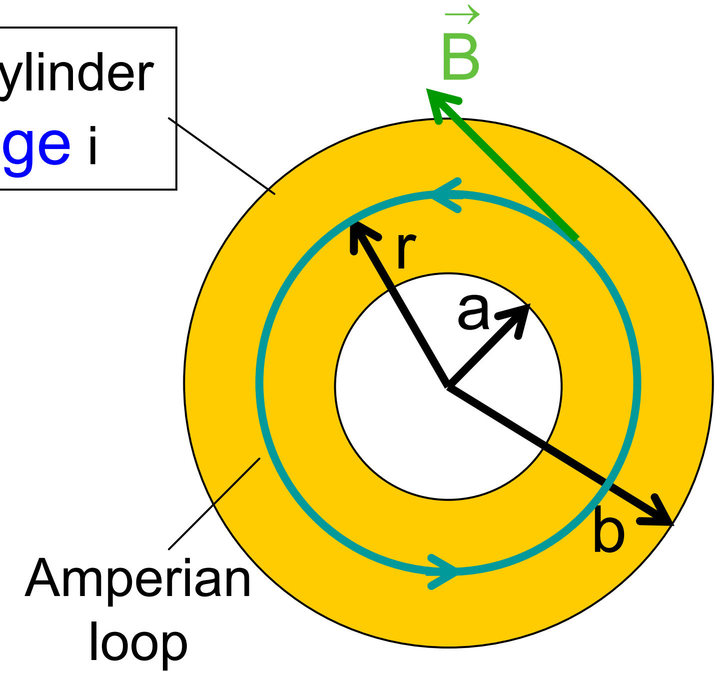
d,
then a and c tie,
then b



30-3 Ampere's Law

Sample problem 30-3

Long straight cylinder
J out of page i



a = 2.0 cm; b= 4.0 cm,
$J = c\, r^2$, where $c = 3.0 \times 10^6$ A/m^2 and r in meters, J is out of page

What is the magnetic field at a point 3.0 cm from the center?

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, i_{enc}$$

B has cylindrical symmetry

$$\oint B\, ds = B \oint ds = B\, 2\pi r$$

$$i_{enc} = \int \vec{J} \cdot d\vec{a} = \int J\, da = \int c\, r^2\, da$$

$$= \int_a^r c\, r^2\, (2\pi\, r dr) = 2\pi c \int_a^r r^3\, dr$$

$$= 2\pi c \left[\frac{r^4}{4} \right]_a^r = 2\pi c \left(\frac{r^4}{4} - \frac{a^4}{4} \right)$$

$$B\, 2\pi r = \mu_0\, 2\pi c \left(\frac{r^4}{4} - \frac{a^4}{4} \right)$$

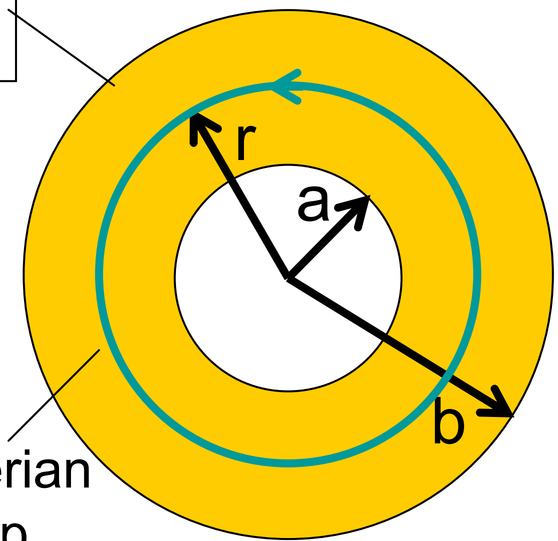
30-3 Ampere's Law

Sample problem 30-3

Long straight cylinder
J out of page i

a = 2.0 cm; b= 4.0 cm,
$J = c\,r^2$, where $c = 3.0 \times 10^6\ A/m^2$ and r in meters, J is out of page

What is the magnetic field at a point 3.0 cm from the center?



$$B\,2\pi r = \mu_0\, 2\pi c\left(\frac{r^4}{4} - \frac{a^4}{4}\right)$$

$$B = \frac{\mu_0\, c}{4\, r}\,(r^4 - a^4)$$

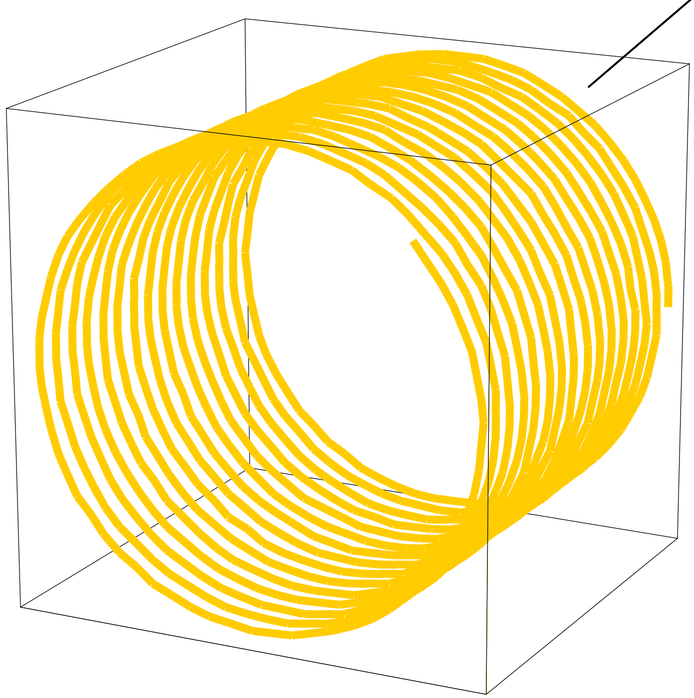
$$= \frac{4\pi \times 10^{-7}\,(3.0 \times 10^6)}{4\,(.03)}\,((.03)^4 - (.02)^4)$$

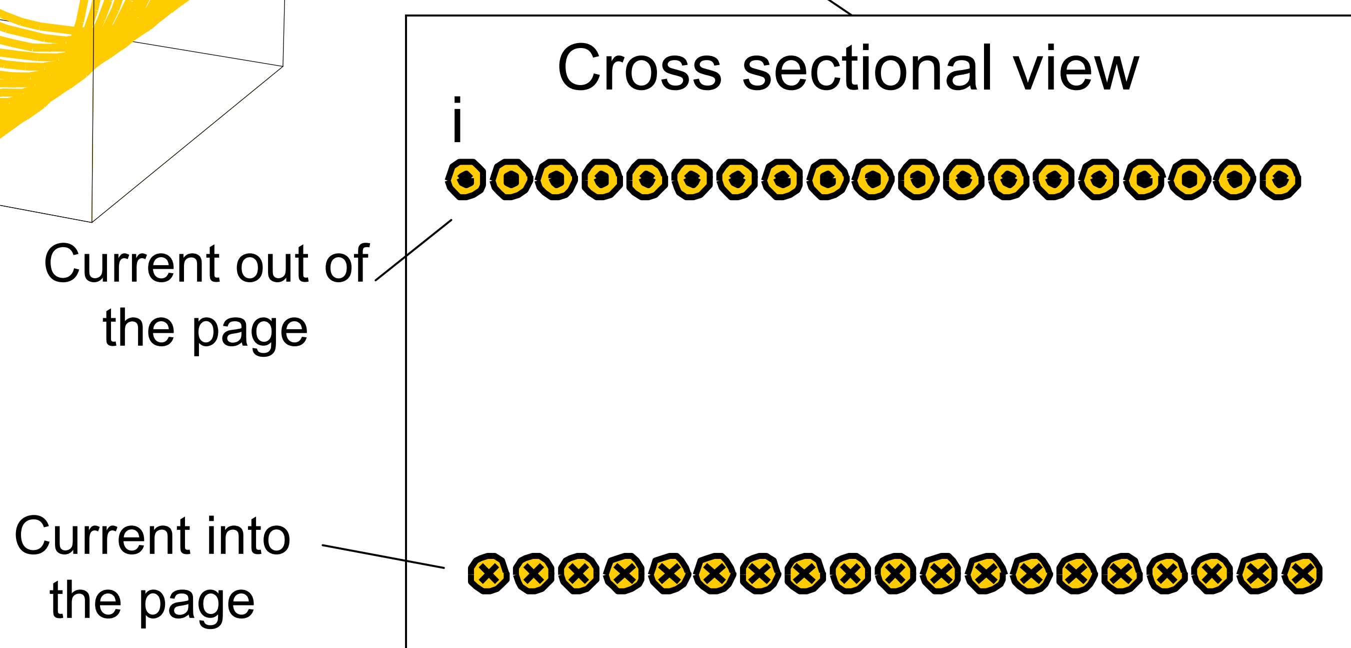
$$= 2.0 \times 10^{-5}\ T$$

If you get a negative value for B, your guess about the direction of B is wrong, the correct direction is opposite of your guess

30-4 Solenoids and Toroids

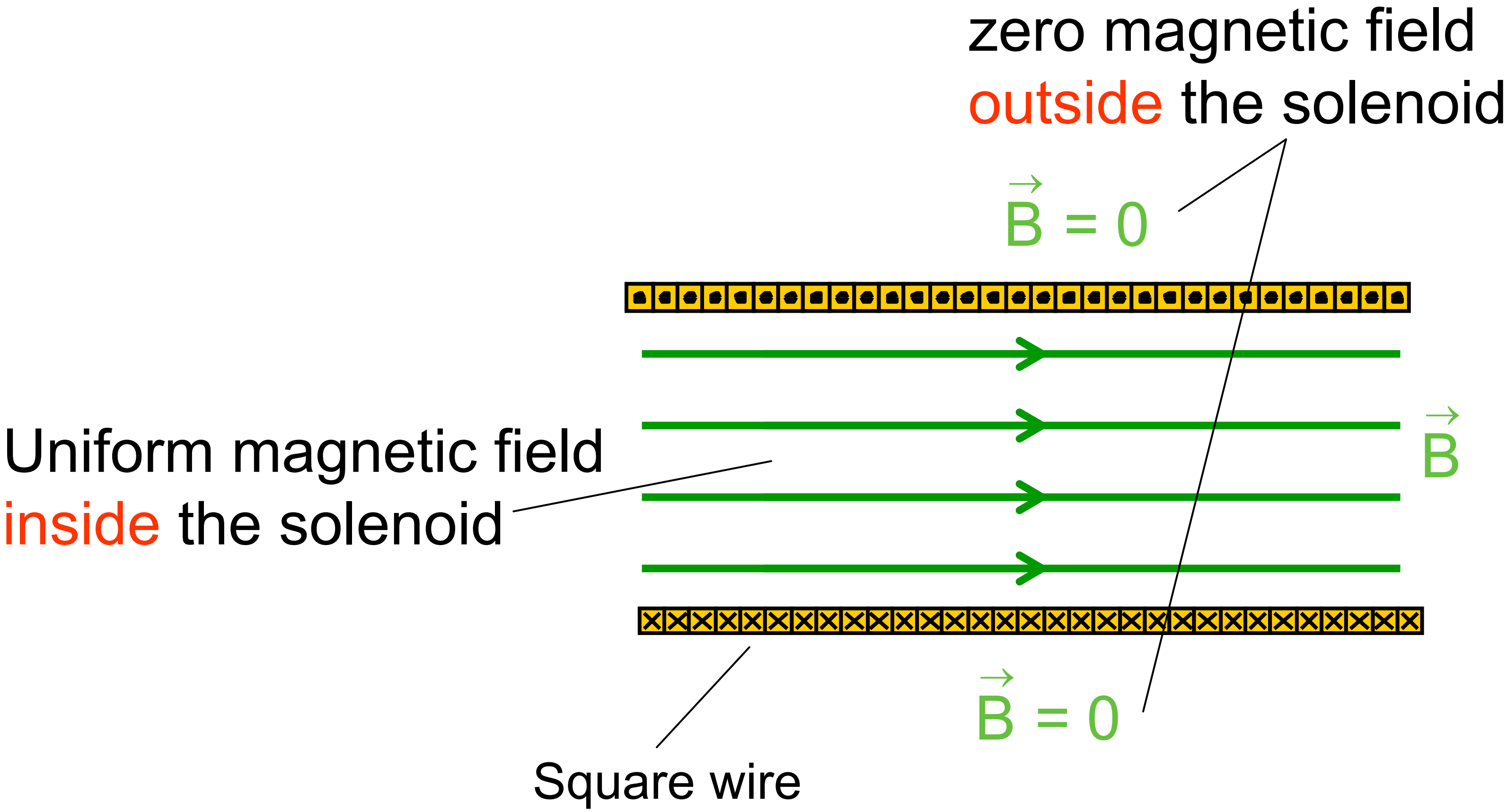
A solenoid is a long highly wound helical coil





30-4 Solenoids and Toroids

Infinitely long ideal solenoid



Cross sectional view

30-4 Solenoids and Toroids

Infinitely long ideal solenoid

Ampere's law $\oint \vec{B} \bullet \vec{ds} = \mu_0 \, i_{enc}$

$$\oint \vec{B} \bullet \vec{ds} = \int_a^b \vec{B} \bullet \vec{ds} + \int_b^c \vec{B} \bullet \vec{ds} + \int_c^d \vec{B} \bullet \vec{ds} + \int_d^a \vec{B} \bullet \vec{ds}$$

$= 0$ since $\vec{B} \perp \vec{ds}$

$= 0$ since $\vec{B} = 0$

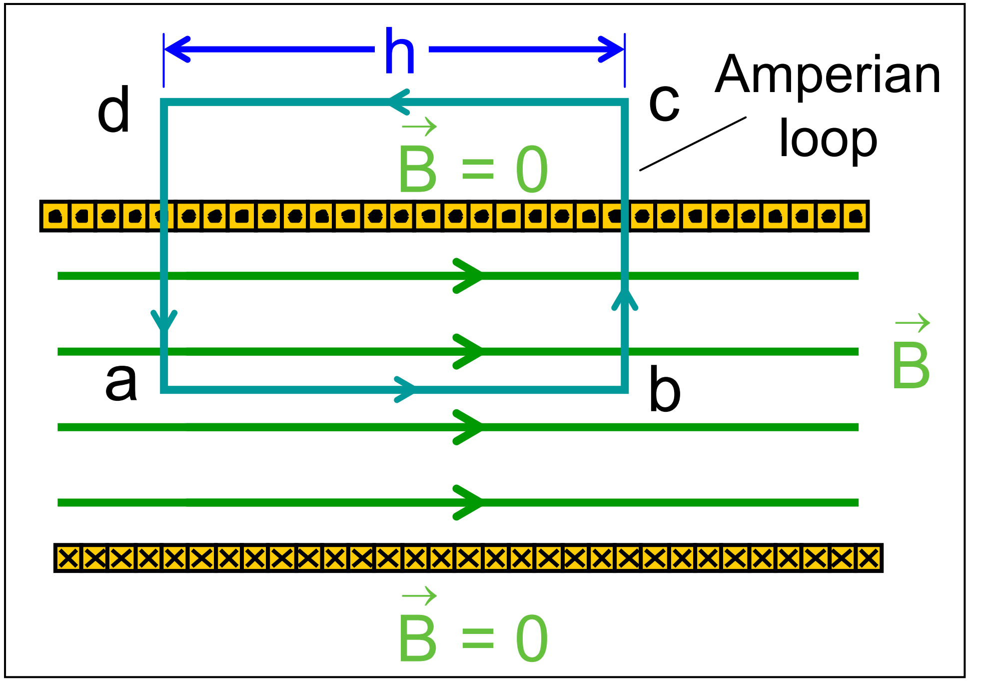
$$= \int_a^b B \, ds = B \int_a^b ds = B \, h$$

Number of turns per unit length

$$i_{enc} = (n \, h) \, i$$

Ampere's law $B \, h = \mu_0 \, n \, h \, i$

$$\boxed{B = \mu_0 \, n \, i}$$



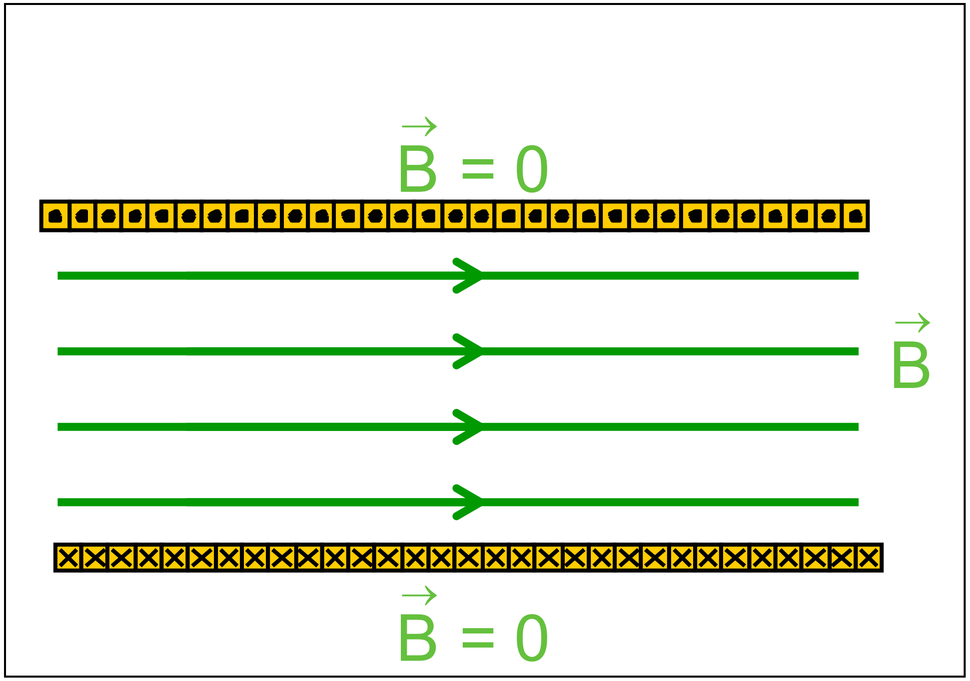
30-4 Solenoids and Toroids

Infinitely long ideal solenoid

Number of turns per unit length

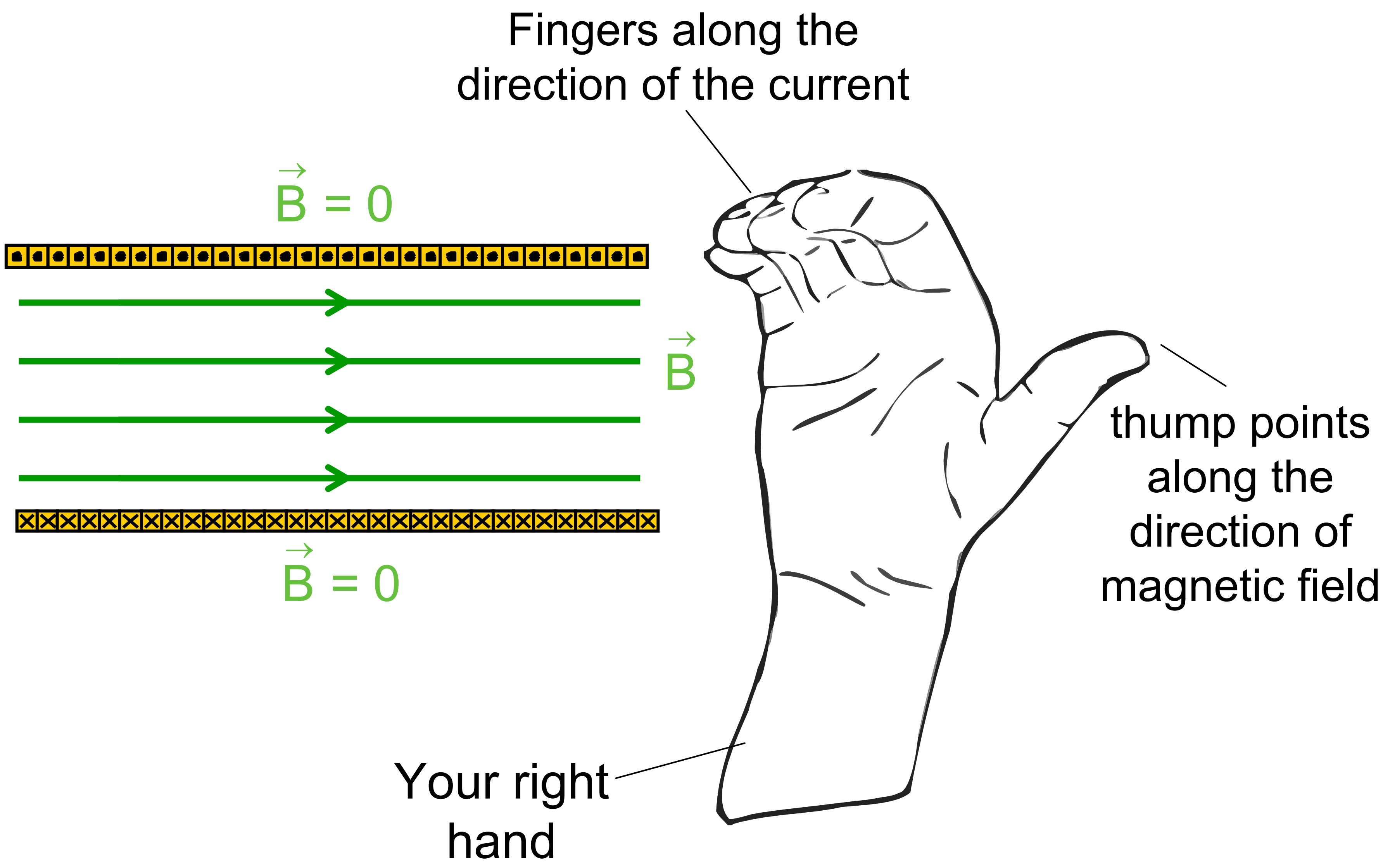
$$B = \mu_0 \, n \, i$$

The magnetic field does not depend on the radius of the solenoid.
The magnetic field is uniform over the cross section of the solenoid.



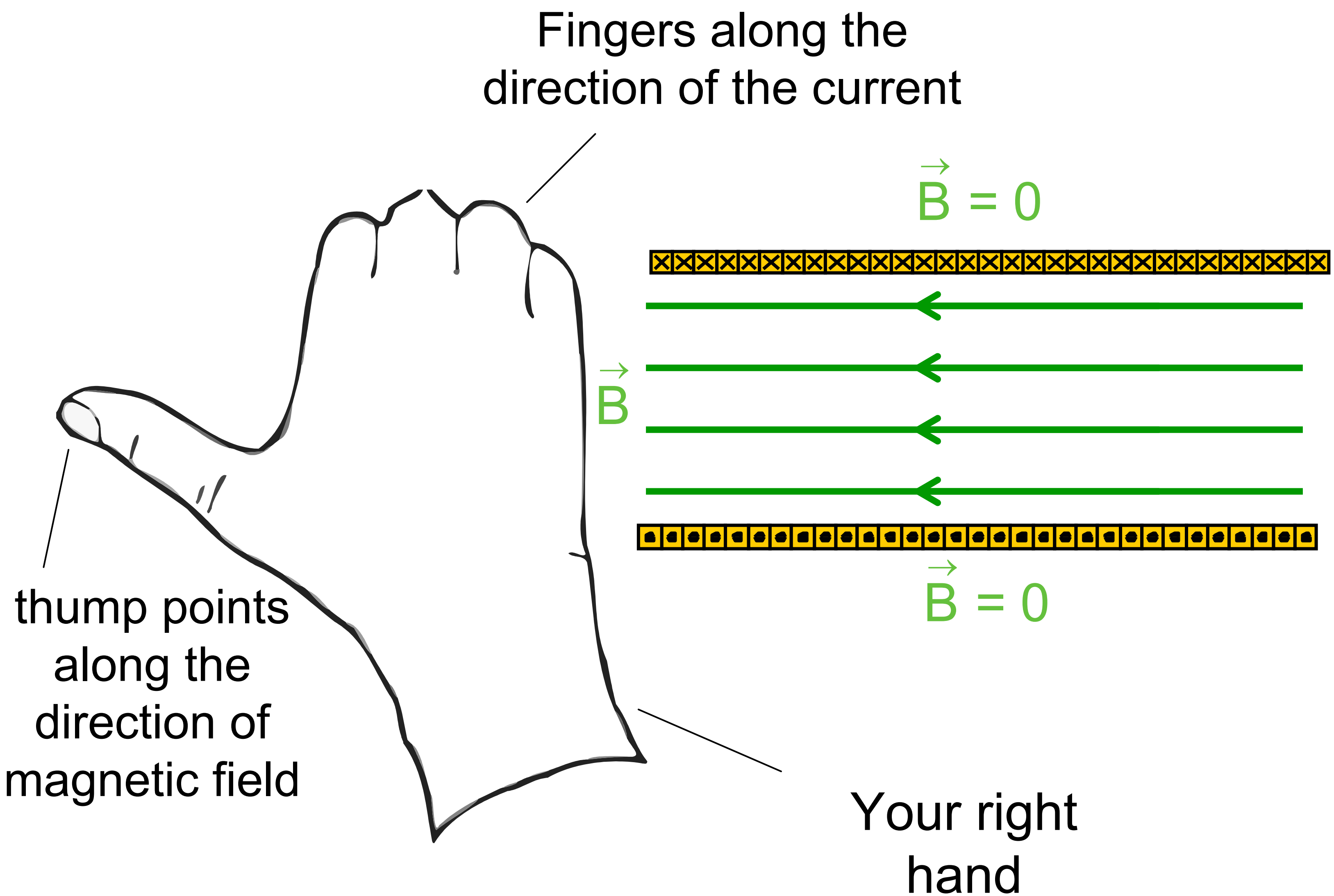
30-4 Solenoids and Toroids

The direction of the magnetic field along the solenoid axis



30-4 Solenoids and Toroids

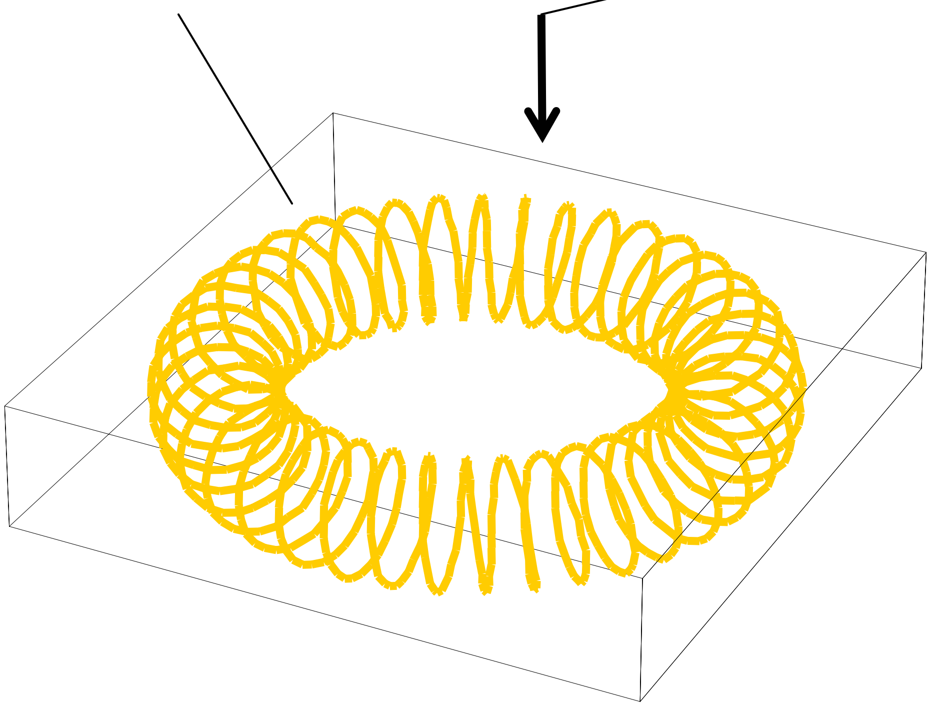
The direction of the magnetic field along the solenoid axis

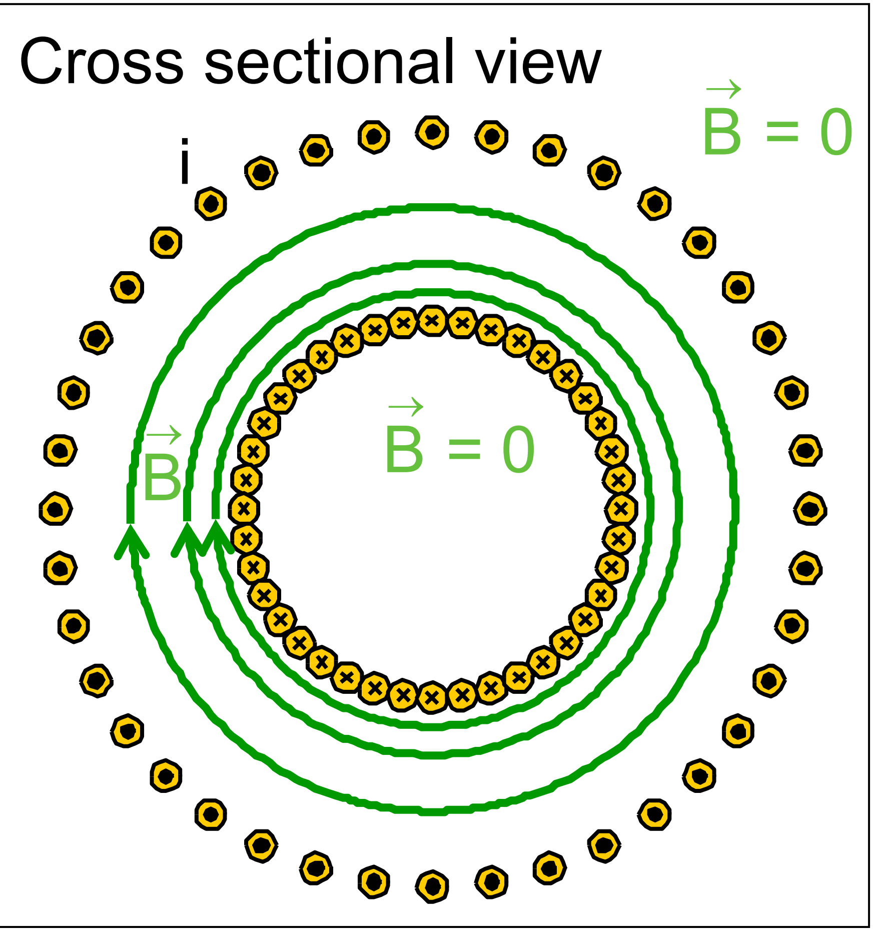


30-4 Solenoids and Toroids

Magnetic field of a toroid

A toroid is similar to a solenoid bent into a hollow doughnut





30-4 Solenoids and Toroids

Magnetic field of a toroid

Ampere's law $\oint \vec{B} \bullet d\vec{s} = \mu_0 \; i_{enc}$

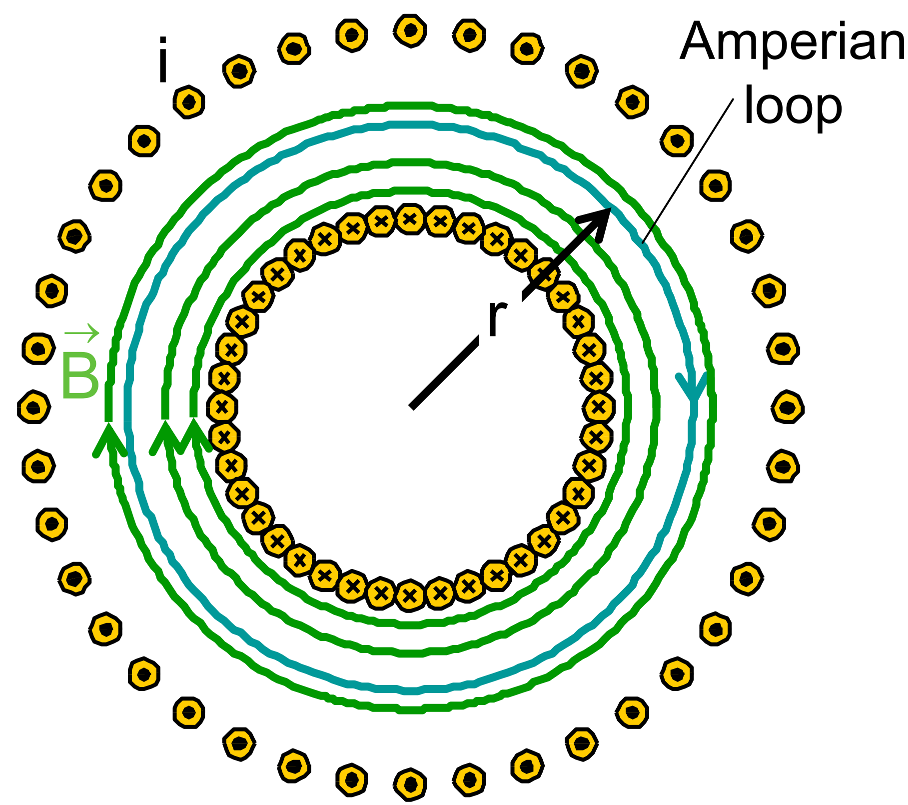
$$\oint \vec{B} \bullet d\vec{s} = \oint B \, ds = B \oint ds = B \, 2 \pi r$$

Total number of turns

$$i_{enc} = N \, i$$

Ampere's law $B \, 2 \pi r = \mu_0 \, N \, i$

$$B = \frac{\mu_0 \, N \, i}{2 \pi} \frac{1}{r}$$



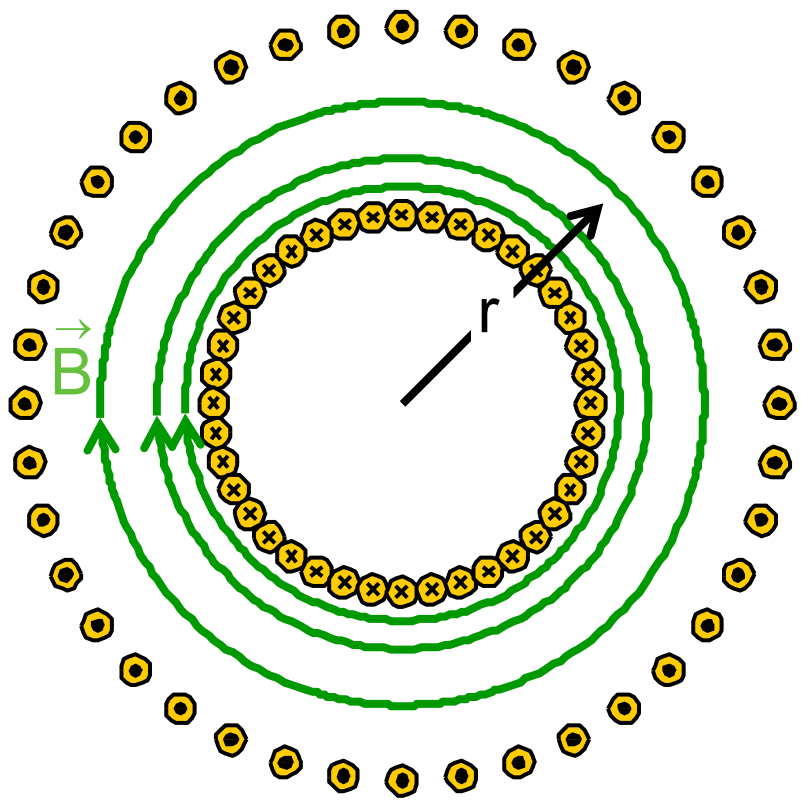
30-4 Solenoids and Toroids

Magnetic field of a toroid

Total number of turns

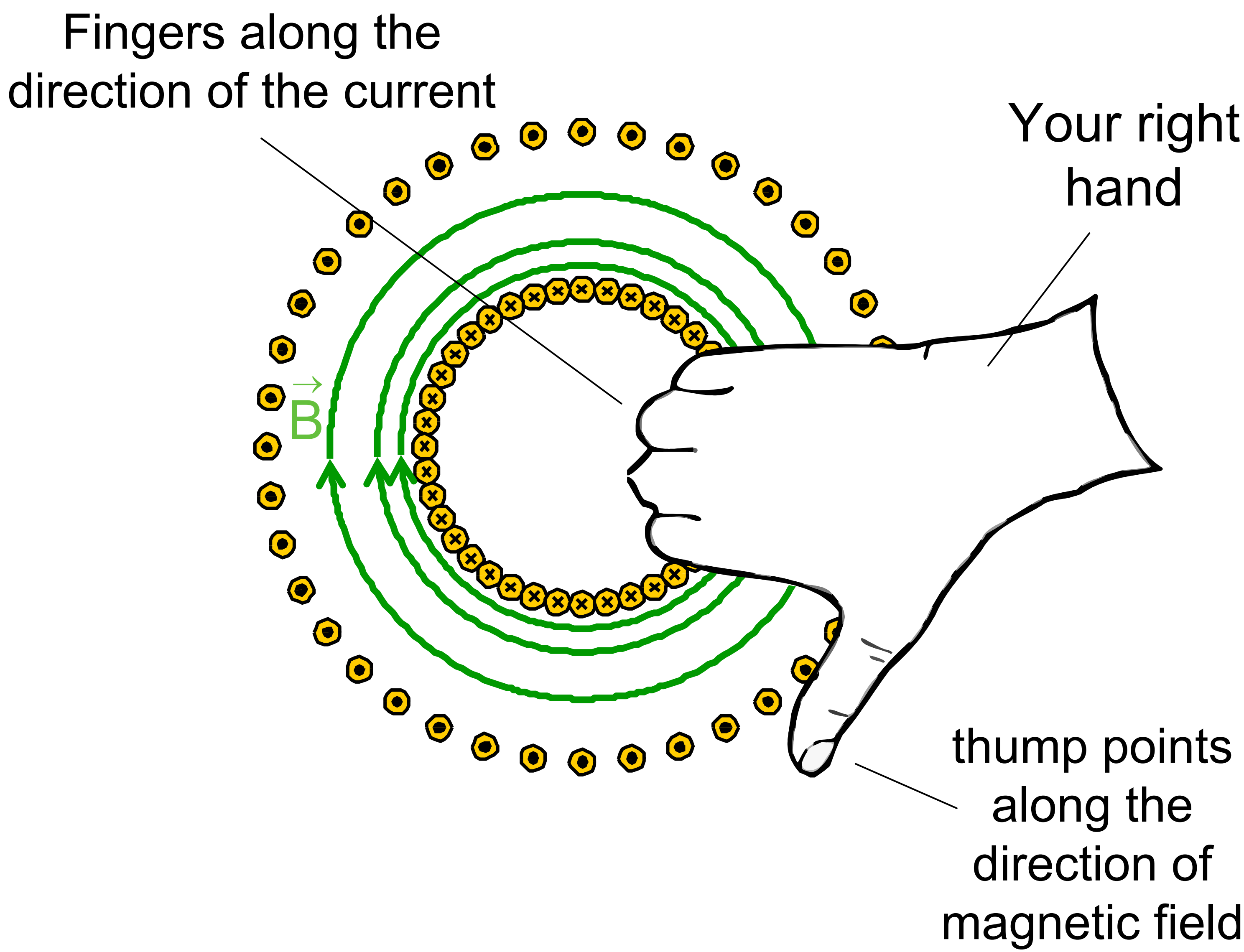
$$B = \frac{\mu_0 N i}{2\pi} \frac{1}{r}$$

The magnetic field is not constant over the cross section of a troroid.



30-4 Solenoids and Toroids

The direction of the magnetic field along the solenoid axis

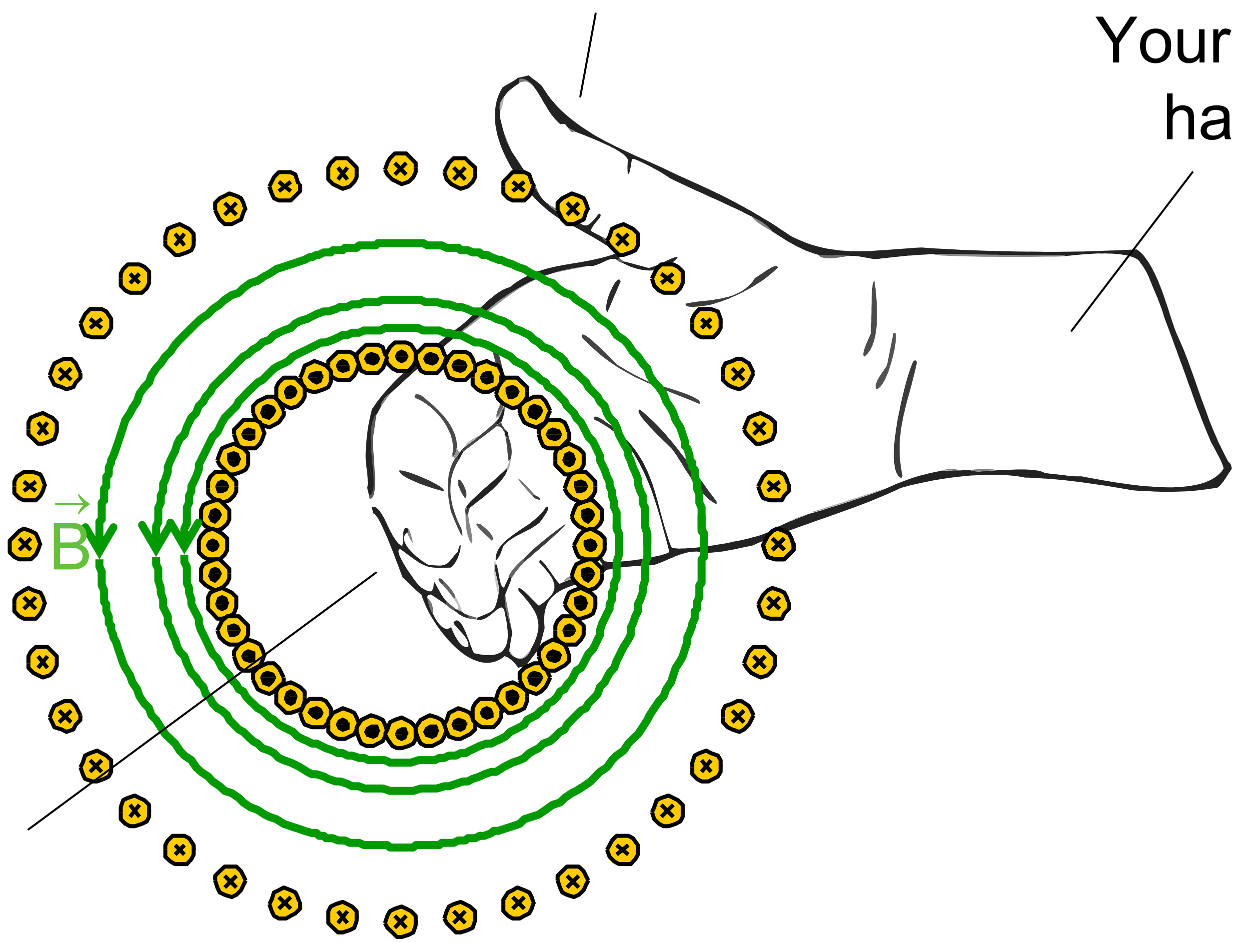


30-4 Solenoids and Toroids

The direction of the magnetic field along the solenoid axis

thump points along the direction of magnetic field

Your right hand



Fingers along the direction of the current

30-4 Solenoids and Toroids

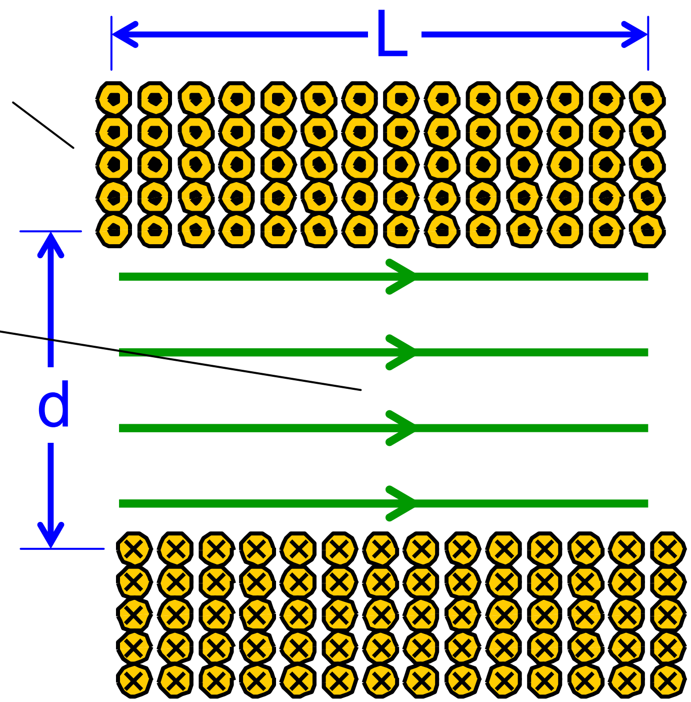
Sample problem 30-4

L = 1.23 m

d =3.55 cm

i =5.57 A

5 closed-packed layers, each with 580 turns along L



What is the magnetic field at the center?

Number of turns per unit length

$$B = \mu_0 \, n \, i$$

$$B = 4\pi \times 10^{-7} \left(\frac{5(580)}{1.23}\right)(5.57)$$

$$= 24.4 \times 10^{-3}\ \text{T} = 24.4\ \text{mT}$$