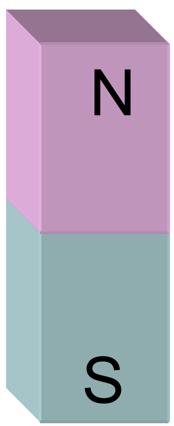
Chapter 29

Magnetic Fields

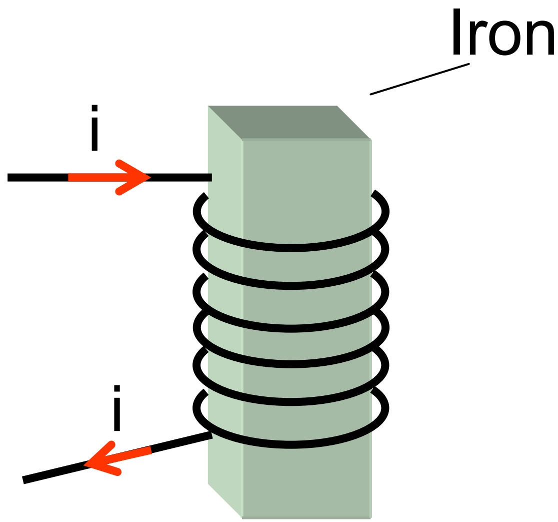
29-1 The magnetic Field

How to produce magnetic fields?



Permanent magnets

An electron has an intrinsic magnetic field. In some materials, magnetic fields from the electrons add together to give a net magnetic field



Electromagnet

Need current

Moving charged particles (electrons) produce magnetic fields

29-2 The Definition of the Magnetic Field

Force on a charged particle due to a magnetic field

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

Charge of the particle

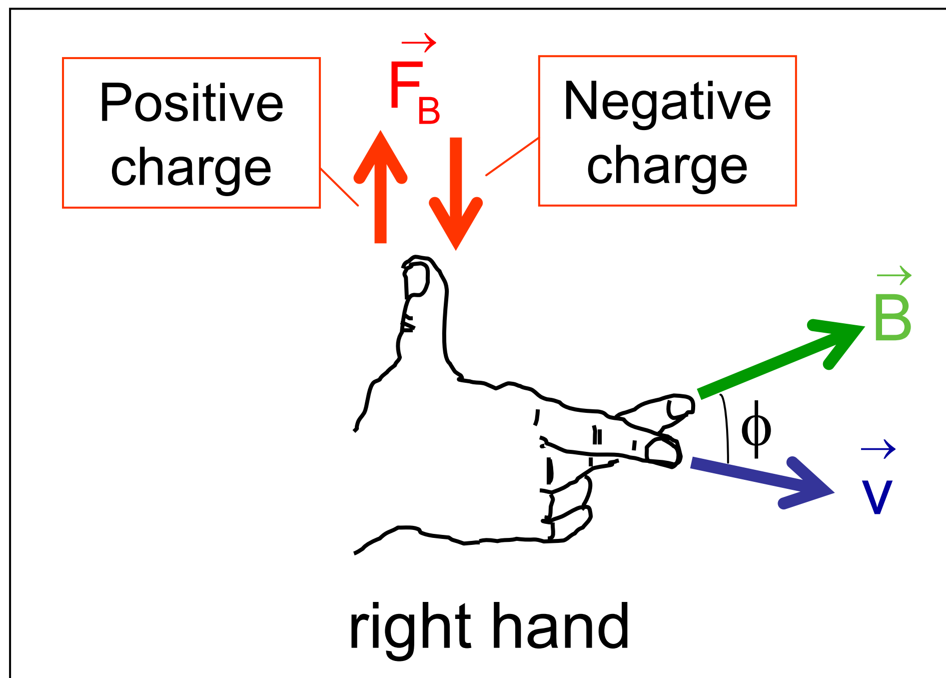
Velocity of the particle

Magnetic field

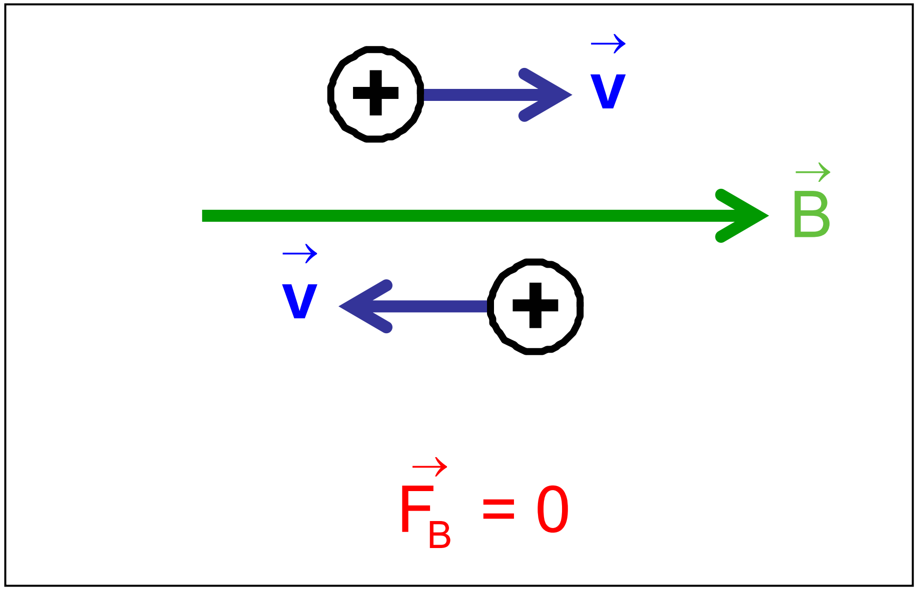
Direction: right-hand rule

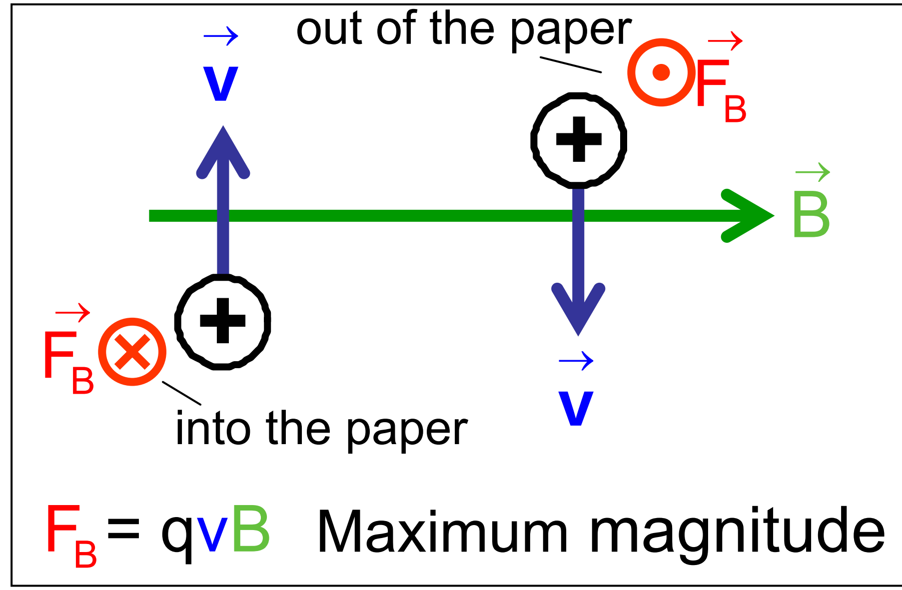
Magnitude: $F_B = |q|\, v\, B \sin \phi$

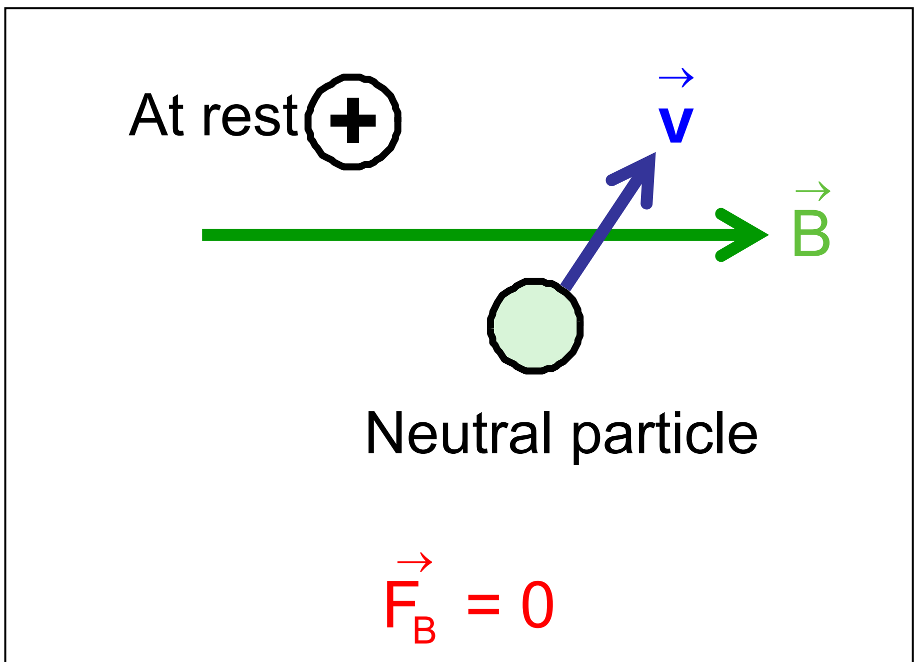
The angle between v and B vectors

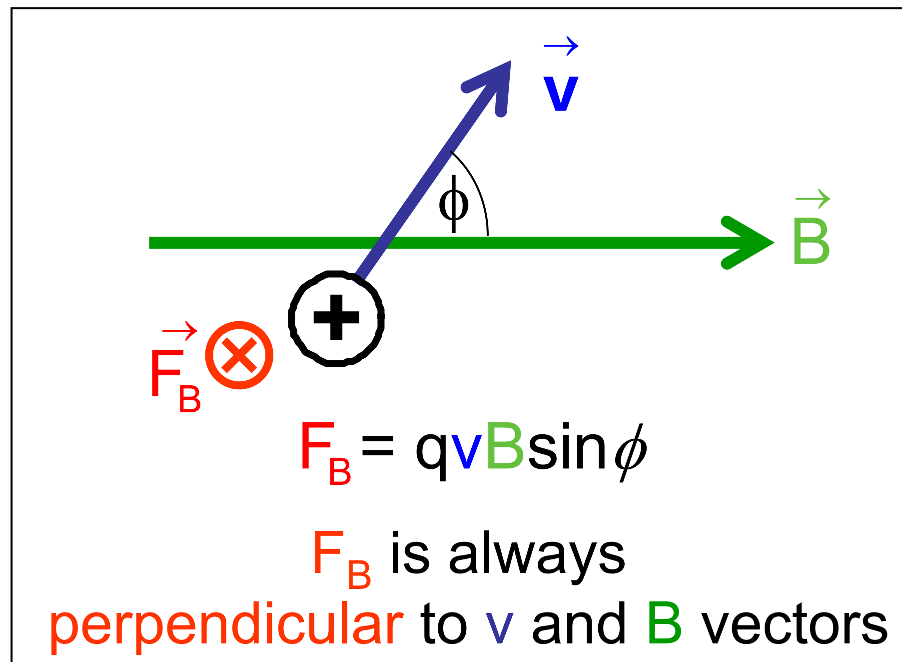


29-2 The Definition of the Magnetic Field

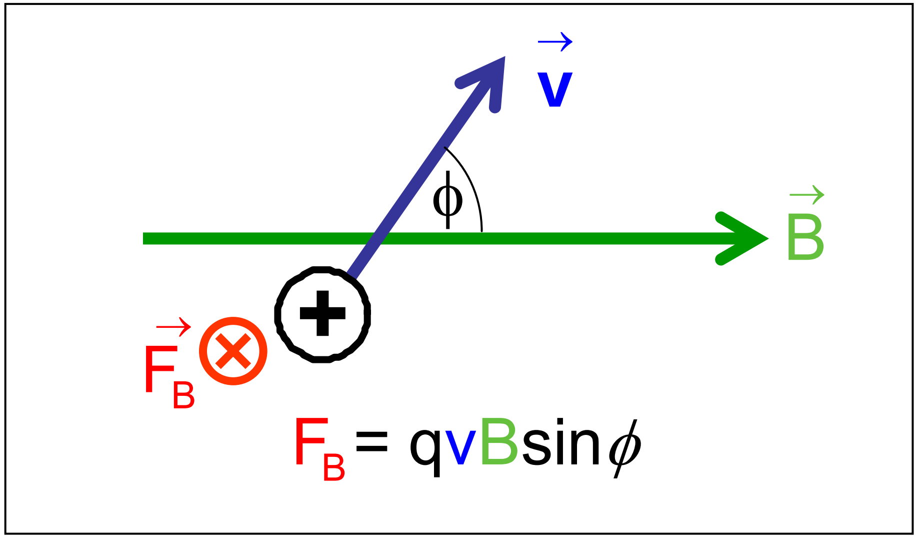








29-2 The Definition of the Magnetic Field



F_B is always perpendicular to v vector

→ F_B does not have a component along v vector

→ F_B cannot change the speed of a particle

→ F_B cannot change the kinetic energy of a particle

→ F_B cannot do work on a particle

→ F_B can accelerate a particle only by changing its direction

29-2 The Definition of the Magnetic Field

SI unit for magnetic fields B

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

Tesla

$$1\ \text{Tesla} = 1\ \frac{\text{Newton}}{(\text{Coulomb})\ (\text{Meter/Second})}$$

Gauss is another unit for measuring magnetic fields B

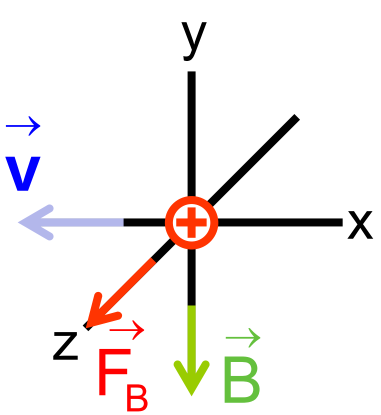
$$1\ \text{Tesla} = 10^4\ \text{Gauss}$$

Small bar magnets	10^{-2} T	100 G
Near Earth's surface	$0.5\text{X}10^{-4}$ T	0.5 G

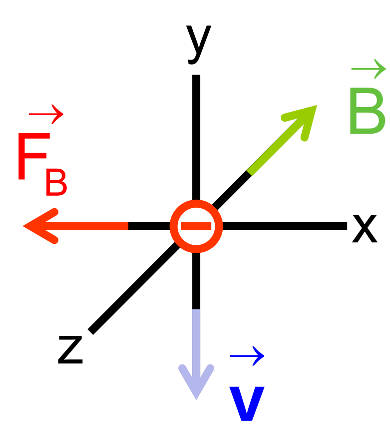
29-2 The Definition of the Magnetic Field

Checkpoint 1

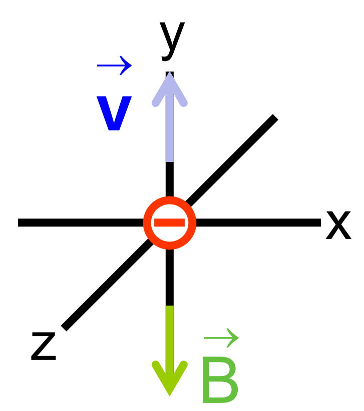
What is the direction of the magnetic force on the particle?



Along positive z-axis



Along negative x-axis

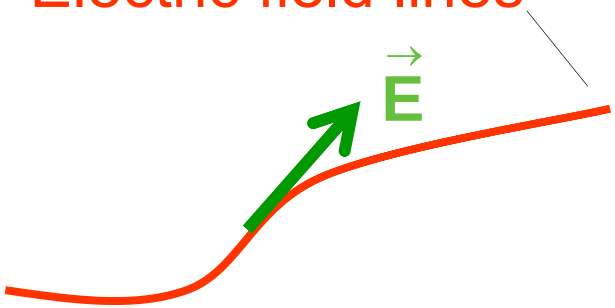


$\vec{F}_B = 0$

29-2 The Definition of the Magnetic Field

Electric Field $\vec{E}$

Electric field lines

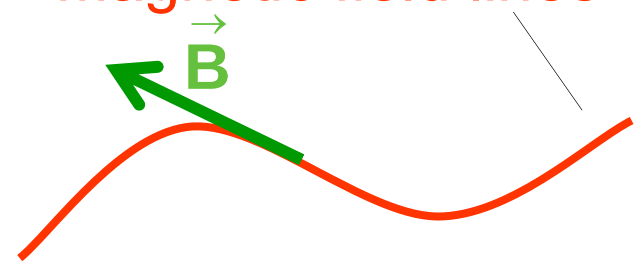


At any point, the tangent of an electric field line gives the direction of the electric field

Number of lines per unit area in a plane perpendicular to the electric field lines is proportional the magnitude of the electric field

Magnetic Field $\vec{B}$

Magnetic field lines

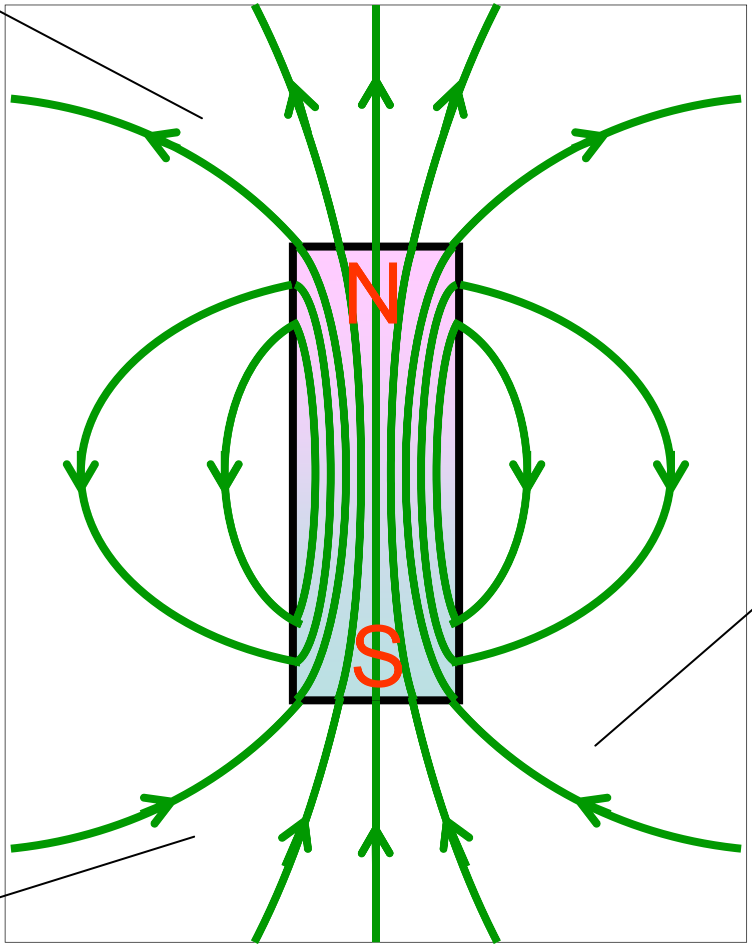


At any point, the tangent of a magnetic field line gives the direction of the magnetic field

Number of lines per unit area in a plane perpendicular to the magnetic field lines is proportional the magnitude of the magnetic field

29-2 The Definition of the Magnetic Field

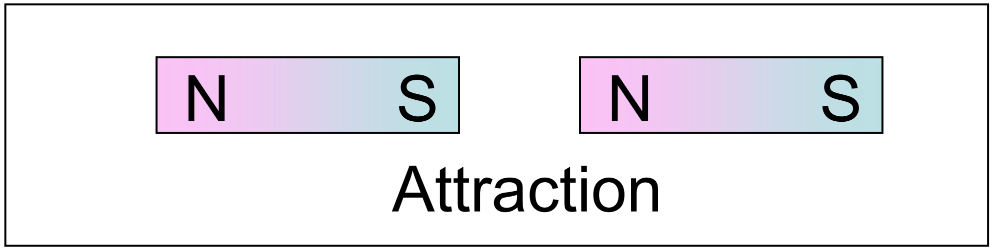
Magnetic lines emerges form the north pole

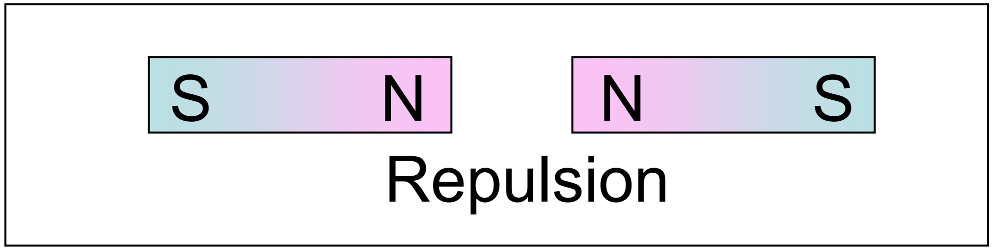


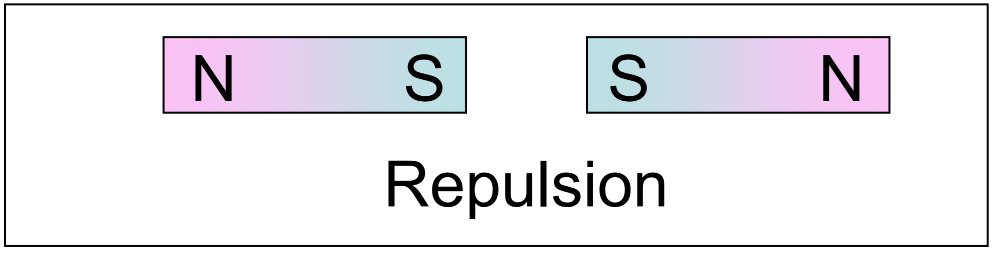
All lines form a closed loops

Magnetic lines enters the south pole

29-2 The Definition of the Magnetic Field

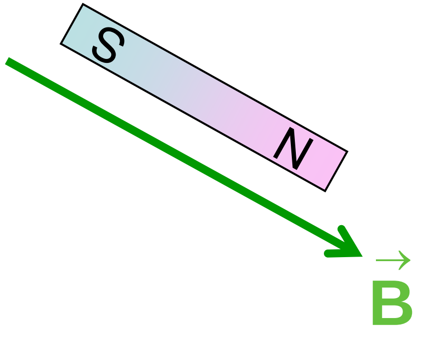


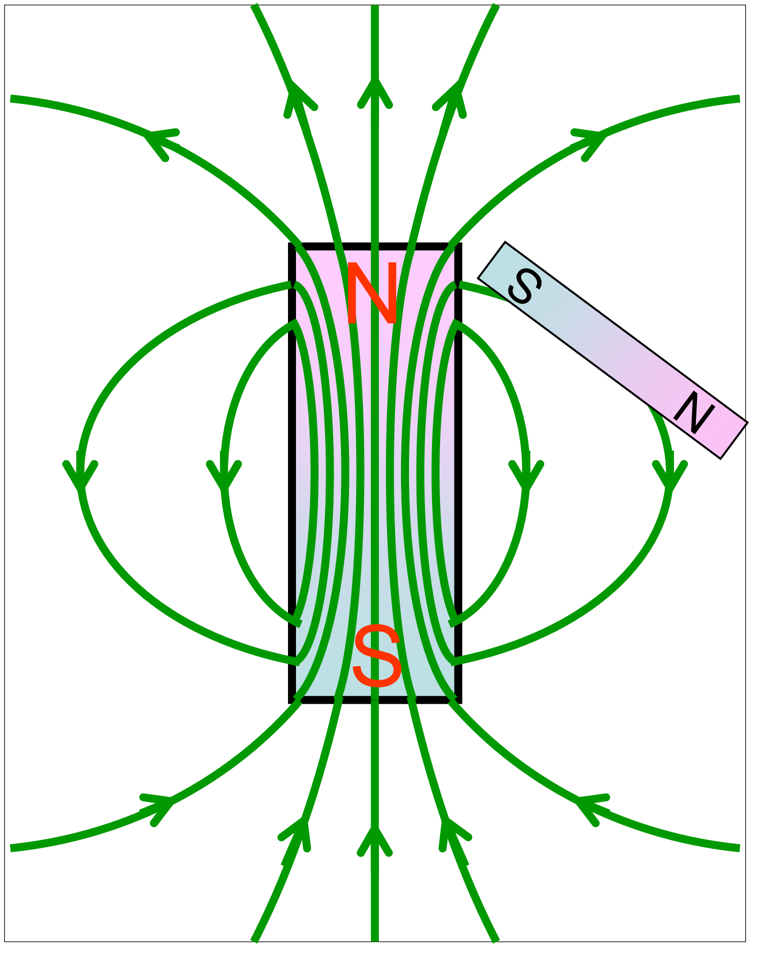




29-2 The Definition of the Magnetic Field

A magnet bar tries to align itself along the magnetic field

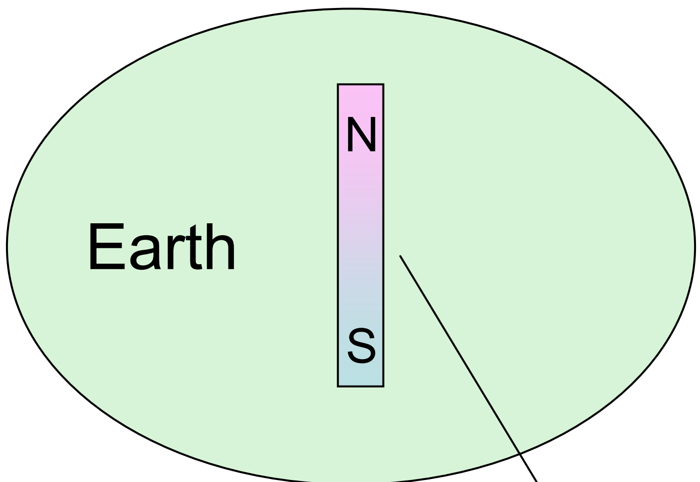




29-2 The Definition of the Magnetic Field

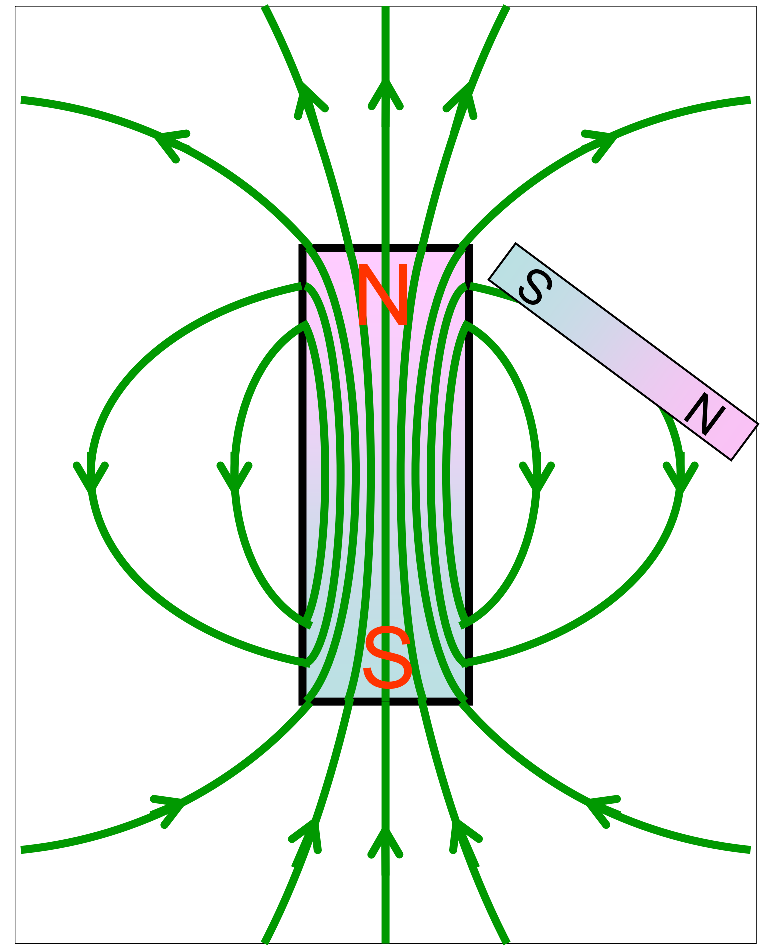
Earth"s south magnetic pole

North Pole



South Pole

The north pole of a magnetic needle (compass) points towards Earth's North pole



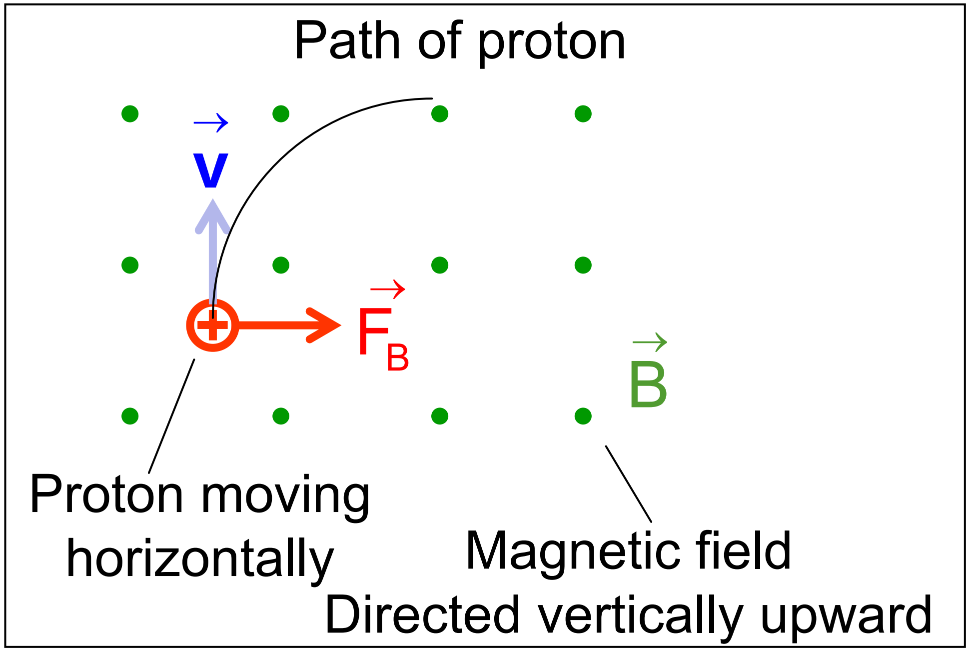
29-2 The Definition of the Magnetic Field

Sample Problem 29-1

Kinetic energy = 1.0 keV

Proton mass = 1.67×10^{-27} Kg

What magnetic deflecting force acts on the proton?



$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

$$K = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.0\times10^{3}\text{eV})\left(\frac{1.6\times10^{-19}\text{J}}{\text{eV}}\right)}{1.67\times10^{-27}\text{kg}}} = 4.4\times10^{5}\ \text{m/s}$$

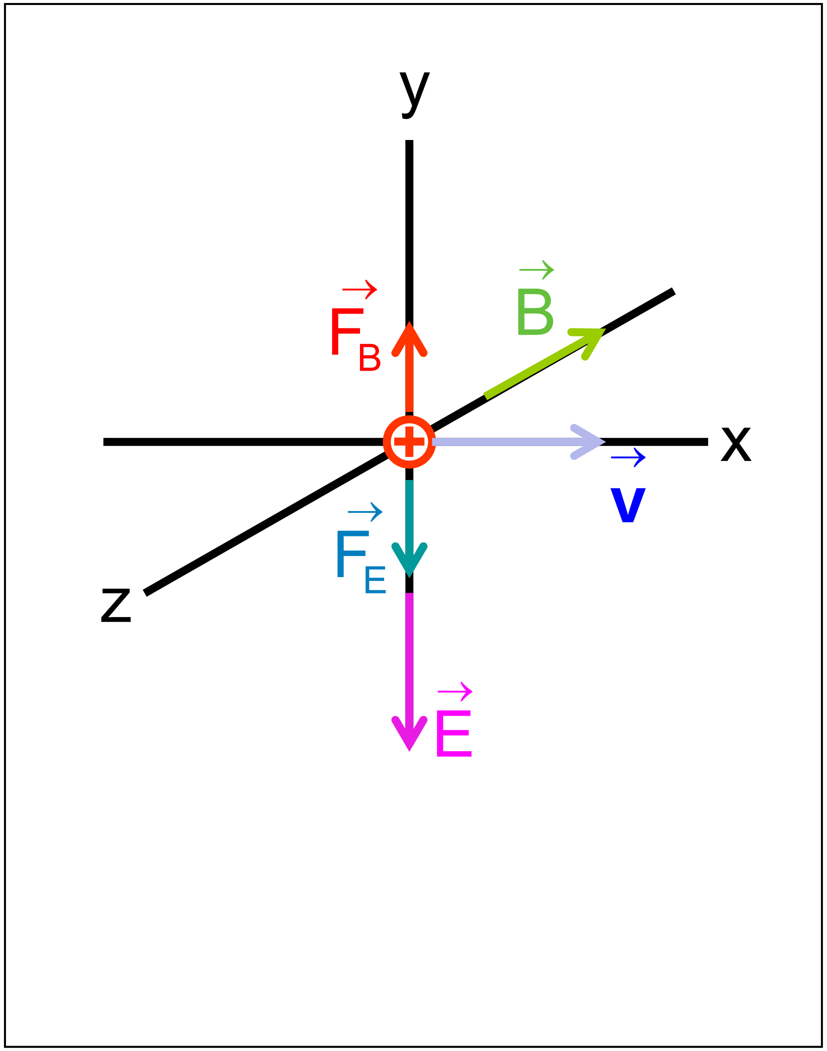
$$F_B = qvB\sin\phi = (1.6\times10^{-19})(4.5\times10^{5})(1.2\times10^{-3})\sin 90^0 = 8.4\times10^{-17}\ \text{N}$$

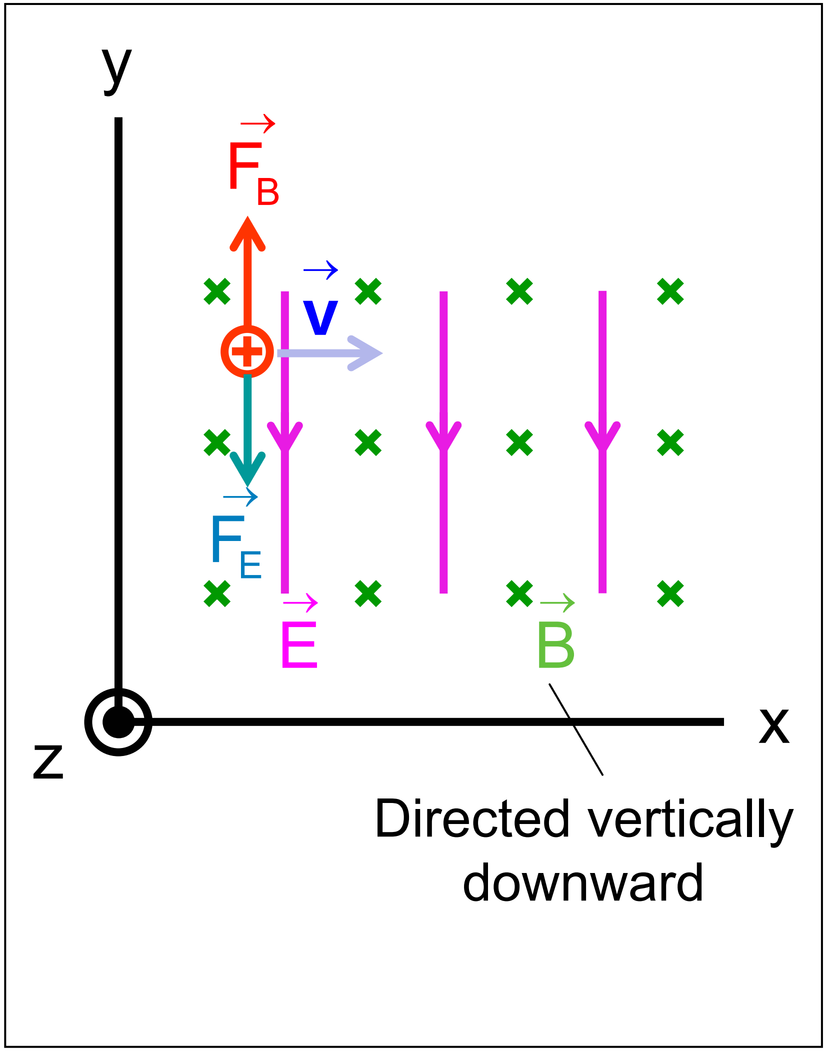
Direction: in the horizontal plane perpendicular to the path of proton

What is the acceleration of the proton? $a = \frac{F_B}{m} = 5.0\times10^{10}\ \text{m/s}^2$

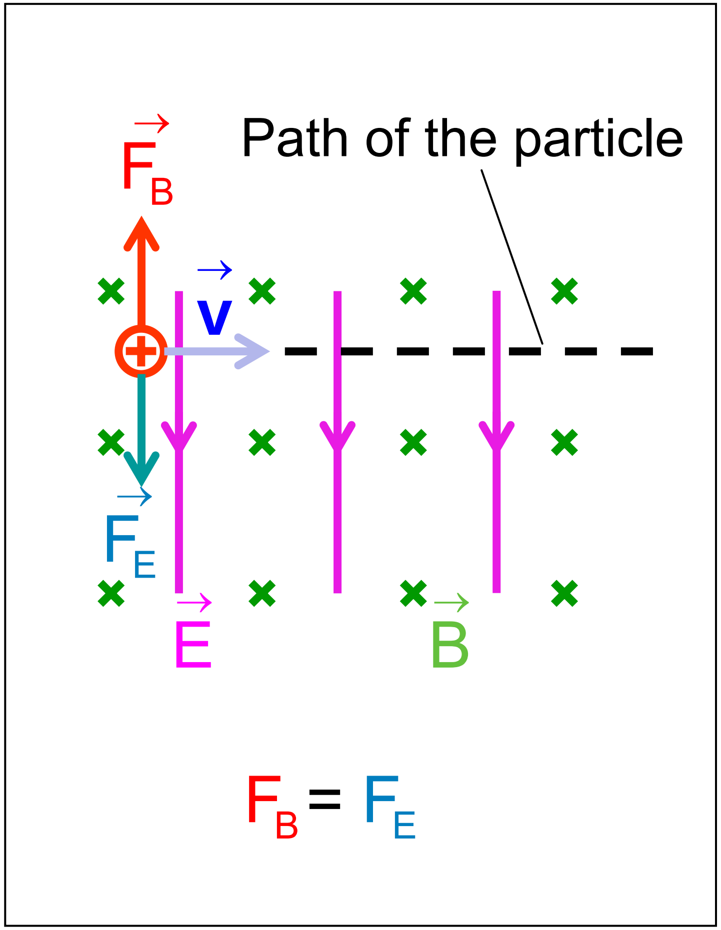
29-3 Crossed Fields: Discovery of the Electron

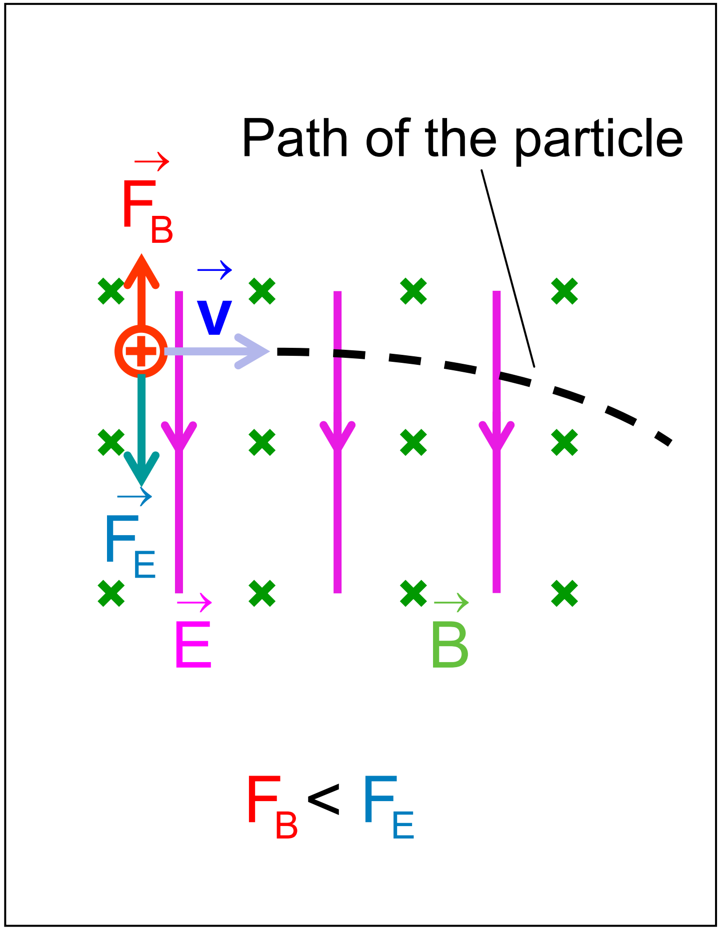
Cross fields $\vec{B} \perp \vec{E}$





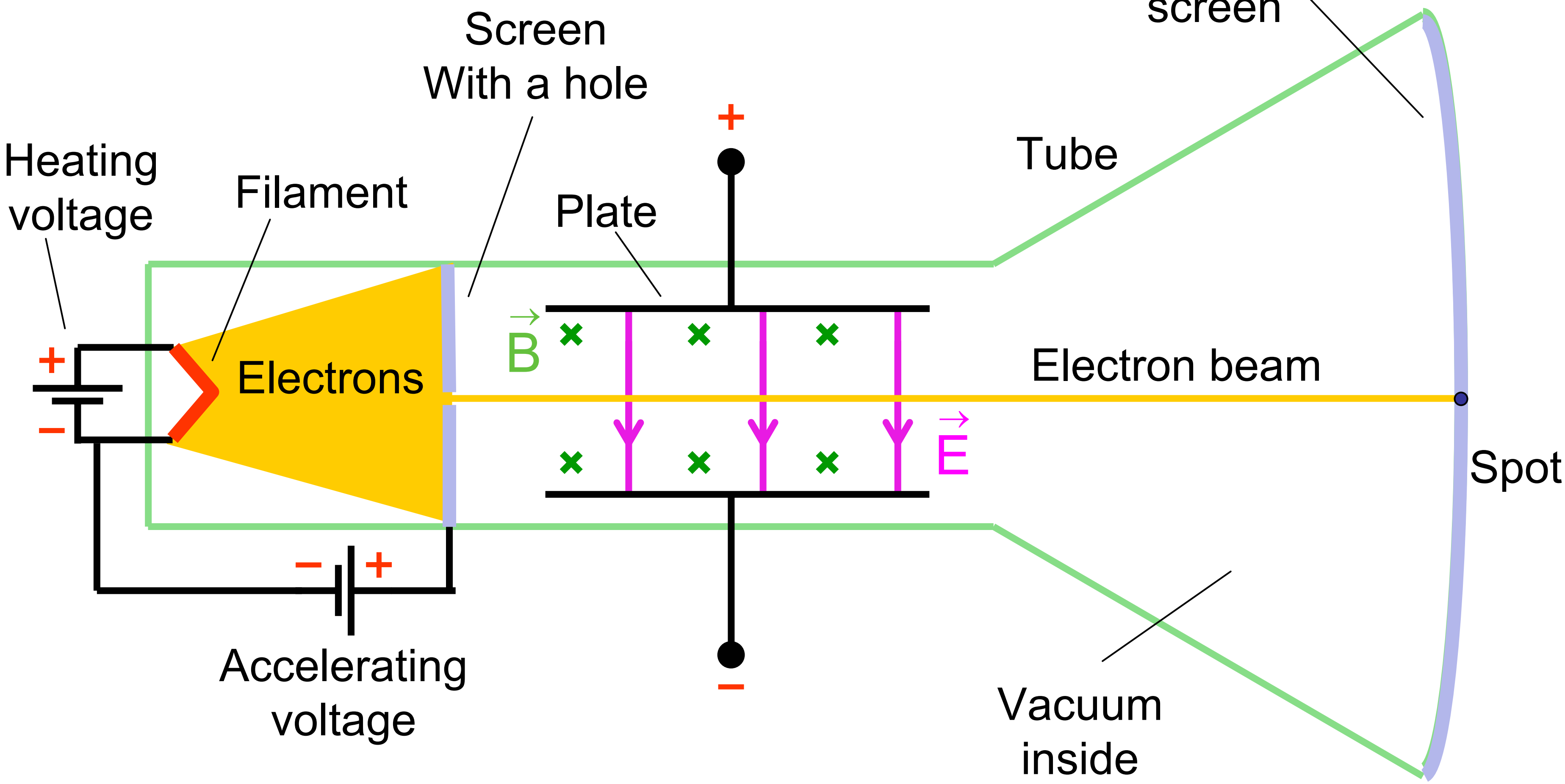
29-3 Crossed Fields: Discovery of the Electron





29-3 Crossed Fields: Discovery of the Electron

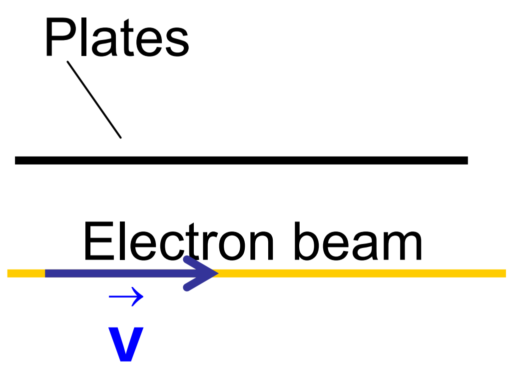
Cathode Ray Tube



29-3 Crossed Fields: Discovery of the Electron

Thomson measured m/q of an electron in 1897

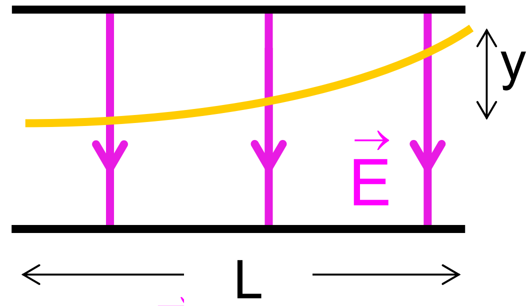
1



$$\vec{E} = 0$$

$$\vec{B} = 0$$

2



$$\vec{E} \neq 0$$

$$\vec{B} = 0$$

$$F_E = q\,E = m\,a$$

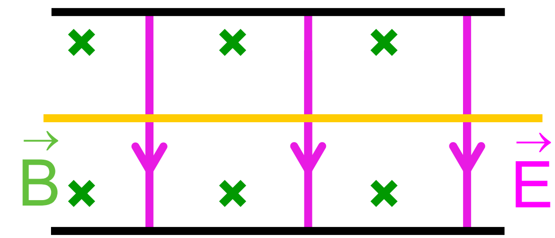
$$a = \frac{q\,E}{m}$$

$$y = \frac{1}{2}\,a\,t^2$$

$$y = \frac{1}{2}\,\frac{q\,E}{m}\left(\frac{L}{v}\right)^2$$

$$v^2 = \frac{1}{2}\,\frac{q\,E}{m}\,\frac{L^2}{y}$$

3



$$\vec{E} \neq 0$$

Increase B till

$$F_B = F_E$$

$$|q|vB = |q|E$$

$$v = \frac{E}{B}$$

$$\left(\frac{E}{B}\right)^2 = \frac{1}{2}\,\frac{q\,E}{m}\,\frac{L^2}{y}$$

$$\boxed{\frac{m}{q} = \frac{B^2L^2}{2Ey}}$$

All quantities on the right can be measured

29-3 Crossed Fields: Discovery of the Electron

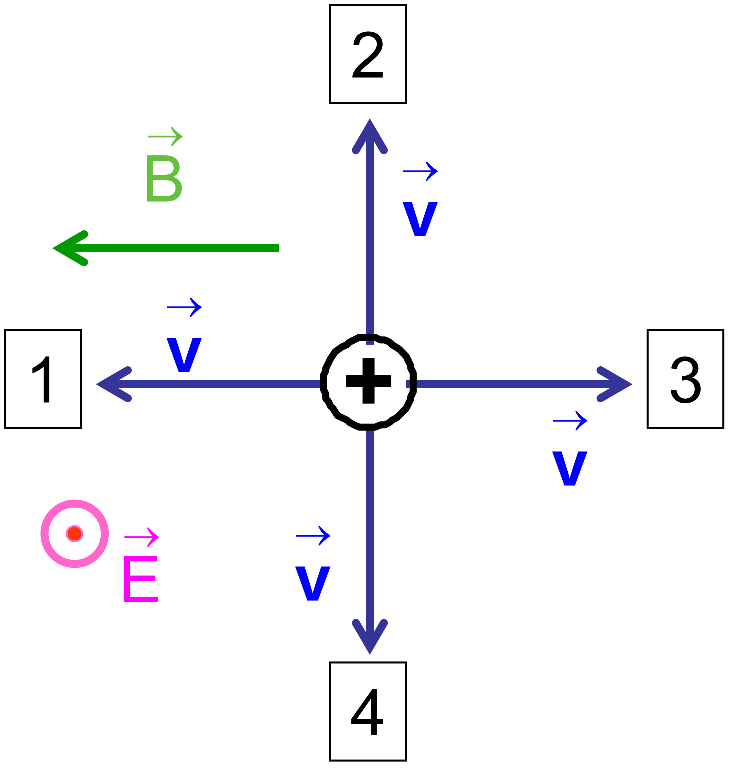
Checkpoint 2

Rank 1, 2, and 3 according to the net force on the particle, greatest first

2
1 and 3 tie

Which direction might result in a net force of zero?

4



29-5 A Circulating Charged Particle

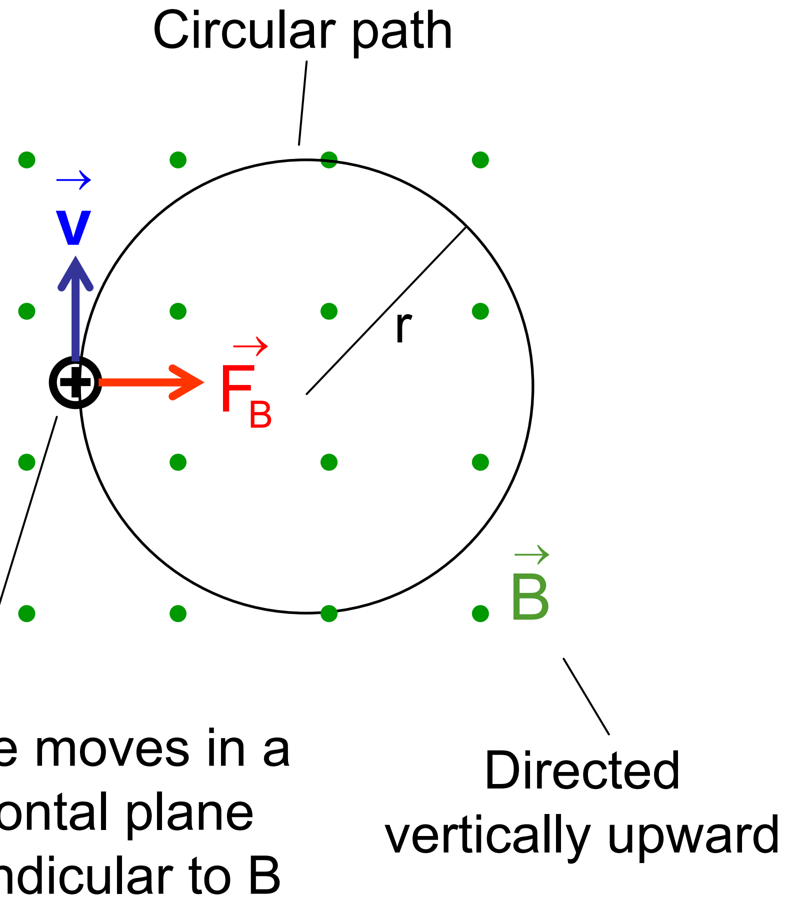
$F_B = q\,v\,B \sin 90^0 = q\,v\,B$

$F_B = m\,a$

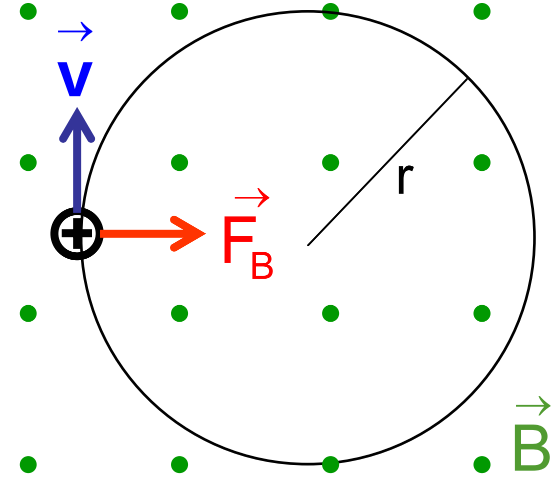
For circular motion $a = \dfrac{v^2}{r}$

$$q\,v\,B = m\,\frac{v^2}{r}$$

Radius $\boxed{r = \dfrac{m\,v}{q\,B}}$



29-5 A Circulating Charged Particle



Radius

$$r = \frac{m\,v}{q\,B}$$

Period T (Time to make one revolution)

$$T = \frac{2\pi\, r}{v}$$

$$T = \frac{2\pi\, m}{q\,B}$$

Frequency (Number of revolutions per unit time)

$$f = \frac{1}{T}$$

$$f = \frac{q\,B}{2\pi\, m}$$

Angular frequency

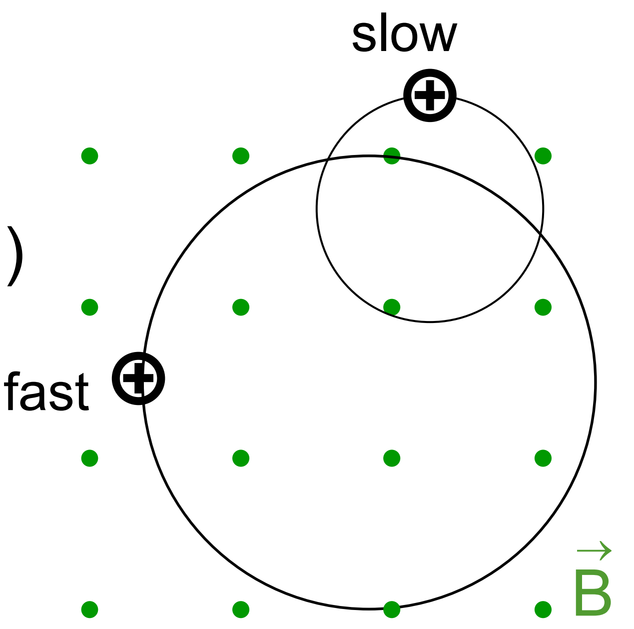
$$\omega = 2\,\pi\, f$$

$$\omega = \frac{q\,B}{m}$$

29-5 A Circulating Charged Particle

Period T (Time to make one revolution)

$$T = \frac{2\pi m}{qB}$$



Periods do not depend on the speed of the particles

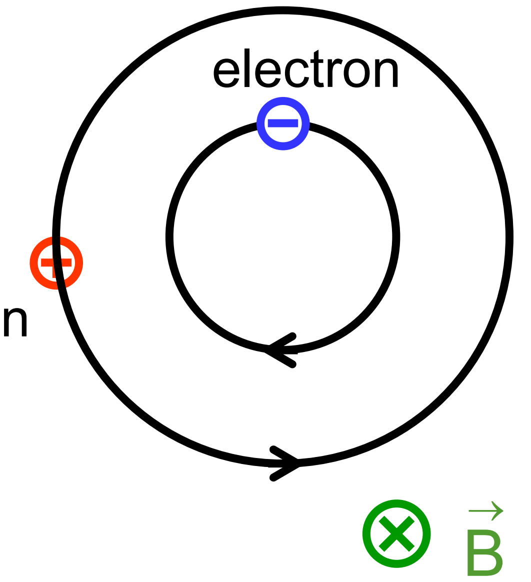
→ All particles with the same charge-to-mass ratio take the same time to make one revolution

→ Fast particles move in big circles and slow particles move in small circles

29-5 A Circulating Charged Particle

Checkpoint 4

An electron and a proton travel at the same speed in a uniform magnetic field



Which particle follows the smaller circle?

$$r = \frac{m\,v}{q\,B}$$

Proton charge = e
Electron charge = - e
Proton mass = 1.67×10^{-27} kg
Electron mass = 9.1×10^{-31} kg

Electron follows the smaller circle

What is the travel direction of the particles ?

Electron follows clockwise direction
Proton follows counterclockwise direction

29-5 A Circulating Charged Particle

Sample problem 29-3

Charge $q = 1.60 \times 10^{-19}$ C
$X = 1.63$ m
$B = 80.0$ mT
$V = 1.00$ kV

What is the mass of an ion?

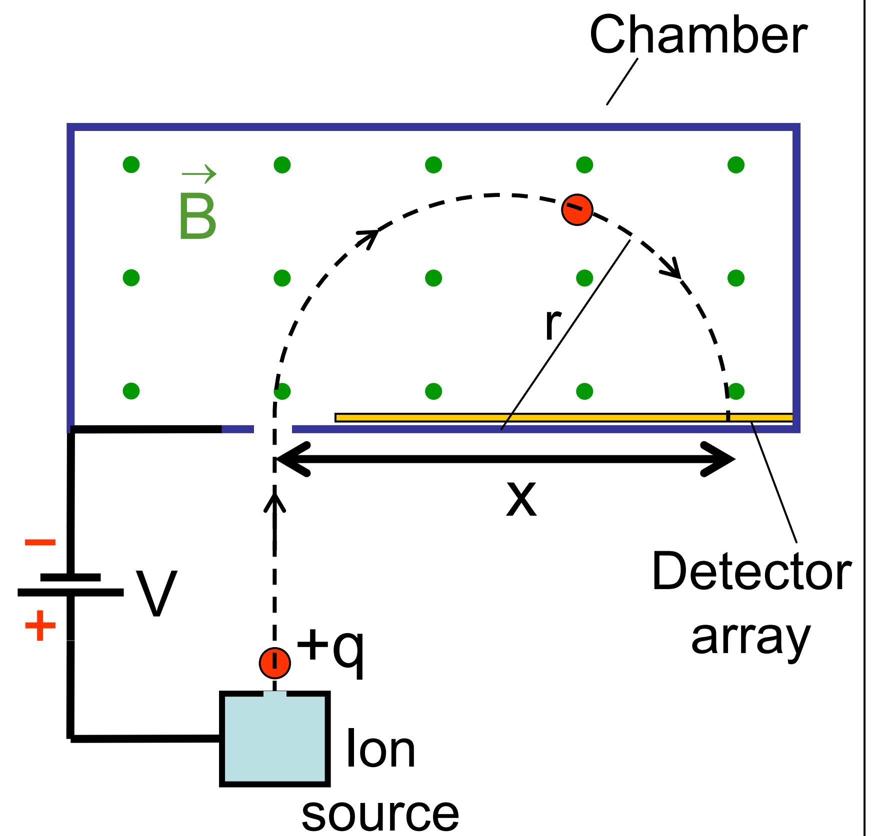
$$r = \frac{m\,v}{q\,B} = \frac{X}{2}$$

$$\Delta K = -\Delta U$$

$$\frac{1}{2} m v^2 - 0 = -(q(-V))$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$r = \frac{m}{q\,B}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$



Mass spectrometer
Device to measure mass of ions

29-5 A Circulating Charged Particle

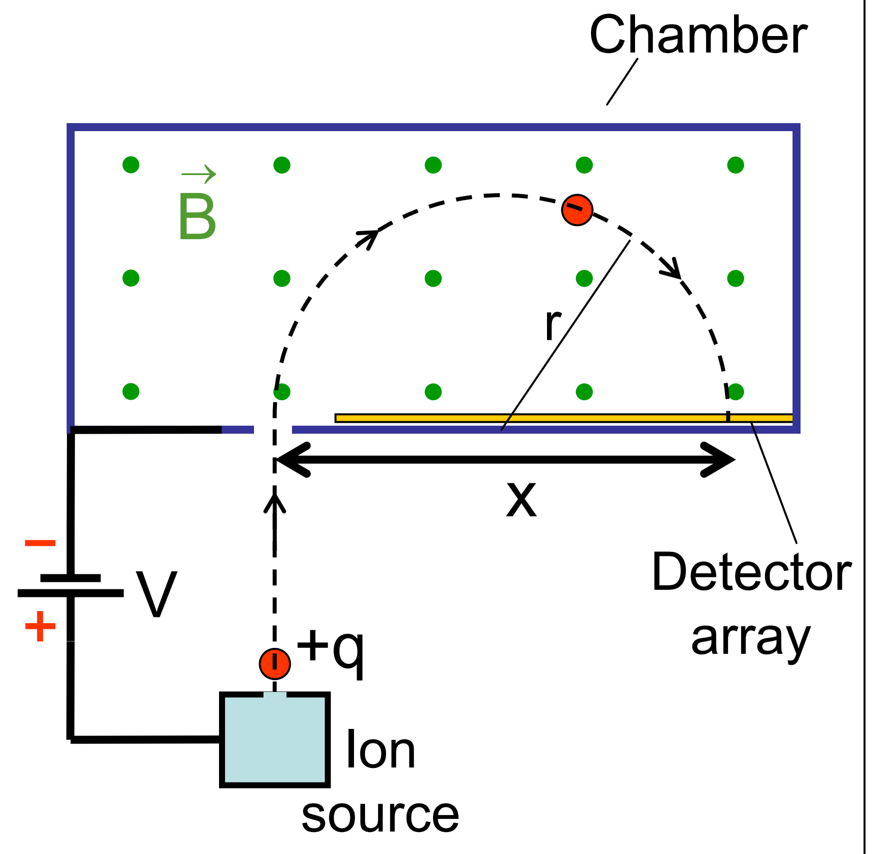
Sample problem 29-3

$$\frac{X}{2} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

$$m = \frac{B^2 q X^2}{8V}$$

$$= \frac{(80\times10^{-3})^2(1.6\times10^{-19})(1.63)^2}{8(10^3)}$$

$$= 3.39\times10^{-27}\ \text{kg}$$



Mass spectrometer

Device to measure mass of ions

What is the mass in atomic mass unit u ($1u = 1.67\times10^{-27}$ kg?

$$m = 3.39\times10^{-27}\ \text{kg}\ \frac{1\ \text{u}}{1.67\times10^{-27}\text{kg}} = 204\ \text{u}$$

29-5 A Circulating Charged Particle

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

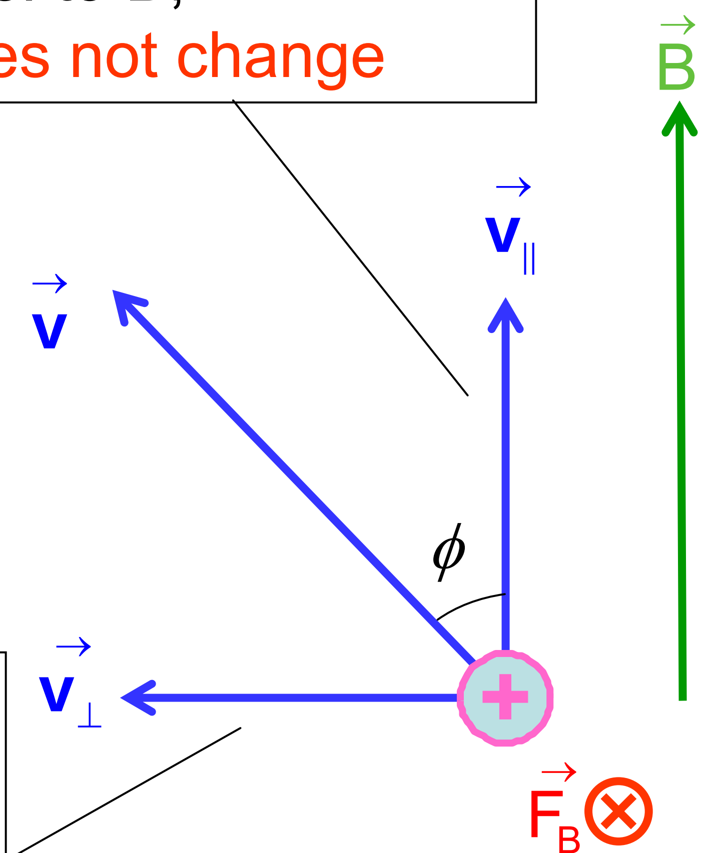
$$F_B = q\,v\,B\,\sin\phi$$

$$F_B = q\,v\,\sin\phi\,B$$

$$F_B = q\,v_\perp B$$

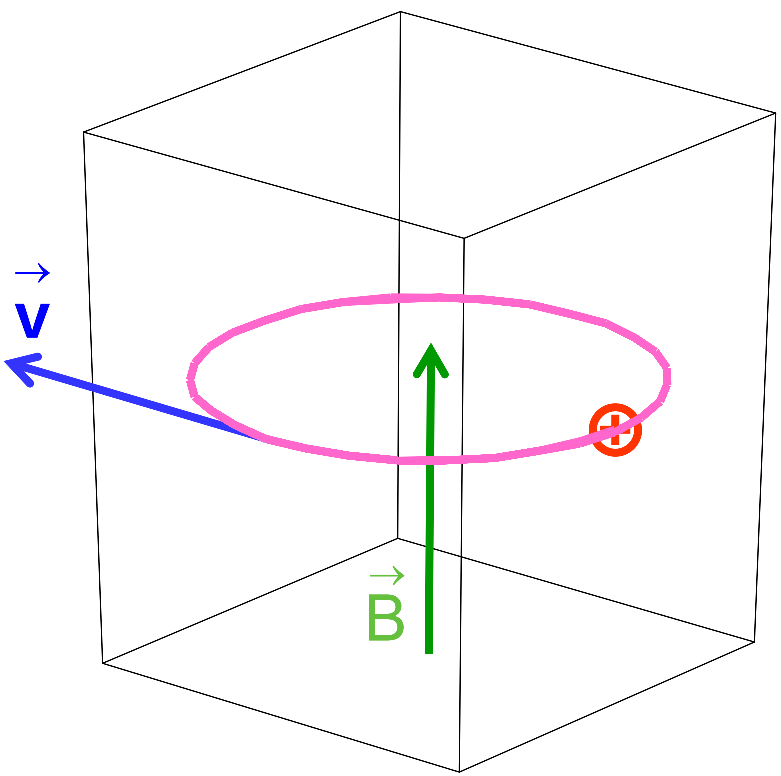
All the force on the particle is produced because of $v_\perp$.

F_B has no component parallel to B,
$V_{||}$ does not change



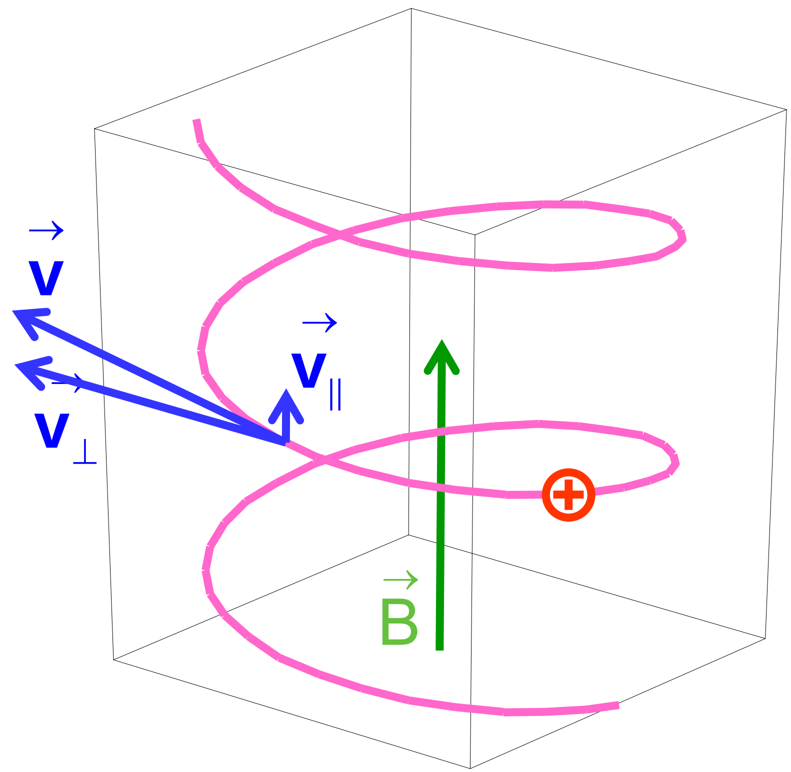
F_B is always perpendicular to $v_\perp$. F_B changes the direction of $v_\perp$ but not its magnitude

29-5 A Circulating Charged Particle



v is perpendicular to B

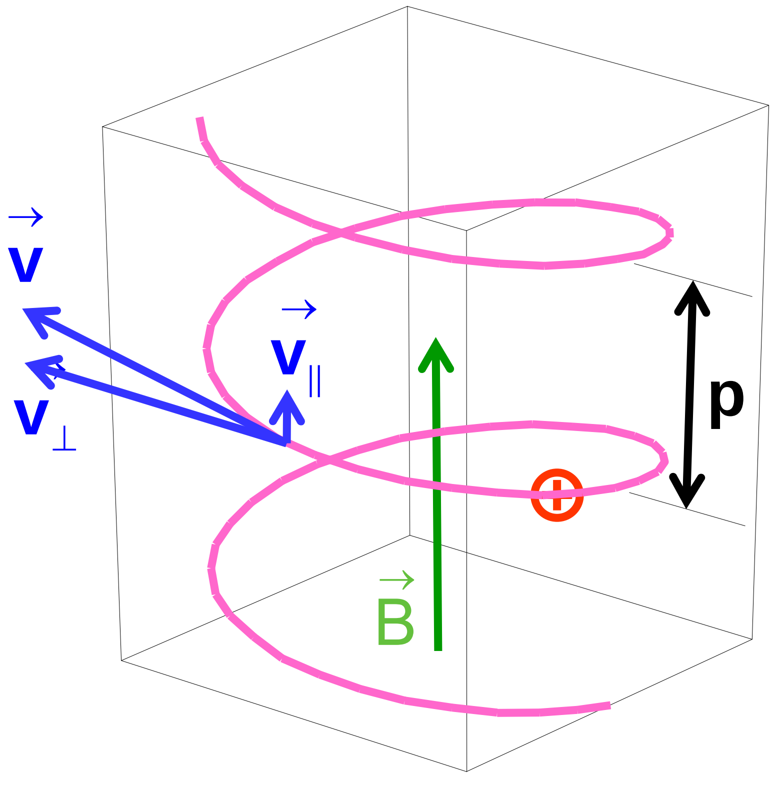
Circular path



v has a component along B

Helical path

29-5 A Circulating Charged Particle



v has a component along B

Pitch p of the helix

$$p = v_{\parallel}\, T$$

Period T
(Time to make one revolution)

$$p = v_{\parallel}\, \frac{2\pi\, m}{q\, B}$$

29-5 A Circulating Charged Particle

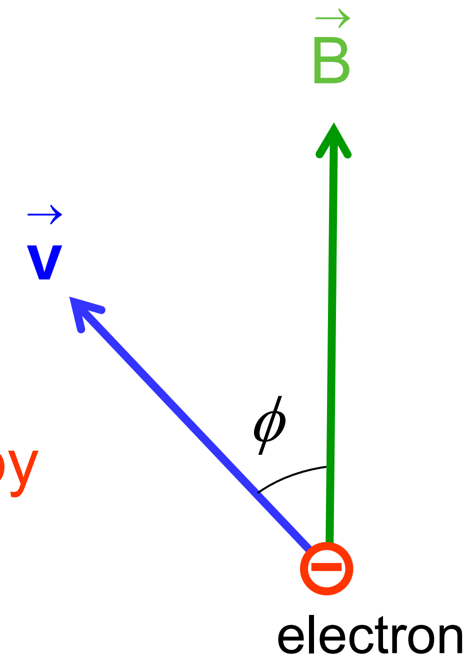
Sample problem 29-4

Kinetic energy = 22.5 eV

B = 45.5 mT

ϕ =65.5^0

1 eV =1.6x10^{-19} J



What is the pitch of the helical path taken by the electron?

$$p = v_{||}\,\frac{2\pi\ m}{q\ B}$$

$$p = v\cos\phi\,\frac{2\pi\ m}{q\ B}$$

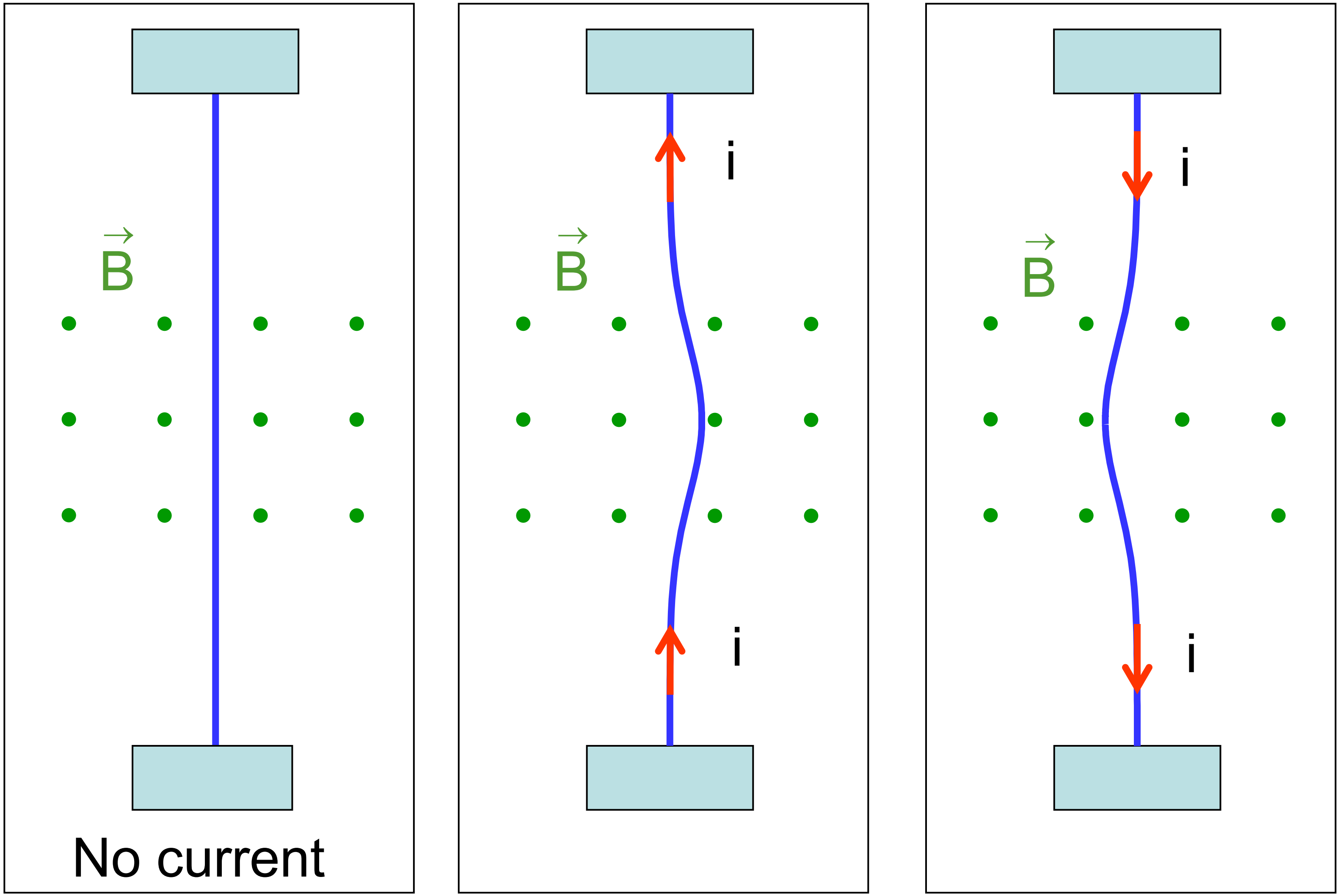
$$K = \frac{1}{2}mv^2$$

$$p = \sqrt{\frac{2K}{m}}\cos\phi\,\frac{2\pi\ m}{q\ B} = \sqrt{2K\,m}\,\cos\phi\,\frac{2\pi}{q\ B}$$

$$= \sqrt{2(22.5x1.6x10^{-19})(9.11x10^{-31})}\ \cos(66.2^0)\ \frac{2\pi}{(1.6x10^{-19})(45.5x10^{-3})}$$

$$= 9.16\ \text{cm}$$

29-7 Magnetic Force on a Current-Carrying Wire

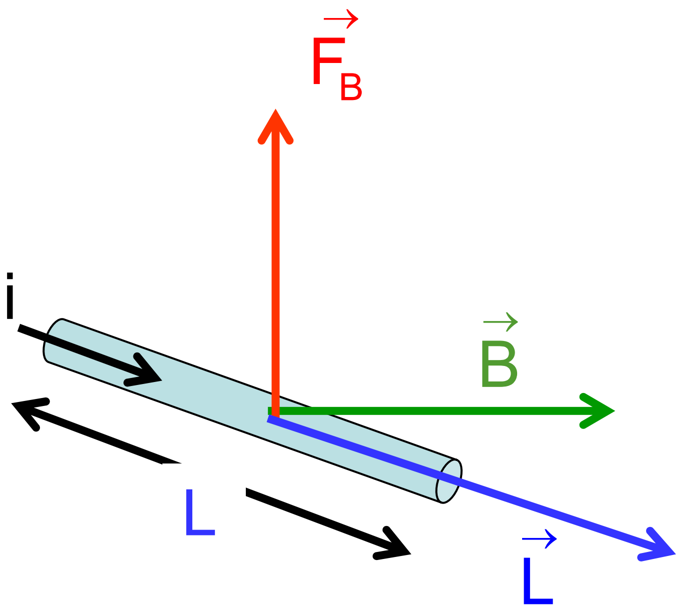


Flexible wire fixed at both ends

29-7 Magnetic Force on a Current-Carrying Wire

Magnetic force of a wire of length L

Current though the wire



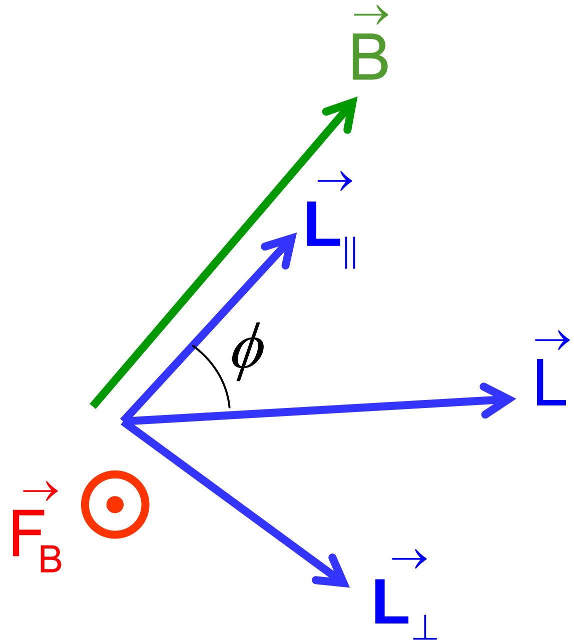
$$\vec{F}_B = i\,\vec{L} \times \vec{B}$$

Length vector

Magnitude: length of the wire
Direction : along the wire segment in the direction of conventional current

F_B is perpendicular to L and B vectors

29-7 Magnetic Force on a Current-Carrying Wire



$$\vec{F}_B = i\,\vec{L} \times \vec{B}$$

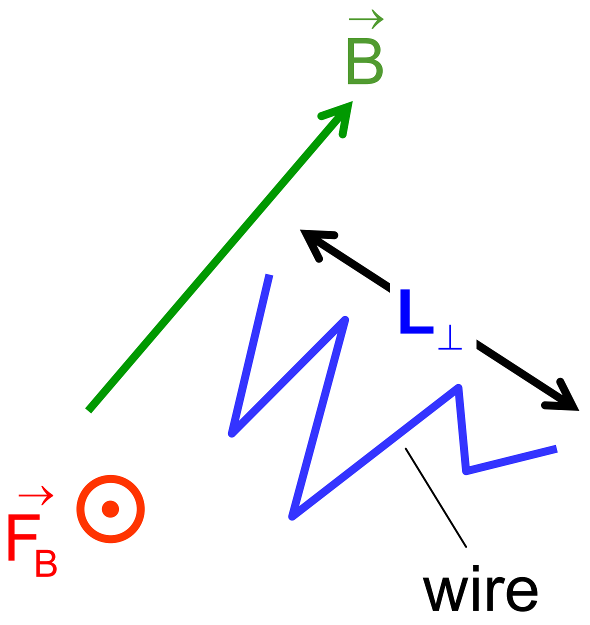
$$F_B = i\,L\,B\,\sin\phi$$

$$F_B = i\,L\sin\phi\,B$$

$$F_B = i\,L_{\perp}B$$

Only the length perpendicular to the magnetic field $L_{\perp}$ contributes to the force.

29-7 Magnetic Force on a Current-Carrying Wire

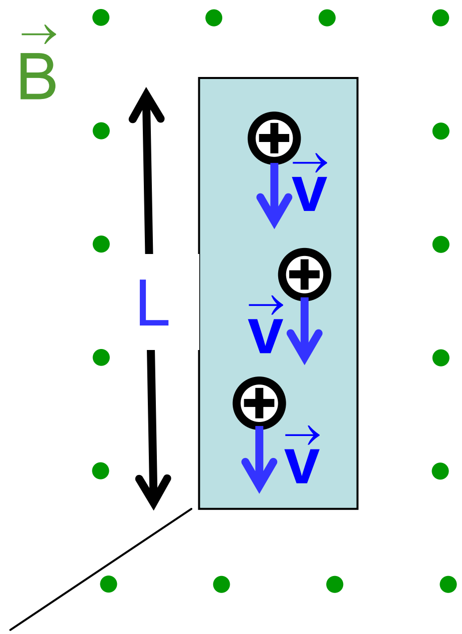


$F_B = i\,L_\perp B$

Only the length perpendicular to the magnetic field $L_\perp$ contributes to the force.

29-7 Magnetic Force on a Current-Carrying Wire

Derivation of $\vec{F}_B = i\,\vec{L} \times \vec{B}$



Section of a wire

All the “moving” ions in a section of length L enter the section during time

$$t = \frac{L}{v}$$

The charge in length L is

$$q = i\,dt = i\,\frac{L}{v}$$

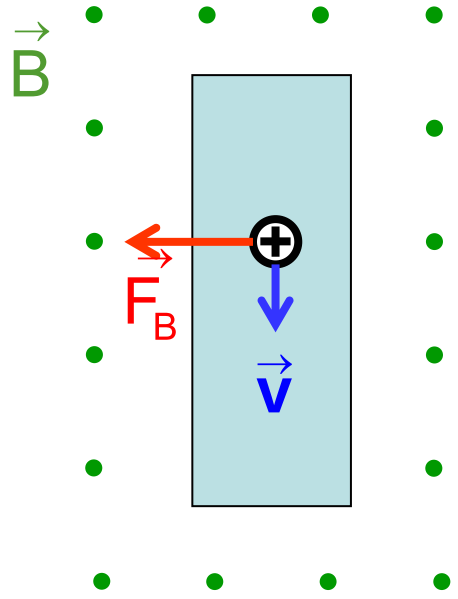
Magnetic force on the section

$$F_B = q\,v\,B\sin\phi = i\,\frac{L}{v}\,v\,B\sin\phi$$

$$= i\,L\,B\sin\phi = i\,\vec{L} \times \vec{B}$$

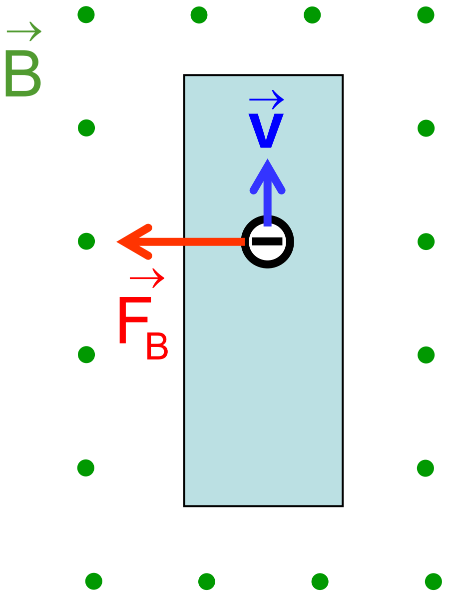
29-7 Magnetic Force on a Current-Carrying Wire

The direction of the magnetic force is the same whether we consider moving ions (convention) or moving electrons (real)



Conventional picture

Moving ions



real situation

Moving electrons

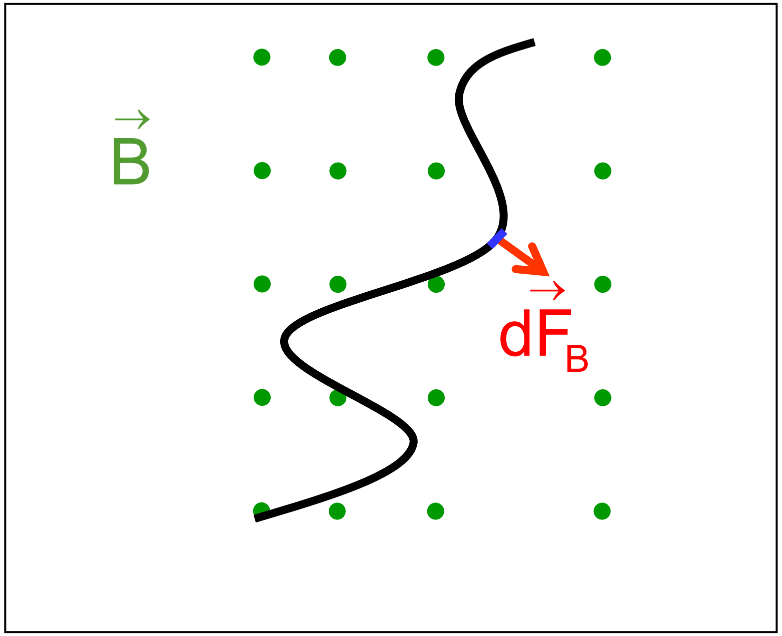
29-7 Magnetic Force on a Current-Carrying Wire

If the wire is not straight or the magnetic field is not uniform,
We can imagine breaking the wire into small segments and apply

$$d\vec{F}_B = i\, d\vec{L} \times \vec{B}$$

The total force on the wire is

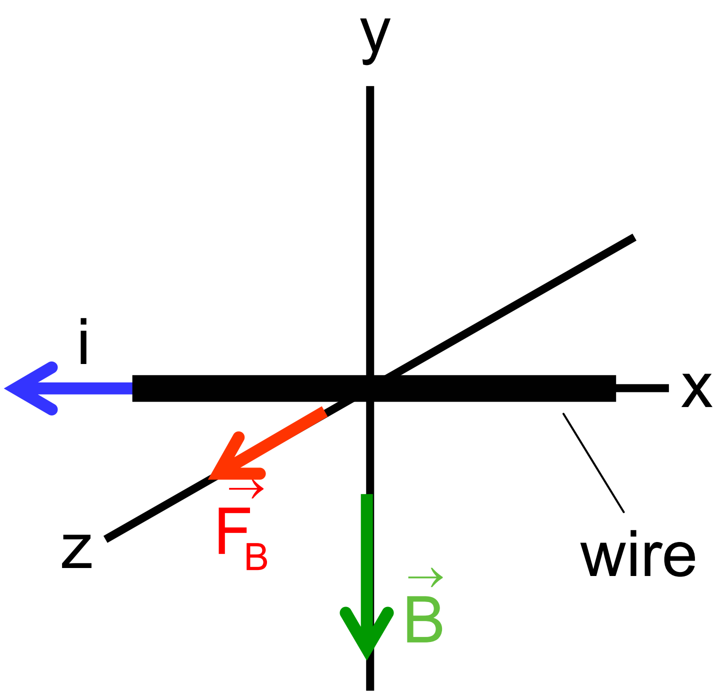
$$\vec{F}_B = \int d\vec{F}_B$$



29-7 Magnetic Force on a Current-Carrying Wire

Checkpoint 5

What is the direction of the field?



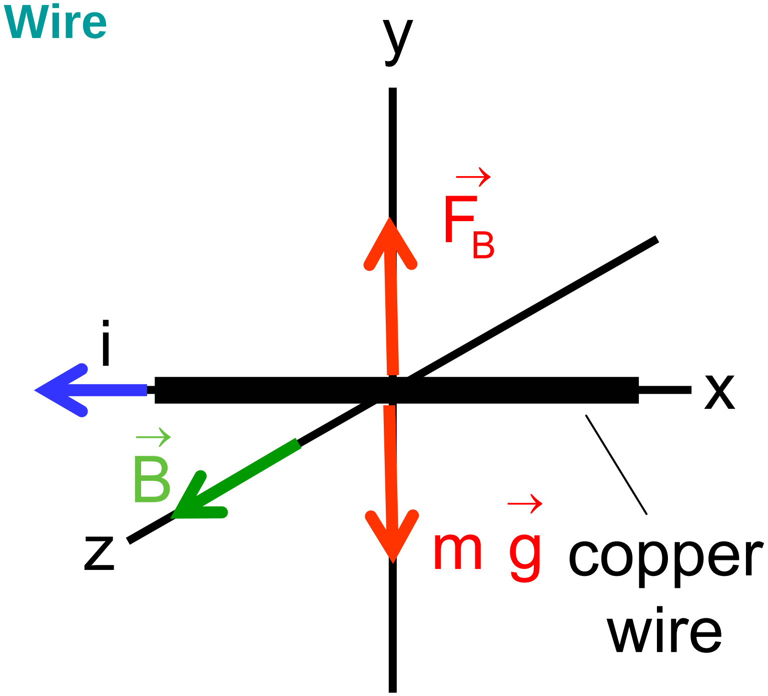
29-7 Magnetic Force on a Current-Carrying Wire

Sample Problem 29-6

$i = 28$ A

Wire linear density = λ = 46.6 g/m

What is the magnitude and direction of the minimum magnetic field to balance the gravitational force on the wire?



$$\vec{F}_B = i\,\vec{L} \times \vec{B}$$

$$F_B = i\,L\,B\,\sin\phi = m\,g$$

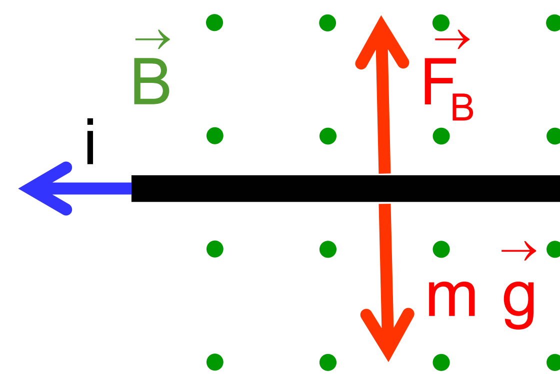
Minimum B

→ Maximum $\sin\phi = 1$

$$i\,L\,B = m\,g = \lambda\,L\,g$$

$$B = \frac{\lambda g}{i} = \frac{(46.6 \times 10^{-3})(9.8)}{28}$$

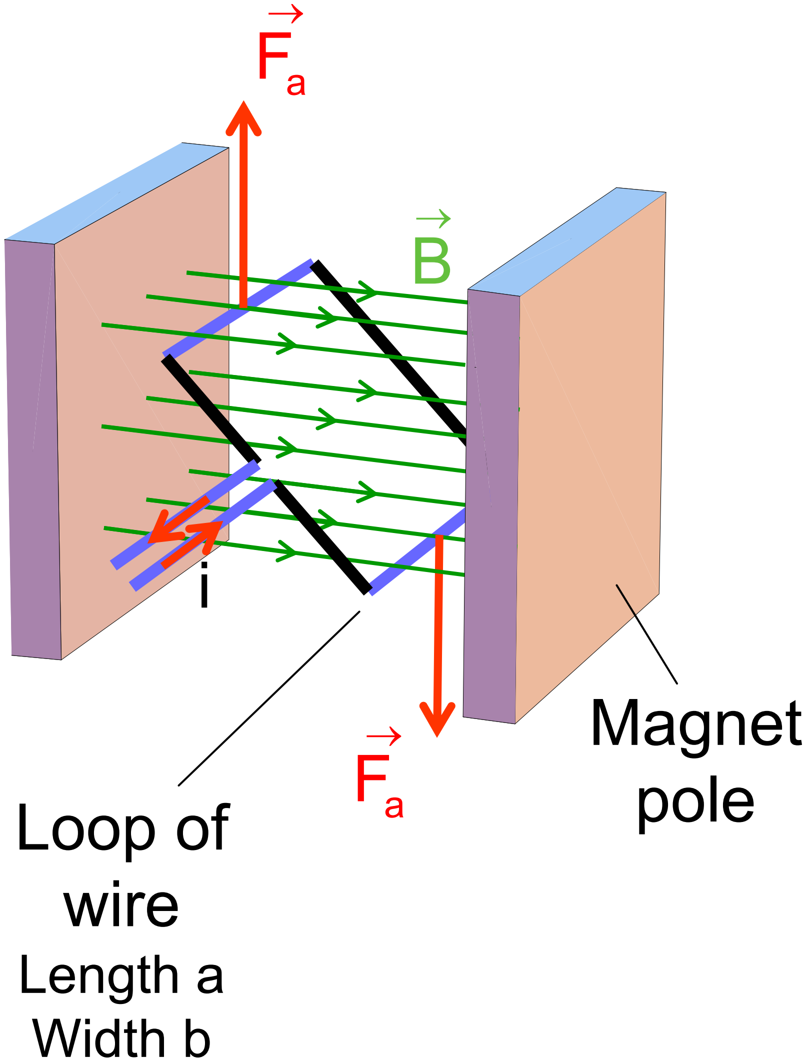
$$= 1.6 \times 10^{-2}\ \text{T}$$



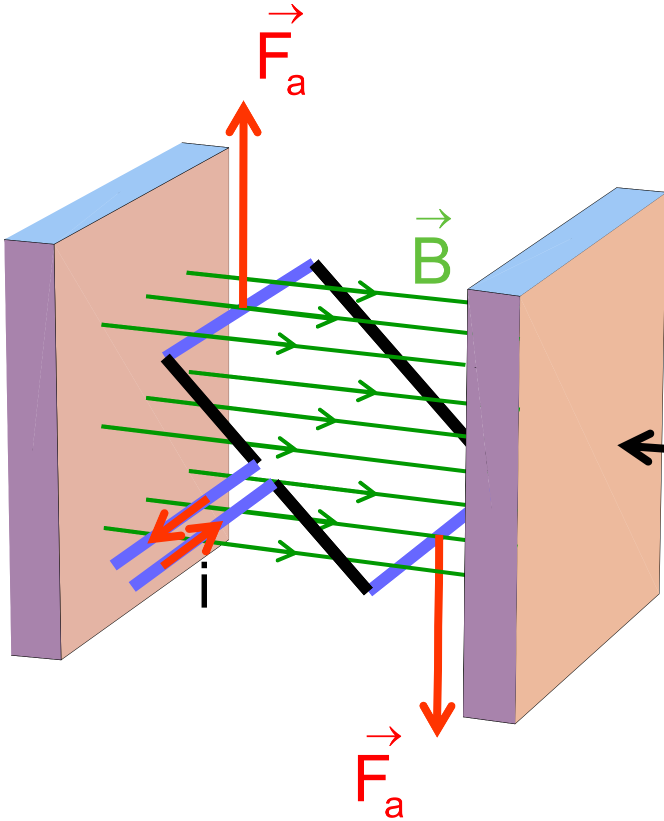
Two-dimensional view

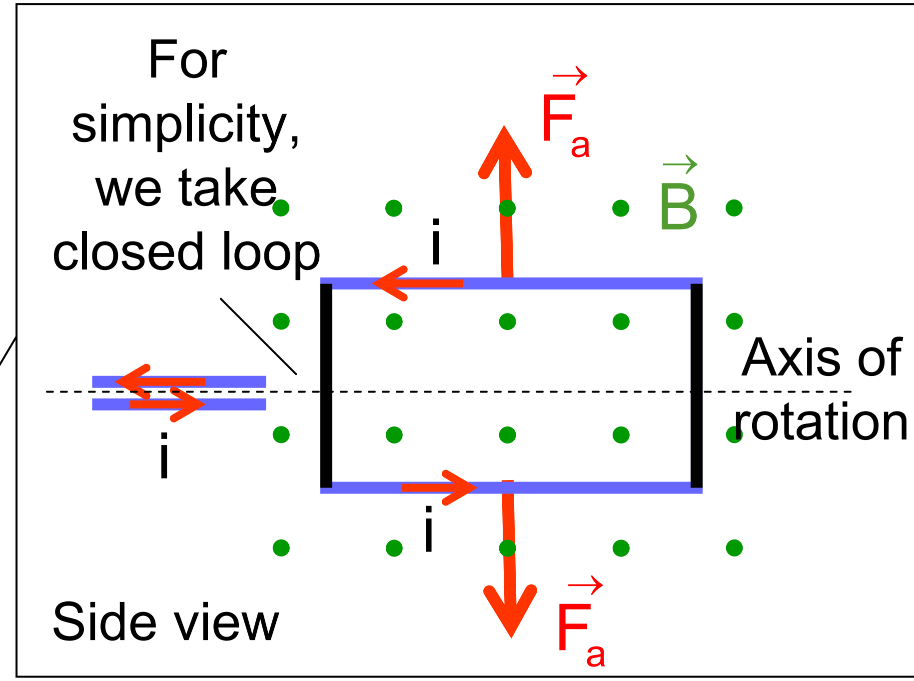
29-8 Torque on a Current Loop

Electric motor

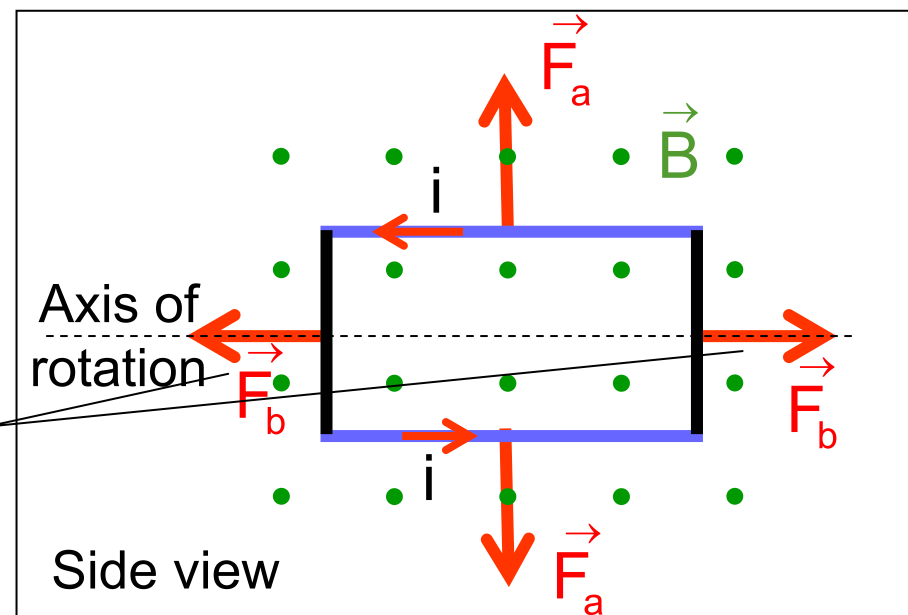


29-8 Torque on a Current Loop



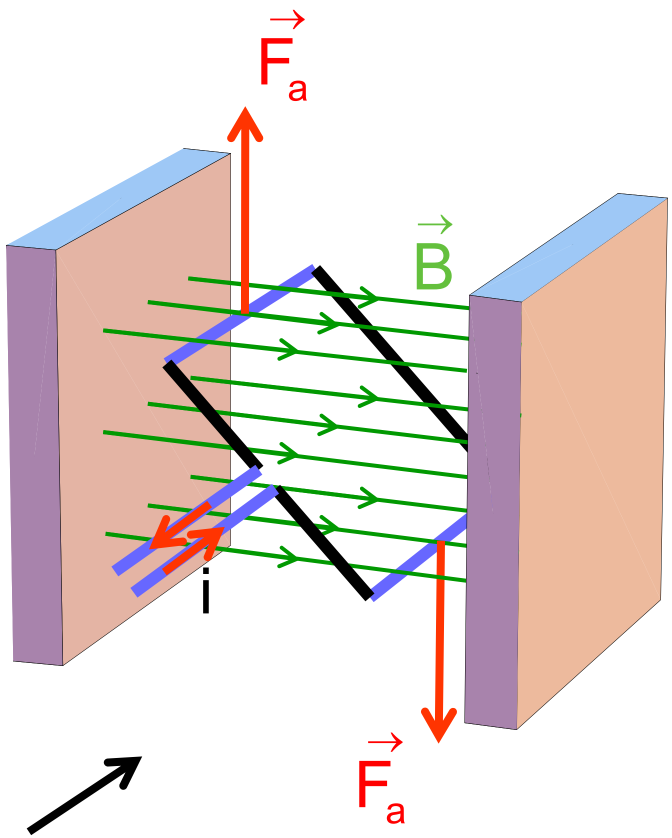


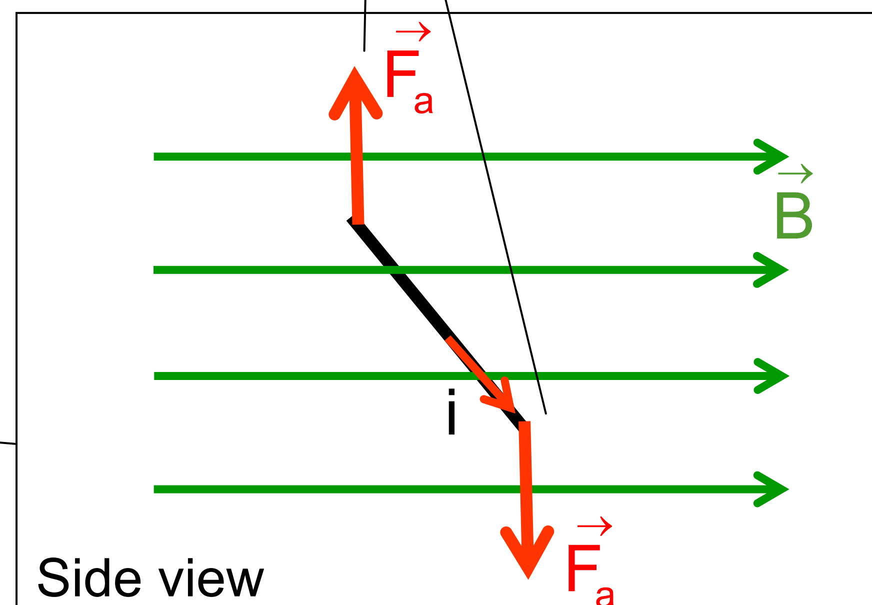
The sum of forces on side b is zero.
They produce no torque because the share the same line of action



29-8 Torque on a Current Loop

The sum of forces on side a is also zero.
But These two forces produce a torque because they do not share the same line of action





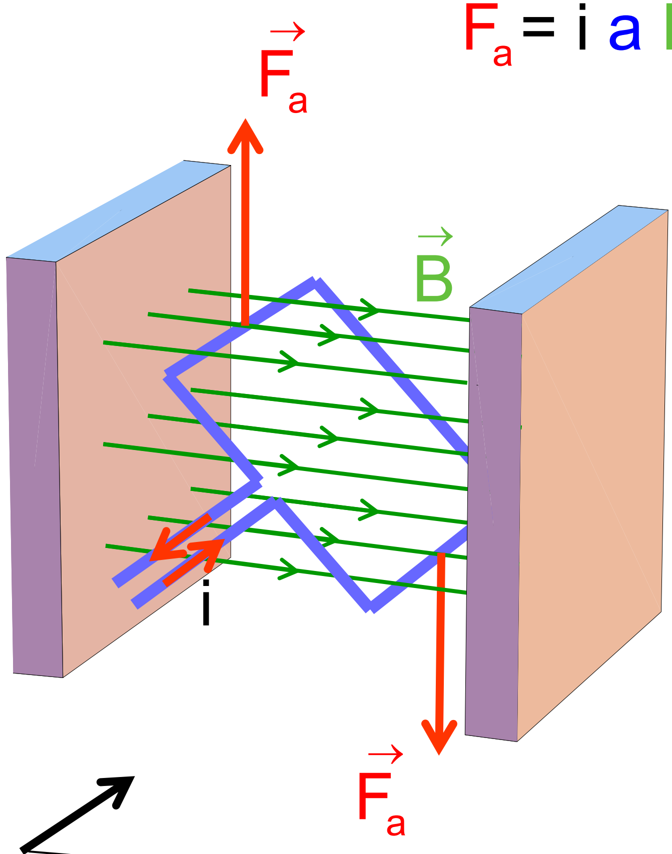
29-8 Torque on a Current Loop

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{F}_a = i\,\vec{L} \times \vec{B}$$

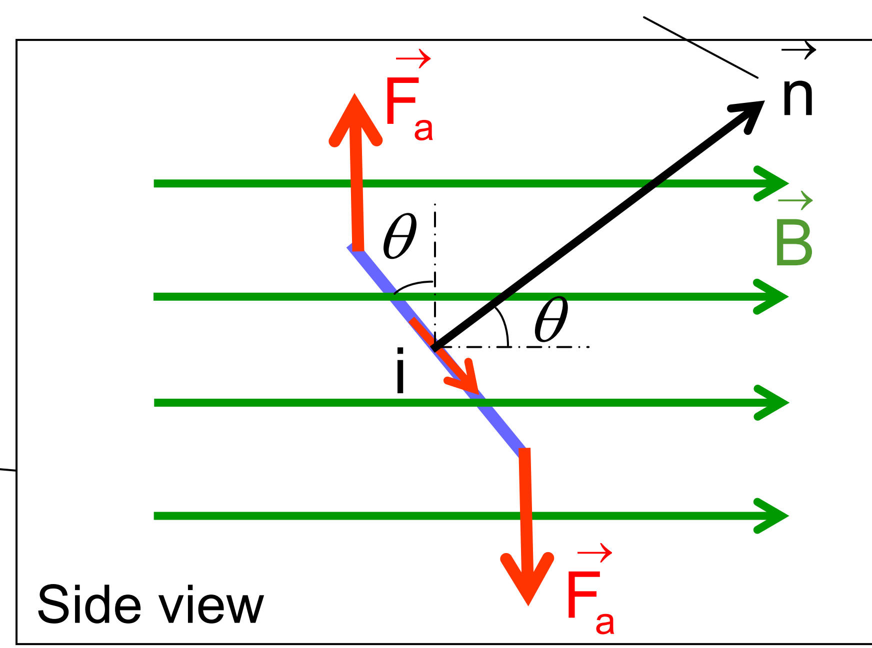
$$F_a = i\,a\,B \sin 90^0$$

$$F_a = i\,a\,B$$



$$\tau = \frac{b}{2}\, i\,a\,B \sin\theta + \frac{b}{2}\, i\,a\,B \sin\theta$$

$$\tau = i\,a\,b\,B \sin\theta$$

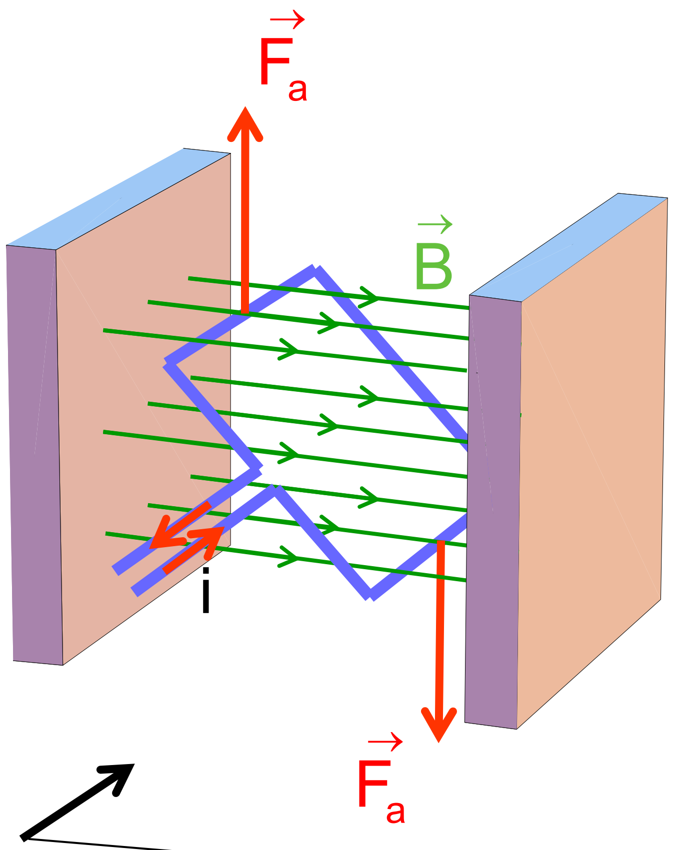


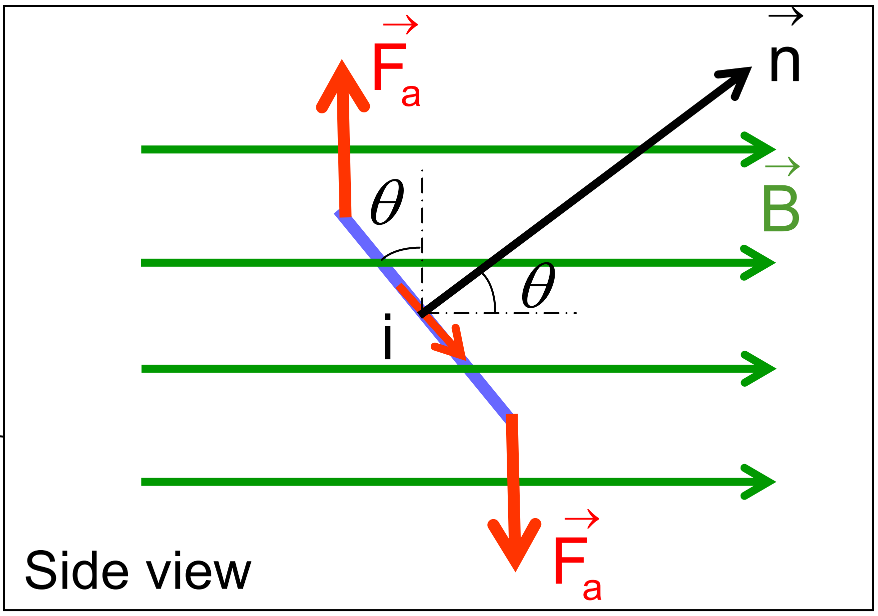
29-8 Torque on a Current Loop

$$\vec{\tau} = \vec{r} \times \vec{F}$$

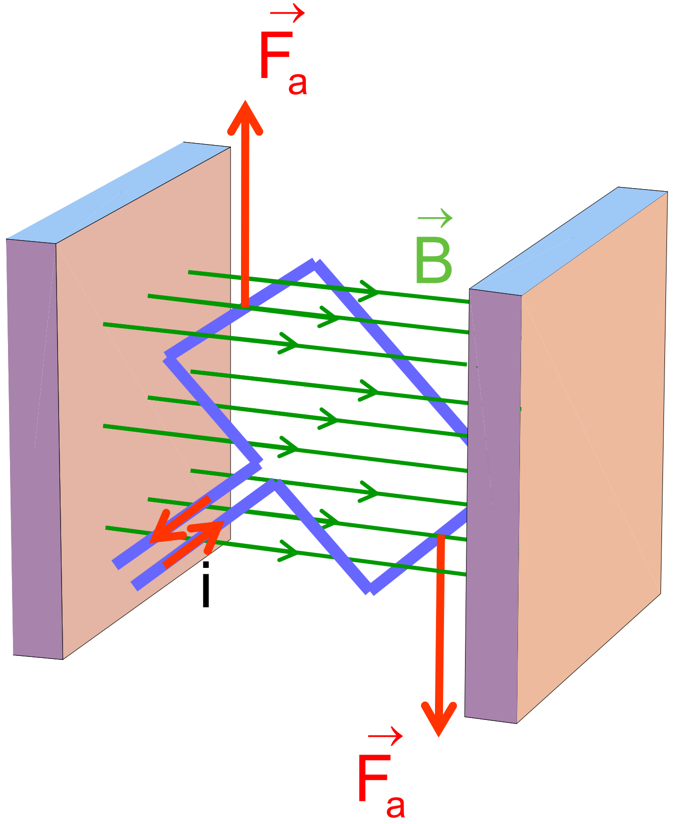
$$\tau = i\,a\,b\,B\sin\theta$$

Torque tries to align n vector with the direction of the magnetic field





29-8 Torque on a Current Loop



τ = i a b B sinθ

τ = i Area B sinθ

True for any loop shape

If you have N turns,

τ = i N Area B sinθ