

Chapter 28

Circuits

28-1 Pumping Charges

Battery
Electric generator
Solar cell

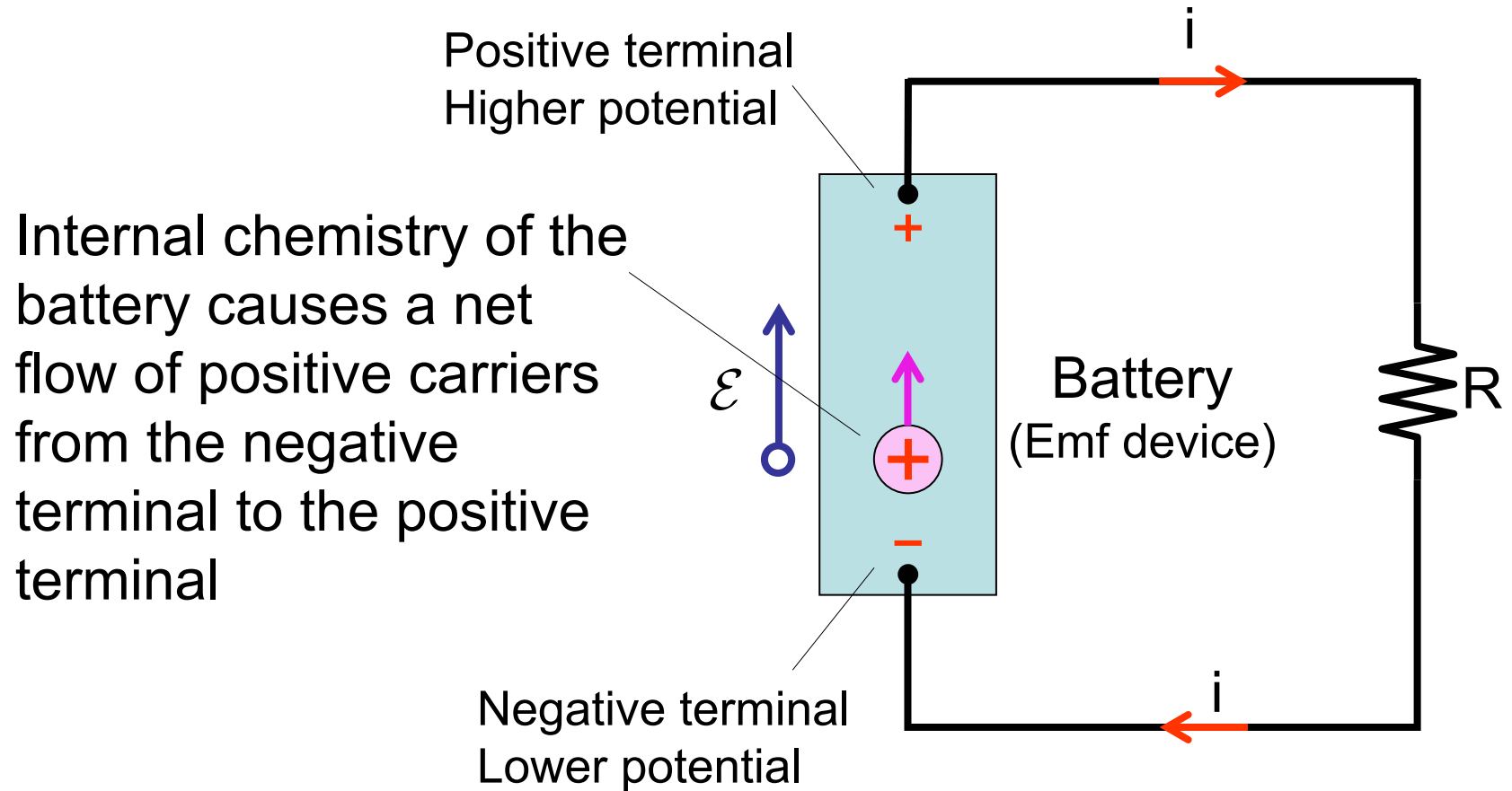
devices that maintain potential difference between their terminals by doing work on charge carriers

These devices are called **emf devices**

Emf is an outdated phrase which stands for electromotive force

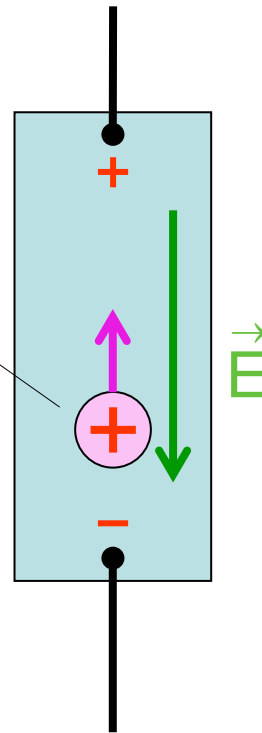
An emf device provides an emf \mathcal{E}

28-2 Work, Energy, and EMF



28-2 Work, Energy, and EMF

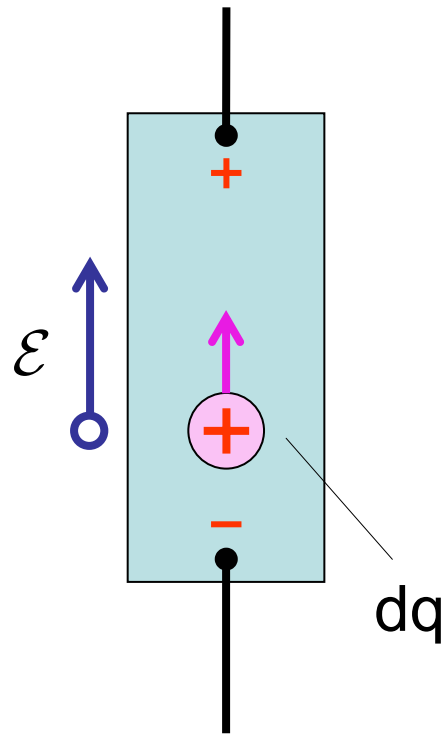
Motion is
opposite to the
electric field
direction



There must be some
source of energy within
the source that enables
it to do work on the
charge carriers

Battery	→ chemical energy
Electric generator	→ mechanical energy
Solar cell	→ light energy

28-2 Work, Energy, and EMF



$$\mathcal{E} = \frac{dW}{dq}$$

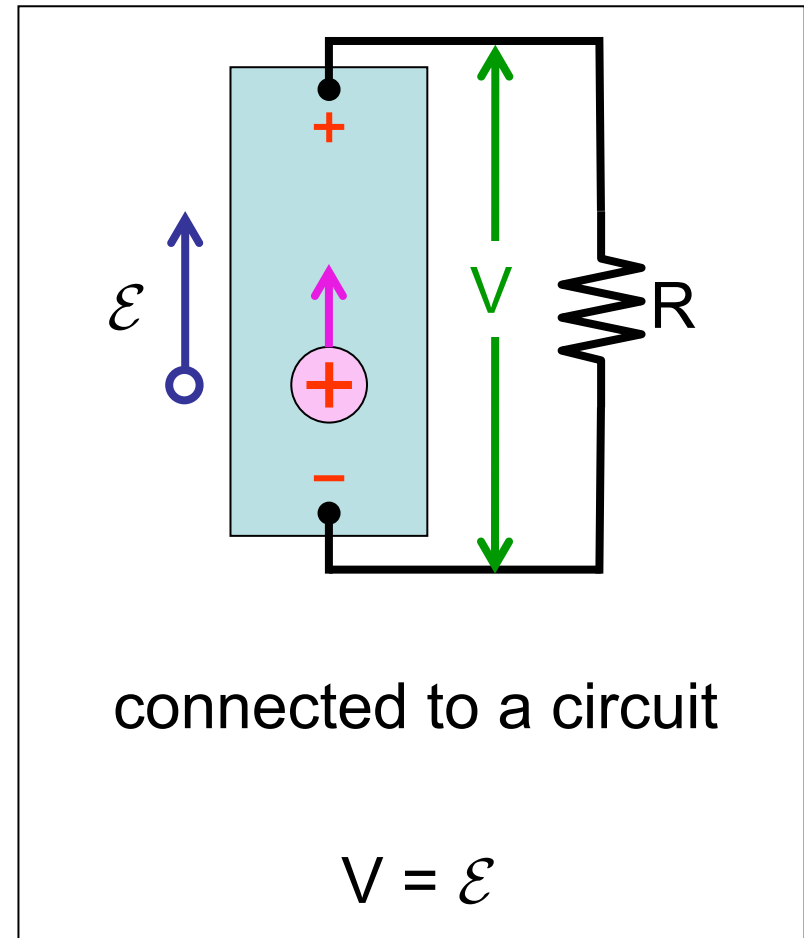
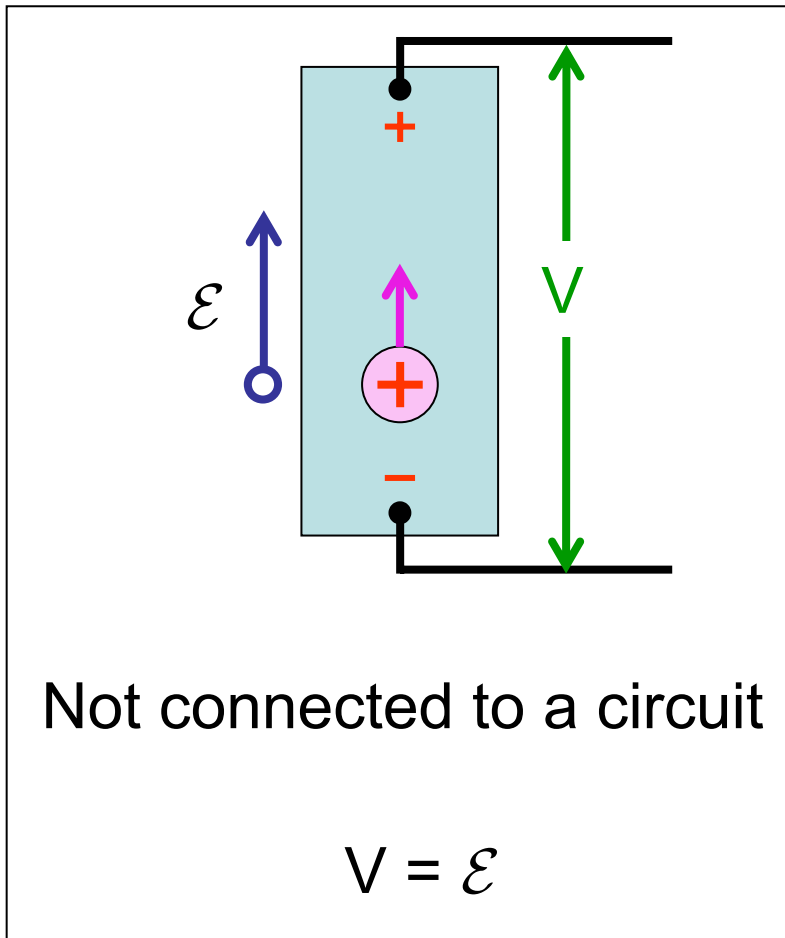
The emf \mathcal{E} of an emf device is the work per unit charge that an emf device does in moving charge from its lower-potential terminal to its higher-potential terminal

\mathcal{E} is measured in $\frac{\text{Joule}}{\text{Coulomb}} = \text{Volt}$

28-2 Work, Energy, and EMF

Ideal emf devices

have **no** internal resistance to the movement of charge

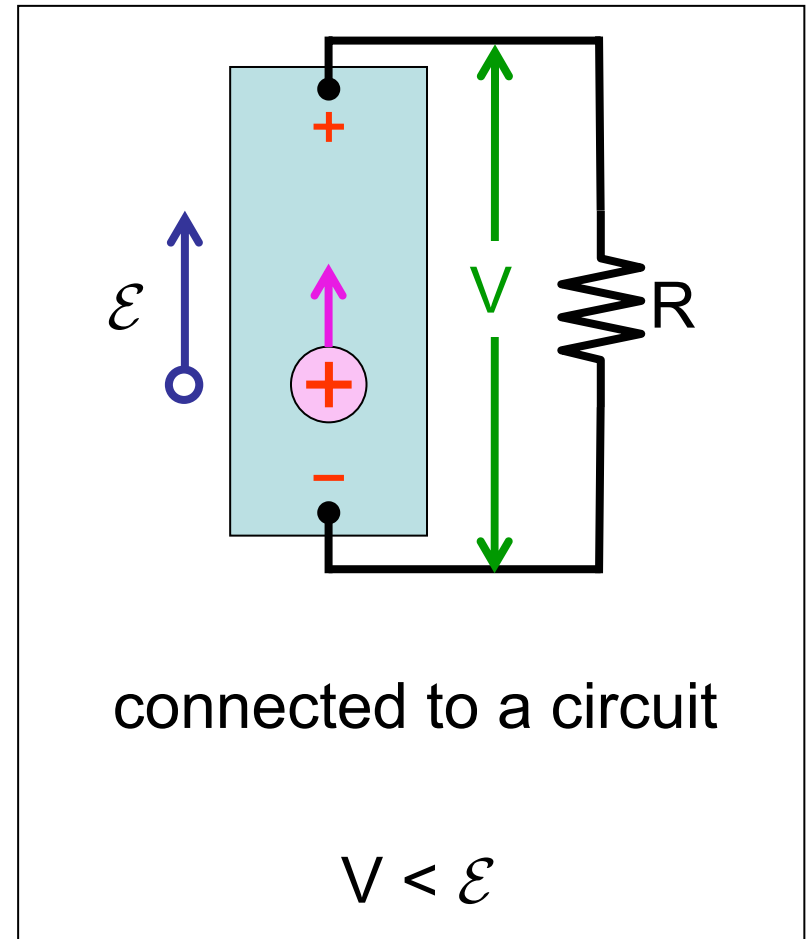
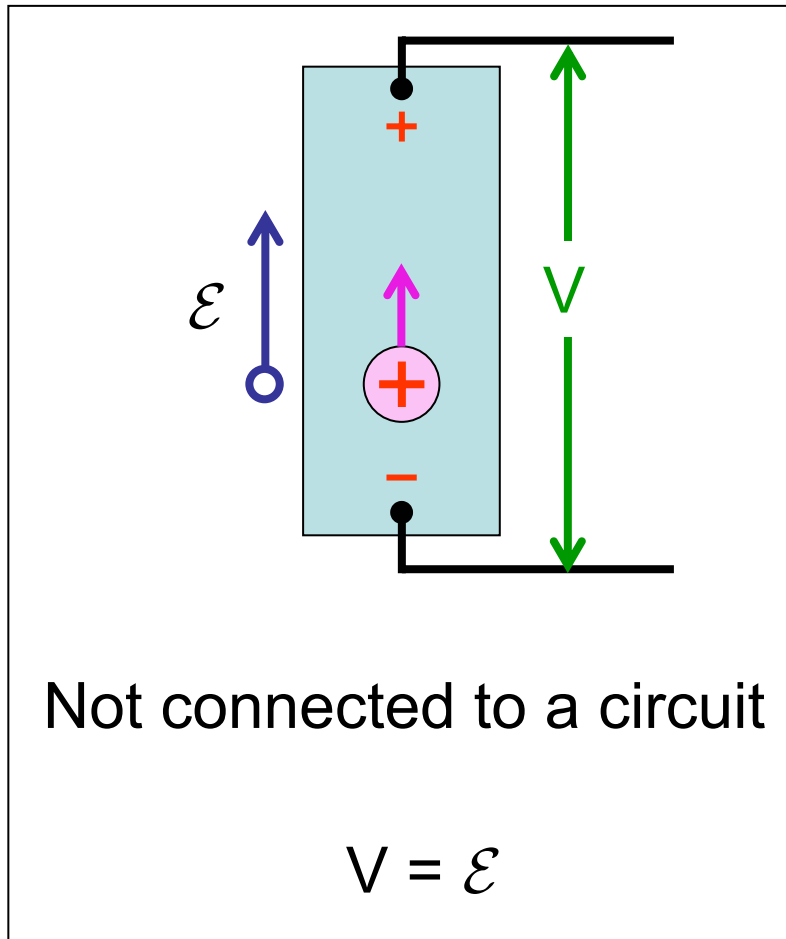


Potential difference between the terminals = the emf of the device

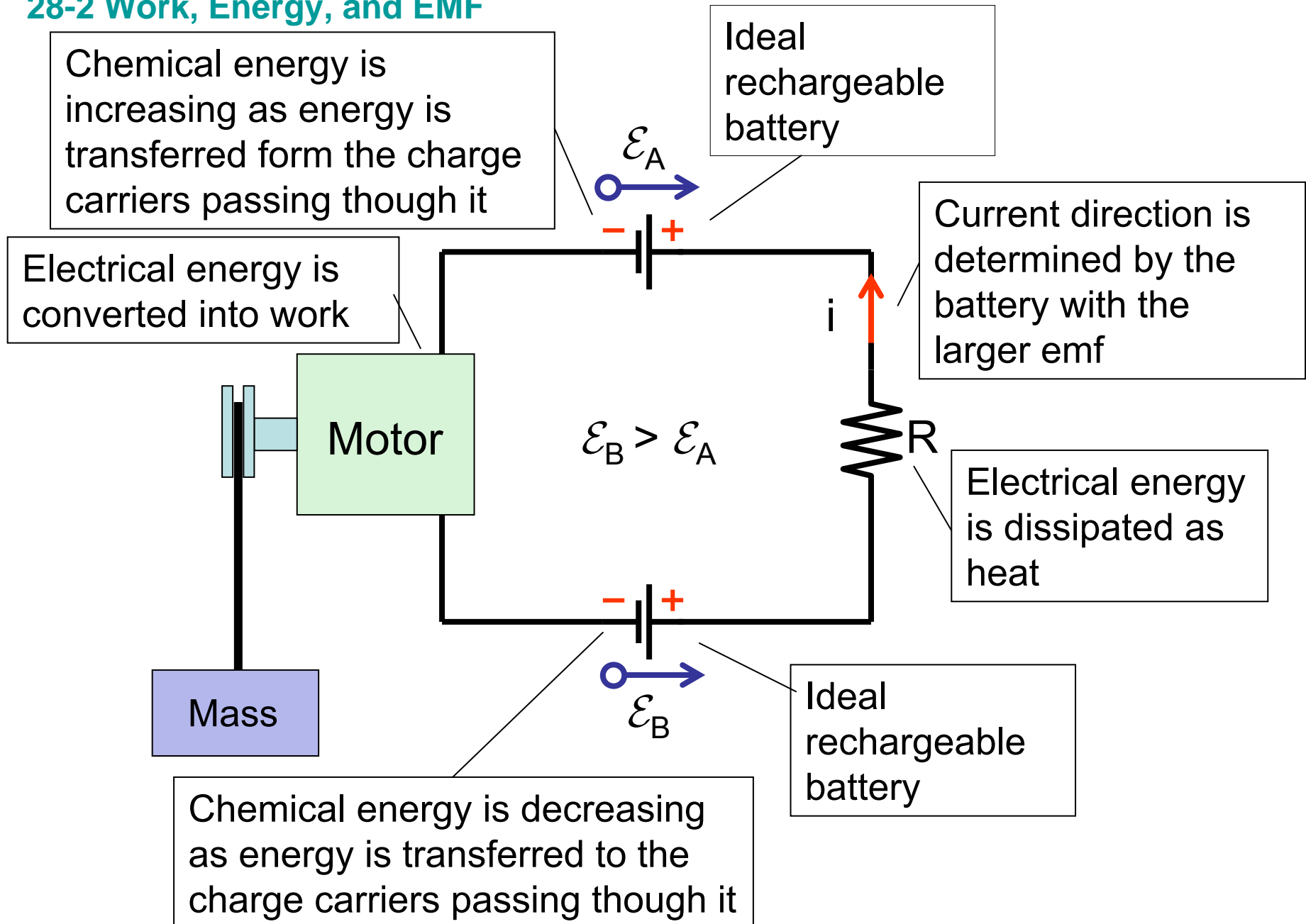
28-2 Work, Energy, and EMF

Real emf devices

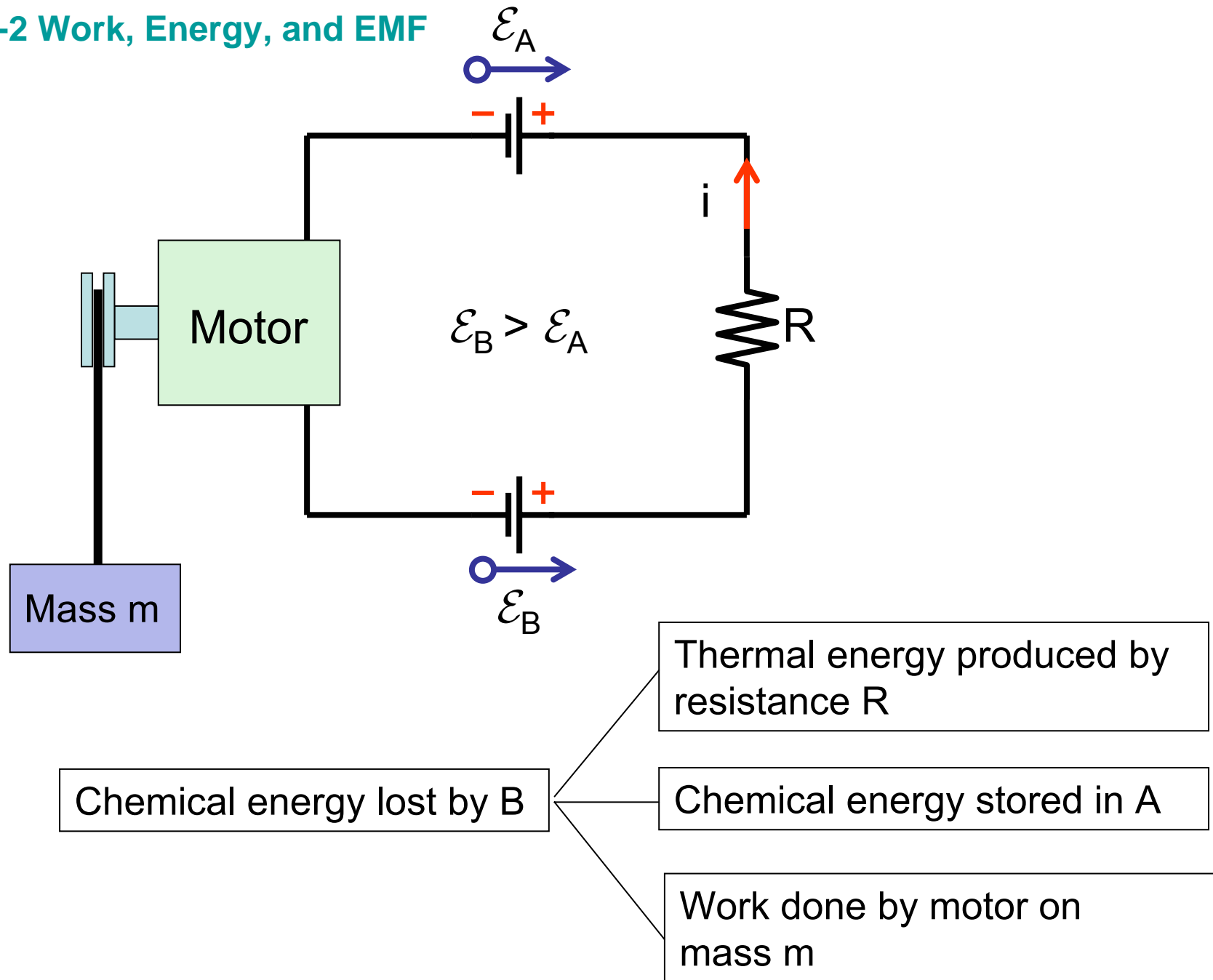
have internal resistance to the movement of charge



28-2 Work, Energy, and EMF

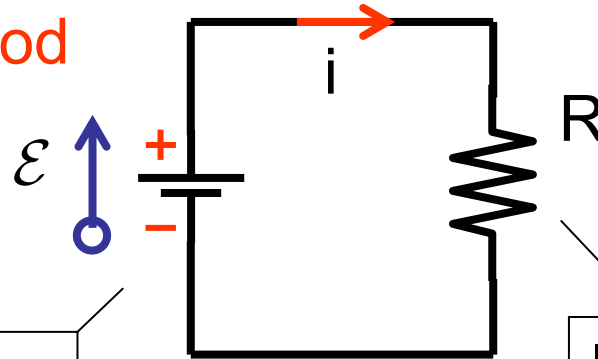


28-2 Work, Energy, and EMF



28-3 Calculating the Current in a Single-Loop Circuit

The energy method



In time dt ,
The battery will do
work of $dW = \mathcal{E} dq$
on dq

In time dt , thermal
energy of
 $dE = i^2 R dt = i R dq$ will
appear in the resistor

Energy is conserved

$$dW = dE$$

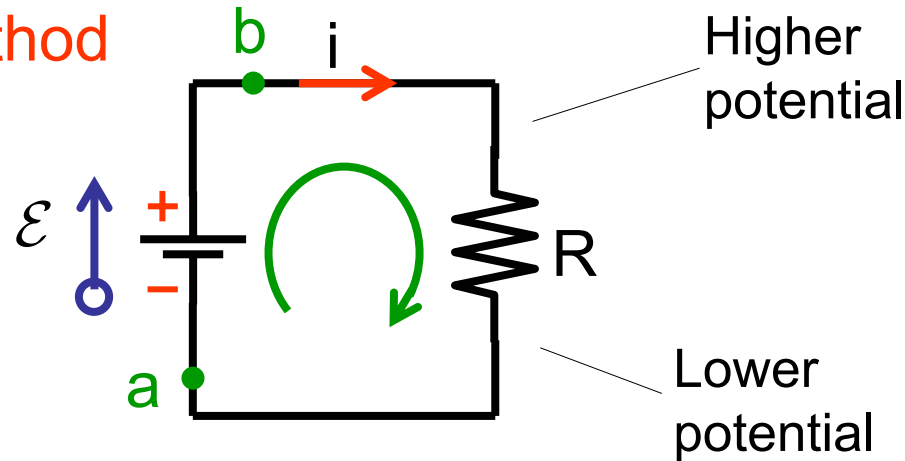
$$\mathcal{E} dq = i R dq$$

$$\mathcal{E} = i R$$

We will **not** use this method to analyze circuits

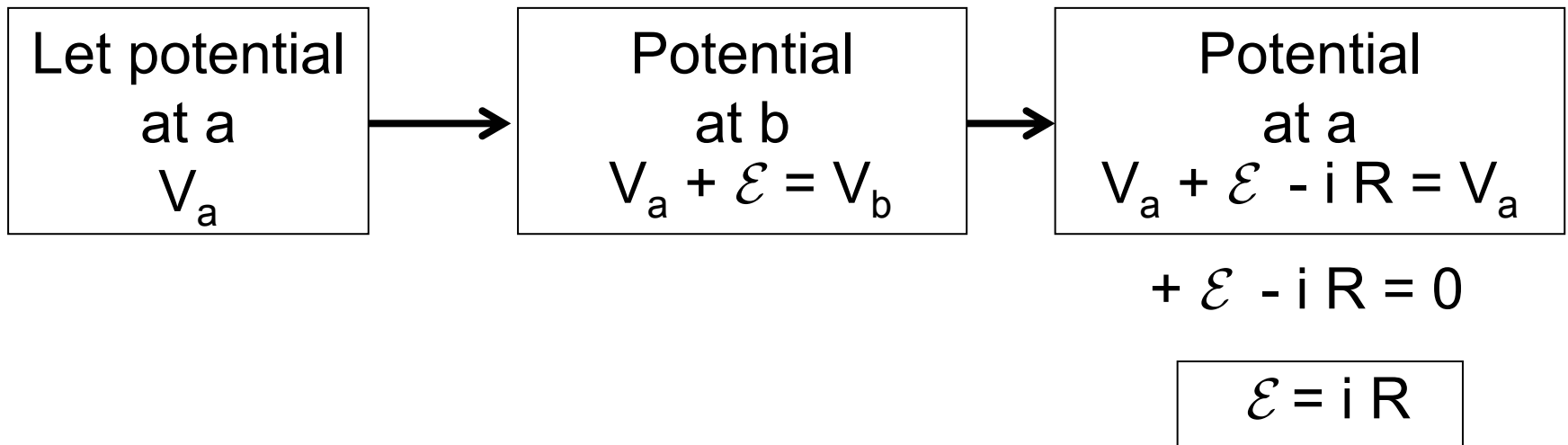
28-3 Calculating the Current in a Single-Loop Circuit

The Potential method



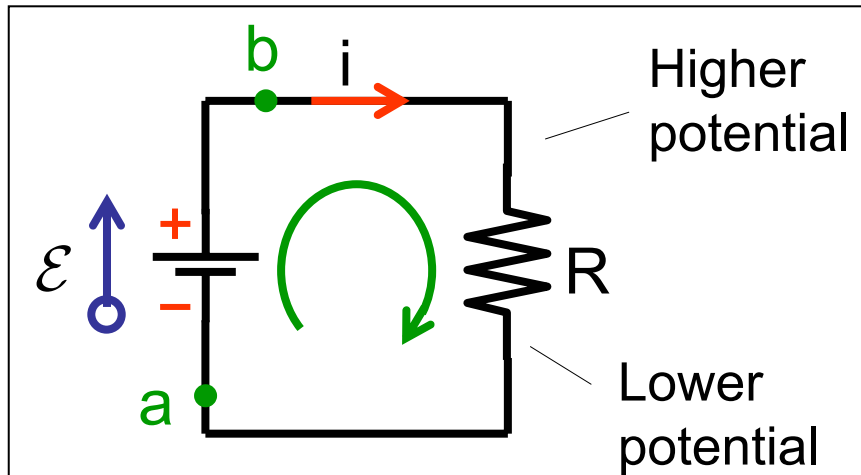
Find the change in the electric potential as you move around the circuit

We will choose the clockwise direction



28-3 Calculating the Current in a Single-Loop Circuit

Direction does not affect the final result

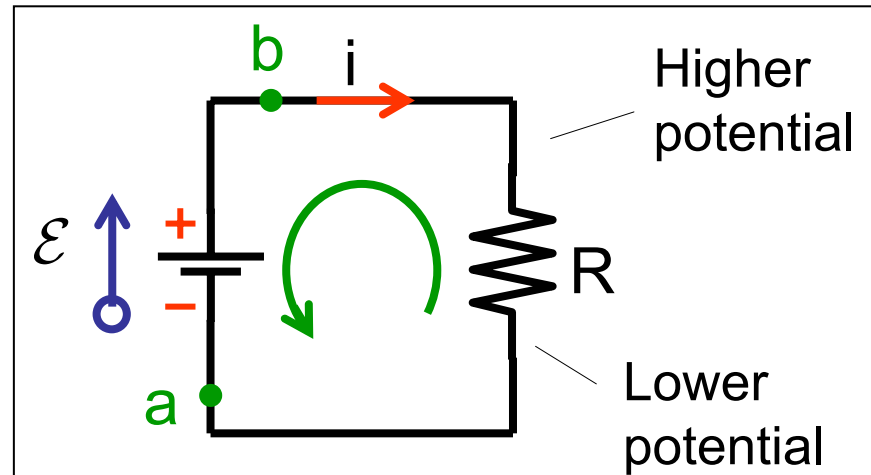


Change in Potential
for clockwise
direction

$$V_a + \mathcal{E} - iR = V_a$$

$$+ \mathcal{E} - iR = 0$$

$$\mathcal{E} = iR$$



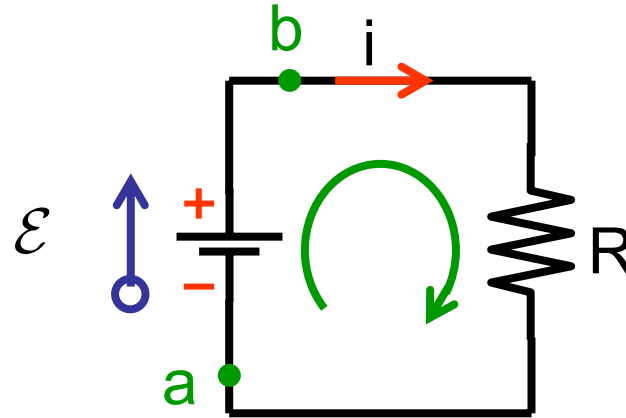
Change in Potential
for counterclockwise
direction

$$V_a + iR - \mathcal{E} = V_a$$

$$+ iR - \mathcal{E} = 0$$

$$\mathcal{E} = iR$$

28-3 Calculating the Current in a Single-Loop Circuit



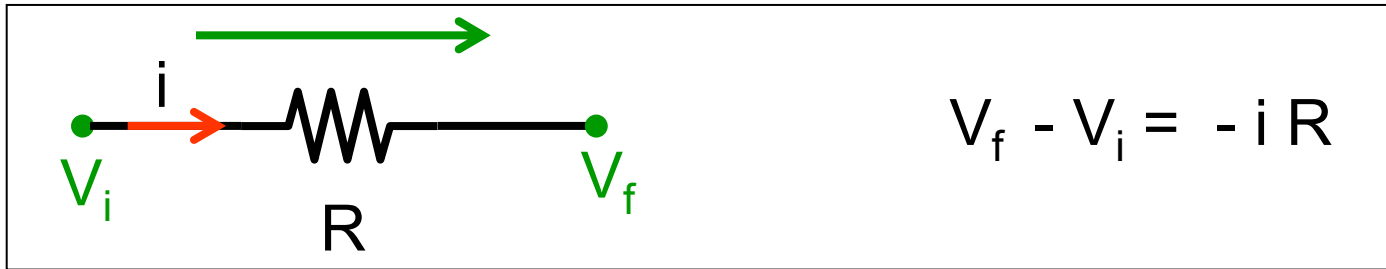
Loop Rule (Kirchhoff's loop rule):

The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero

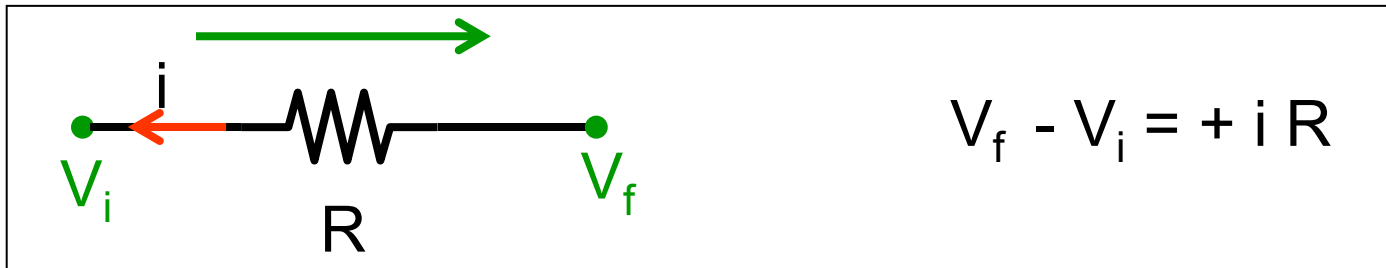
$$\sum_{\text{loop}} \Delta V = 0$$

Conservation of energy

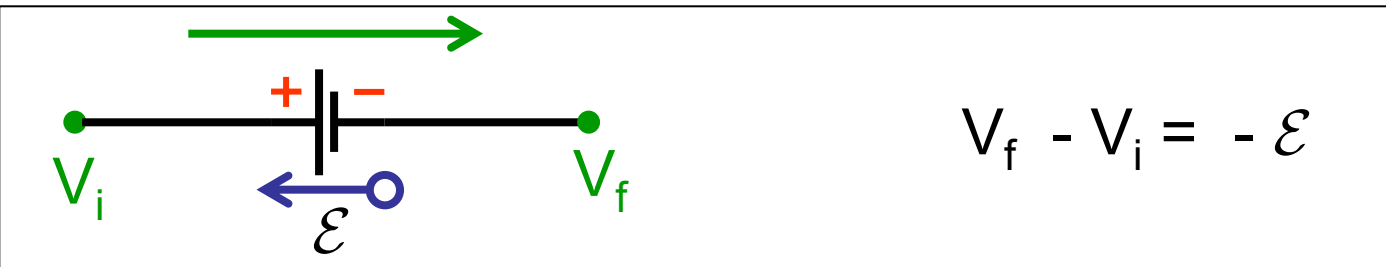
28-3 Calculating the Current in a Single-Loop Circuit



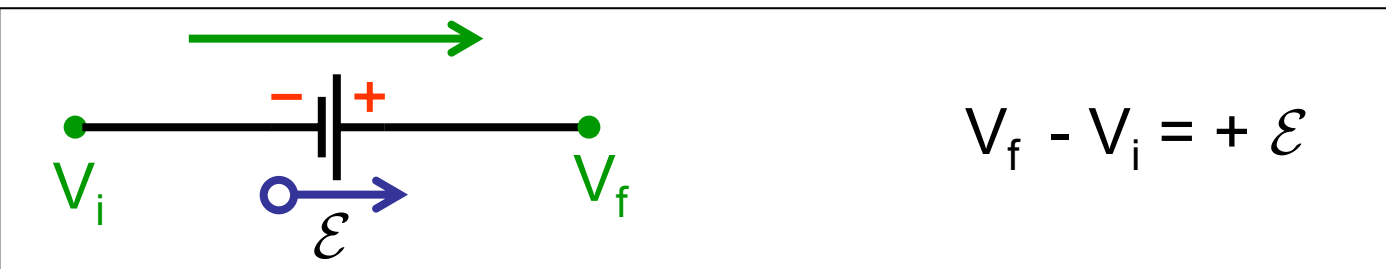
$$V_f - V_i = -iR$$



$$V_f - V_i = +iR$$



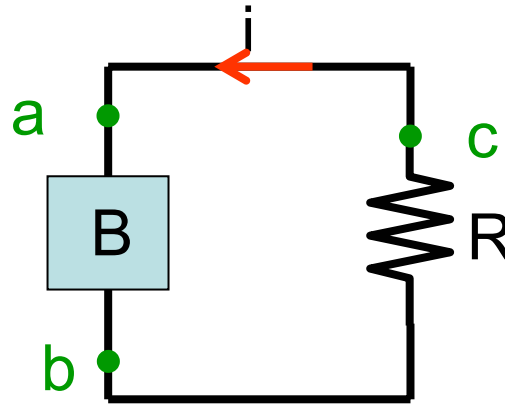
$$V_f - V_i = -\mathcal{E}$$



$$V_f - V_i = +\mathcal{E}$$

28-3 Calculating the Current in a Single-Loop Circuit

Checkpoint 1



Draw the emf arrow



At points a, b, and c rank ...

The magnitude of current

All tie

The electric potential

$$V_b > V_a = V_c$$

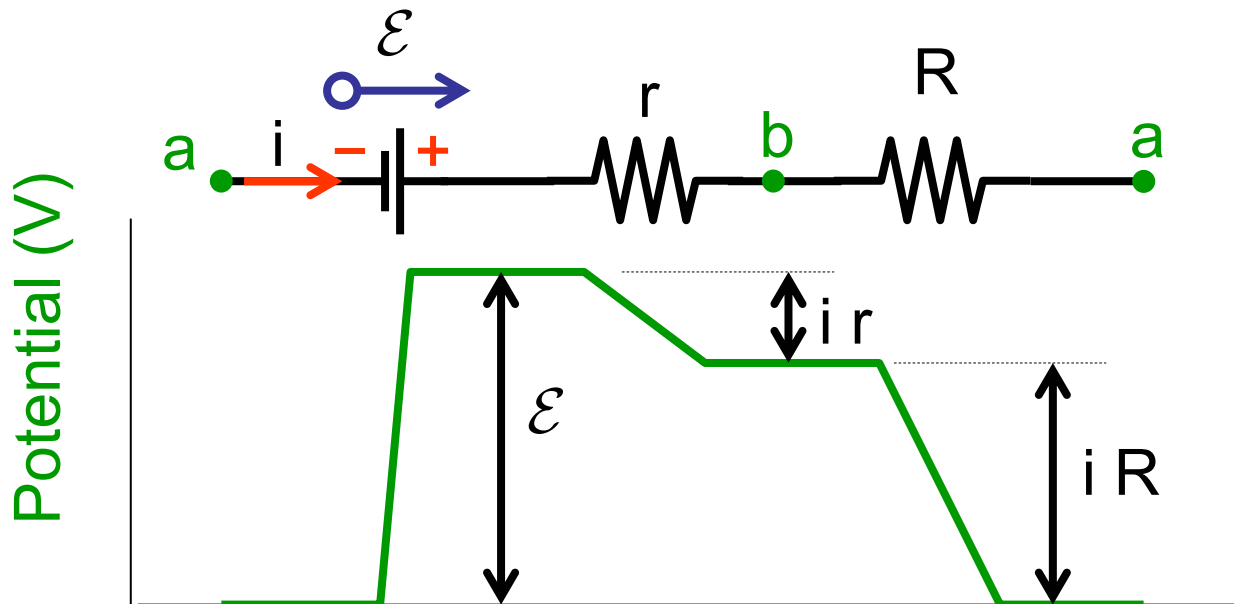
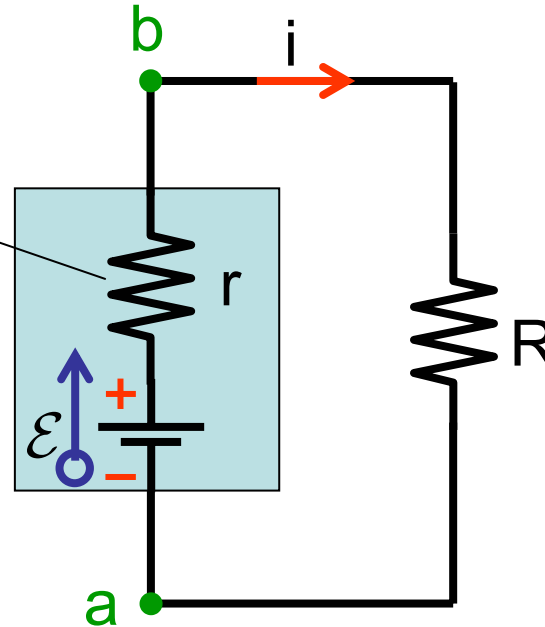
The electric potential energy

$$U_b > U_a = U_c$$

28-4 Other Single-Loop Circuits

Internal Resistance

Real battery

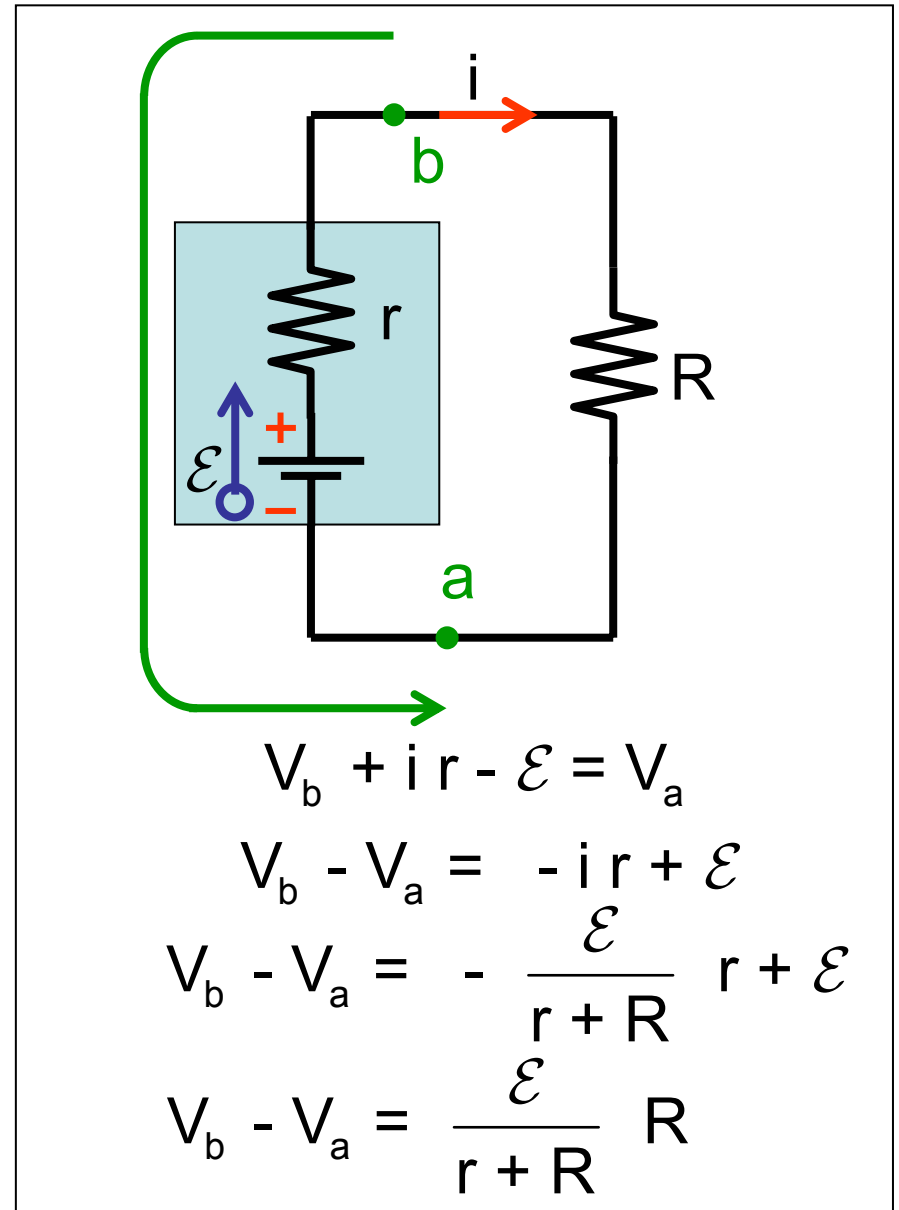
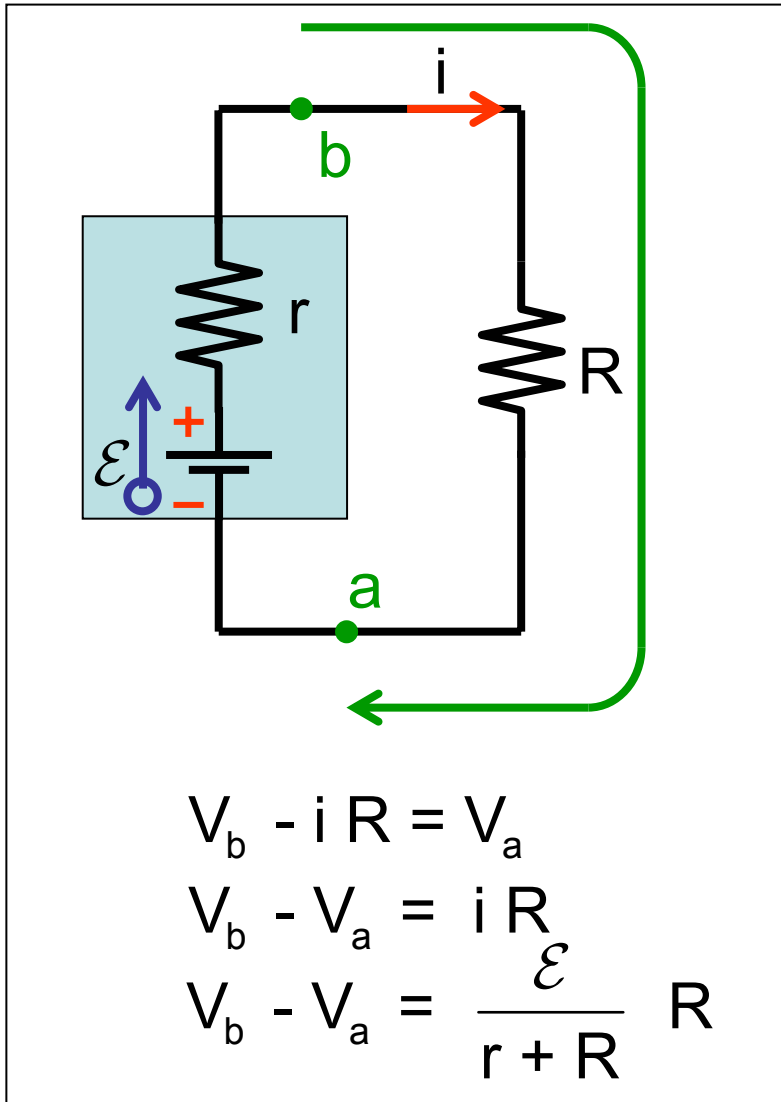


$$\mathcal{E} - ir - iR = 0$$

$$i = \frac{\mathcal{E}}{r + R}$$

28-5 Potential Differences

The potential difference between any two points in a circuit does not depend on the path

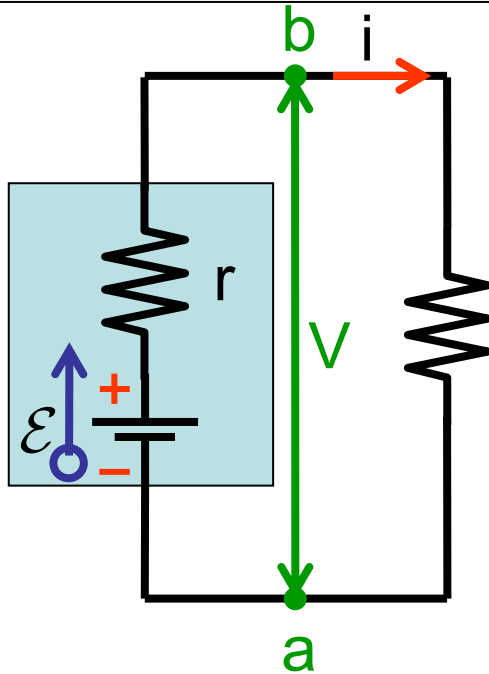


28-4 Other Single-Loop Circuits

$$\mathcal{E} = 12 \text{ V}$$

$$r = 2 \Omega$$

$$R = 10 \Omega$$

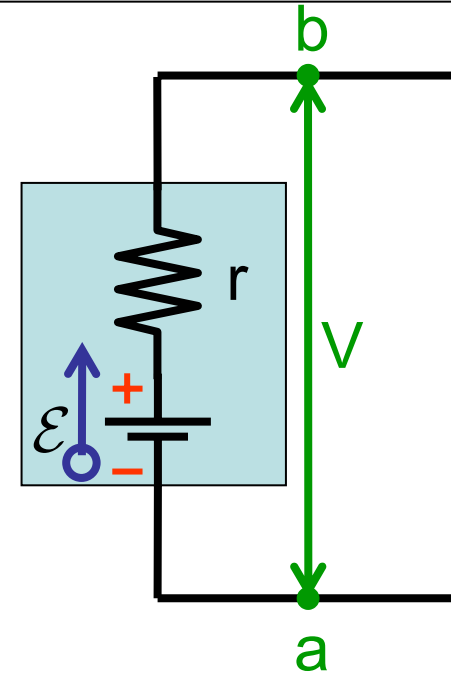


connected to a circuit

$$i = \frac{\mathcal{E}}{r + R} = \frac{12}{2 + 10} = 1 \text{ A}$$

$$V = \mathcal{E} - i r$$

$$= 12 - 1(2) = 10 \text{ V} < \mathcal{E}$$



Not connected to a circuit

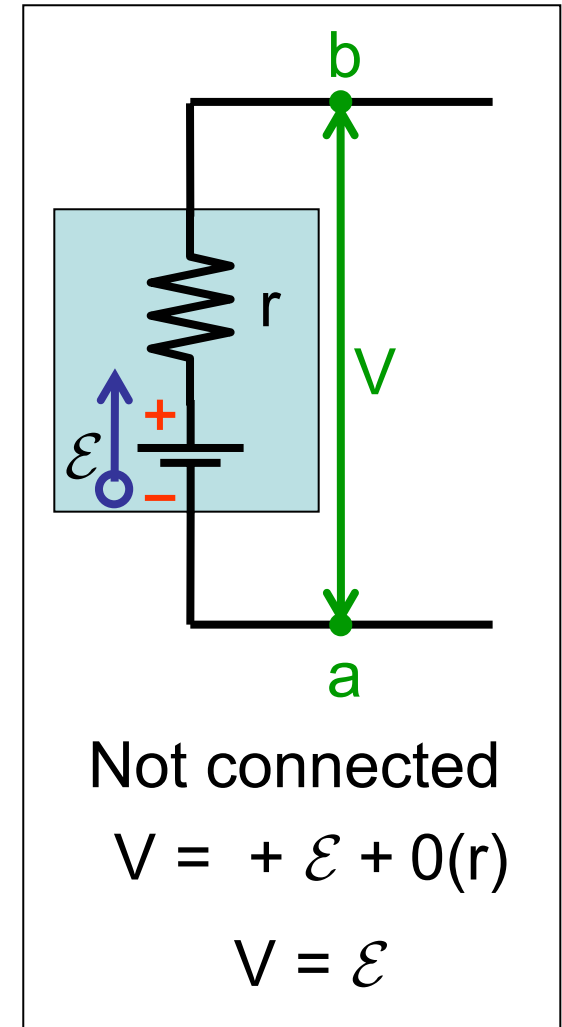
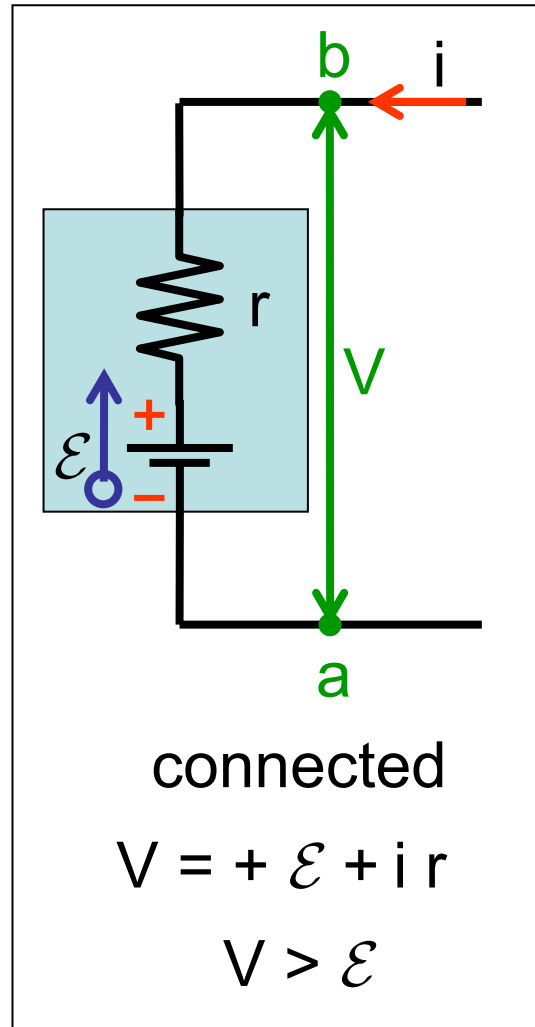
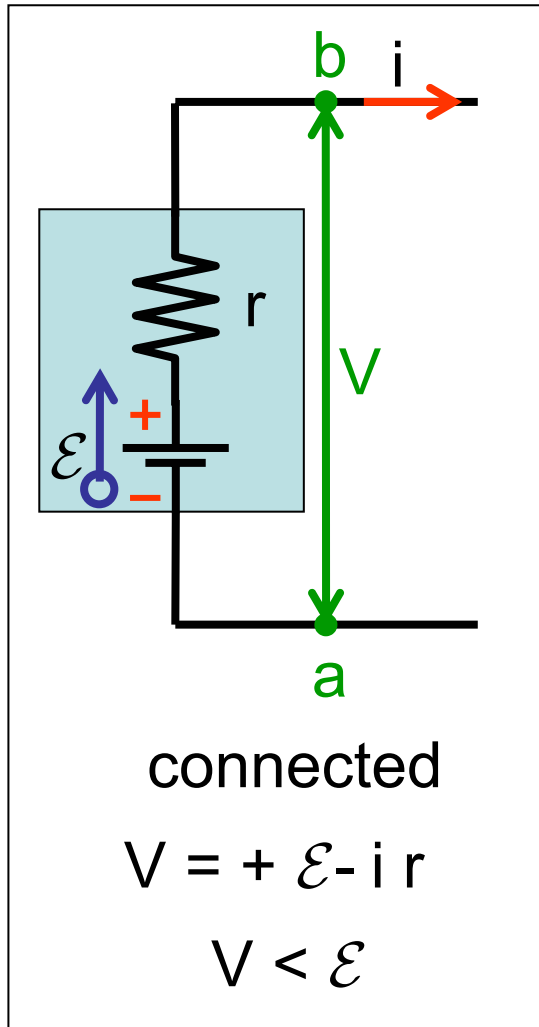
$$i = 0$$

$$V = \mathcal{E} - i r = \mathcal{E} = 12 \text{ V}$$

28-5 Potential Differences

Checkpoint 3

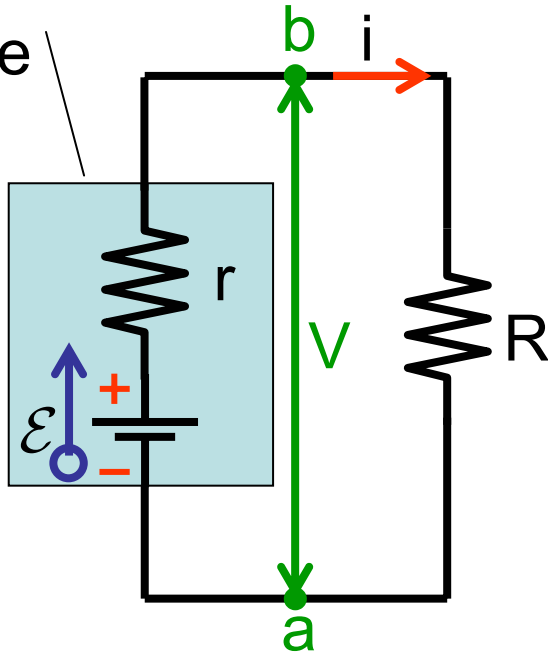
Compare \mathcal{E} and V



28-5 Potential Differences

Power, Potential, and Emf

Real emf device



Rate of energy transfer from the emf device to the charge carriers

$$P = iV$$

$$P = i(\mathcal{E} - ir)$$

$$P = i\mathcal{E} - i^2 r$$

$$P = P_{\text{emf}} - P_r$$

Rate of energy transfer from the emf device to the charge carriers and to internal thermal energy

Rate of energy transfer to internal thermal energy

28-5 Potential Differences

Sample Problem 28-1

$$\mathcal{E}_1 = 4.4 \text{ V}, \mathcal{E}_2 = 2.1 \text{ V},$$

$$r_1 = 2.3 \text{ } \Omega, r_2 = 1.8 \text{ } \Omega, R = 5.5 \text{ } \Omega.$$

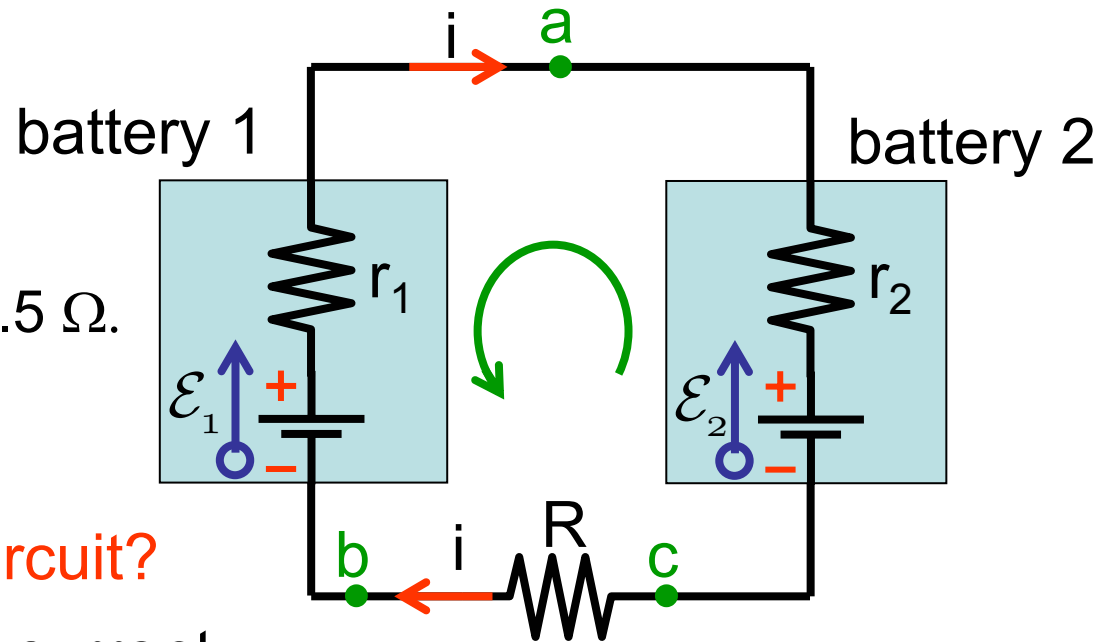
What is current i in the circuit?

Pick any direction for the current

Pick any direction for the loop rule

$$i r_1 - \mathcal{E}_1 + i R + \mathcal{E}_2 + i r_2 = 0$$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + R + r_2} = 0.240 \text{ A} = 240 \text{ mA}$$



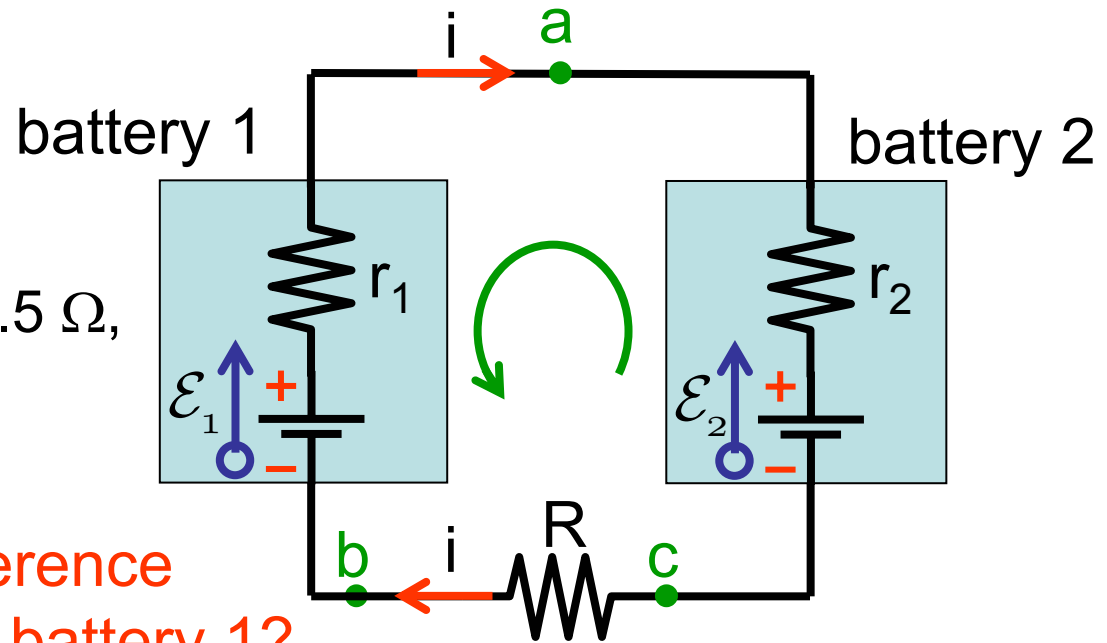
If your guess is wrong,
your current will be
negative

28-5 Potential Differences

Sample Problem 28-1

$$\begin{aligned}\mathcal{E}_1 &= 4.4 \text{ V}, \quad \mathcal{E}_2 = 2.1 \text{ V}, \\ r_1 &= 2.3 \, \Omega, \quad r_2 = 1.8 \, \Omega, \quad R = 5.5 \, \Omega, \\ I &= 240 \text{ mA}.\end{aligned}$$

What is the potential difference between the terminals of battery 1?



$$V_a + i r_1 - \mathcal{E}_1 = V_b$$

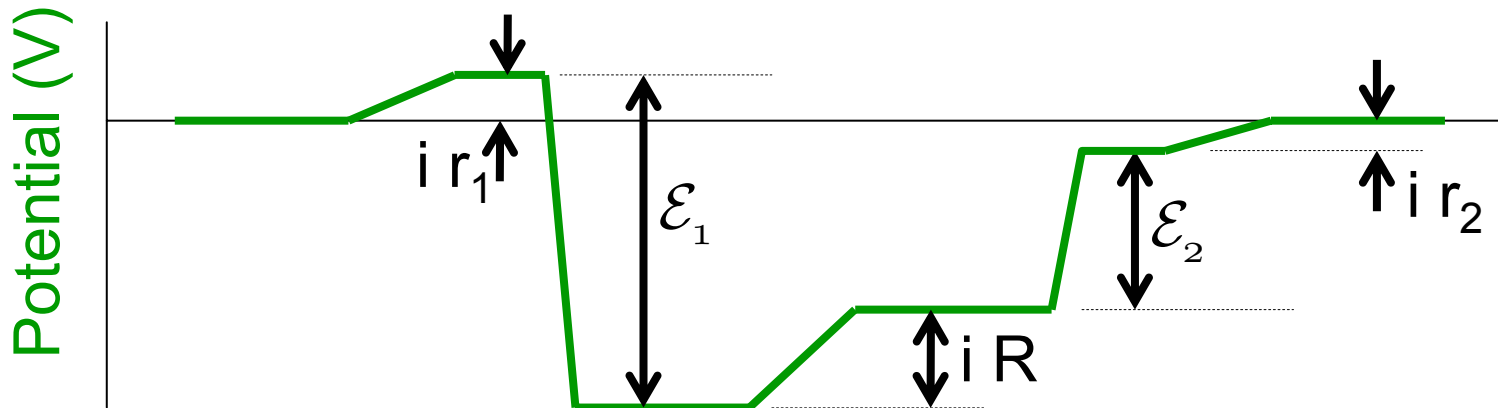
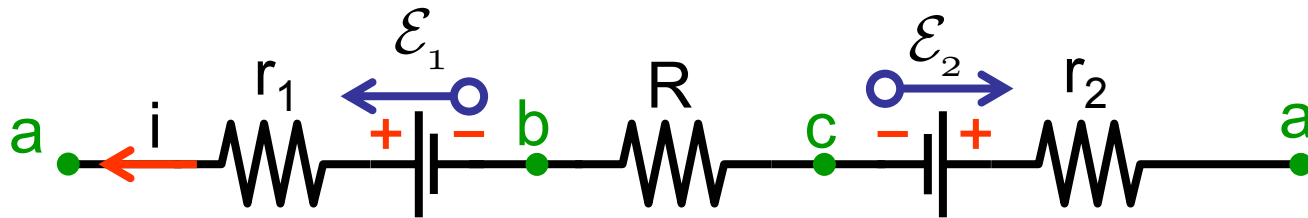
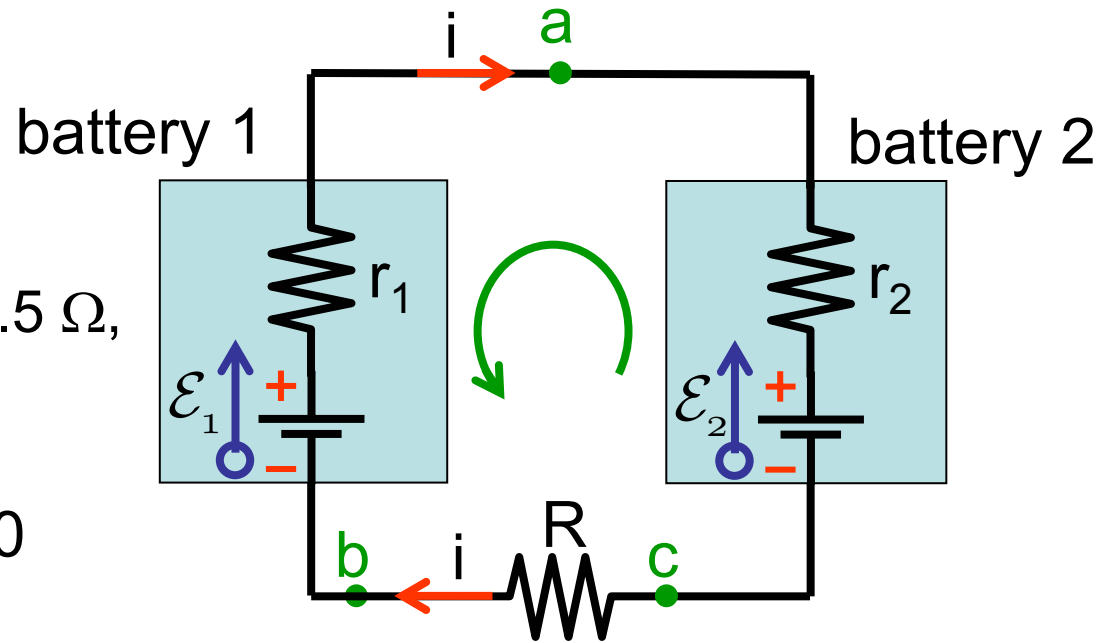
$$V_a - V_b = -i r_1 + \mathcal{E}_1 = 3.84 \text{ V}$$

28-5 Potential Differences

Sample Problem 28-1

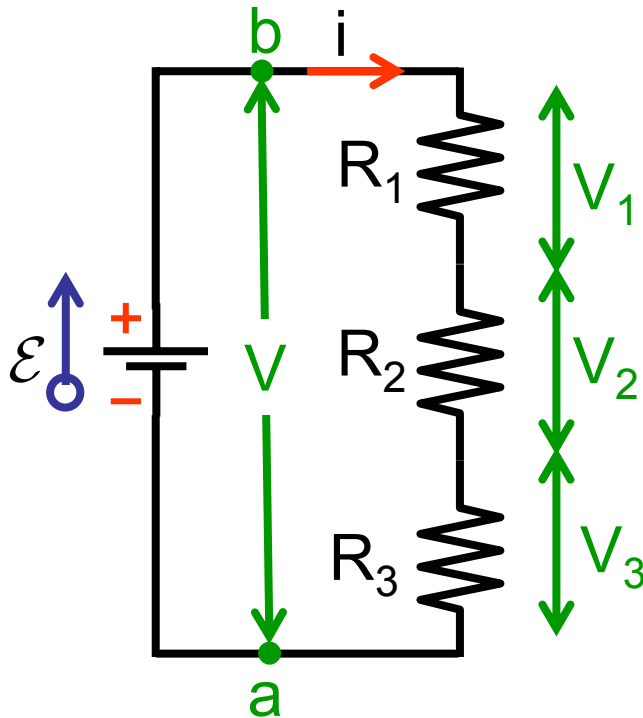
$$\begin{aligned} \mathcal{E}_1 &= 4.4 \text{ V}, \mathcal{E}_2 = 2.1 \text{ V}, \\ r_1 &= 2.3 \Omega, r_2 = 1.8 \Omega, R = 5.5 \Omega, \\ i &= 240 \text{ mA}. \end{aligned}$$

$$i r_1 - \mathcal{E}_1 + i R + \mathcal{E}_2 + i r_2 = 0$$



28-4 Other Single-Loop Circuits

Resistances in Series



Resistances are wired one after another and a potential difference is applied to the two ends of the series

The applied potential difference V is equal to the sum of the potential differences across all the resistances

$$V = V_1 + V_2 + V_3$$

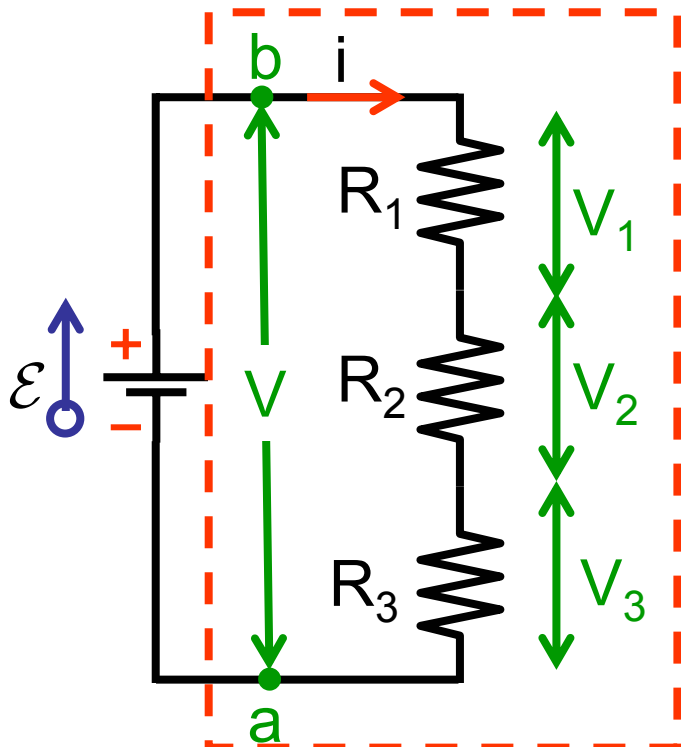
Since the charge is conserved, all the resistances have the same current i

$$i = i_1 = i_2 = i_3$$

For steady flow of charge

28-4 Other Single-Loop Circuits

Resistances in Series



R_{eq} has the same current

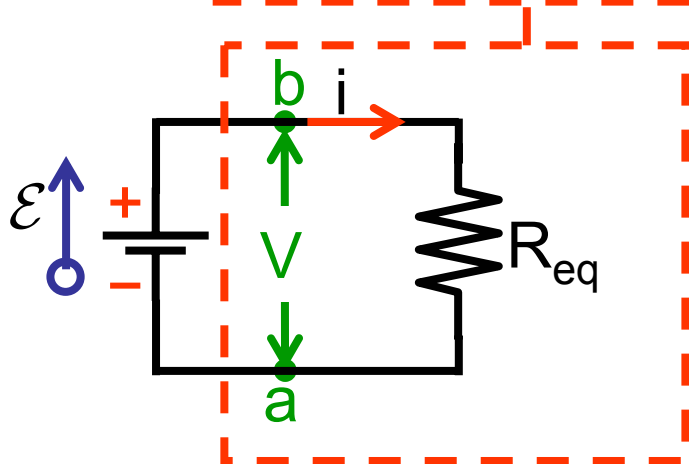
$$i = i_1 = i_2 = i_3$$

and the same total potential difference

$$V = V_1 + V_2 + V_3$$

as the actual resistances

$$R_{\text{eq}} = R_1 + R_2 + R_3$$

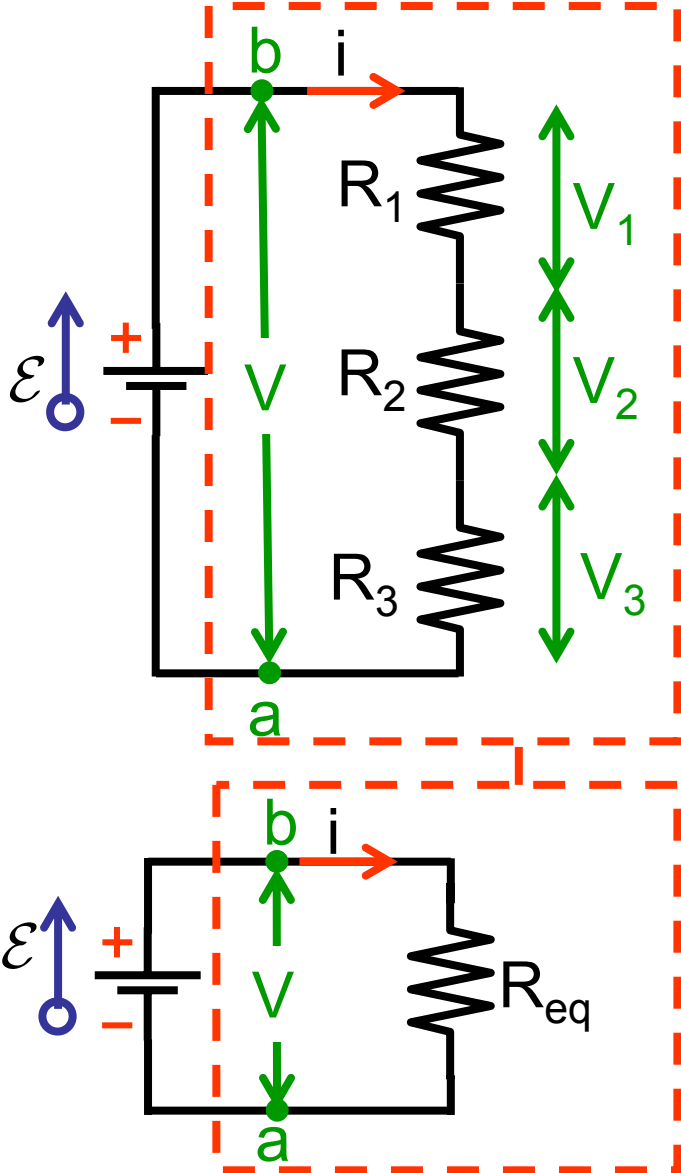


$$R_{\text{eq}} = \sum_{j=1}^n R_j$$

n resistances in series

28-4 Other Single-Loop Circuits

Derivation of $R_{\text{eq}} = R_1 + R_2 + R_3$



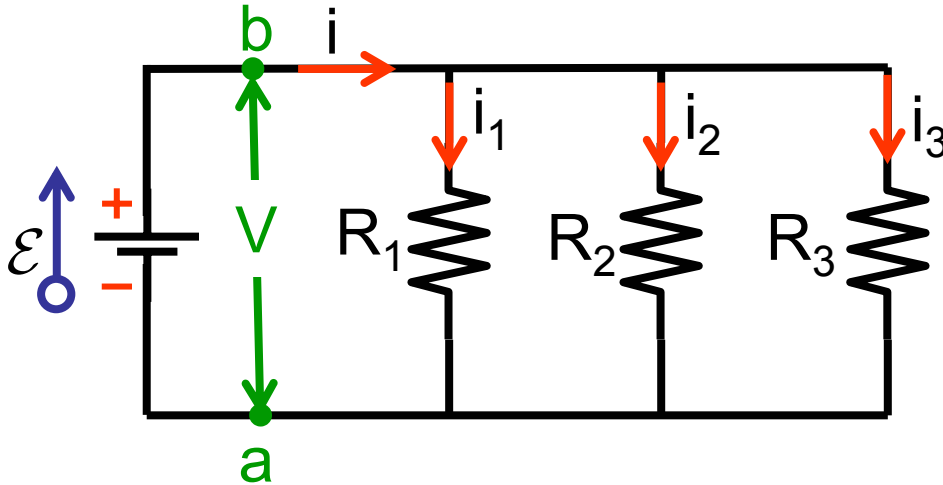
$$V = V_1 + V_2 + V_3$$

$$i R_{\text{eq}} = i R_1 + i R_2 + i R_3$$

$$R_{\text{eq}} = R_1 + R_2 + R_3$$

28-4 Other Single-Loop Circuits

Resistances in Parallel



Resistances are directly wired together on one side and directly wired together on the other side and a potential difference is applied across the pair of connected sides

All the resistances have the same potential difference

$$V = V_1 = V_2 = V_3$$

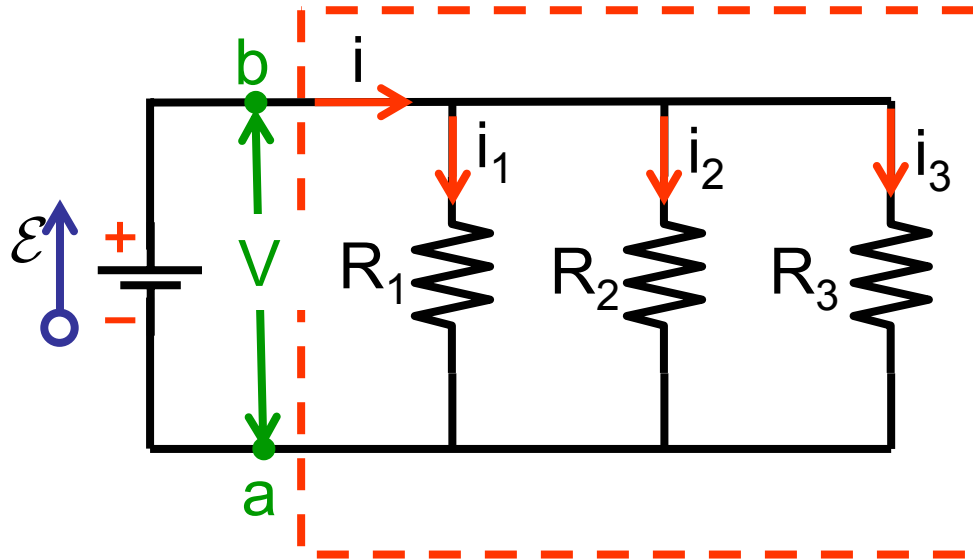
Since the charge is conserved, the total current passing through the resistances is equal to the sum of the current passing through each resistance

$$i = i_1 + i_2 + i_3$$

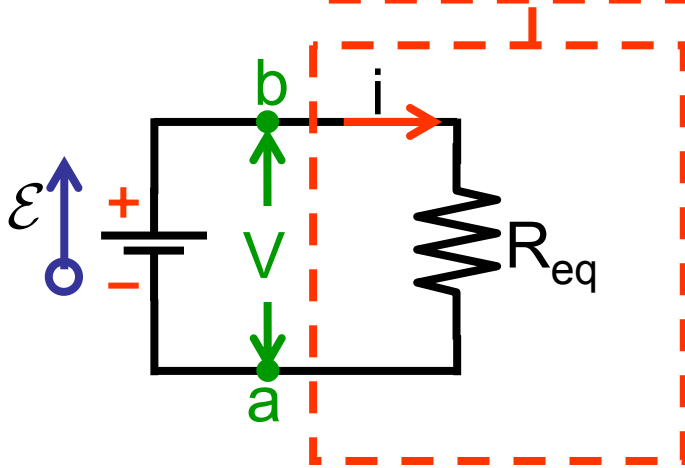
For steady flow of charge

28-4 Other Single-Loop Circuits

Resistances in Parallel



R_{eq} has the same total current
 $i = i_1 + i_2 + i_3$
 and the same potential difference
 $V = V_1 = V_2 = V_3$
 as the actual resistances



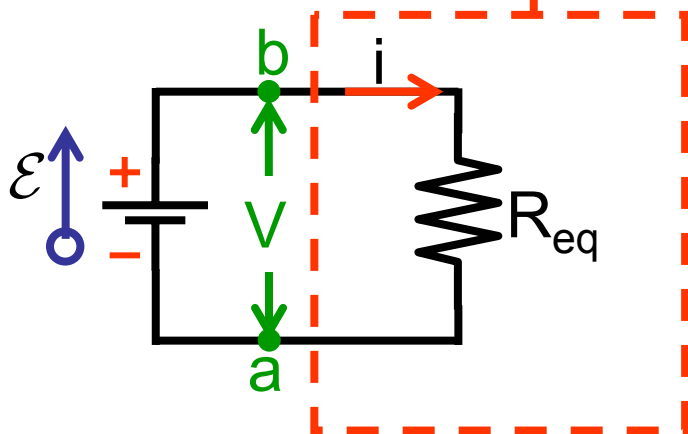
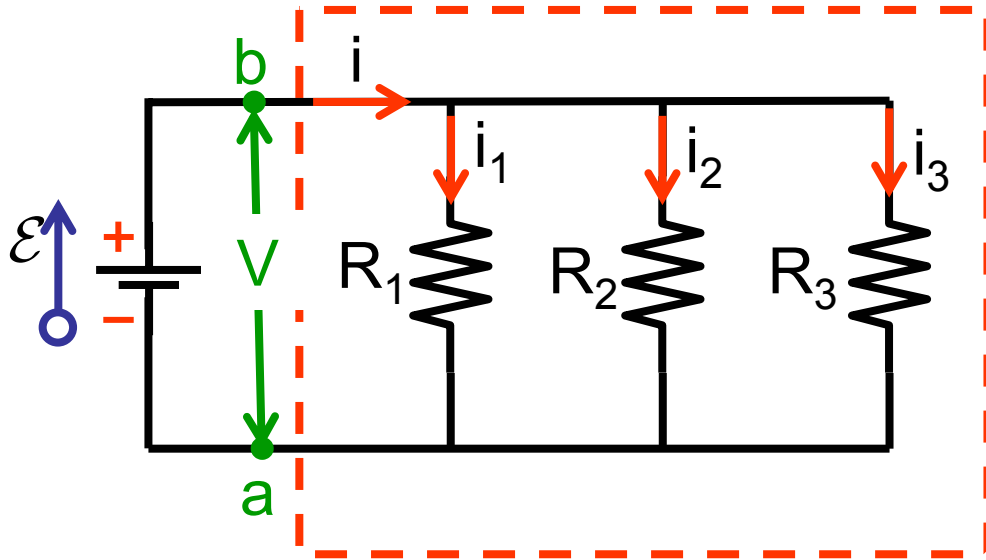
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{\text{eq}}} = \sum_{j=1}^n \frac{1}{R_j} \quad \text{n resistances in series}$$

R_{eq} is smaller than any of the actual resistances

28-4 Other Single-Loop Circuits

Derivation of
$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

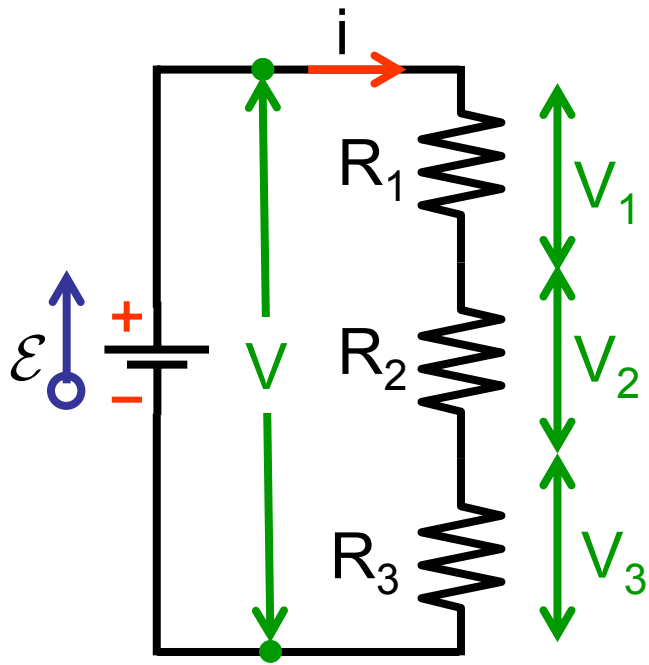


$$i = i_1 + i_2 + i_3$$

$$\frac{V}{R_{\text{eq}}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

28-4 Other Single-Loop Circuits

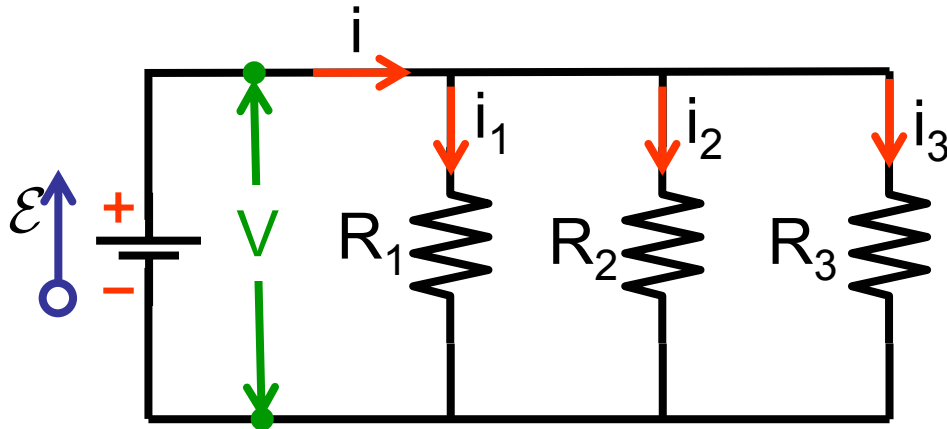


Resistances in Series

$$i = i_1 = i_2 = i_3$$

$$V = V_1 + V_2 + V_3$$

$$R_{\text{eq}} = R_1 + R_2 + R_3$$



Resistances in Parallel

$$i = i_1 + i_2 + i_3$$

$$V = V_1 = V_2 = V_3$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

28-4 Other Single-Loop Circuits

Sample Problem 2

$$\mathcal{E} = 12 \text{ V},$$

$$R_1 = 20 \ \Omega, \ R_2 = 20 \ \Omega,$$

$$R_3 = 30 \ \Omega, \ R_4 = 8.0 \ \Omega,$$

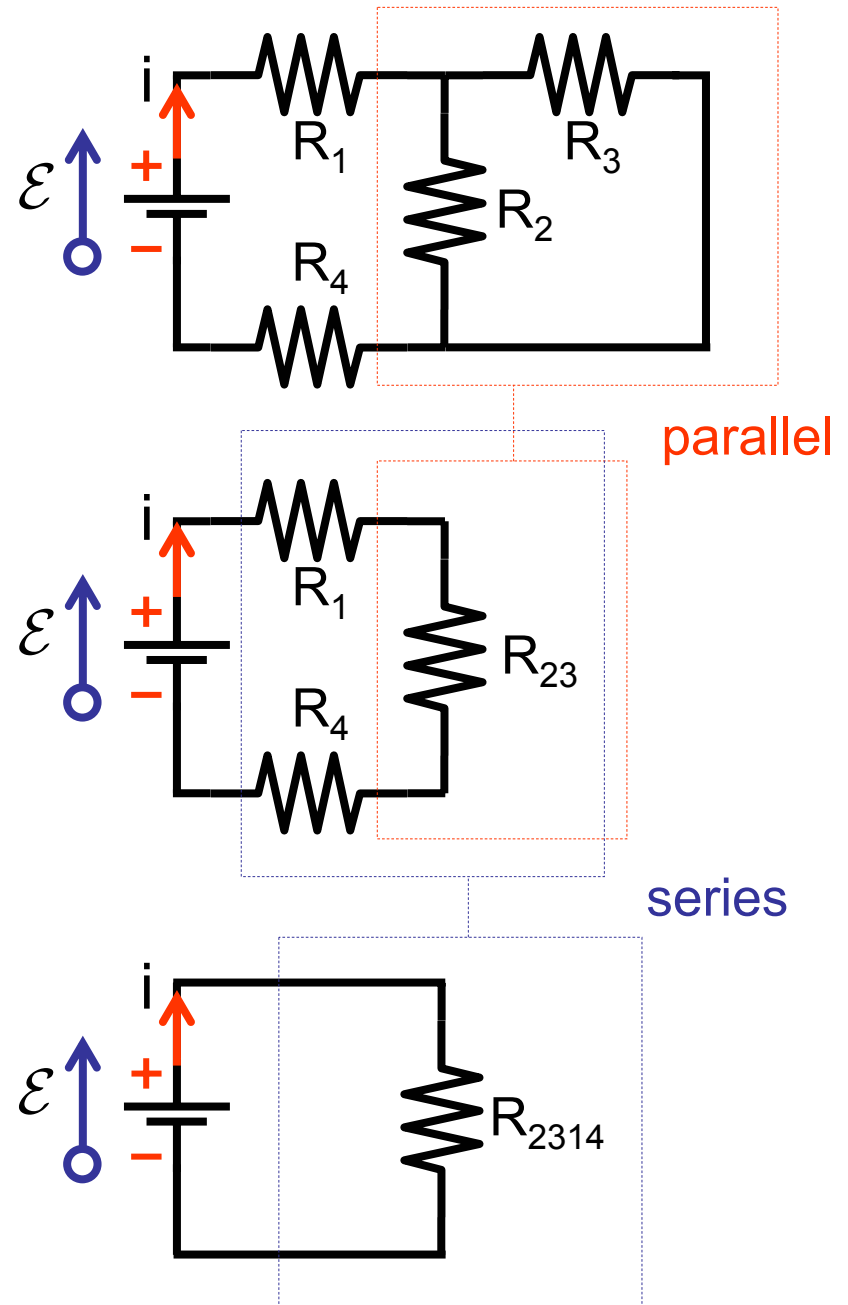
What is current i through the battery?

Simplify the circuit

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = 12 \ \Omega$$

$$R_{2314} = R_1 + R_4 + R_{23} = 40 \ \Omega$$

$$i = \frac{\mathcal{E}}{R_{2314}} = 0.3 \text{ A}$$



28-4 Other Single-Loop Circuits

Sample Problem 2

$$\mathcal{E} = 12 \text{ V},$$

$$R_1 = 20 \ \Omega, \ R_2 = 20 \ \Omega,$$

$$R_3 = 30 \ \Omega, \ R_4 = 8.0 \ \Omega,$$

$$R_{23} = 12 \ \Omega, \ i = 0.3 \text{ A}$$

What is current i_2 through R_2 ?

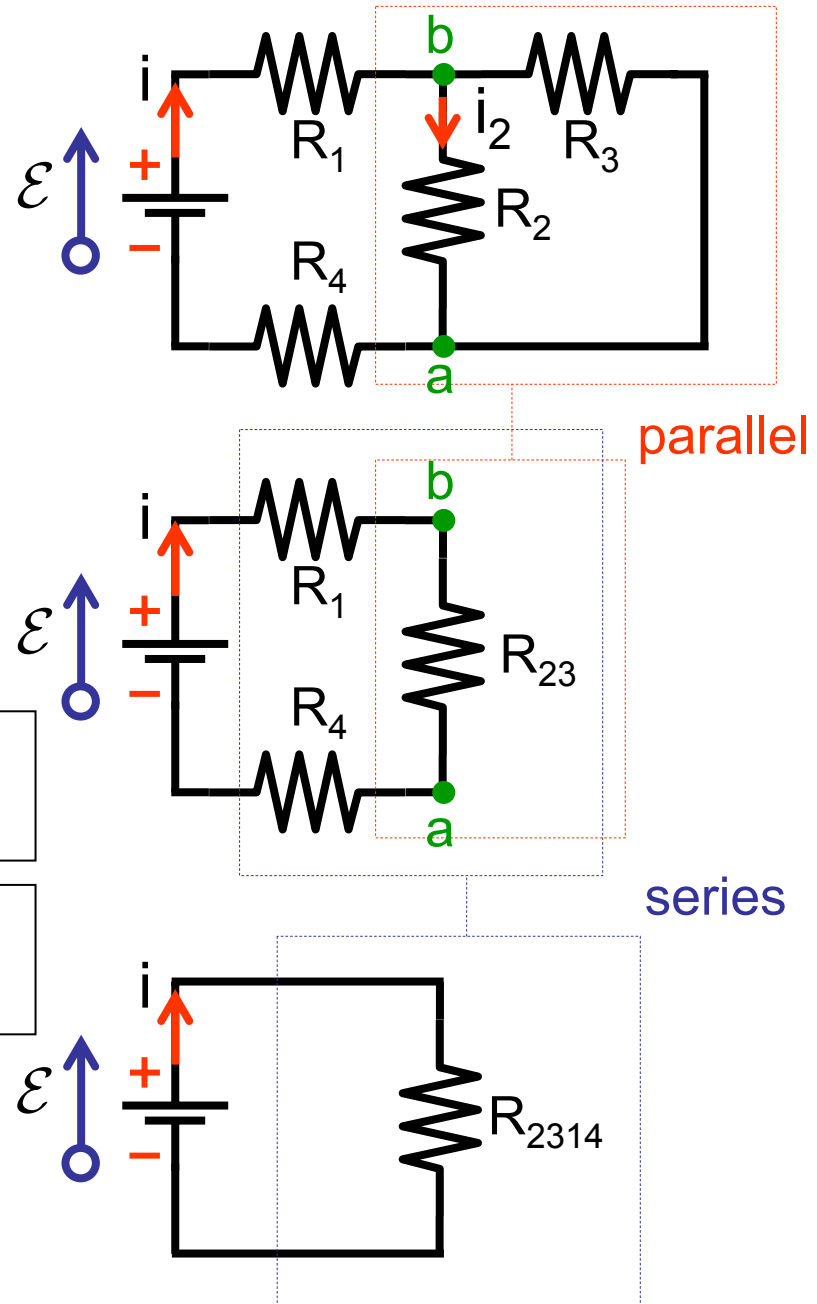
$$i_2 = \frac{V_{ba}}{R_2}$$

R_2 and R_3 are in parallel, potential differences across R_2 and R_{23} are equal

R_1 , R_{23} and R_4 are in series, currents thorough R_{2314} and R_{23} are equal

$$V_{ba} = R_{23} i = 3.6 \text{ V}$$

$$i_2 = \frac{V_{ba}}{R_2} = 0.18 \text{ A}$$



28-4 Other Single-Loop Circuits

Sample Problem 2

$$\mathcal{E} = 12 \text{ V},$$

$$R_1 = 20 \ \Omega, \ R_2 = 20 \ \Omega,$$

$$R_3 = 30 \ \Omega, \ R_4 = 8.0 \ \Omega,$$

$$R_{23} = 12 \ \Omega, \ i = 0.3 \text{ A},$$

$$i = 0.18 \text{ A}, \ V_{ab} = 3.6 \text{ V}$$

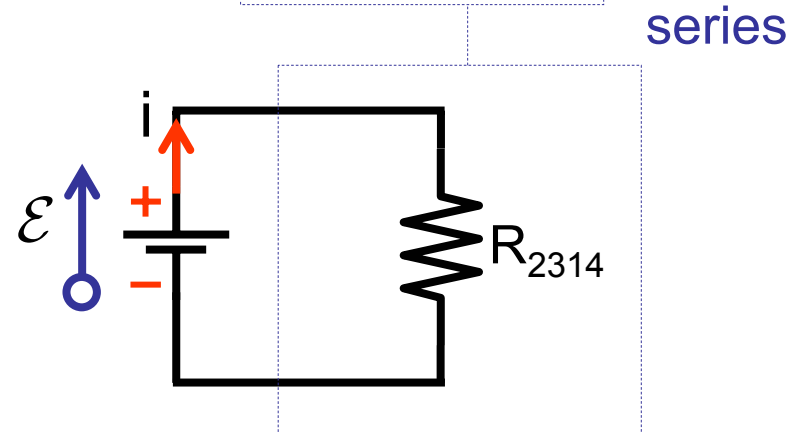
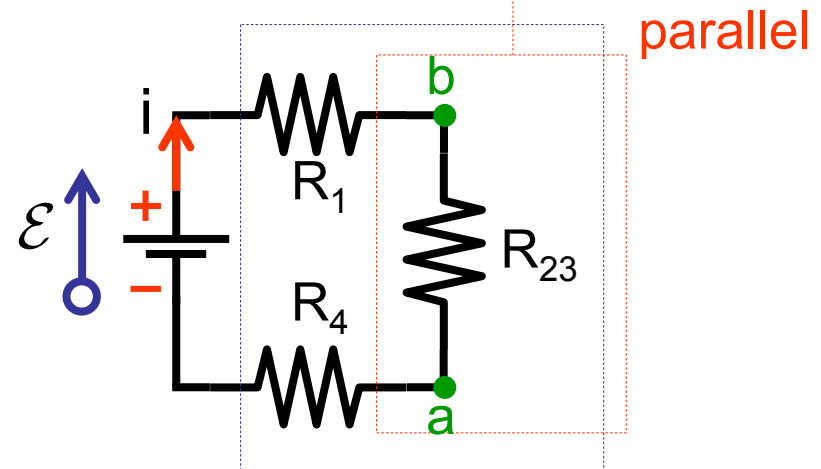
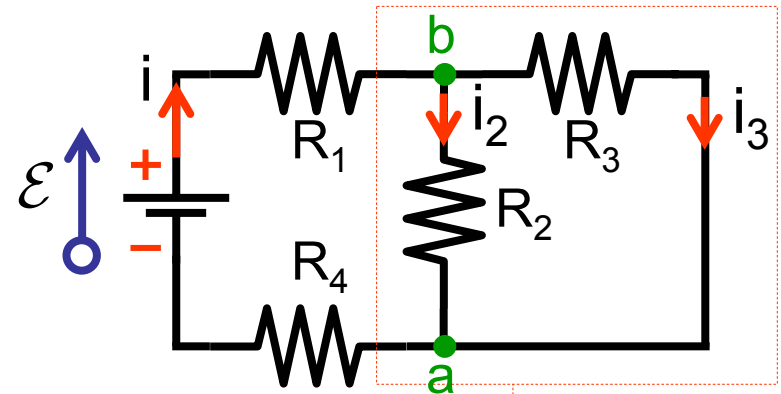
What is current i_3 through R_3 ?

$$i_3 = \frac{V_{ba}}{R_3} = 0.12 \text{ A}$$

Another way

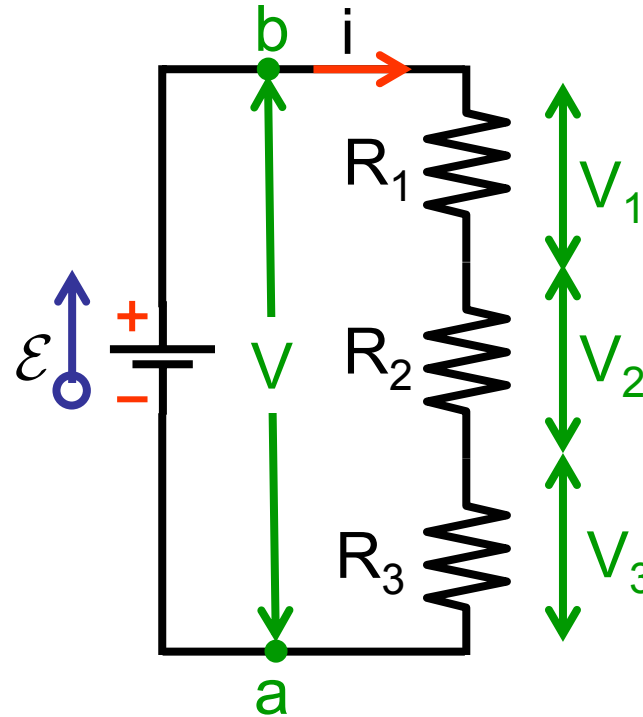
$$i = i_2 + i_3$$

$$i_3 = i - i_2 = 0.12 \text{ A}$$



28-4 Other Single-Loop Circuits

Checkpoint 2



Let $R_1 > R_2 > R_3$

Rank ...

the current through the resistances

All tie

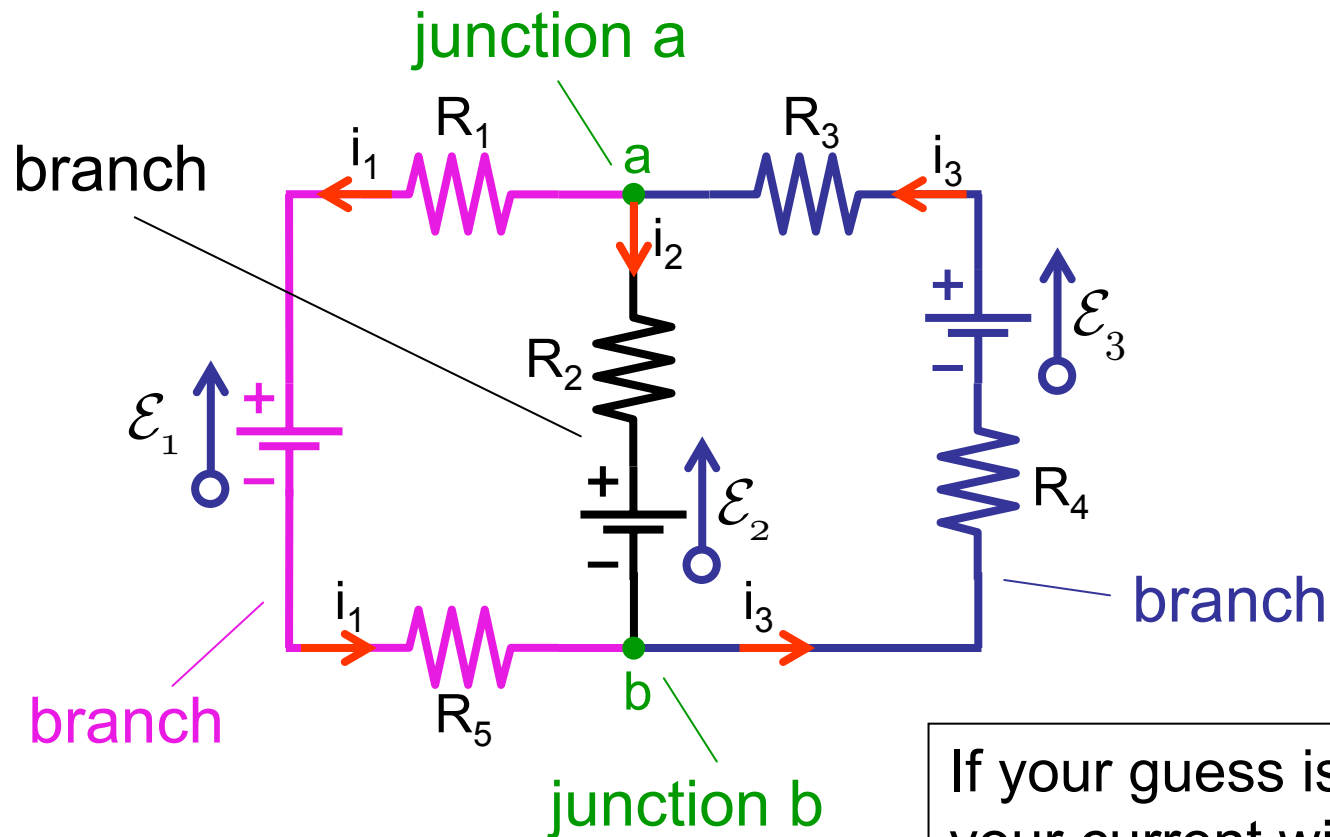
the potential difference across them

$$R_1 > R_2 > R_3$$

$$i R_1 > i R_2 > i R_3$$

$$V_1 > V_2 > V_3$$

28-6 Multiloop Circuits

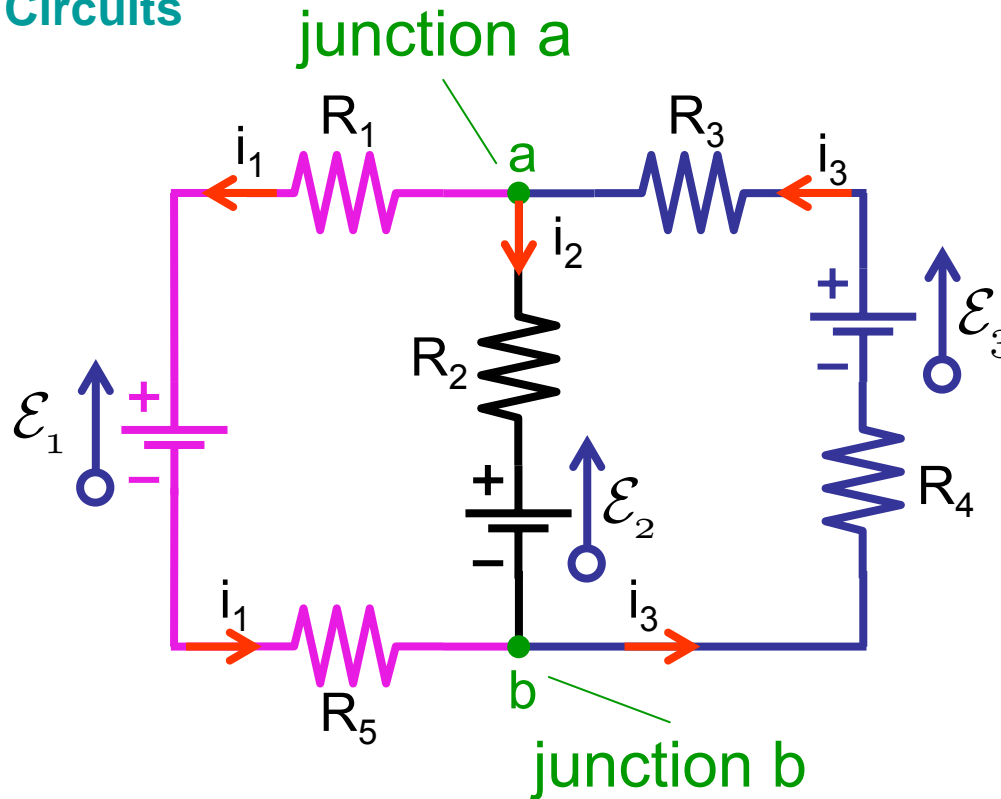


Pick any direction for the currents

If your guess is wrong, your current will be negative. The actual current is moving opposite to your guess

Any point in a branch has the same current.

28-6 Multiloop Circuits



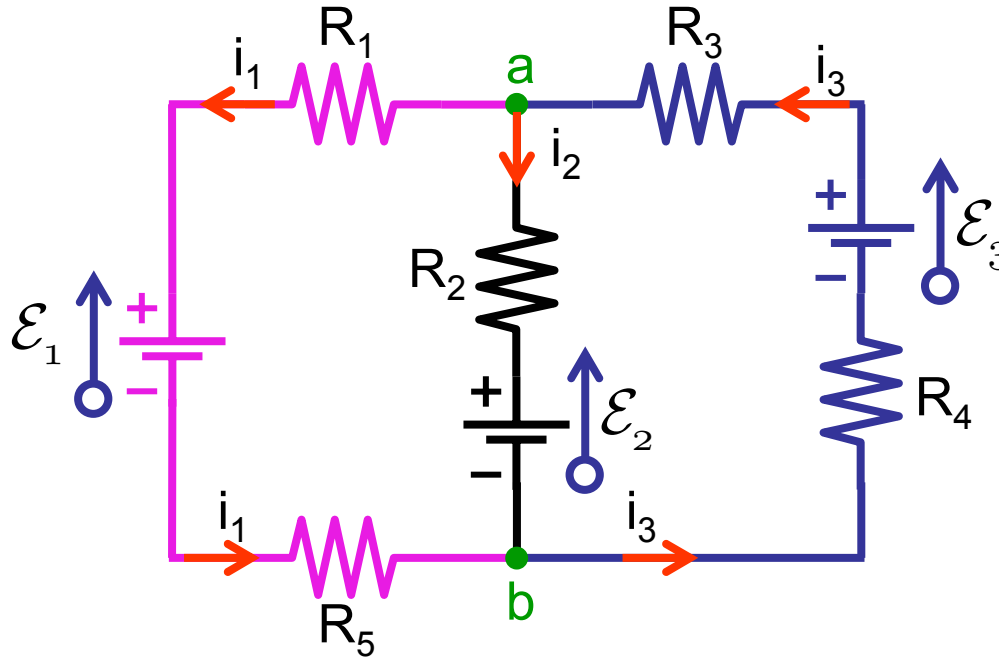
Junction Rule (Kirchhoff's junction rule):

The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction

$$\sum_{\text{in}} i = \sum_{\text{out}} i$$

Conservation of charge
for steady flow of charge

28-6 Multiloop Circuits



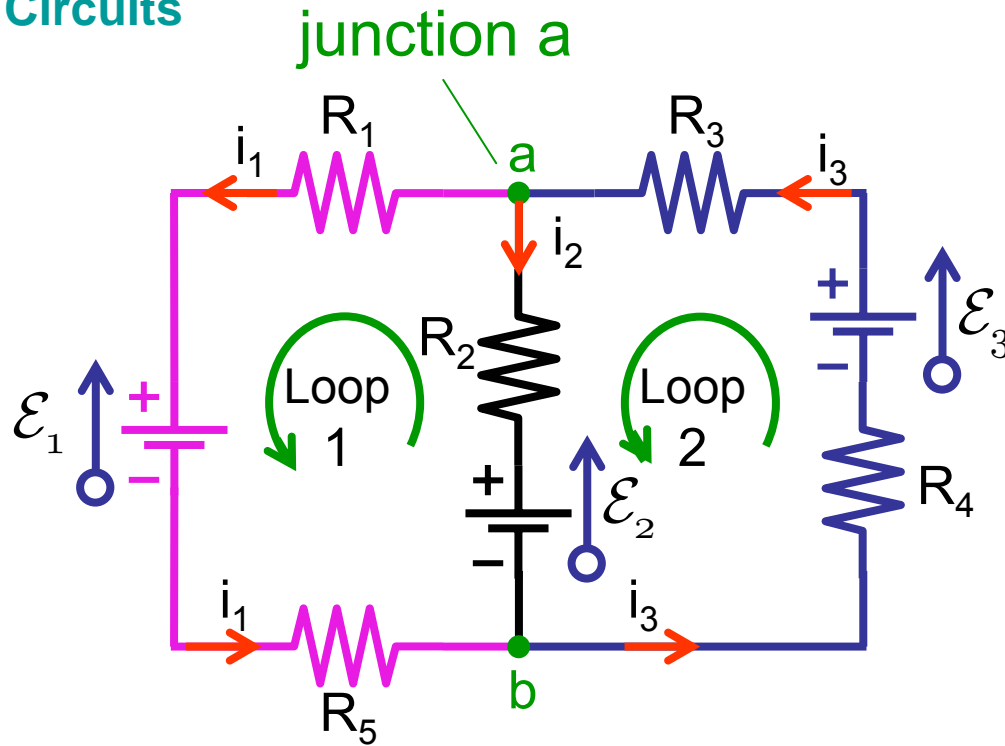
To analyze a circuit, we need to find as many **independent** equations as the number of unknowns in the circuit

Suppose R 's and \mathcal{E} 's are given. we need to find i_1 , i_2 , and i_3

Number of unknowns = 3

Number of **independent** equations = 3

28-6 Multiloop Circuits



Junction rule at junction a

$$i_3 = i_1 + i_2$$

Loop rule on loop 1

$$-i_1 R_1 - \mathcal{E}_1 - i_1 R_5 + \mathcal{E}_2 + i_2 R_2 = 0$$

Loop rule on loop 2

$$-i_2 R_2 - \mathcal{E}_2 - i_3 R_4 + \mathcal{E}_3 - i_3 R_3 = 0$$

28-6 Multiloop Circuits

Sample Problem 28-3

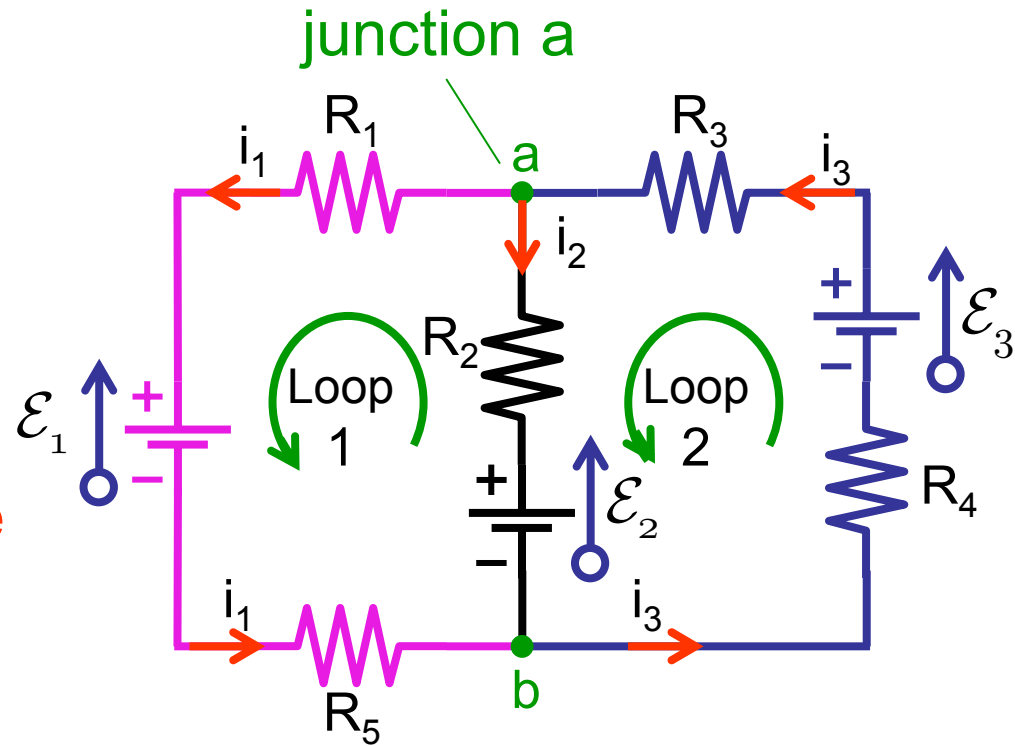
$$\mathcal{E}_1 = 3.0 \text{ V},$$

$$\mathcal{E}_2 = \mathcal{E}_3 = 6.0 \text{ V},$$

$$R_1 = R_3 = R_4 = R_5 = 2.0 \ \Omega,$$

$$R_2 = 4.0 \ \Omega.$$

Find the magnitude and the direction of the current in each branch.



Junction rule at junction a

$$i_3 = i_1 + i_2$$

Loop rule on loop 1

$$-i_1 R_1 - \mathcal{E}_1 - i_1 R_5 + \mathcal{E}_2 + i_2 R_2 = 0$$

$$-4 i_1 + 3 + 4 i_2 = 0$$

Loop rule on loop 2

$$-i_2 R_2 - \mathcal{E}_2 - i_3 R_4 + \mathcal{E}_3 - i_3 R_3 = 0$$

$$-4 i_2 - 4 i_3 = 0$$

28-6 Multiloop Circuits

Sample Problem 28-3

$$i_3 = i_1 + i_2$$

$$-4 i_1 + 3 + 4 i_2 = 0$$

$$-4 i_2 - 4 i_3 = 0$$

Eliminate one variable, say i_3

$$-4 i_1 + 3 + 4 i_2 = 0$$

$$-4 i_2 - 4 (i_1 + i_2) = 0$$

$$-4 i_1 + 3 + 4 i_2 = 0$$

$$-8 i_2 - 4 i_1 = 0$$

$$-4 i_1 + 3 + 4 i_2 = 0$$

$$i_2 = -\frac{i_1}{2}$$

Eliminate another variable, say i_2

$$-4 i_1 + 3 + 4 \left(-\frac{i_1}{2}\right) = 0$$

$$-6 i_1 + 3 = 0$$

$$i_1 = 0.5 \text{ A}$$

$$i_2 = -0.25 \text{ A}$$

$$i_3 = 0.25 \text{ A}$$

28-6 Multiloop Circuits

Sample Problem 28-3

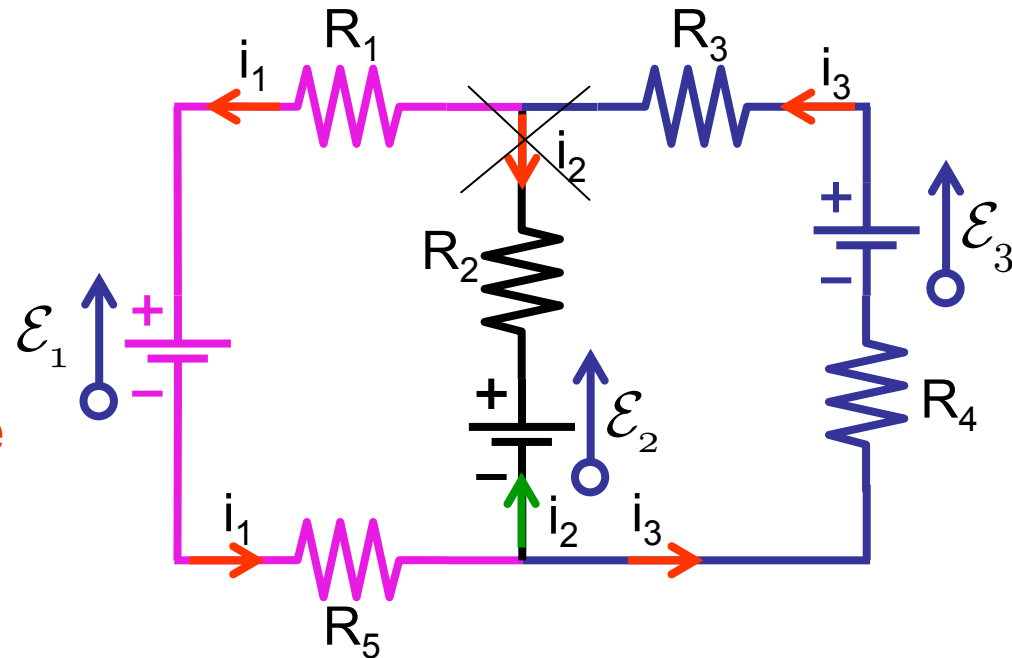
$$\mathcal{E}_1 = 3.0 \text{ V},$$

$$\mathcal{E}_2 = \mathcal{E}_3 = 6.0 \text{ V},$$

$$R_1 = R_3 = R_4 = R_5 = 2.0 \ \Omega,$$

$$R_2 = 4.0 \ \Omega.$$

Find the magnitude and the direction of the current in each branch.



$$i_1 = 0.5 \text{ A}$$

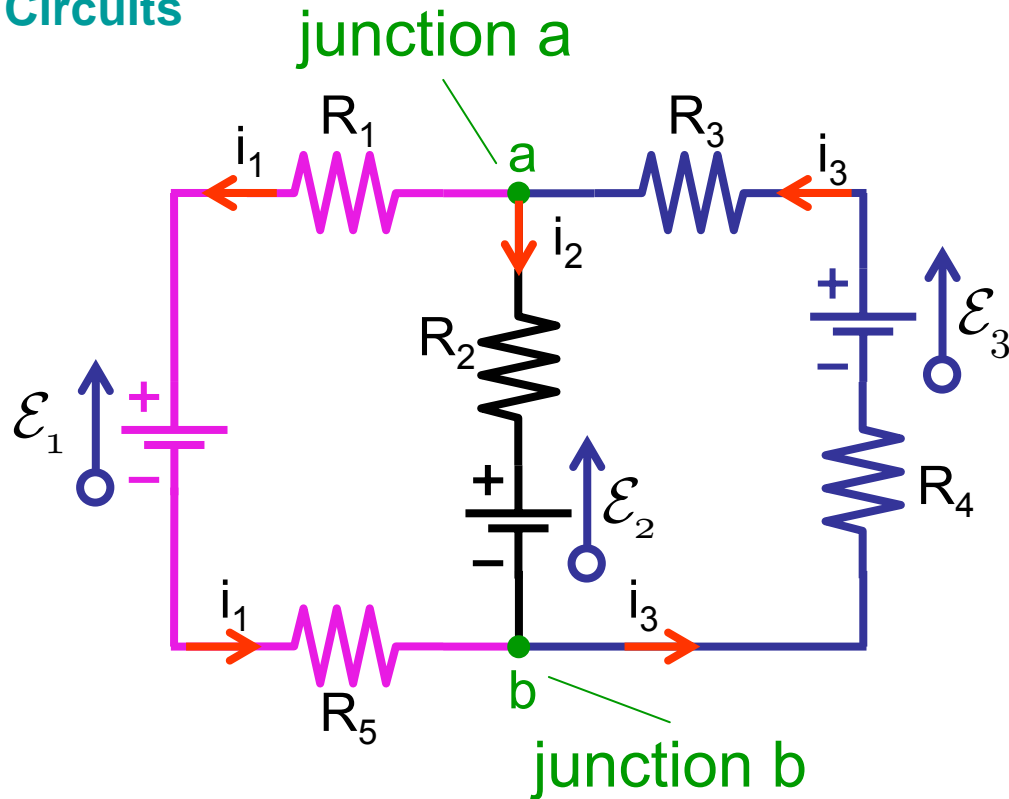
$$i_2 = -0.25 \text{ A}$$

$$i_3 = 0.25 \text{ A}$$

Our guess for the direction of i_2 is wrong, it should be in the opposite direction.

Only after you finish finding all currents in a circuit, you can correct your guesses about directions

28-6 Multiloop Circuits



Junction rule at junction a

$$i_3 = i_1 + i_2$$

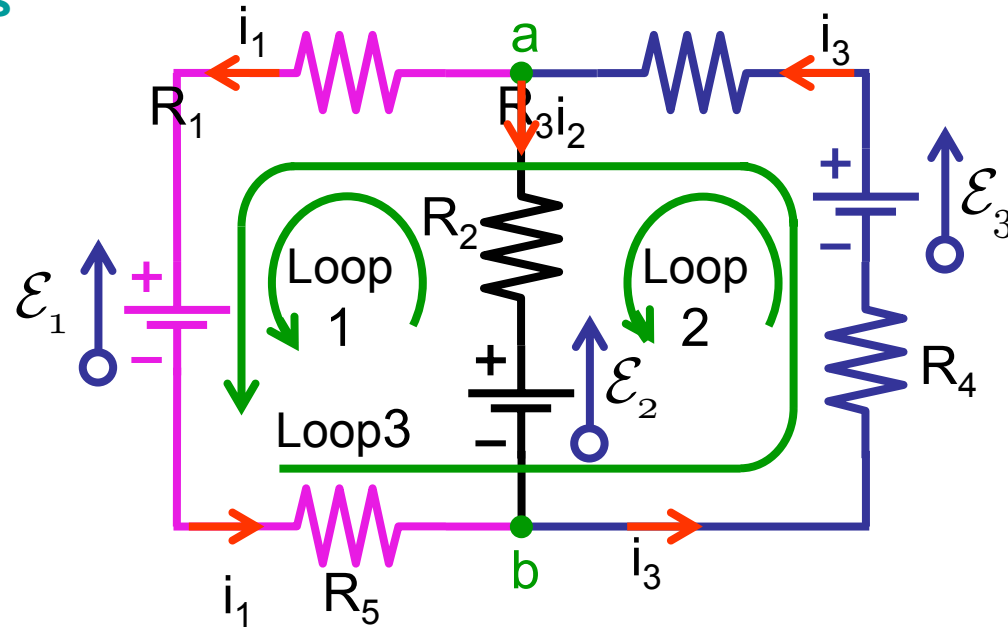
Junction rule at junction b

$$i_1 + i_2 = i_3$$

Same

No new information

28-6 Multiloop Circuits



Loop rule on loop 1

$$-i_1 R_1 - \mathcal{E}_1 - i_1 R_5 + \mathcal{E}_2 + i_2 R_2 = 0$$

Loop rule on loop 2

$$-i_2 R_2 - \mathcal{E}_2 - i_3 R_4 + \mathcal{E}_3 - i_3 R_3 = 0$$

Add the two
equations

$$-i_1 R_1 - \mathcal{E}_1 - i_1 R_5 - i_3 R_4 + \mathcal{E}_3 - i_3 R_3 = 0$$

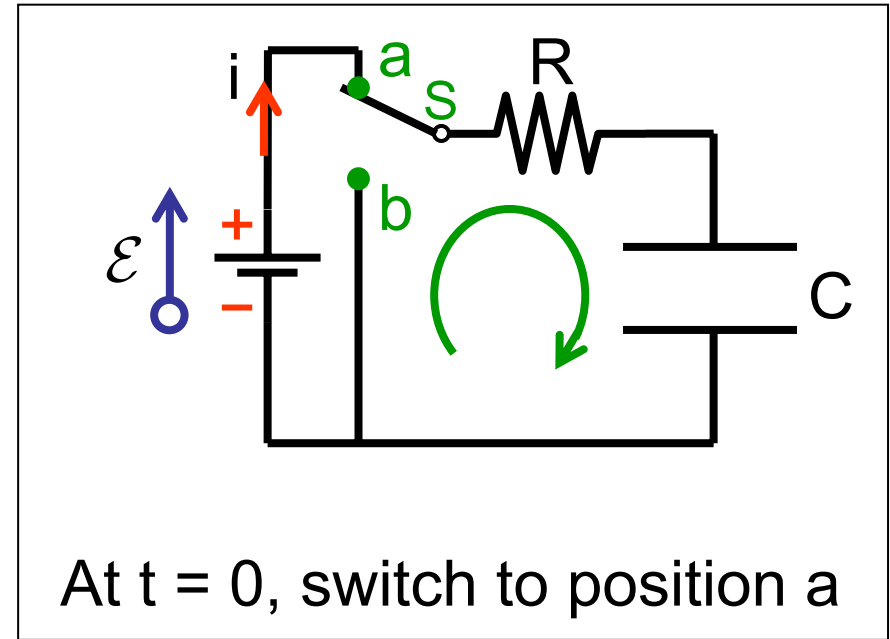
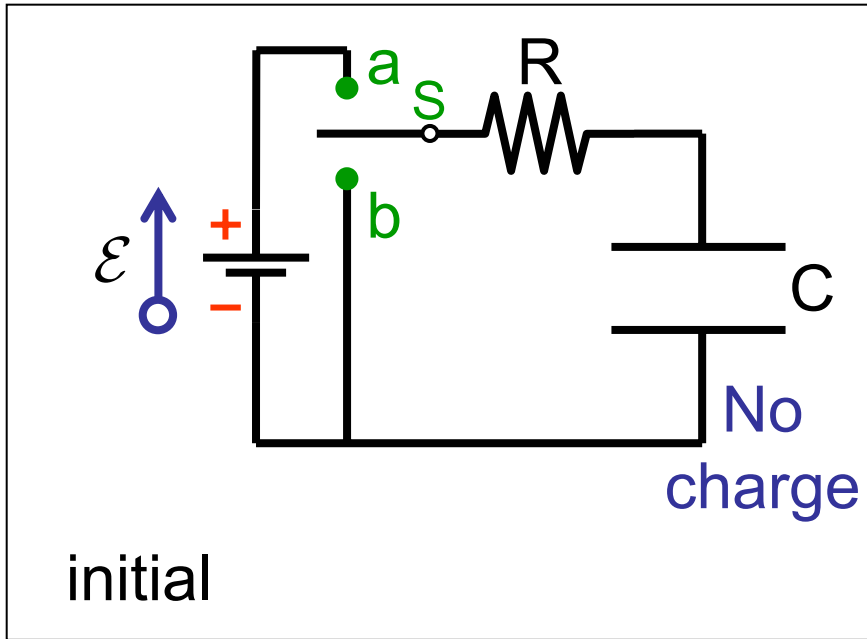
Loop rule on loop 3

Same as loop1 + loop2
No new information

You can use only two of the three loops

28-8 RC Circuits

Charging a capacitor



What is the charge on the capacitor as a function of time?

Loop rule $\mathcal{E} - iR - \frac{q}{C} = 0$

$$\mathcal{E} - \frac{dq}{dt}R - \frac{q}{C} = 0$$

Differential equation

28-8 RC Circuits

Differential equation

$$\mathcal{E} - \frac{dq}{dt} R - \frac{q}{C} = 0$$

At $t = 0$, $q = 0$

At $t = \infty$, $q = C\mathcal{E}$

Solution

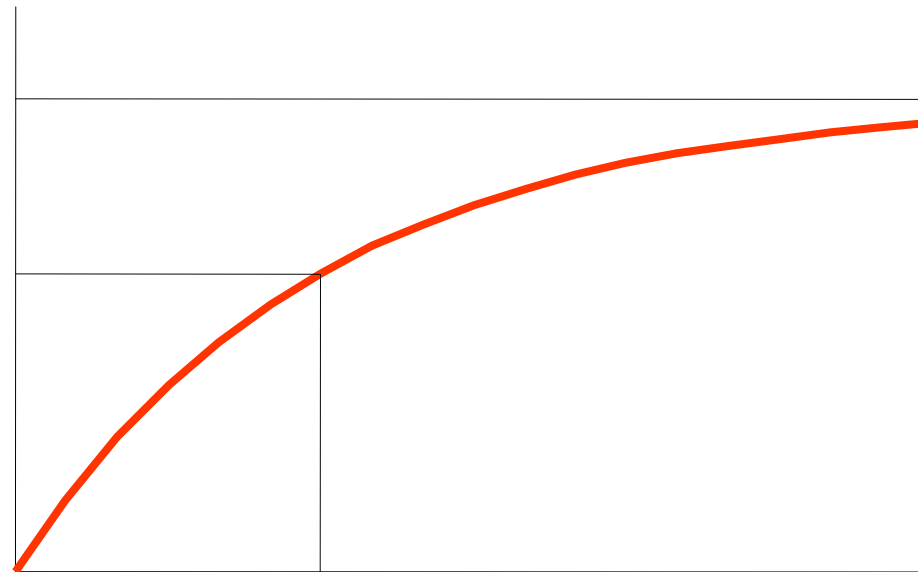
$$q = C\mathcal{E}(1 - e^{-t/RC})$$

Charge q

$$q = C\mathcal{E}$$

$$q = C\mathcal{E}(1 - e^{-1})$$

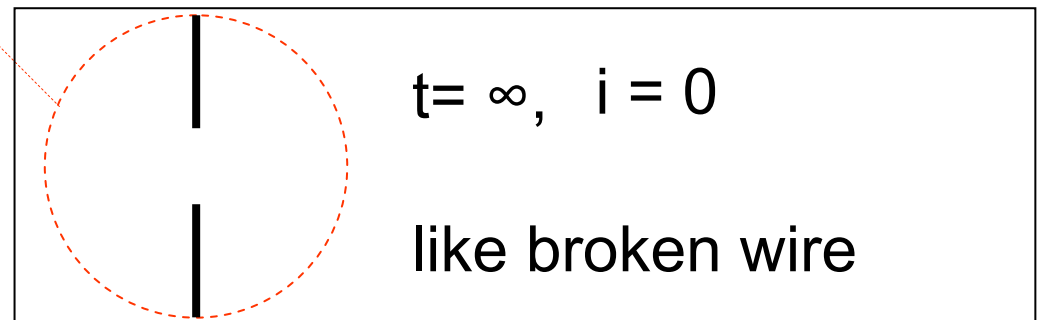
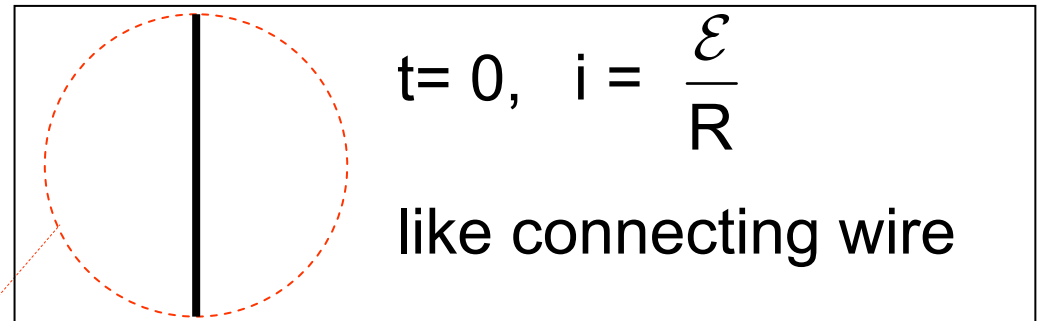
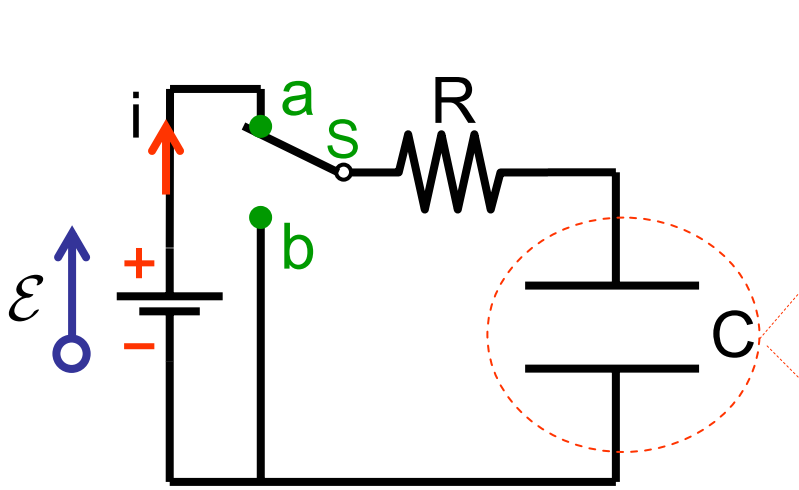
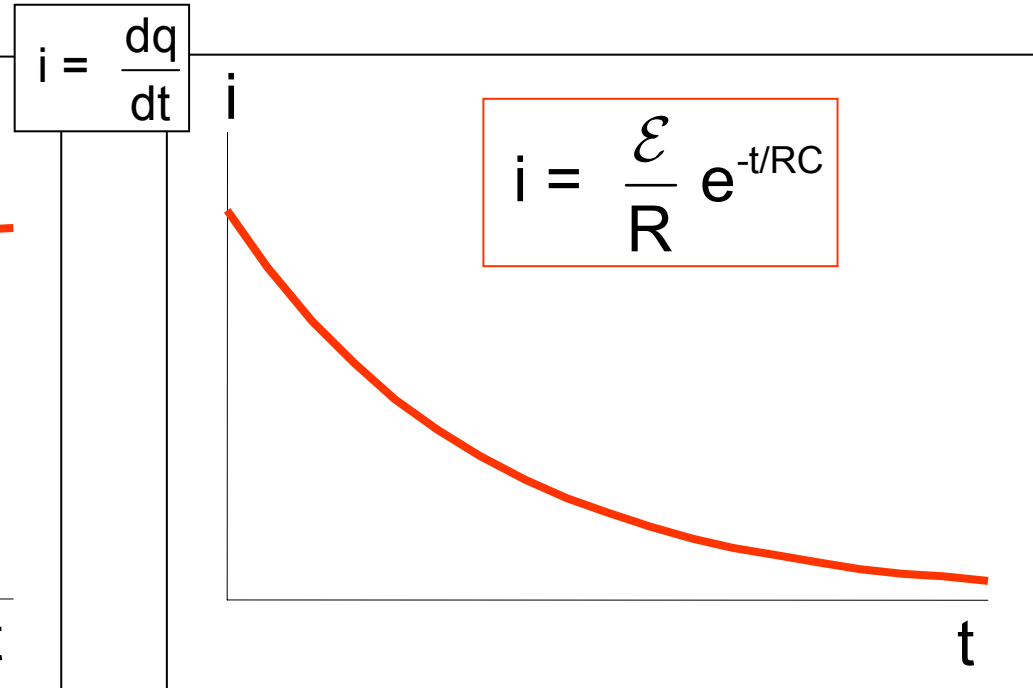
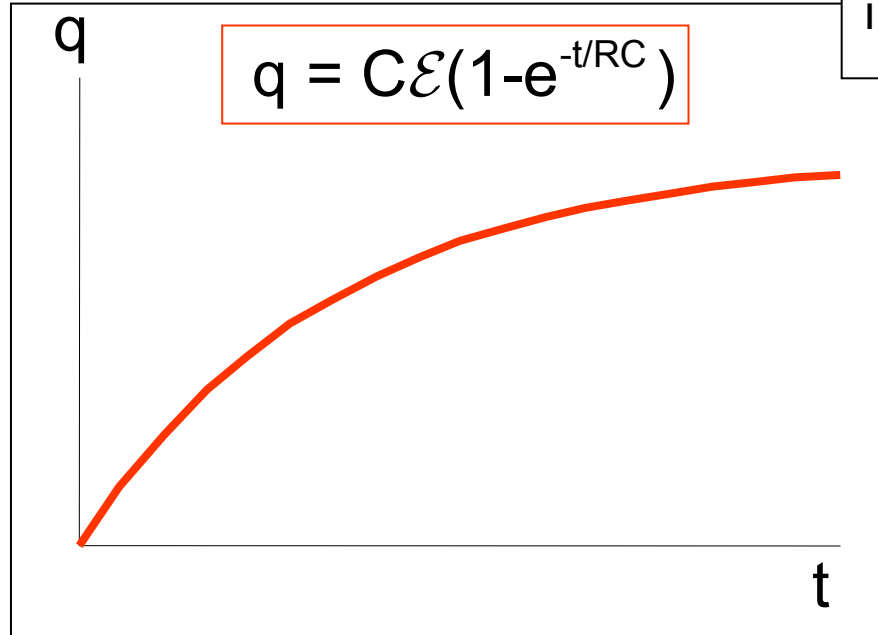
$$= 0.63 C\mathcal{E}$$



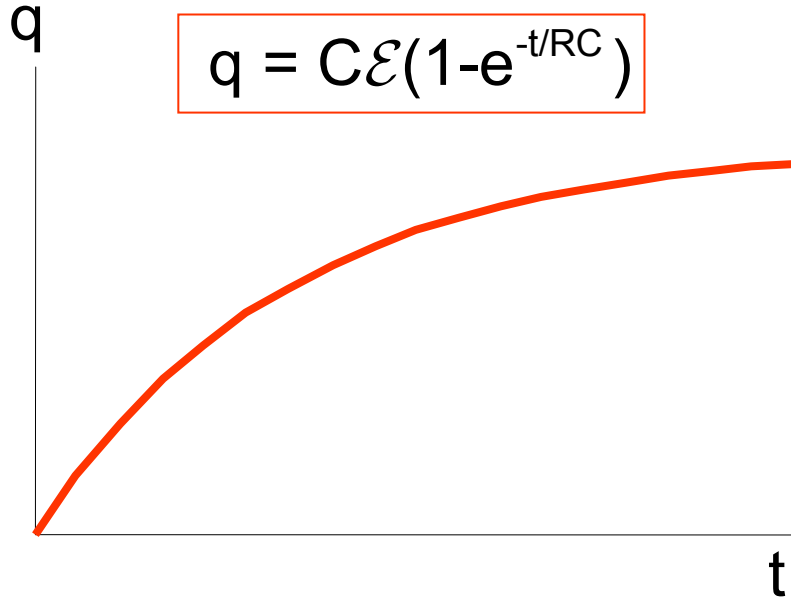
Time t

$$t = RC$$

28-8 RC Circuits

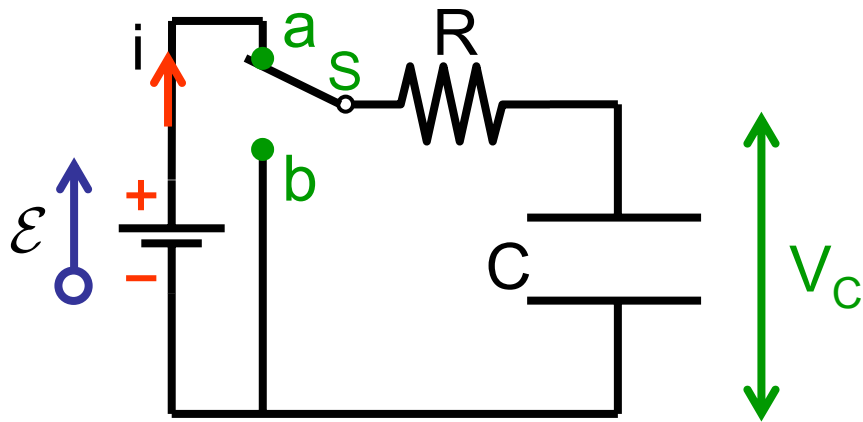


28-8 RC Circuits



$$V = \frac{q}{C}$$

$V_C = \mathcal{E}(1 - e^{-t/RC})$



$$t = 0, V_C = 0$$

$$t = \infty, V_C = \mathcal{E}$$

28-8 RC Circuits

RC has dimension of time

$$RC = \tau$$

RC = capacitive time constant

RC

Dimension like

$$\frac{V}{i} \frac{Q}{V} = \frac{Q}{i} = \frac{Q}{\frac{Q}{t}} = t$$

$$q = C\mathcal{E}(1 - e^{-t/RC}) = C\mathcal{E}(1 - e^{-t/\tau})$$

After one time constant

$$q = C\mathcal{E}(1 - e^{-1}) = 0.63 C\mathcal{E}$$

$$i = \frac{\mathcal{E}}{R} e^{-1} = 0.37 \frac{\mathcal{E}}{R}$$

$$V_C = \mathcal{E}(1 - e^{-1}) = 0.63 \mathcal{E}$$

28-8 RC Circuits

Show that

$$q = C\mathcal{E}(1 - e^{-t/RC})$$

is a solution of $\mathcal{E} - \frac{dq}{dt} R - \frac{q}{C} = 0$

Check the solution

$$\text{At } t = 0, \quad q = C\mathcal{E}(1 - e^{0/RC}) = C\mathcal{E}(1 - 1) = 0$$

$$\text{At } t = \infty, \quad q = C\mathcal{E}(1 - e^{-\infty/RC}) = C\mathcal{E}(1 - 0) = C\mathcal{E}$$

$$\frac{dq}{dt} = C\mathcal{E} \left(\frac{1}{RC} e^{-t/RC} \right) = \frac{\mathcal{E}}{R} e^{-t/RC}$$

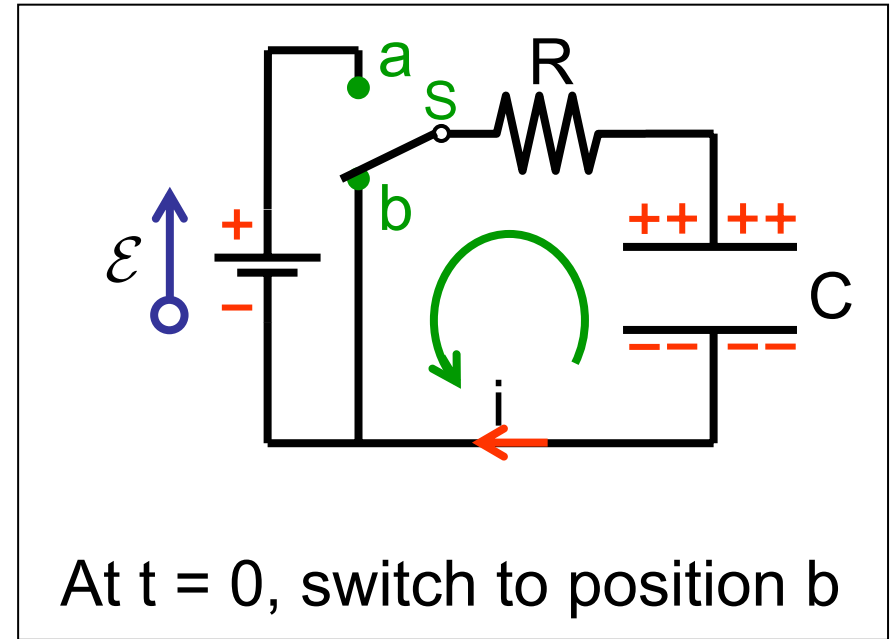
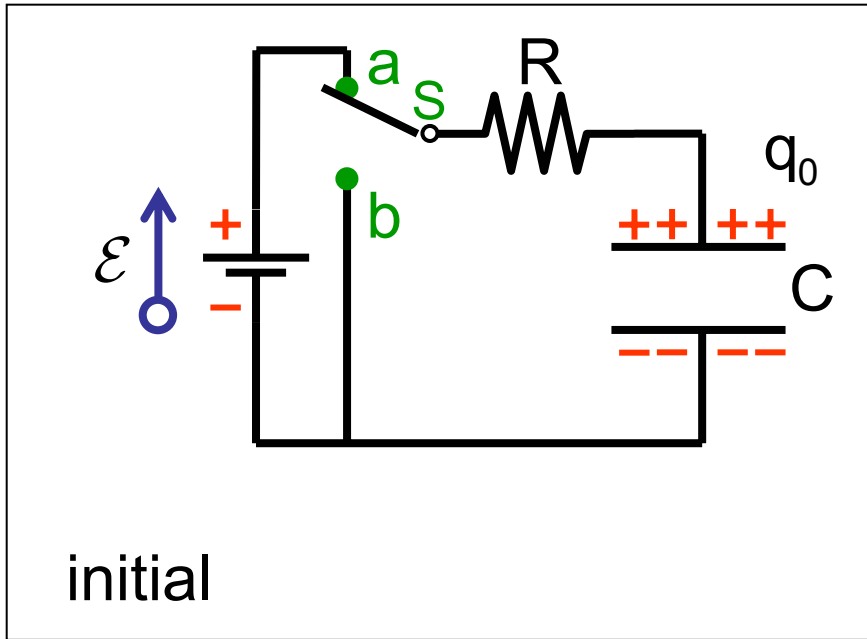
$$\mathcal{E} - \frac{dq}{dt} R - \frac{q}{C}$$

$$= \mathcal{E} - \frac{\mathcal{E}}{R} e^{-t/RC} R - \frac{C\mathcal{E}(1 - e^{-t/RC})}{C}$$

$$= \mathcal{E} - \mathcal{E} e^{-t/RC} - \mathcal{E}(1 - e^{-t/RC}) = 0$$

28-8 RC Circuits

Discharging a capacitor



What is the charge on the capacitor as a function of time?

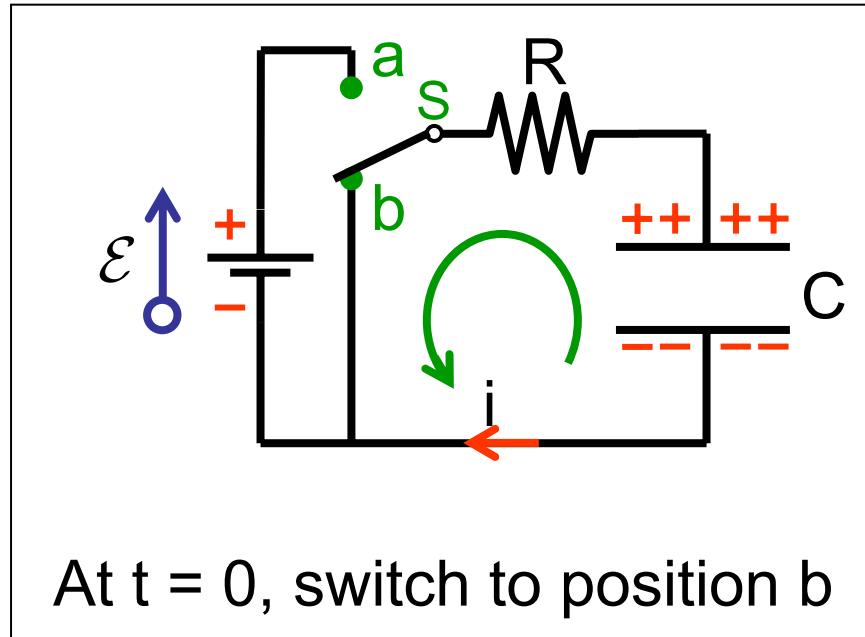
$$\frac{q}{C} + iR = 0$$

$$\frac{q}{C} + \frac{dq}{dt}R = 0$$

Differential equation

28-8 RC Circuits

Discharging a capacitor



Differential equation

$$\frac{dq}{dt} R + \frac{q}{C} = 0$$

$$\text{At } t = 0, \quad q = q_0$$

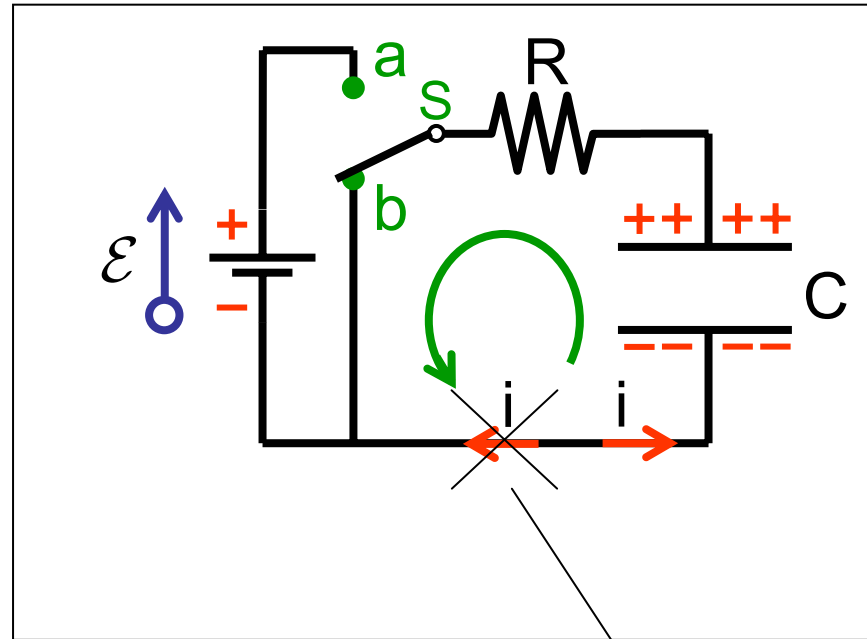
$$\text{At } t = \infty, \quad q = 0$$

Solution

$$q = q_0 e^{-t/RC}$$

28-8 RC Circuits

Discharging a capacitor



$$q = q_0 e^{-t/RC}$$

$$i = \frac{dq}{dt}$$

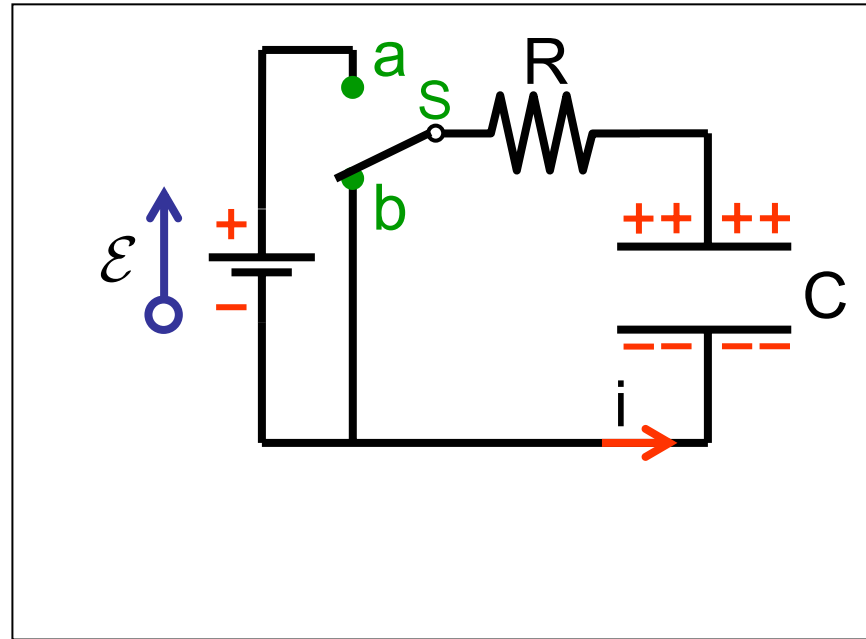
$$i = -\frac{q_0}{RC} e^{-t/RC}$$

Our guess for the direction of the current is wrong

28-8 RC Circuits

$$q = q_0 e^{-t/RC}$$

$$i = \frac{q_0}{RC} e^{-t/RC}$$



After one time constant

$$q = q_0 e^{-1} = 0.37 q_0$$

$$i = - \frac{q_0}{RC} e^{-1} = 0.37 \frac{q_0}{RC}$$

$$V_C = \frac{q_0}{C} e^{-1} = 0.37 \frac{q_0}{C}$$

28-8 RC Circuits

Sample Problem 28-5

In terms of RC , when will the charge on the capacitor be half its initial value?

$$q = q_0 e^{-t/RC}$$

$$\frac{1}{2} q_0 = q_0 e^{-t/RC}$$

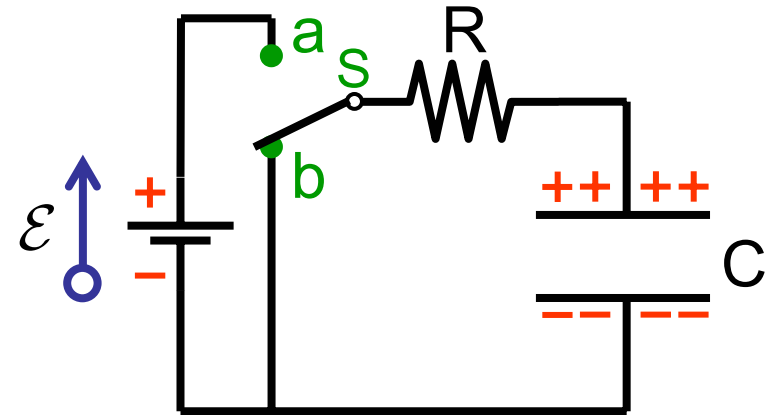
$$\frac{1}{2} = e^{-t/RC}$$

$$\ln\left(\frac{1}{2}\right) = \ln(e^{-t/RC})$$

$$\ln(2) = \frac{t}{RC}$$

$$t = \ln(2)RC = 0.69 RC = 0.69 \tau$$

Discharging a capacitor



At $t = 0$, switch to position b

28-8 RC Circuits

Sample Problem 28-5

In terms of RC , when will the energy stored in the capacitor be half its initial value?

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2C} e^{-2t/RC} = U_0 e^{-2t/RC}$$

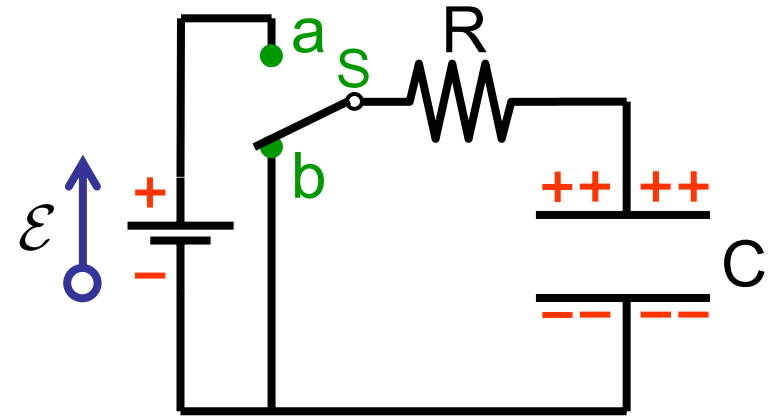
$$\frac{1}{2} U_0 = U_0 e^{-2t/RC}$$

$$\frac{1}{2} = e^{-2t/RC}$$

$$\ln(2) = \frac{2t}{RC}$$

$$t = \frac{\ln(2)}{2} RC = 0.35 RC = 0.35 \tau$$

Discharging a capacitor



At $t = 0$, switch to position b

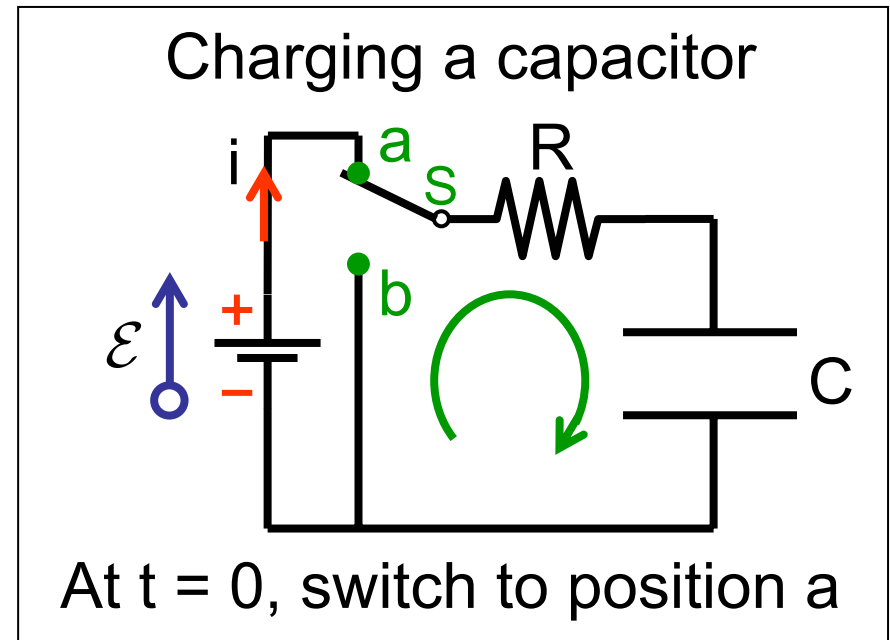
28-8 RC Circuits

Checkpoint 5

Rank the sets according to ...

A - initial current

B - time required for the current to decrease to its half its initial value



$\mathcal{E}(\text{V})$	$R(\Omega)$	$C(\mu\text{F})$	A	B
12	2	3	1	2
12	3	2	2	2
10	10	0.5	4	4
10	5	2	3	1

$$i = \frac{\mathcal{E}}{R}$$

$$t = 0.69 RC$$