Chapter 26

Capacitance

26-1 The Uses of Capacitors

A Capacitor

is a device that stores electric potential energy

Uses

Many applications

Computer memory

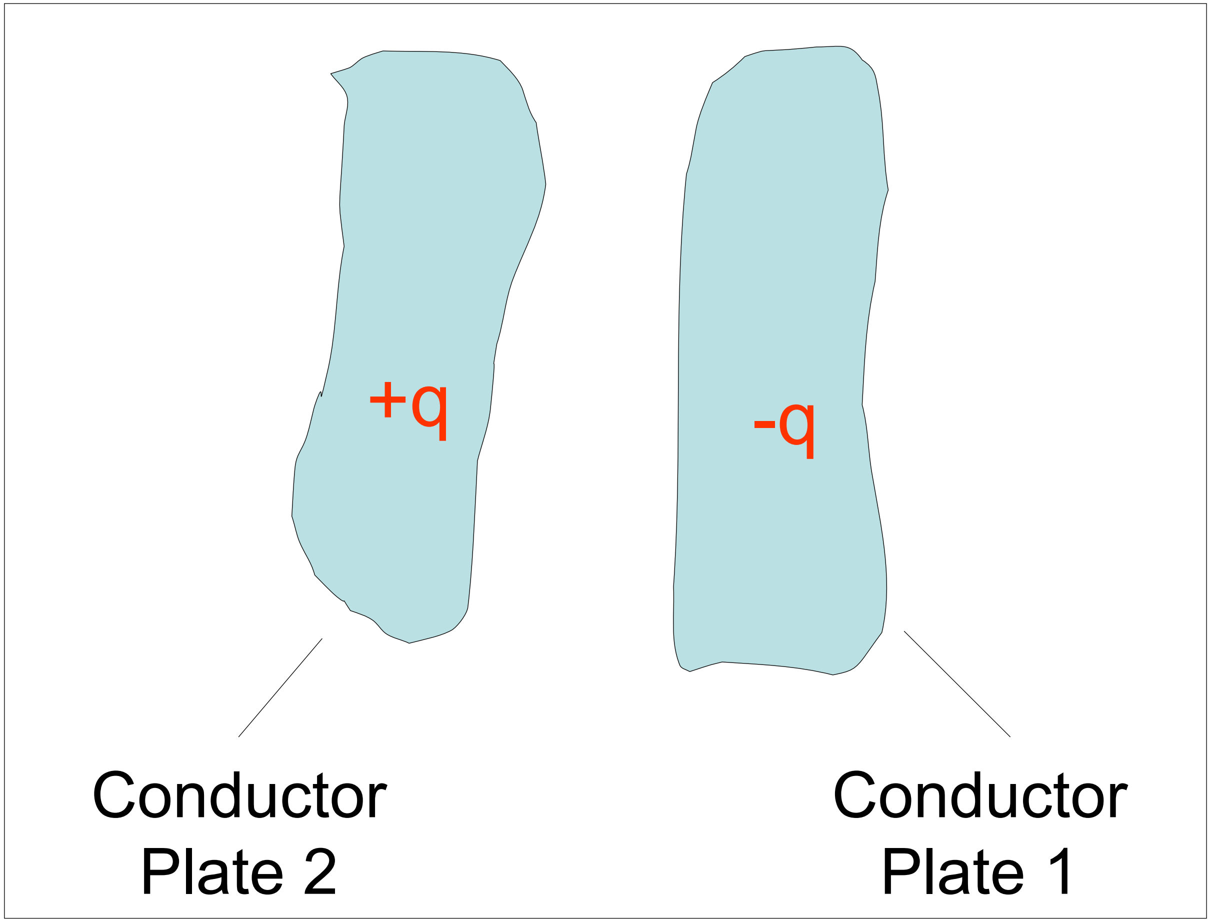
Tune radio and TV transmitter
and receiver

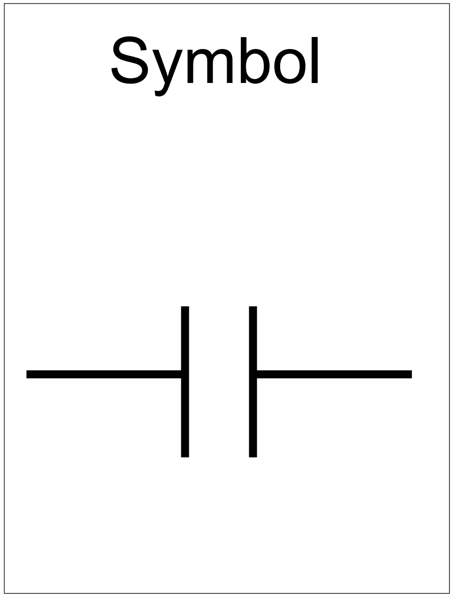
Operate photoflash unit

26-2 Capacitance

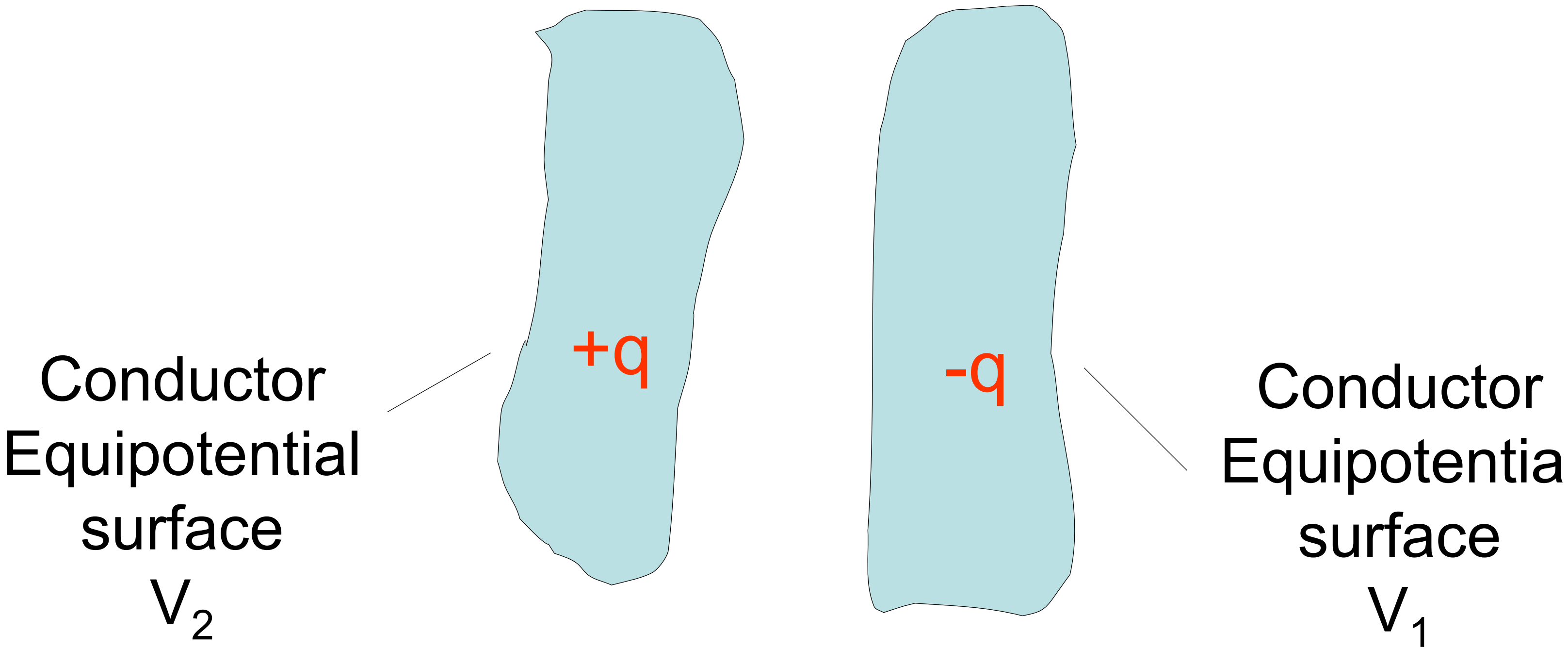
A Capacitor

Is any two isolated conductors of any shape





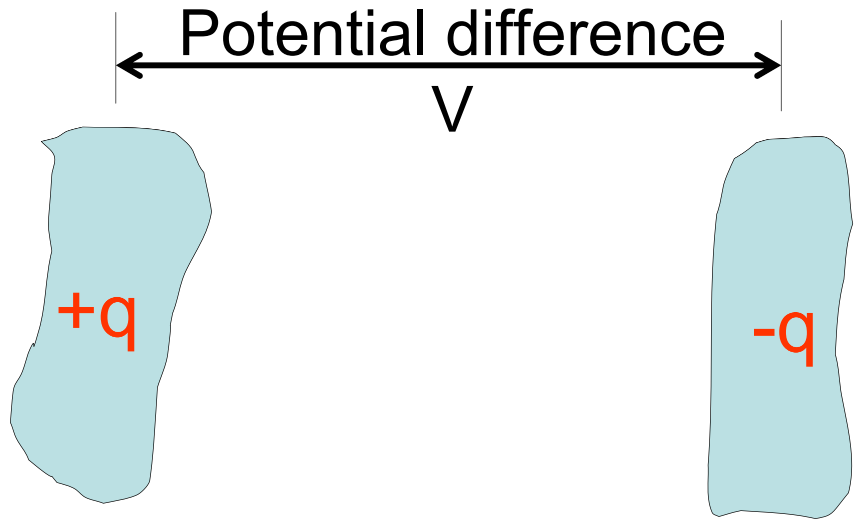
26-2 Capacitance



Potential difference between the two plates $\Delta V = V_2 - V_1$

ΔV — In this chapter, we call it → V

26-2 Capacitance



$$q = C\,V$$

Absolute value of the charge on one plate

Capacitance of the capacitor
Proportionality constant

The capacitance of a capacitor indicates how much charge on the pates is needed to produce a certain potential difference

26-2 Capacitance

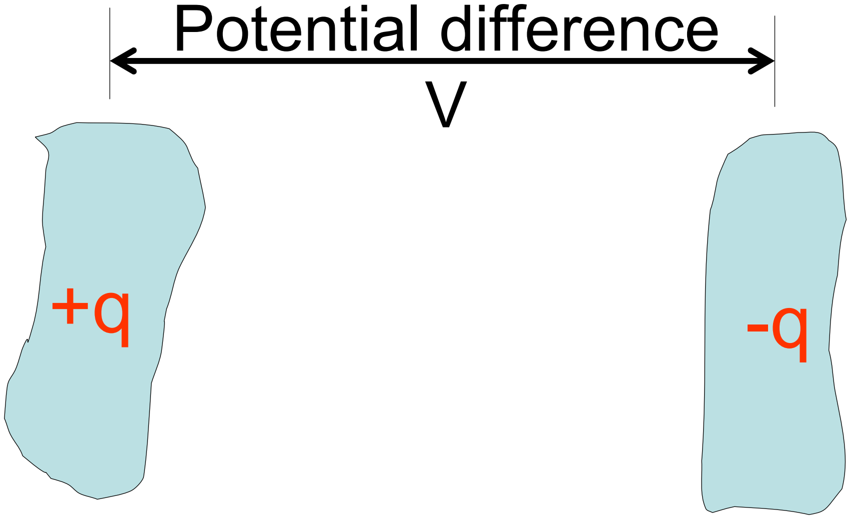
SI unit for capacitance

Farad

$$q = C\,V$$

$$1\ \text{Farad} = \frac{1\ \text{Coulomb}}{1\ \text{Volt}}$$

26-2 Capacitance



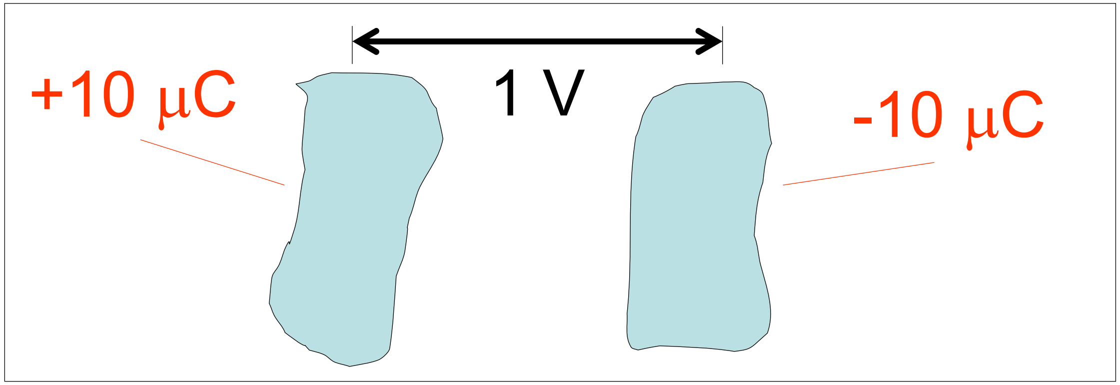
$$q = C\,V$$

The capacitance of a capacitor does not depend on the potential difference nor on the charge of the capacitor

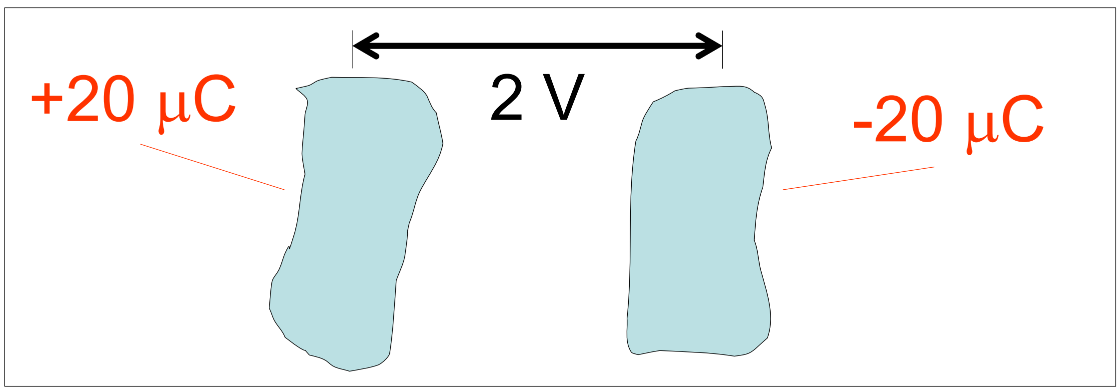
The capacitance depends only on the geometry (shape) of the capacitor

26-2 Capacitance

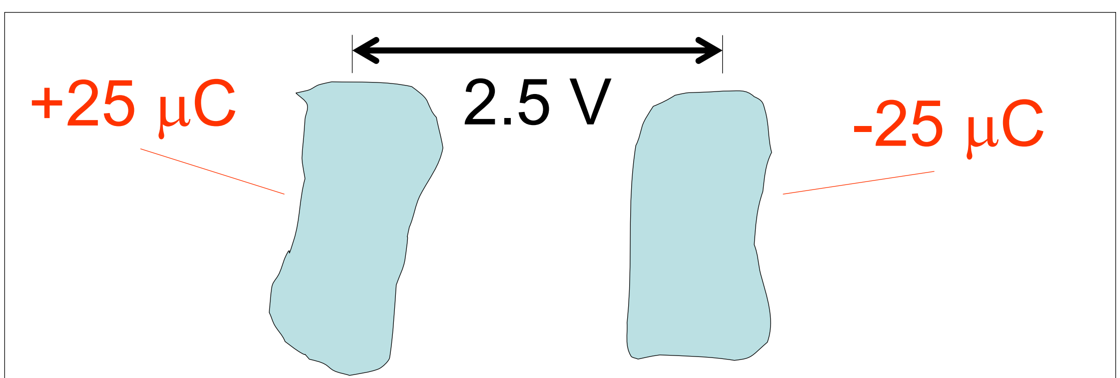
The capacitance depends only on the geometry of the capacitor



$$C = \frac{10\ \mu C}{1\ V} = 10\ \mu F$$



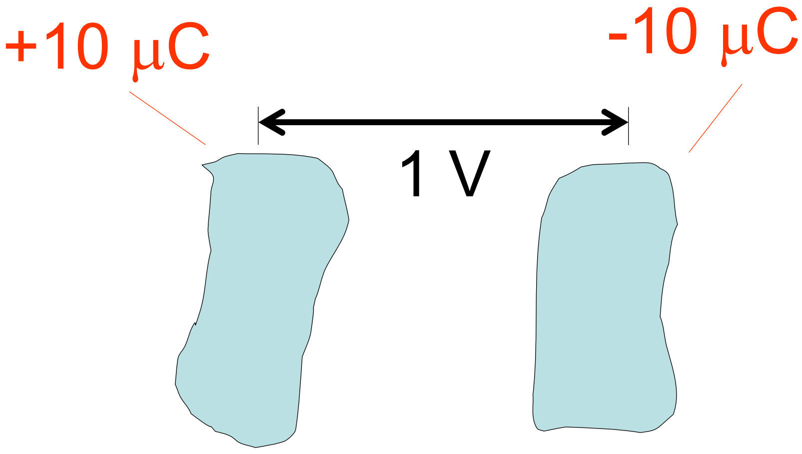
$$C = \frac{20\ \mu C}{2\ V} = 10\ \mu F$$



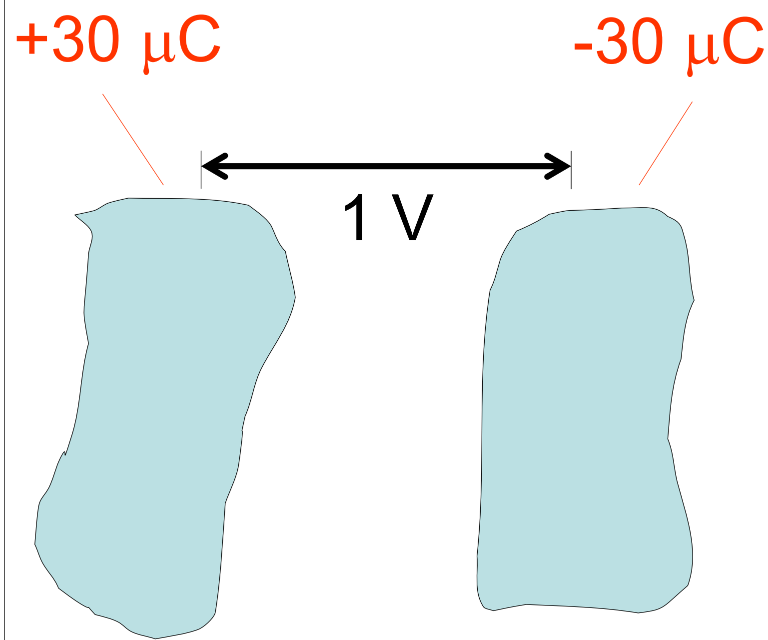
$$C = \frac{25\ \mu C}{2.5\ V} = 10\ \mu F$$

26-2 Capacitance

The capacitance depends only on the geometry of the capacitor



$$C = \frac{10\ \mu C}{1\ V} = 10\ \mu F$$

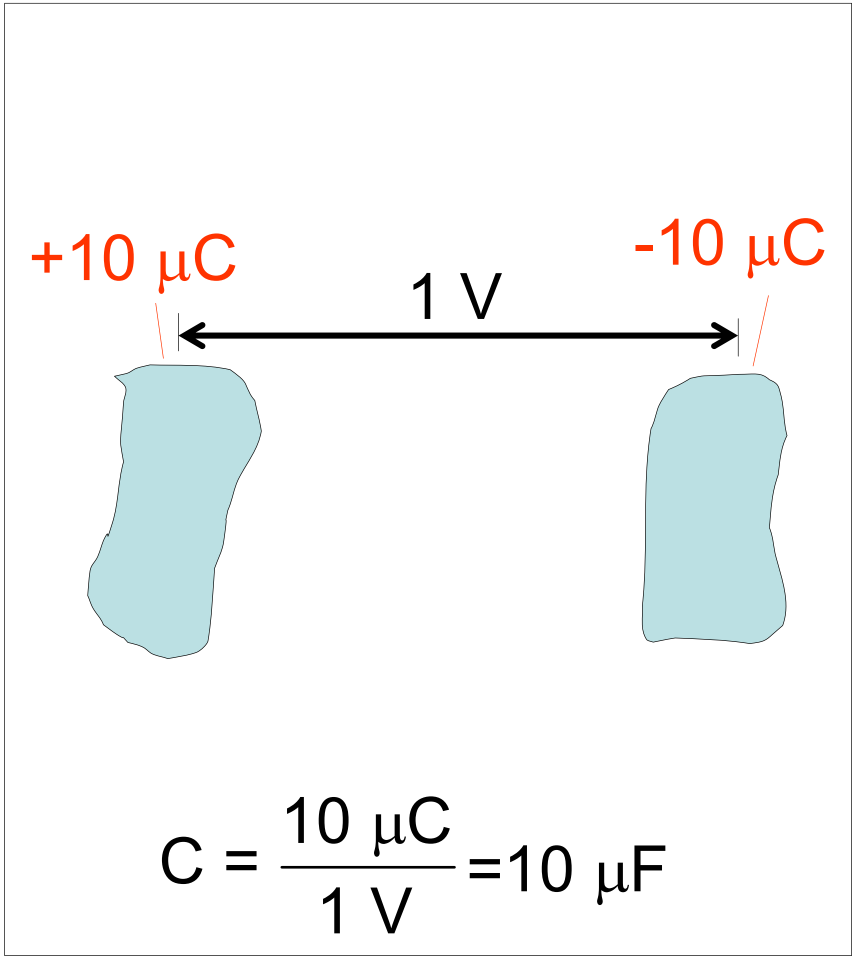


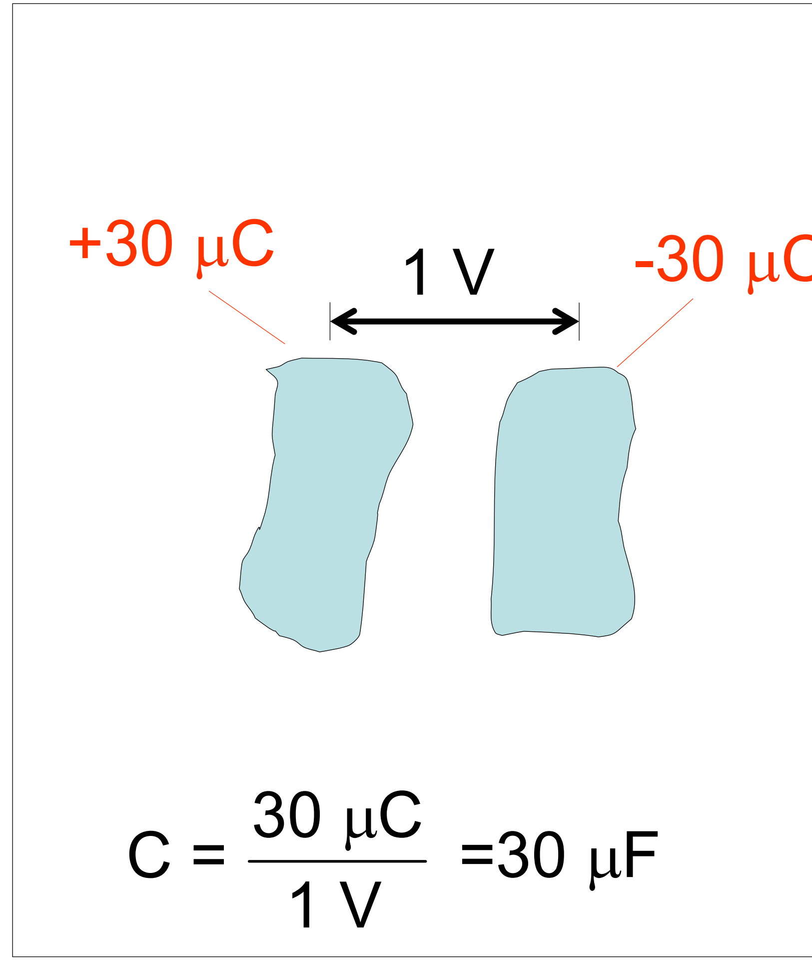
$$C = \frac{30\ \mu C}{1\ V} = 30\ \mu F$$

Larger plates area → Larger capacitance

26-2 Capacitance

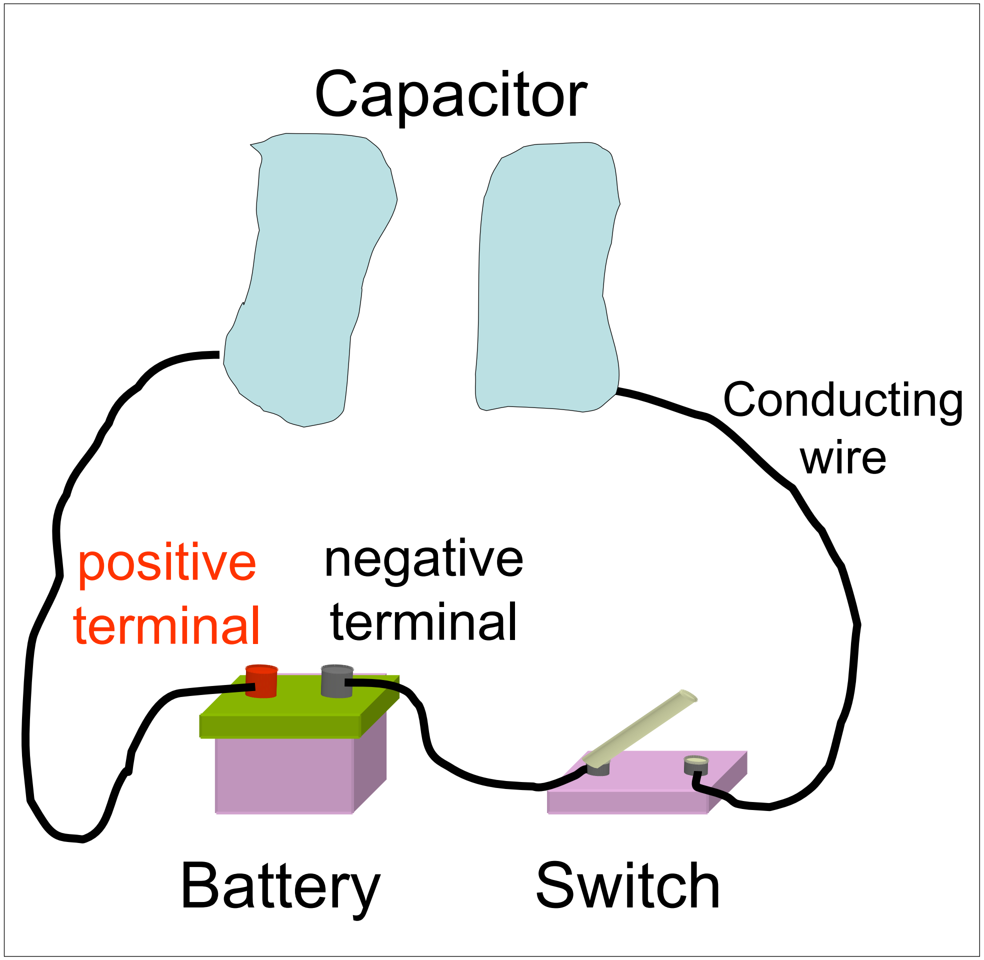
The Capacitance depends only on the geometry of the capacitor

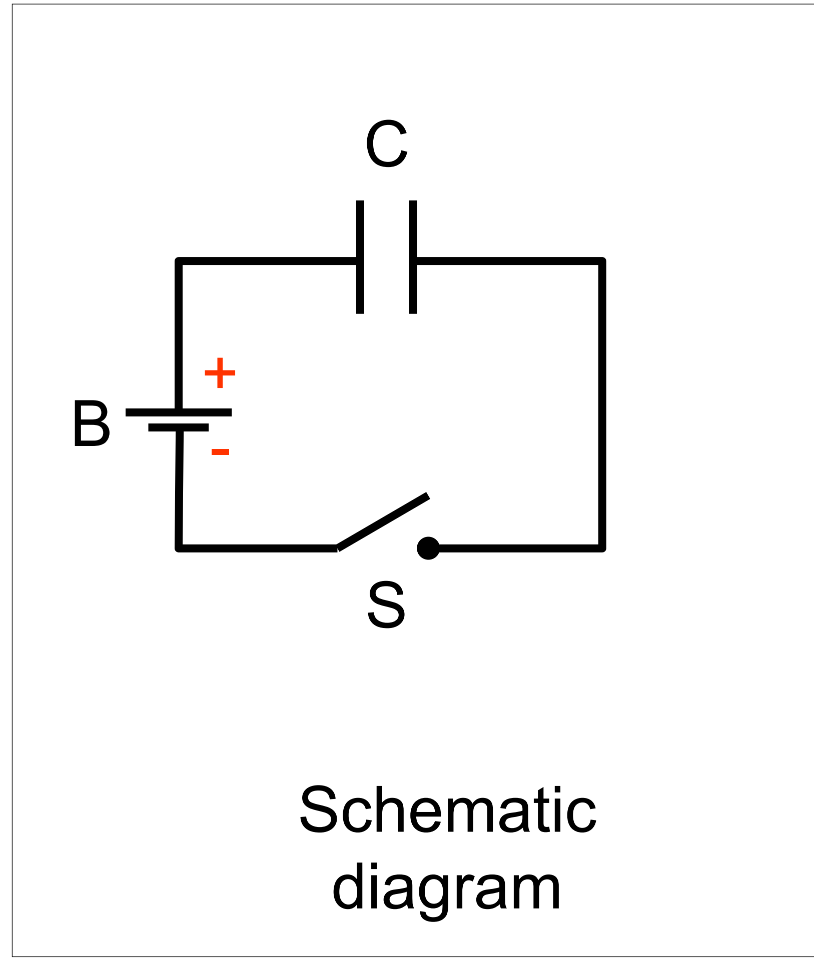




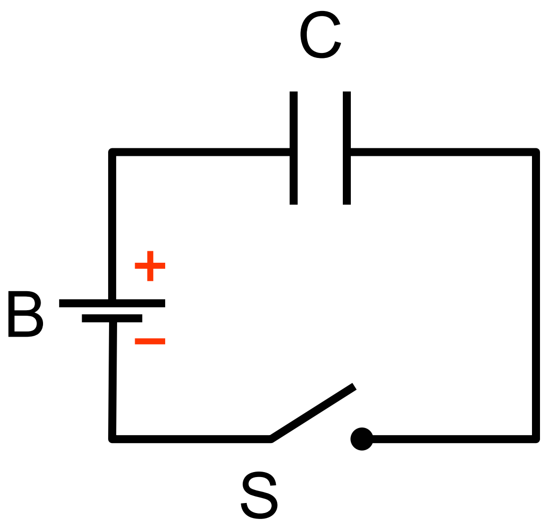
Smaller distance between plates → Larger capacitance

26-3 Capacitance

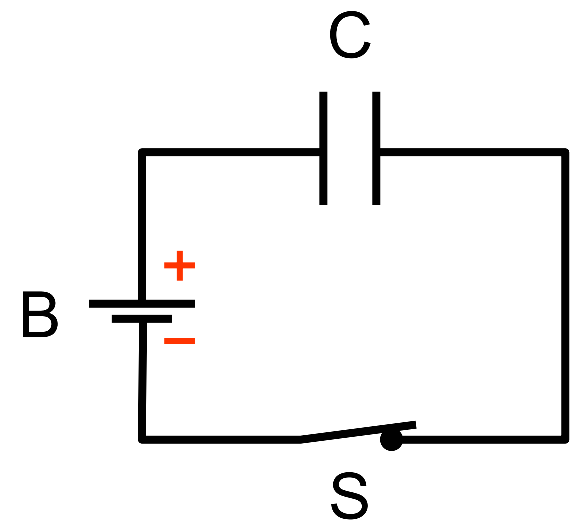




26-3 Capacitance



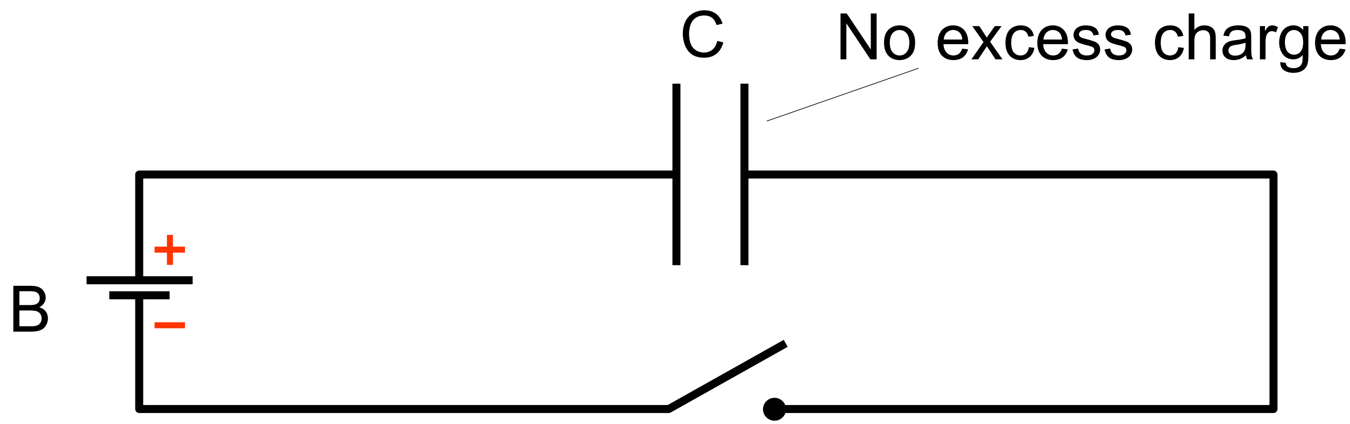
Switch is open
Circuit is incomplete



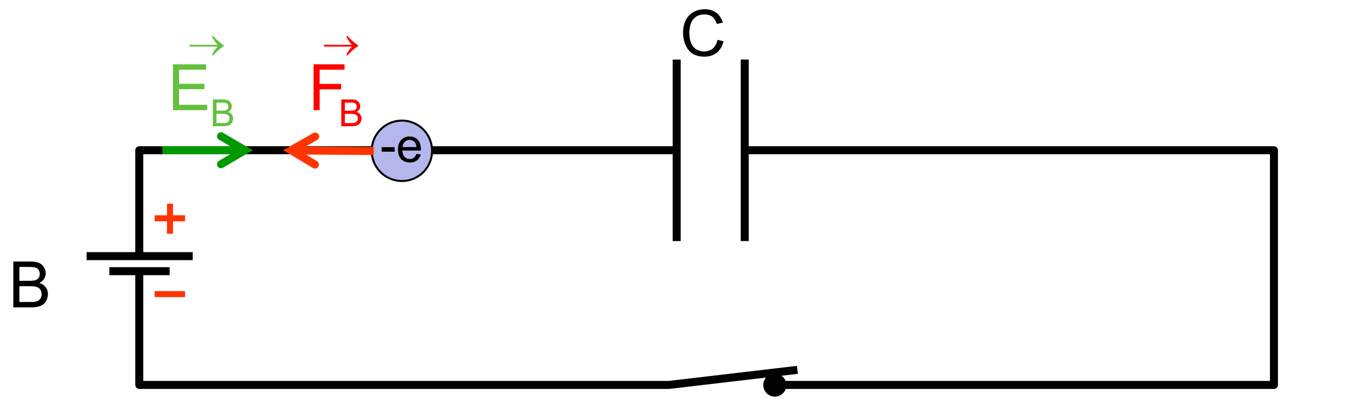
Switch is closed
Circuit is complete
Electrons can flow from one capacitor plate to another through the wires

26-2 Capacitance

Charging a capacitor



At the beginning

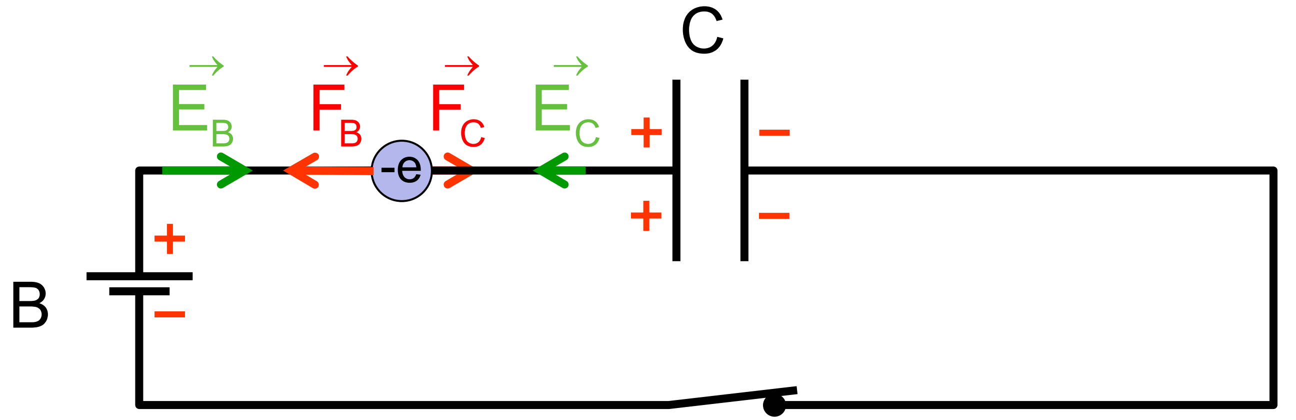


Just after closing the switch, electrons start flowing

because of the electric field of the battery

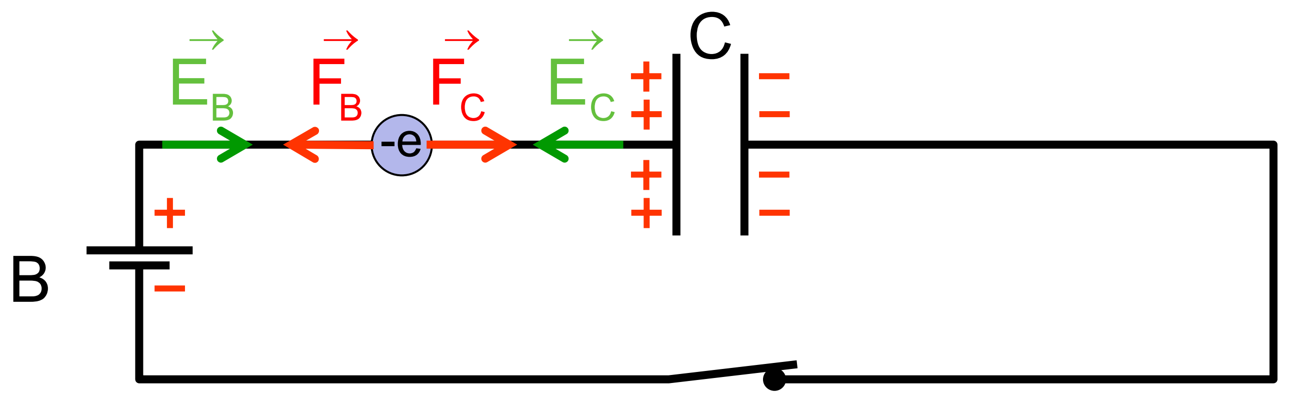
26-2 Capacitance

Charging a capacitor



After some time, electrons continue flowing

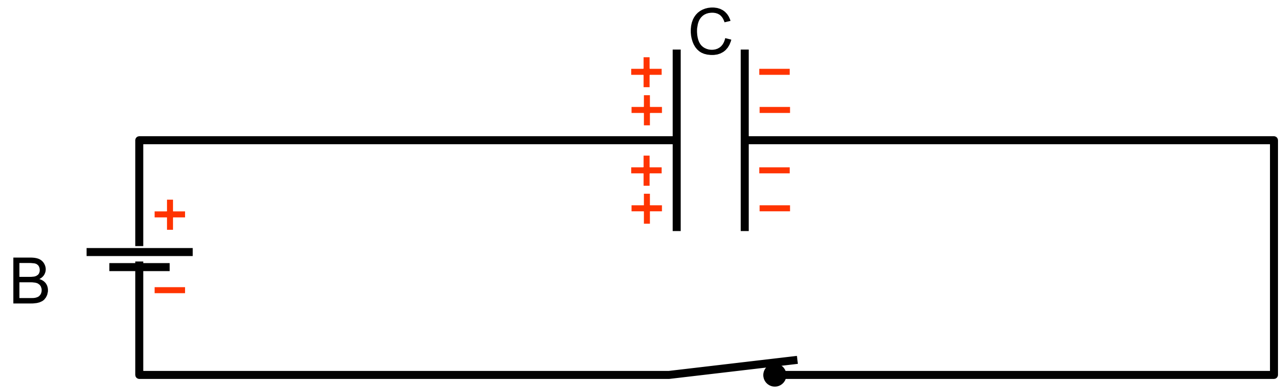
Electric field due to the potential difference between the plates increases



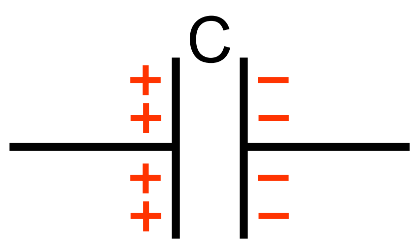
After very long time, the capacitor is fully charged

Electrons stop flowing when the electric fields from the battery and capacitor are equal

26-2 Capacitance



After very long time, the capacitor is fully charged



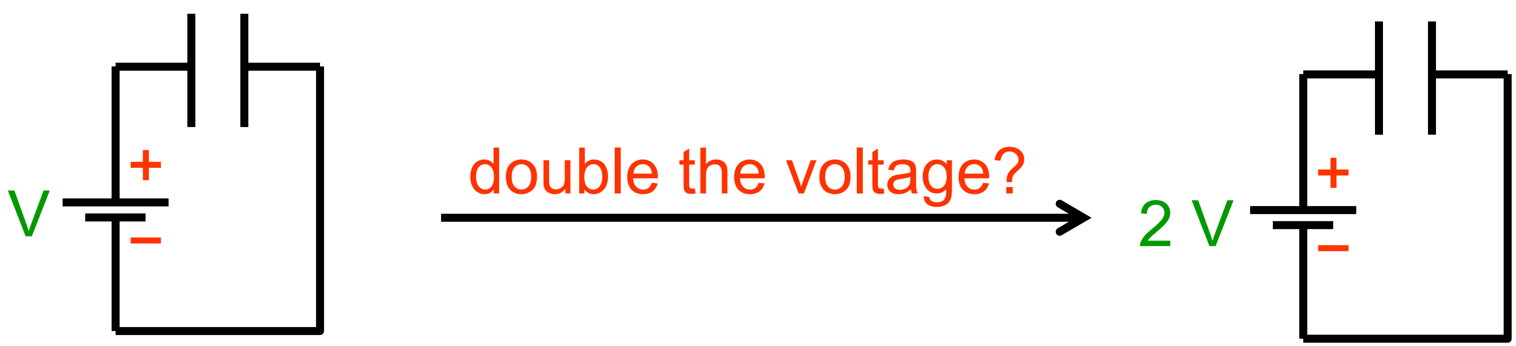
When the battery is disconnected, the charges remain on the capacitor

26-2 Capacitance

Checkpoint 1

What happens to the capacitance if you …

Capacitance remains the same



Capacitance remains the same

26-3 Calculating the Capacitance

We can calculate the capacitance of a capacitor form its shape

We will find the capacitance of

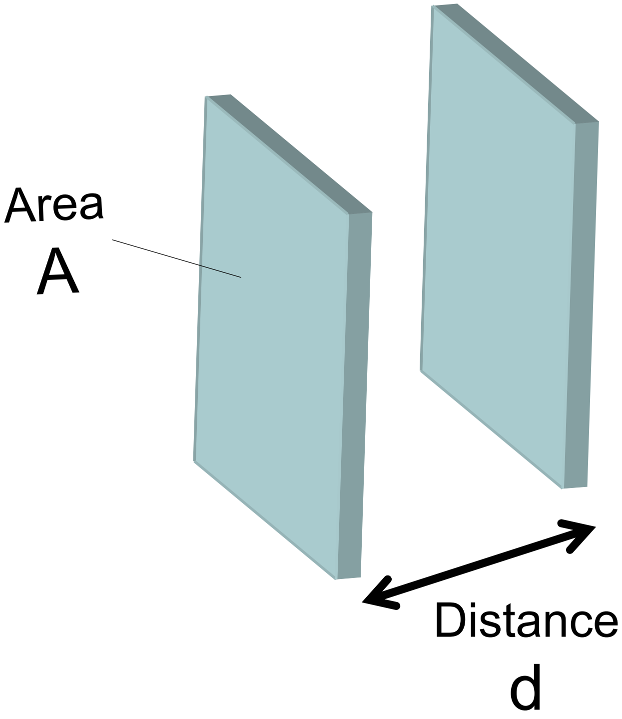
Parallel-plate capacitor

Cylindrical capacitor

Spherical capacitor

26-3 Calculating the Capacitance

Parallel-plate capacitor



$$C = \frac{\varepsilon_0 A}{d}$$

26-3 Calculating the Capacitance

Derivation of $C = \dfrac{\varepsilon_0 A}{d}$

Assume there is charge q on the capacitor plates

$$C = \frac{q}{V}$$

$$V = -\int_i^f \vec{E} \cdot \vec{ds}$$

Gauss' Law

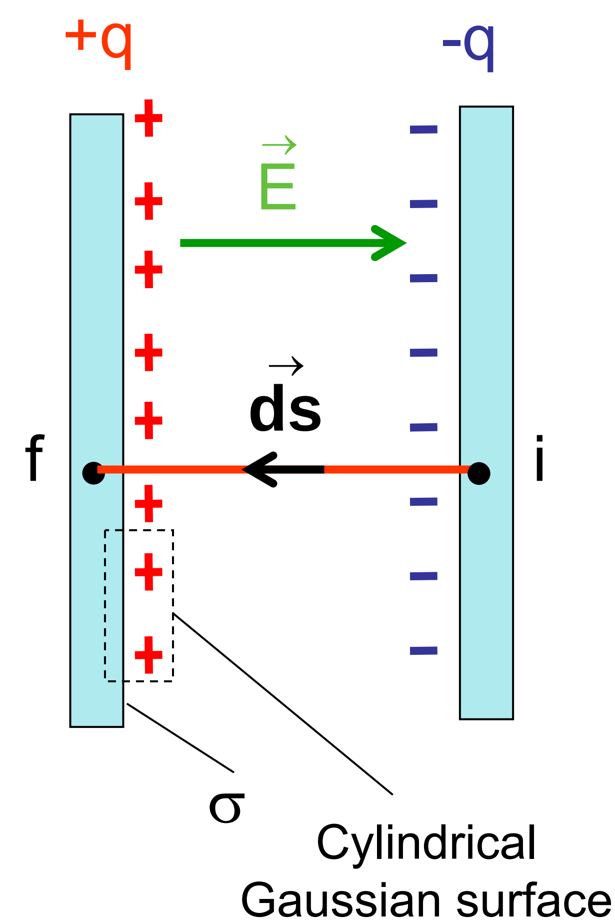
$$\varepsilon_0 (E A_G) = \sigma A_G$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{q}{A}$$

$$V = E d = \frac{1}{\varepsilon_0}\frac{q}{A} d$$

$$C = \frac{q}{V} = \frac{q}{\frac{1}{\varepsilon_0}\frac{q}{A} d} = \frac{\varepsilon_0 A}{d}$$

Cross section

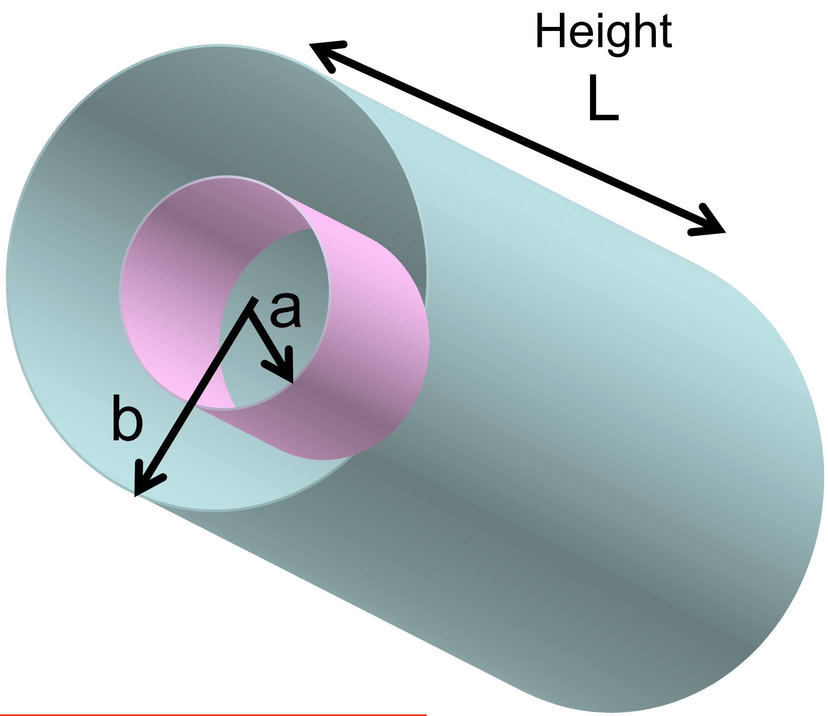


Cap area = A_G

Flux = $E A_G$

26-3 Calculating the Capacitance

Cylindrical capacitor



$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

26-3 Calculating the Capacitance

Derivation of $C = 2\pi\varepsilon_0 \dfrac{L}{\ln(b/a)}$

Assume there is charge q on the capacitor plates

$$C = \frac{q}{V}$$

$$V = -\int_i^f \vec{E} \cdot \vec{ds}$$

Gauss' Law

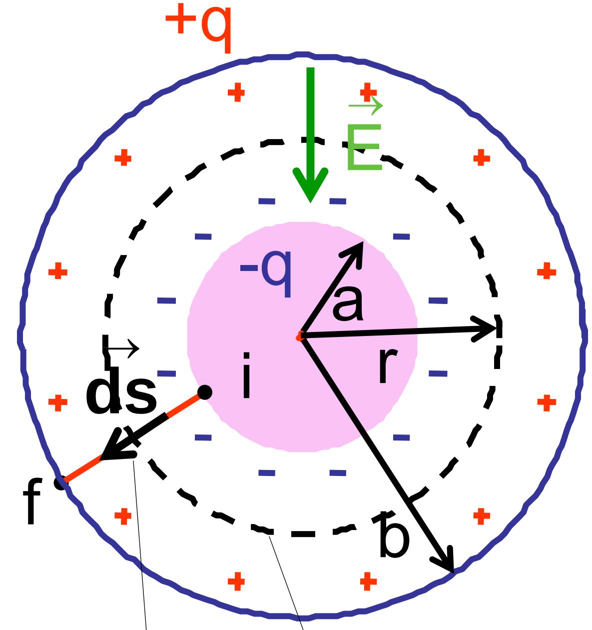
$$\varepsilon_0 (E\, 2\, \pi\, r\, L) = q$$

$$E = \frac{q}{2\, \pi\, \varepsilon_0\, r\, L}$$

$$V = \int_a^b E\, dr = \int_a^b \frac{q}{2\, \pi\, \varepsilon_0\, r\, L} dr = \frac{q}{2\, \pi\, \varepsilon_0\, L} \ln(b/a)$$

$$C = \frac{q}{V} = \frac{q}{\dfrac{q}{2\, \pi\, \varepsilon_0\, L} \ln(b/a)} = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

Cross section



ds=dr

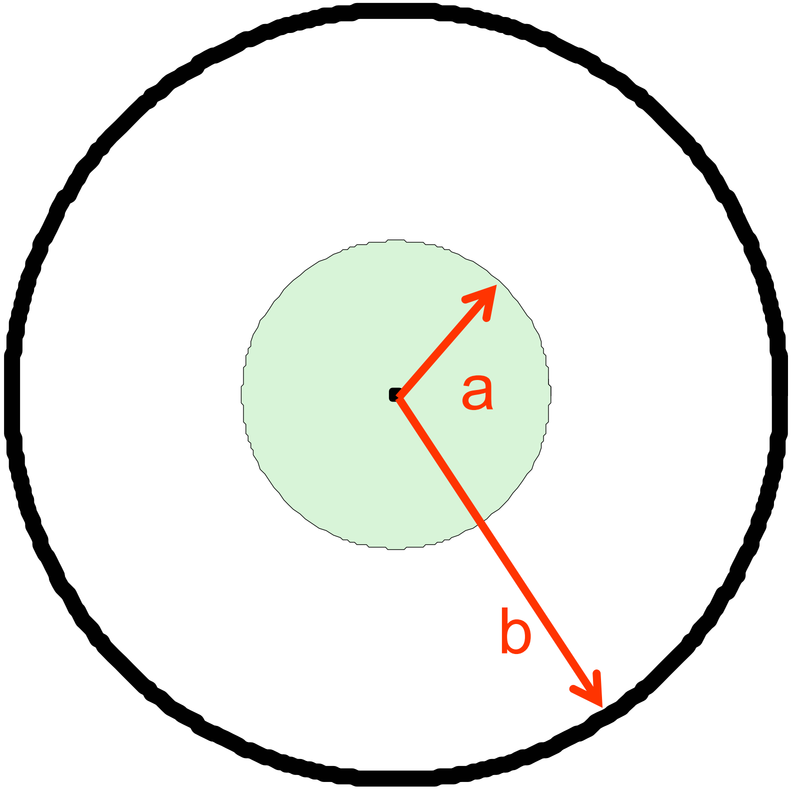
Cylindrical Gaussian surface

Radius = r Height = L

Flux = E (2 π r L)

26-3 Calculating the Capacitance

Spherical capacitor



$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

26-3 Calculating the Capacitance

Derivation of $C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$

Assume there is charge q on the capacitor plates

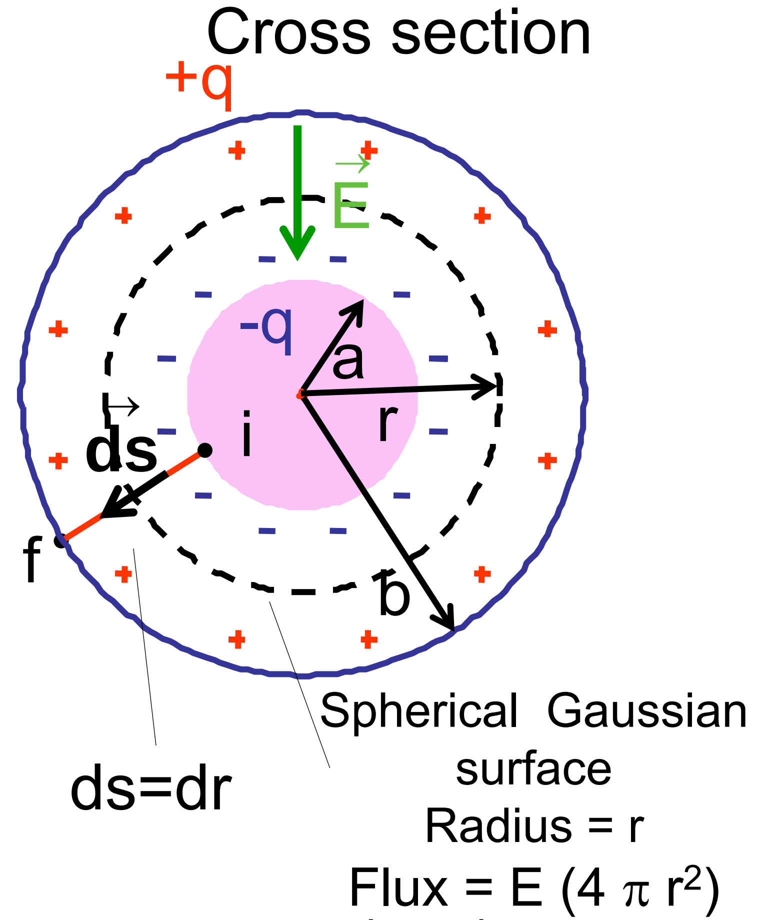
$$C = \frac{q}{V}$$

$$V = -\int_i^f \vec{E} \bullet \vec{ds}$$

Gauss' Law

$$\varepsilon_0 (E\, 4\pi r^2) = q$$

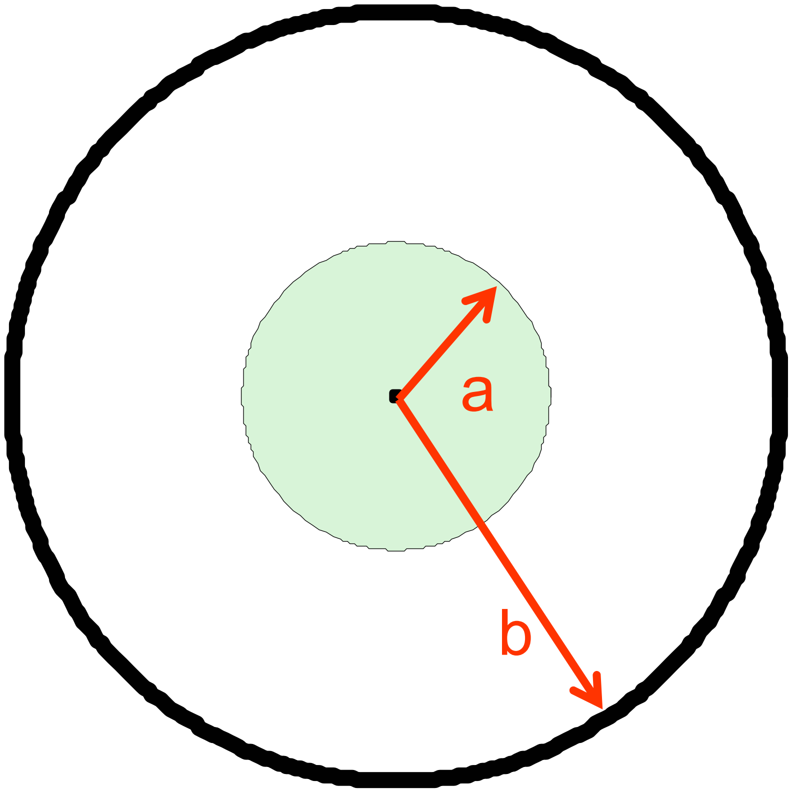
$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$



$$V = \int_a^b E\, dr = \int_a^b \frac{q}{4\pi\varepsilon_0 r^2} dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

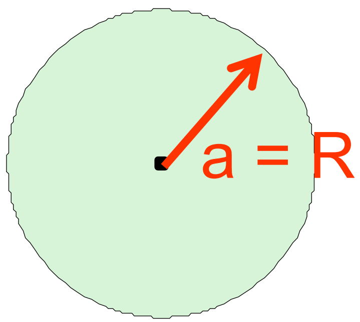
$$C = \frac{q}{V} = \frac{q}{\dfrac{q}{4\pi\varepsilon_0}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)} = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

26-3 Calculating the Capacitance



$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Isolated Sphere



$$b \rightarrow \infty$$

$$C = 4\pi\varepsilon_0 a$$

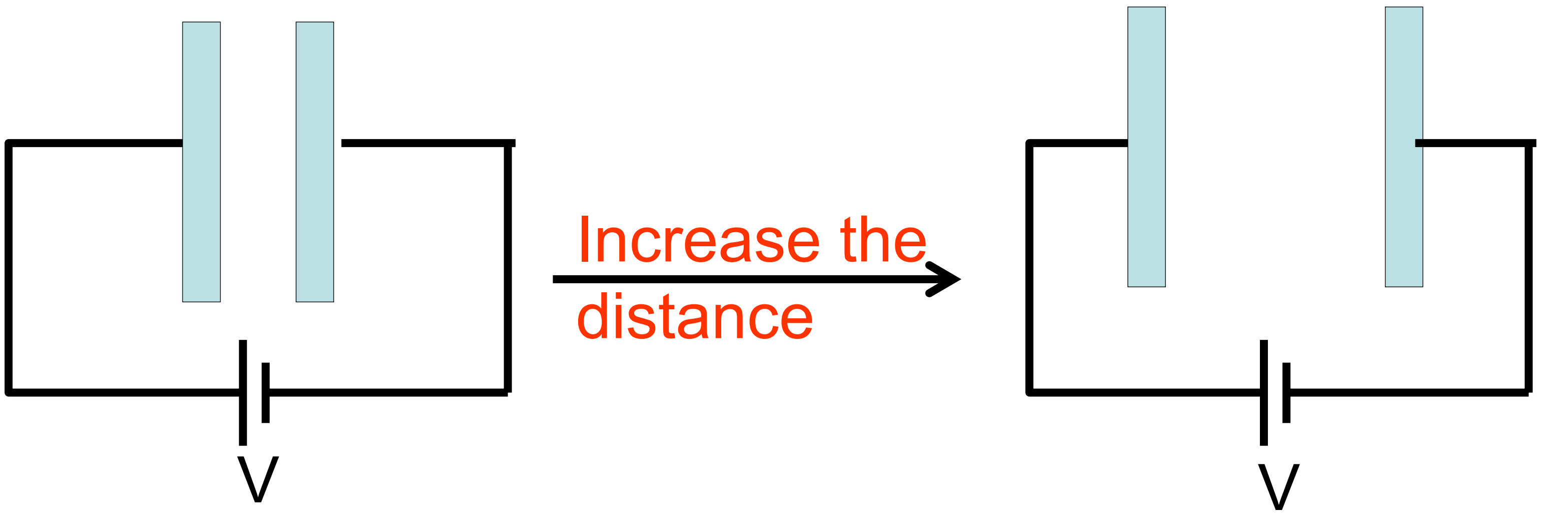
$$C = 4\pi\varepsilon_0 R$$

26-3 Calculating the Capacitance

Checkpoint 2

What happens to the charge on the capacitor?

Parallel-plate capacitor



$q = C V$

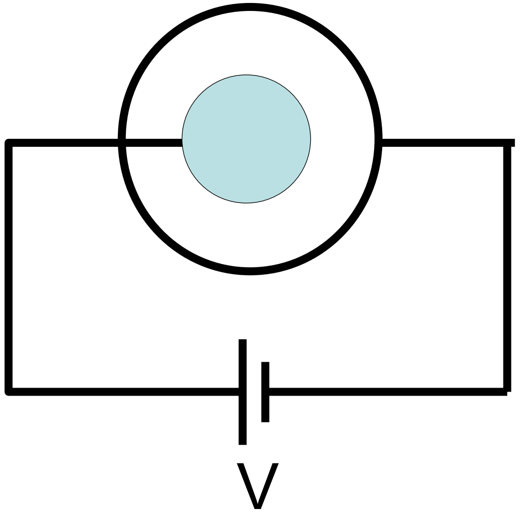
$C = \frac{\varepsilon_0 A}{d}$ decreases → Charge decreases

26-3 Calculating the Capacitance

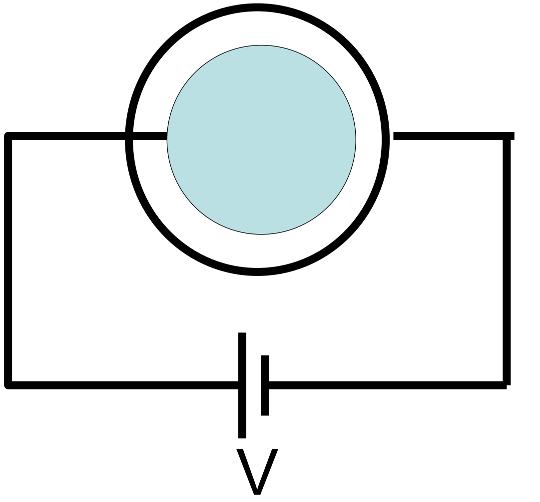
Checkpoint 2

What happens to the charge on the capacitor?

Cylindrical capacitor



Increase the radius of inner cylinder



$q = C\,V$

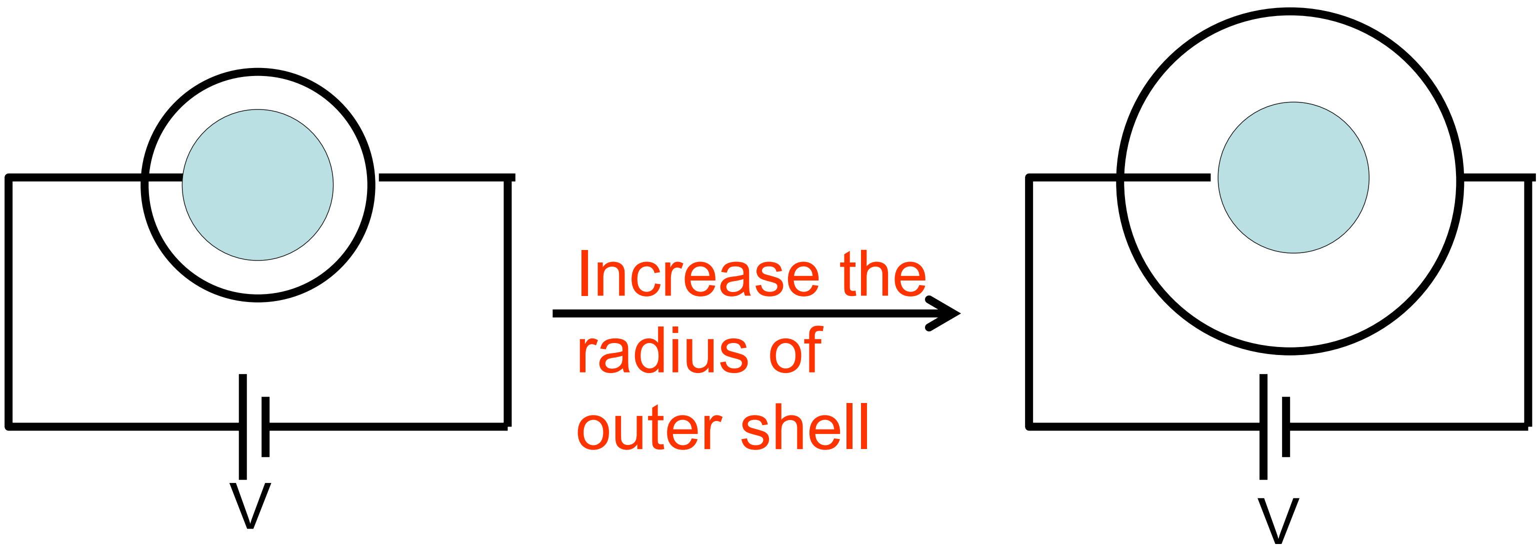
$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$ increases → Charge increases

26-3 Calculating the Capacitance

Checkpoint 2

What happens to the charge on the capacitor?

Spherical capacitor



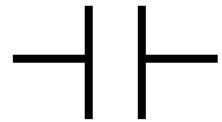
$q = C\,V$

$C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$ decreases → Charge decreases

26-3 Calculating the Capacitance

Sample Problem 26-1

Capacitor of Random Access Memory (RAM)



$C = 55$ fF

$V = 5.3$ V

Femtofarad = 10^{-15} F

How many electrons on each plate?

$$q = CV = (55 \times 10^{-15})(5.3) \quad C$$

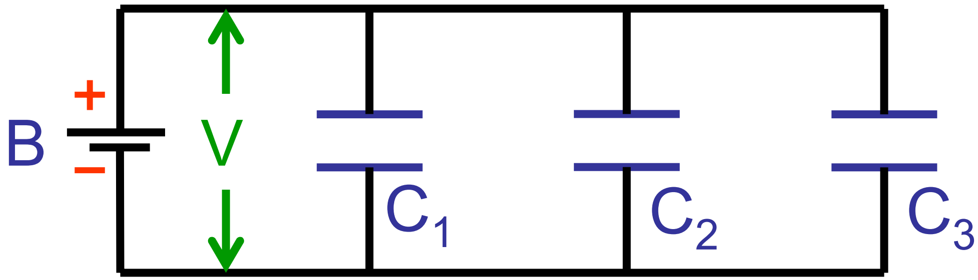
$$n = \frac{q}{e} = 1.8 \times 10^{6} \text{ electrons}$$

Charge of an electron

26-4 Capacitors in Parallel and in Series

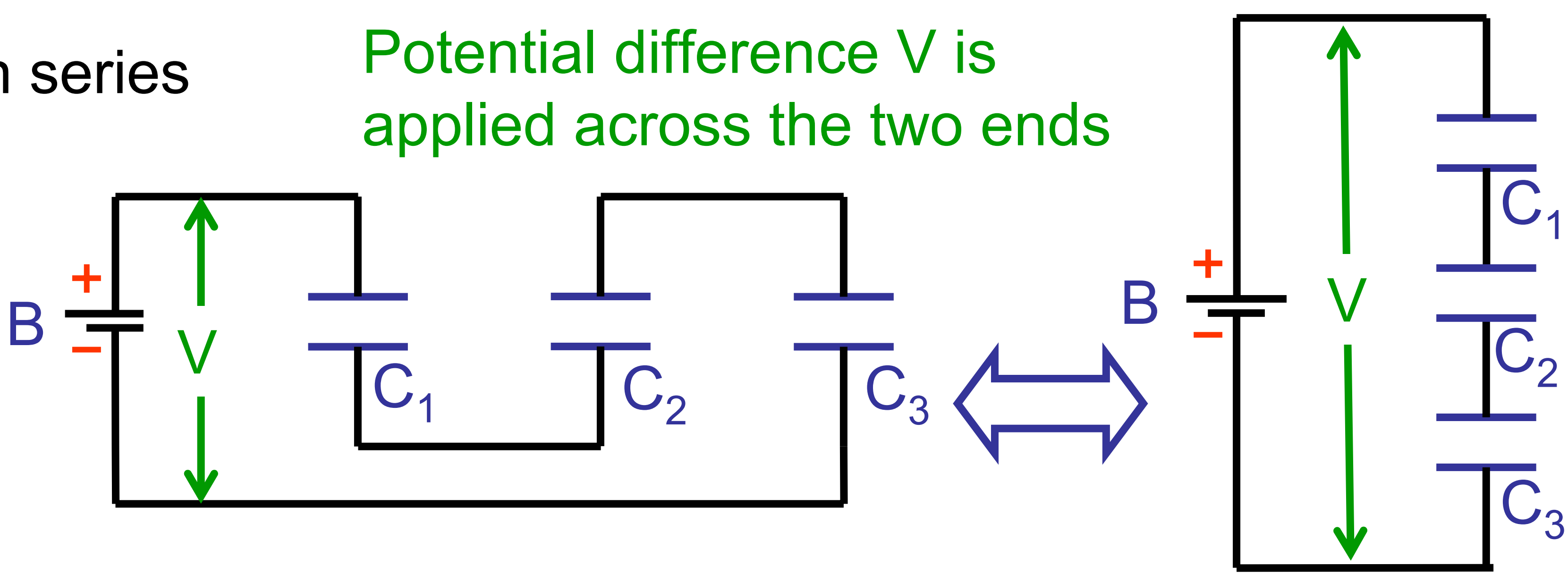
In parallel

Potential difference V is applied to each capacitor



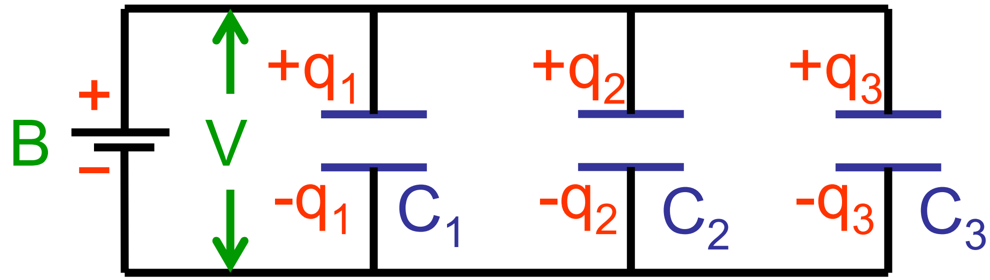
In series

Potential difference V is applied across the two ends



26-4 Capacitors in Parallel and in Series

In parallel



Each capacitor has the same potential difference V across it

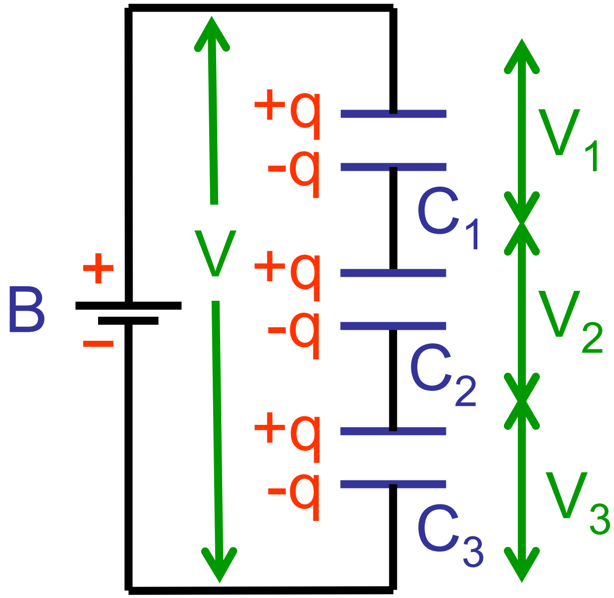
$$V = V_1 = V_2 = V_3$$

Total charge q stored on the capacitors is the sum of the charges stored on all capacitor

$$q = q_1 + q_2 + q_3$$

26-4 Capacitors in Parallel and in Series

In series



The applied potential difference V is equal to the sum of the potential differences across all the capacitors

$$V = V_1 + V_2 + V_3$$

The capacitors have identical charges q

$$q = q_1 = q_2 = q_3$$

26-4 Capacitors in Parallel and in Series

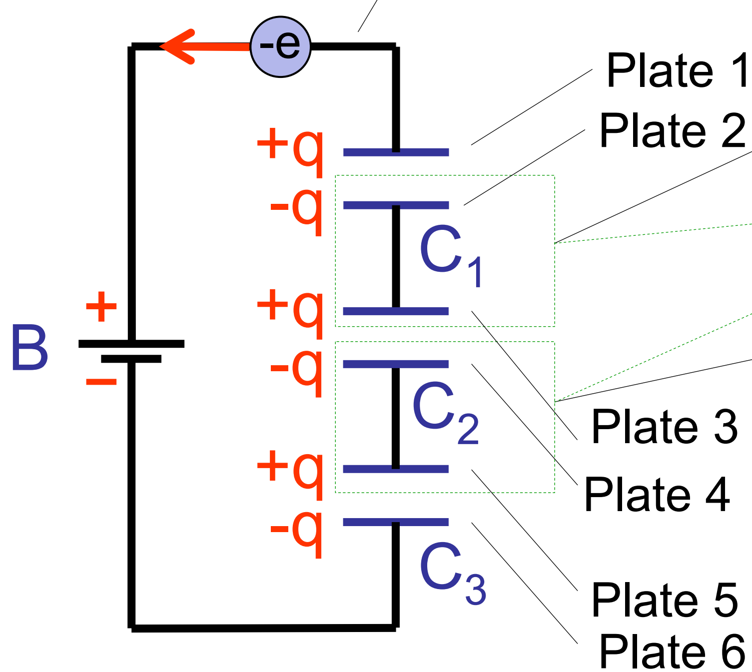
Why do capacitors connected in series have identical charges?

1

The battery moves electrons form plate1 to plate 6.
Plate1 → + q
Plate 6 → - q

3

Electrons in Plate 4 are repelled by electrons on plate 3
Plate 3 → +q
Plate 2 → - q



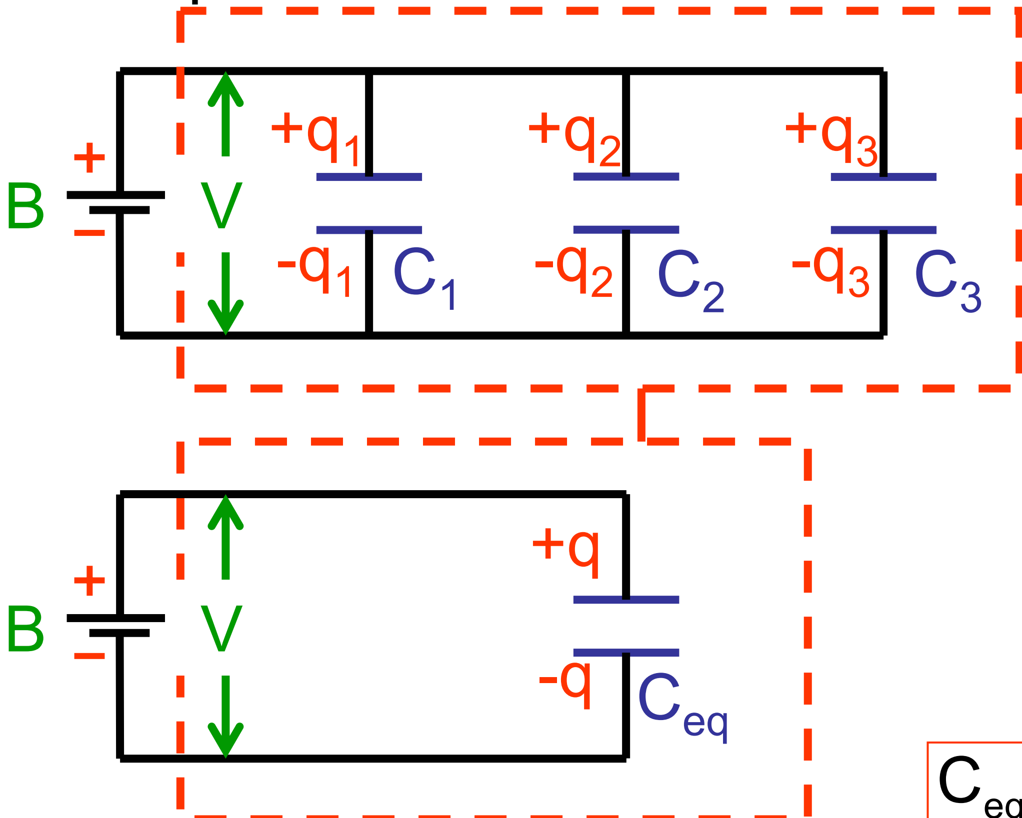
Only charges redistribution

Electrons in Plate 5 are repelled by electrons on plate 6
Plate 5 → +q
Plate 4 → - q

2

26-4 Capacitors in Parallel and in Series

In parallel



C_{eq} has the same total charge

$q = q_1+q_2+q_3$

and the same potential difference

$V = V_1= V_2= V_3$

as the actual parallel capacitors

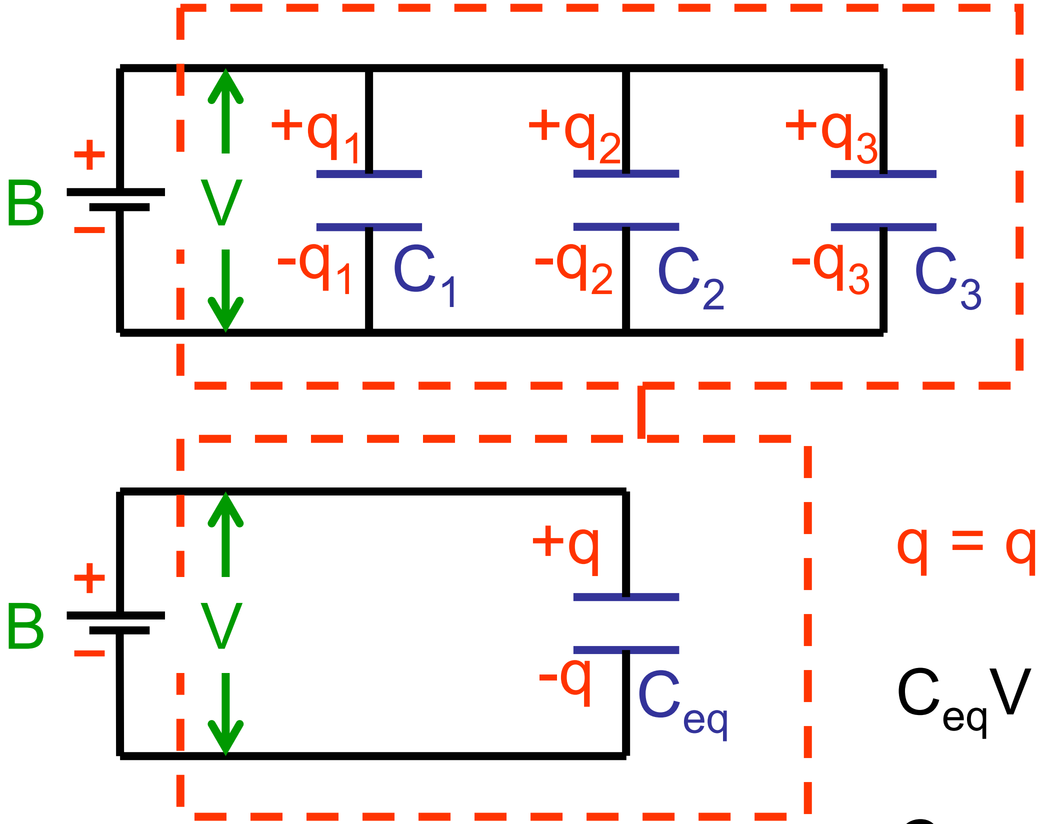
$$C_{eq} = C_1 + C_2 + C_3$$

$$C_{eq} = \sum_{j=1}^{n} C_j$$

n capacitors in parallel

26-4 Capacitors in Parallel and in Series

Derivation of $C_{eq} = C_1 + C_2 + C_3$



Using $q = C\,V$

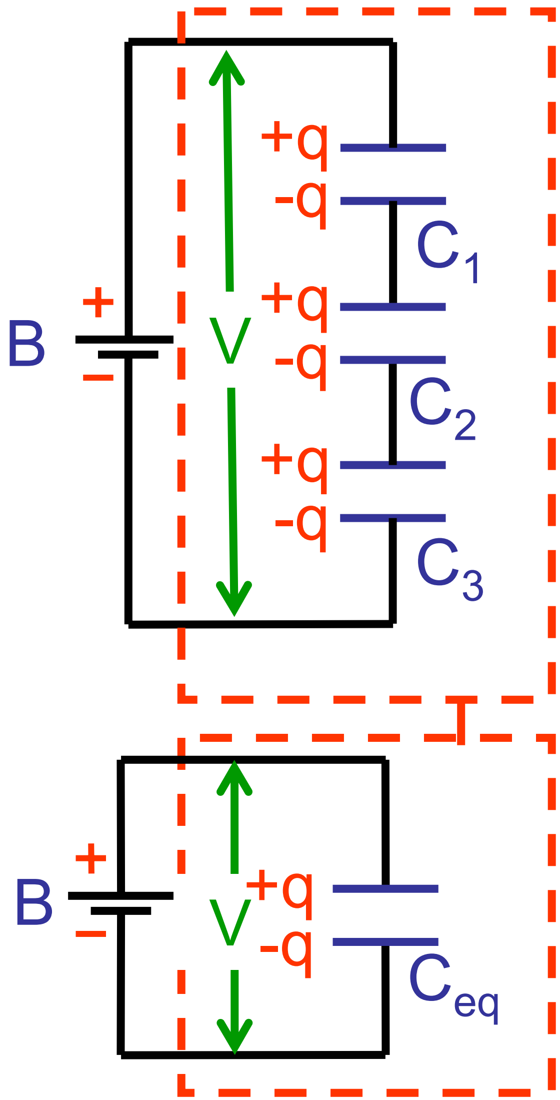
$q = q_1 + q_2 + q_3$

$C_{eq}V = C_1V + C_2V + C_3V$

$C_{eq} = C_1 + C_2 + C_3$

26-4 Capacitors in Parallel and in Series

In series



C_{eq} has the same charge

$q = q_1 = q_2 = q_3$

and the same total potential difference

$V = V_1 + V_2 + V_3$

as the actual series capacitors

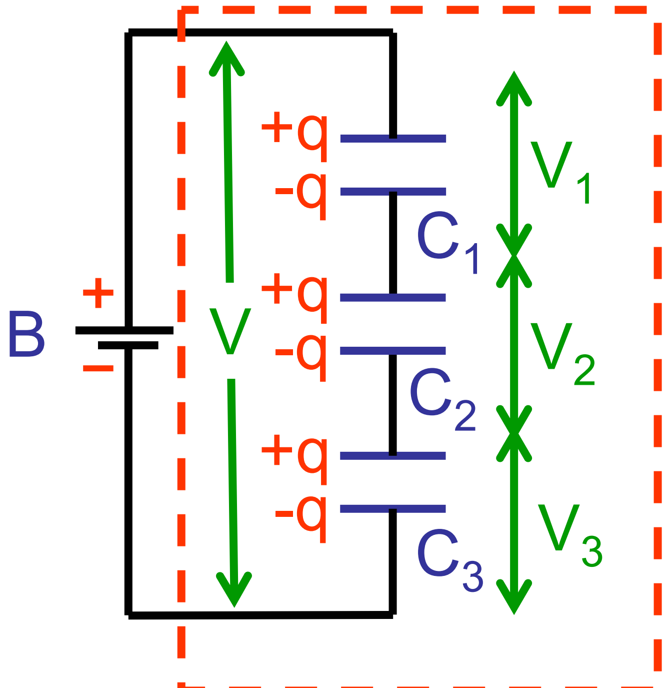
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

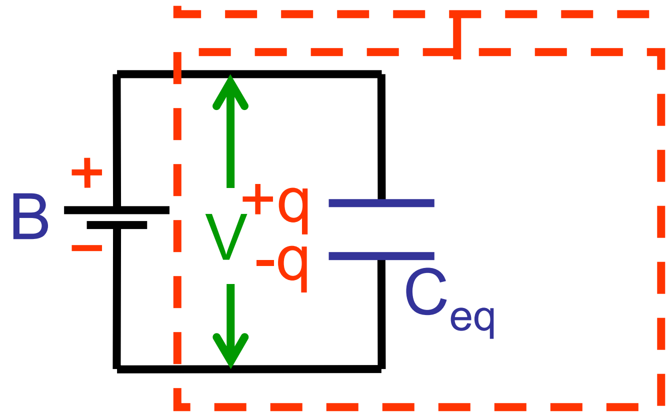
$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

n capacitors in series

26-4 Capacitors in Parallel and in Series

Derivation of $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$





Using $q = C\,V$

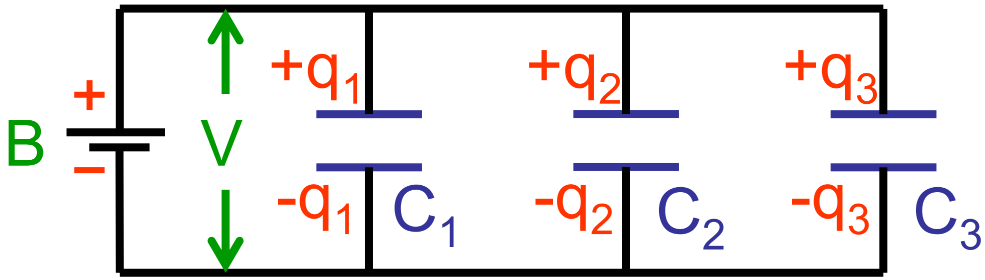
$$V = V_1 + V_2 + V_3$$

$$\frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

26-4 Capacitors in Parallel and in Series

In parallel

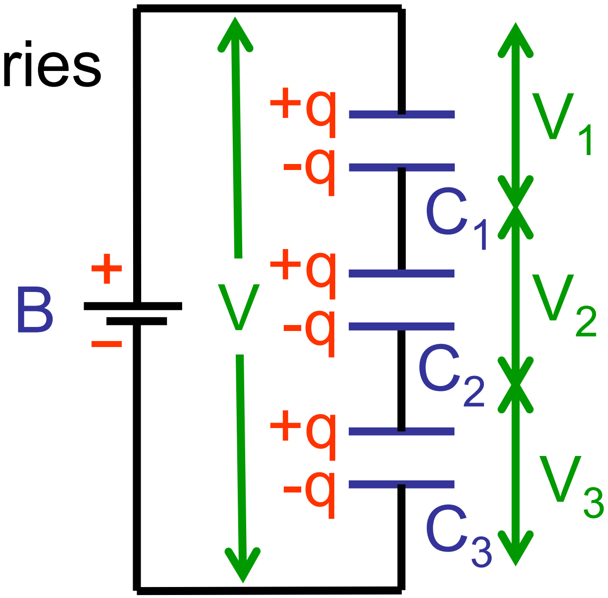


$V = V_1 = V_2 = V_3$

$q = q_1 + q_2 + q_3$

$C_{eq} = C_1 + C_2 + C_3$

In series



$V = V_1 + V_2 + V_3$

$q = q_1 = q_2 = q_3$

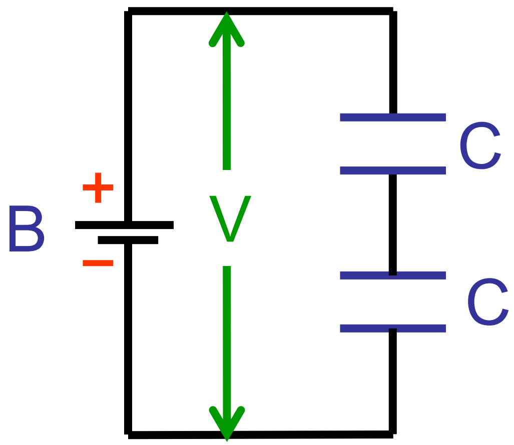
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

26-4 Capacitors in Parallel and in Series

Checkpoint 3

A battery of potential V stores charge q on a combination of two capacitors.

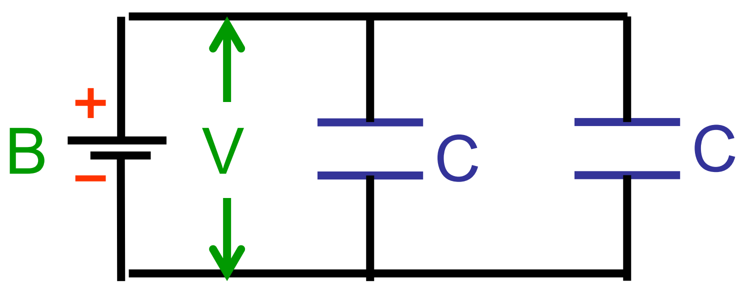
What is the potential difference cross and the charge on each capacitor?



For each capacitor,

$$\text{Potential difference} = \frac{V}{2}$$

$$\text{Charge} = q = \frac{C\,V}{2}$$



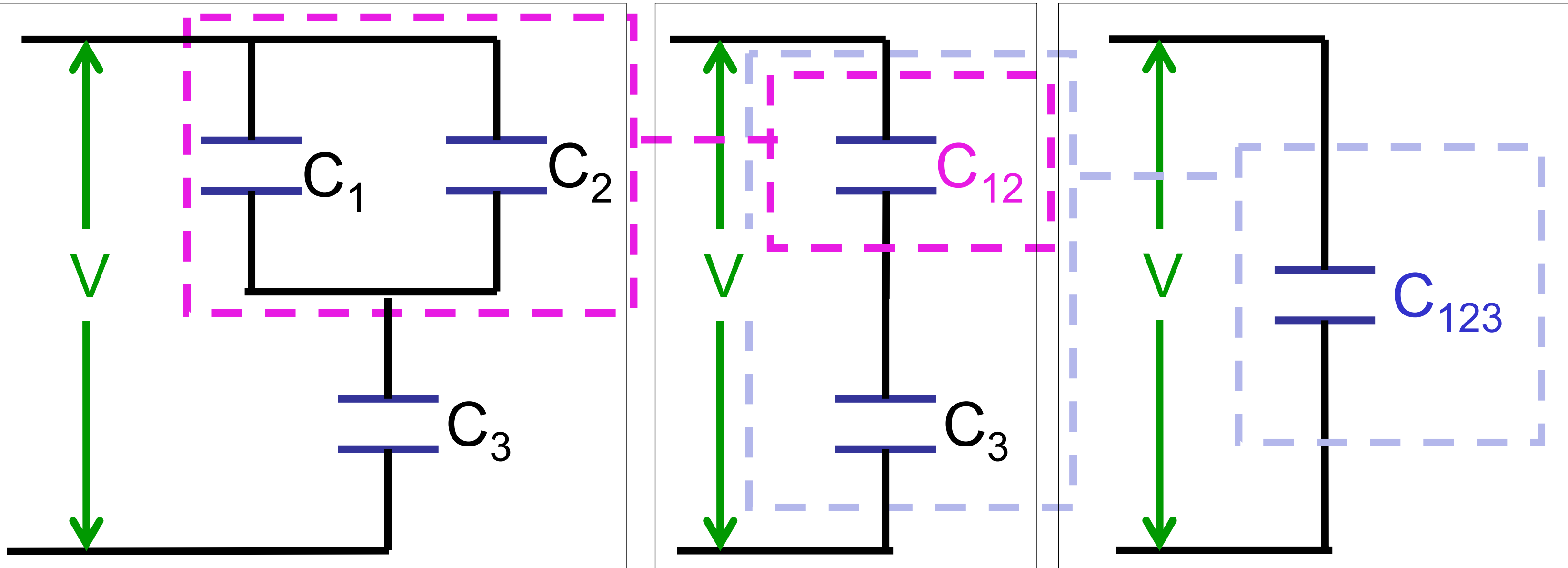
For each capacitor,

$$\text{Potential difference} = V$$

$$\text{Charge} = \frac{q}{2} = C\,V$$

26-4 Capacitors in Parallel and in Series

Sample Problem 26-2



$C_1 = 12.0\ \mu F$

$C_2 = 5.3\ \mu F$

$C_3 = 4.50\ \mu F$

Find the equivalent capacitance

$C_{12} = C_1 + C_2$

$C_{12} = 17.3\ \mu F$

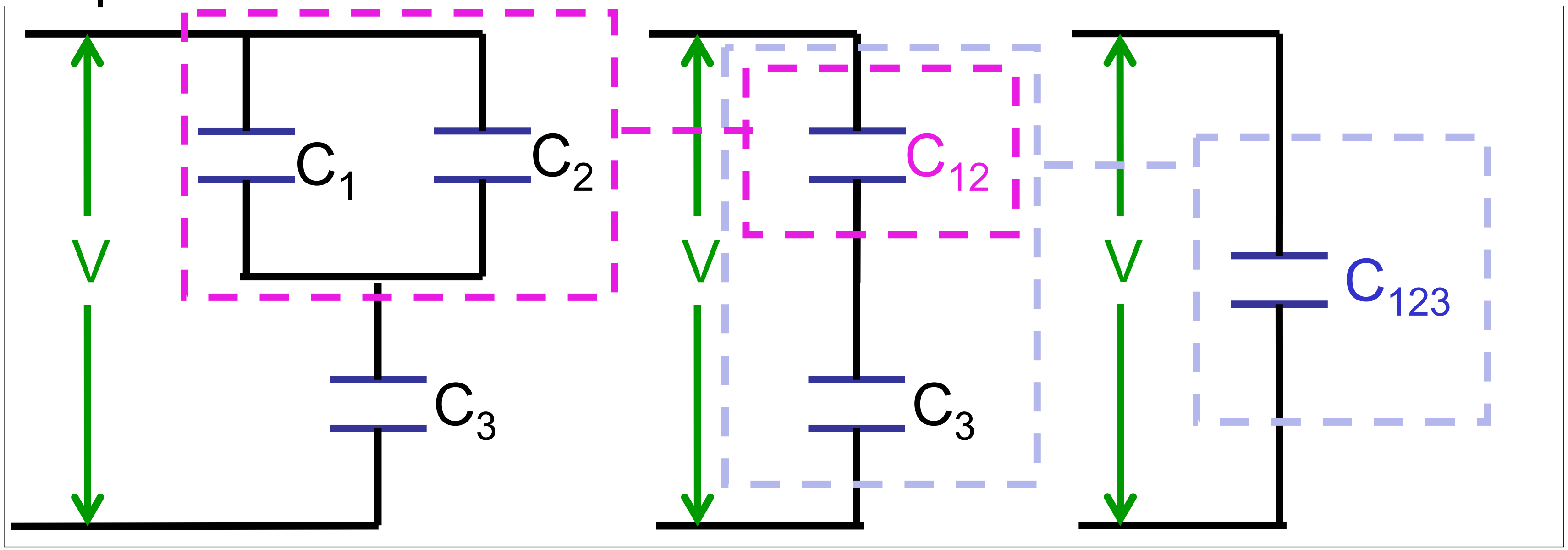
$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$\frac{1}{C_{123}} = \frac{C_3 + C_{12}}{C_{12}\ C_3}$$

$$C_{123} = \frac{C_{12}\ C_3}{C_3 + C_{12}} = 3.57\ \mu F$$

26-4 Capacitors in Parallel and in Series

Sample Problem 26-2



If the potential difference V =12.5 V, what is the charge on C_1?

$q_1 = C_1 V_1 = 31.0\ \mu C$

$V_1 = V_{12} = \frac{q_{12}}{C_{12}} = 2.58\ V$

$q_{12} = q_{123} = C_{123} V = 44.6\ \mu C$

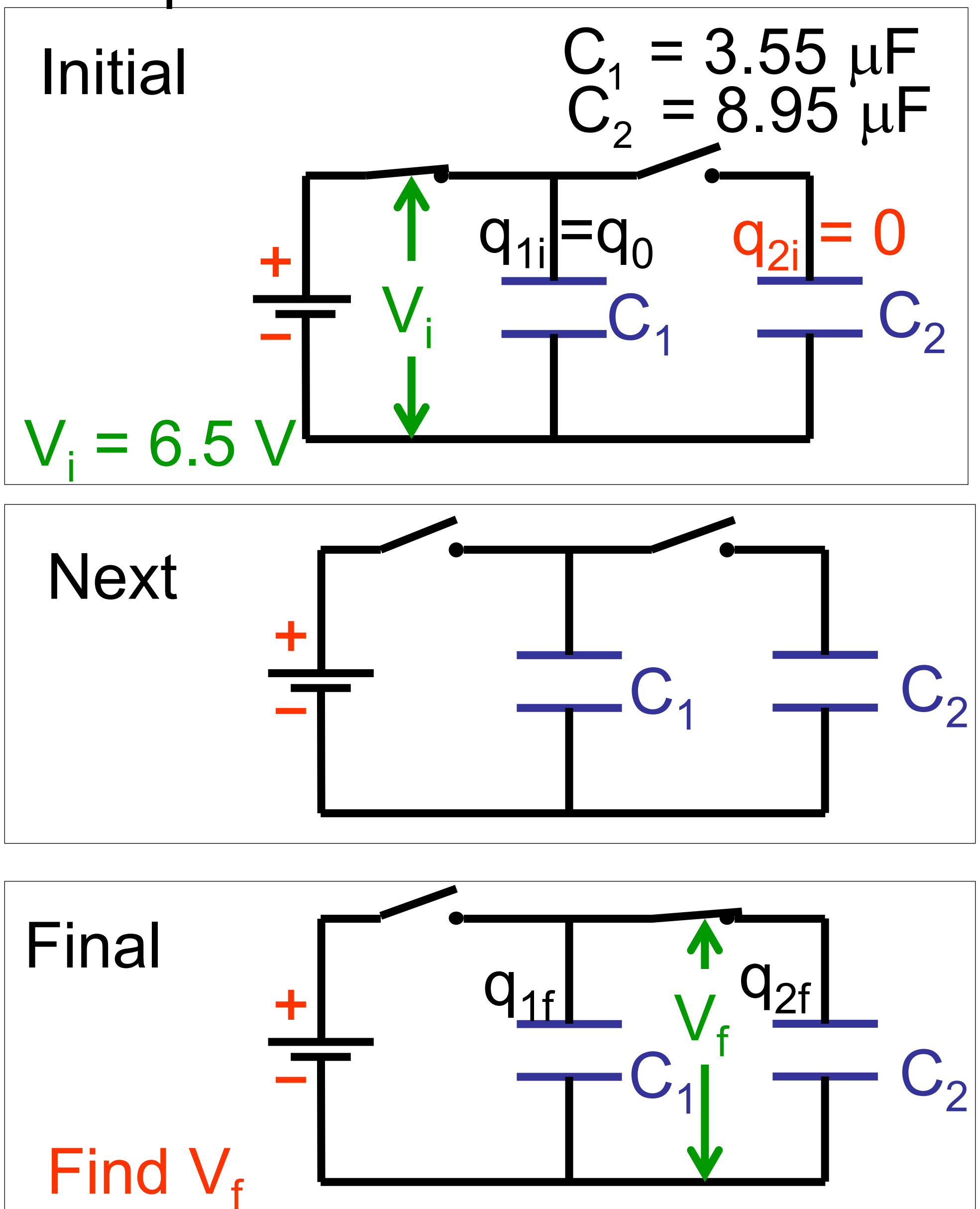
$C_1 = 12.0\ \mu F$

$C_{12} = 17.3\ \mu F$

$C_{123} = 3.57\ \mu F$

26-4 Capacitors in Parallel and in Series

Sample Problem 26-3



Total final charges on C_1 and C_2 are equal to the initial charge q_0

$$q_{1f} + q_{2f} = q_0$$

Using $q = C\,V$

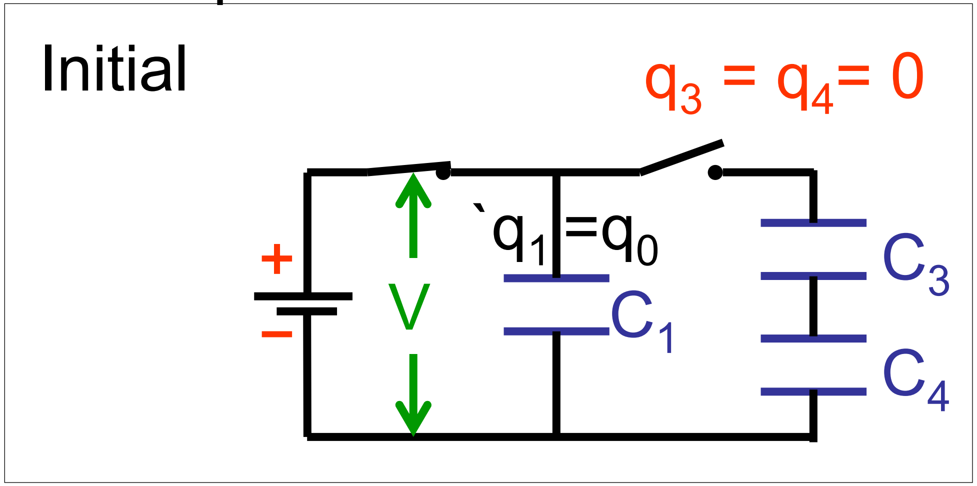
$$C_1 V_f + C_2 V_f = C_1 V_i$$

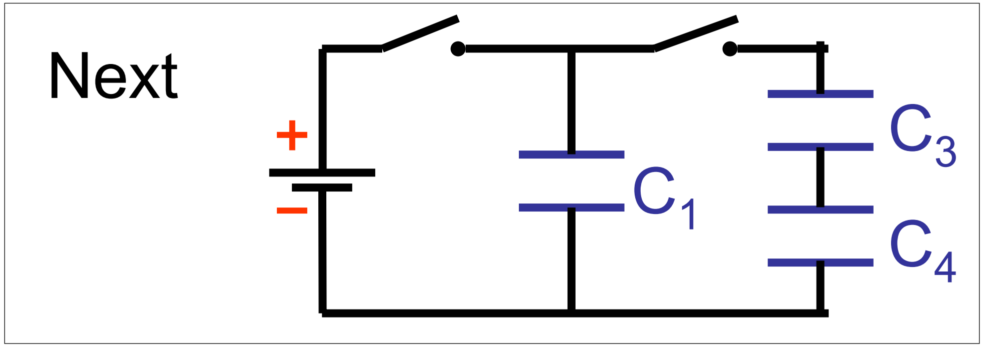
$$V_f = \frac{C_1}{C_1 + C_2} V_i$$

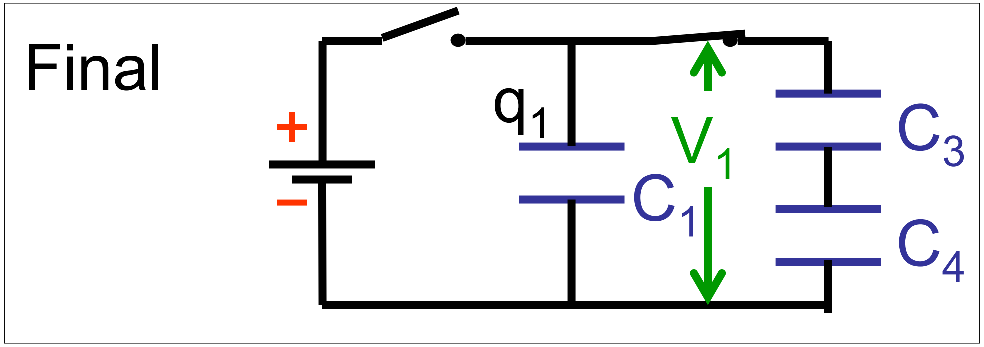
$$V_f = 1.79\ V$$

26-4 Capacitors in Parallel and in Series

Checkpoint 4







What is the relation between q_0 and the final charge on C_1 and C_{34} (equivalent capacitance of C_3 and C_4) ?

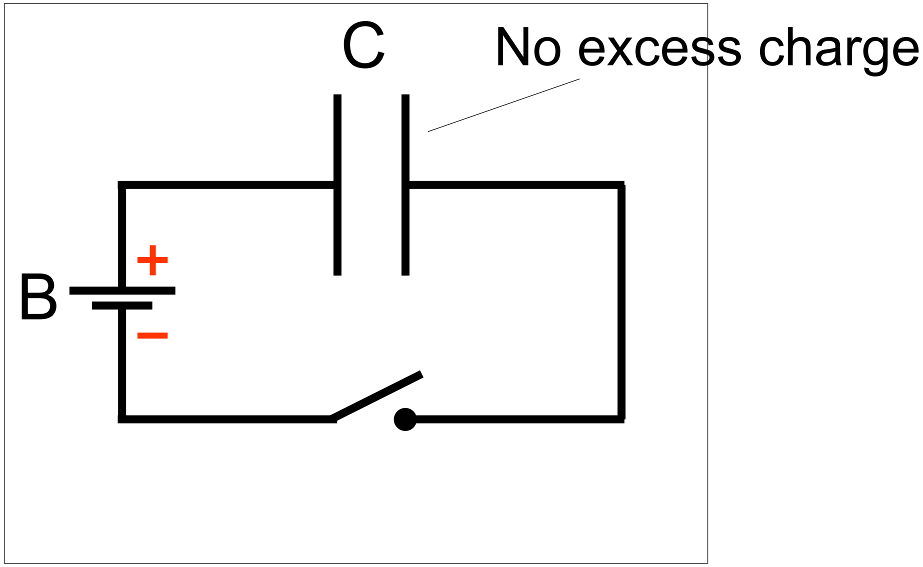
$$q_0 = q_1 + q_{34}$$

If $C_3 > C_4$, what the relation between q_3 and q_4?

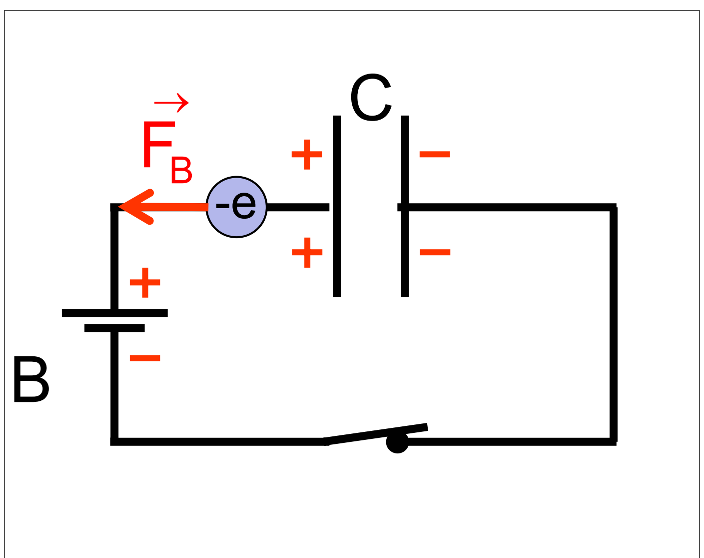
$$q_3 = q_4$$

Because they are in series

26-5 Energy Stored in an electric field

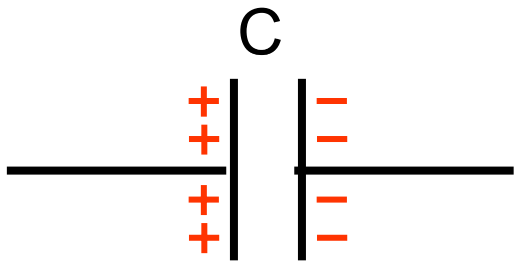


Battery does work to transfer electrons from one plate to another

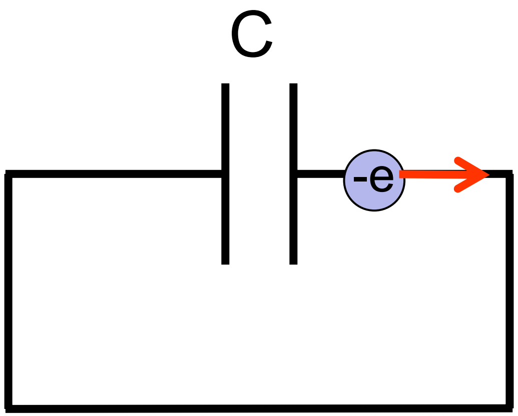


Work required to charge the capacitor is stored in the form of electric potential energy U in the electric field of the capacitor

26-5 Energy Stored in an electric field



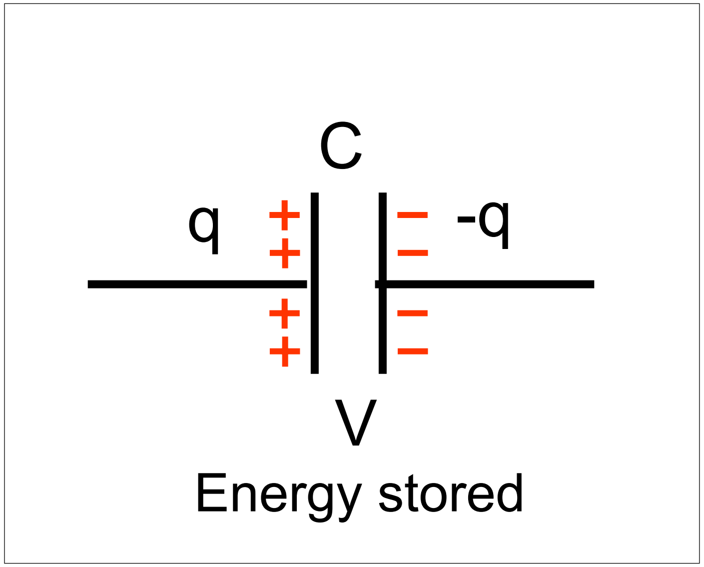
Energy stored



discharging

You can recover the energy stored in the capacitor by discharging the capacitor in a circuit

26-5 Energy Stored in an electric field



Electric potential energy U stored in the capacitor

$$U = \frac{q^2}{2C}$$

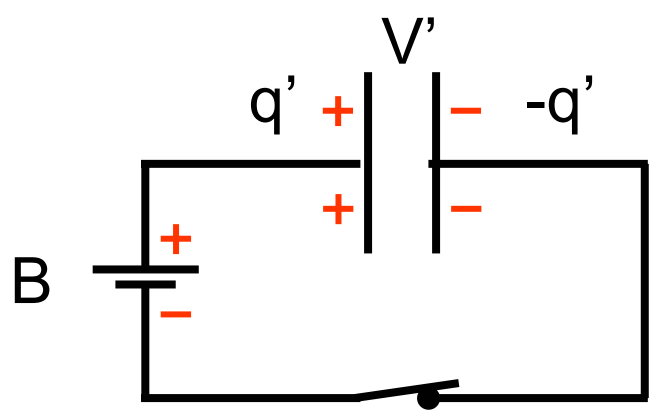
Using $q = CV$

$$U = \frac{1}{2} C V^2$$

26-5 Energy Stored in an electric field

Derivation of $U = \dfrac{q^2}{2C}$

Suppose the capacitor charge is q' and the potential difference between its plate is V'



The work done by the battery to add dq' to the capacitor

$$dW_{app} = V' \, dq'$$

Using q = C V

$$dW_{app} = \frac{q'}{C} \, dq'$$

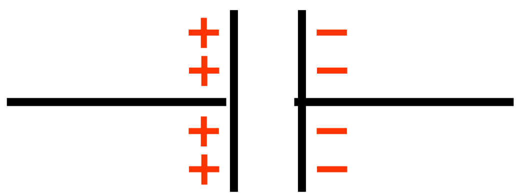
The work required to bring the total capacitor charge form 0 up to q

$$W_{app} = \int_0^q \frac{q'}{C} \, dq' = \frac{q^2}{2C}$$

$$W_{app} = U = \frac{q^2}{2C}$$

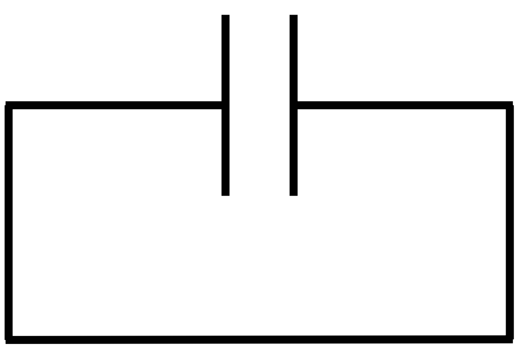
26-5 Energy Stored in an electric field

q = 75 μF



V = 500 V

What is the power delivered by the capacitor?



Discharge in t = 2 ms

$$P = \frac{U}{t} = \frac{\frac{1}{2} C V^2}{t} = 100 \text{ kW}$$

26-5 Energy Stored in an electric field

The potential energy of a charged capacitor is stored in the electric field between its plates

$$u = \frac{1}{2}\varepsilon_0 E^2$$

For any electric field

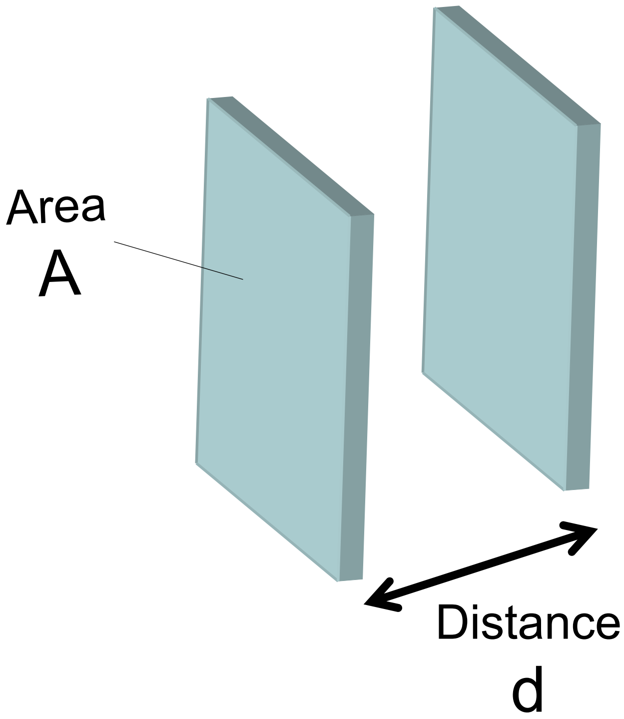
Energy density $u = \dfrac{U}{\text{Volume}}$

Potential energy stored in an electric field per unit volume

26-5 Energy Stored in an electric field

Derivation of $u = \frac{1}{2}\varepsilon_0 E^2$ for the case of parallel-plate capacitor

$$u = \frac{U}{\text{Volume}} = \frac{\frac{1}{2} C V^2}{A\, d} = \frac{\frac{1}{2}\left(\varepsilon_0 \frac{A}{d}\right) V^2}{A\, d} = \frac{\frac{1}{2}\left(\varepsilon_0 \frac{A}{d}\right)(Ed)^2}{A\, d} = \frac{1}{2}\varepsilon_0 E^2$$



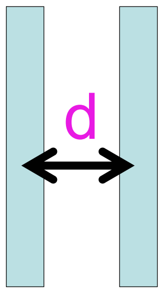
26-5 Energy Stored in an electric field

Parallel-plate capacitor

$A_1 = A$

$d_1 = d$

$q_1 = q$



$$E_1 = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\,\varepsilon_0} = E$$

$$u_1 = \frac{1}{2}\varepsilon_0 E^2$$

$$U_1 = (d_1\ A_1)\ u_1 = U$$

$$C_1 = \frac{\varepsilon_0\ A_1}{d_1} = C$$

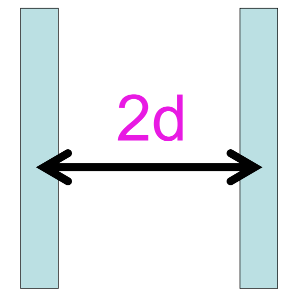
$$U_1 = \frac{q^2}{2C_1} = U$$

Parallel-plate capacitor

$A_2 = A$

$d_2 = 2\,d$

$q_2 = q$



$$E_2 = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\,\varepsilon_0} = E$$

$$u_2 = \frac{1}{2}\varepsilon_0 E^2$$

$$U_2 = (d_2\ A_2)\ u_2 = 2\ U$$

$$C_2 = \frac{\varepsilon_0\ A_2}{d_2} = \frac{1}{2}\ C$$

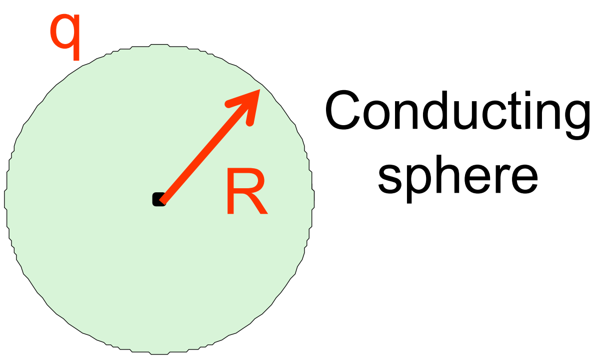
$$U_2 = \frac{q^2}{2C_2} = 2\ U$$

26-5 Energy Stored in an electric field

Sample Problem 26- 4

R = 6.85 cm

q = 1.25 nC



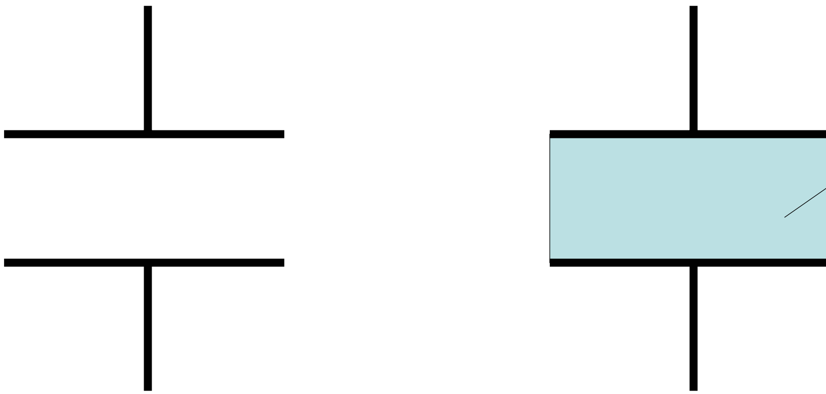
How much potential energy is stored in the electric field of this charged capacitor?

$$U = \frac{q^2}{2C} = \frac{q^2}{2(2\pi\varepsilon_0 R)} = 103 \text{ nJ}$$

What is the electric energy density at the surface of the sphere?

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{q}{4\pi\varepsilon_0 R^2}\right)^2 = 25.4 \ \mu\text{J/m}^3$$

26-6 Capacitor with dielectric



Dielectric material (insulating material like plastic and oil)

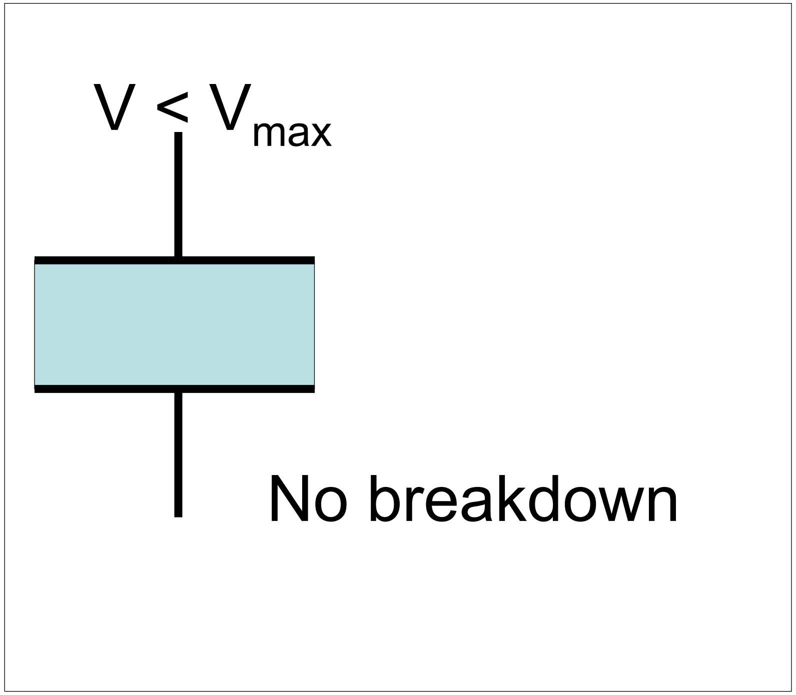
Capacitance C

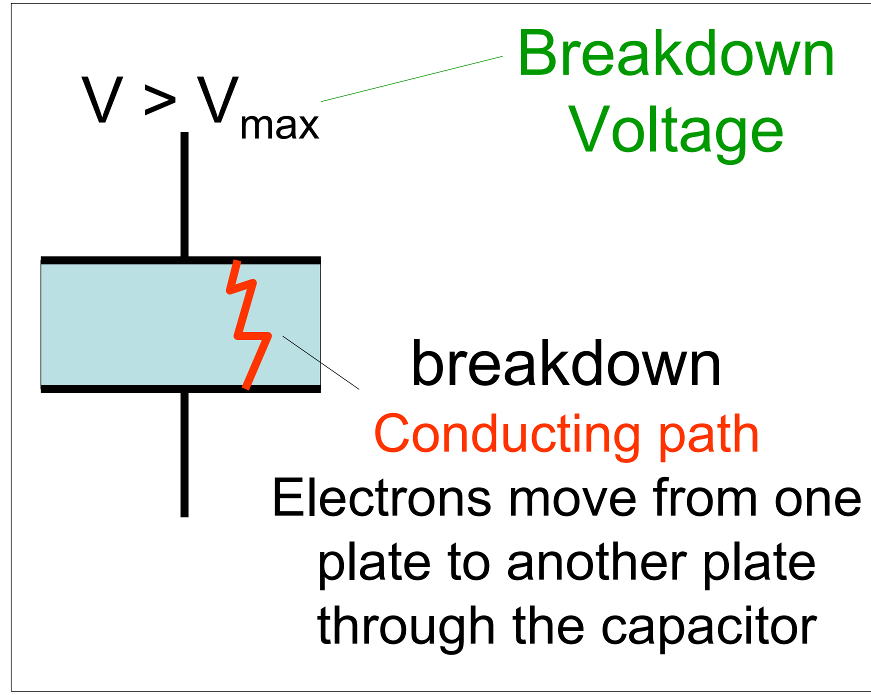
Capacitance κC

Dielectric constant of the insulating material

Material	κ
Vacuum	1
Air	1.0005
Paper	3.5
Water (20^0C)	80.4
Strontium titanate	310

26-6 Capacitor with dielectric

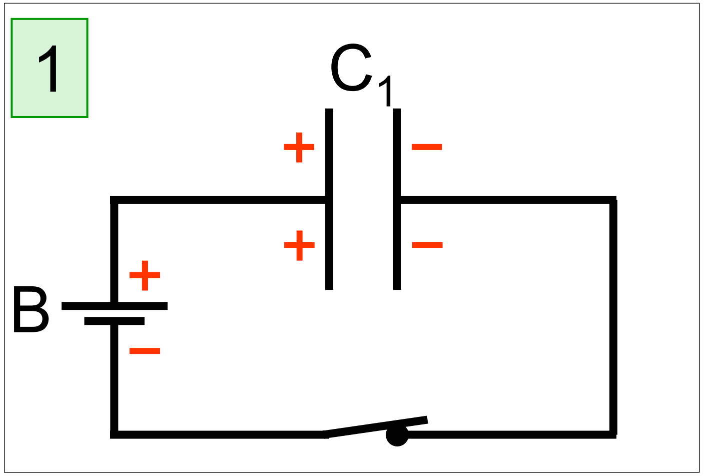


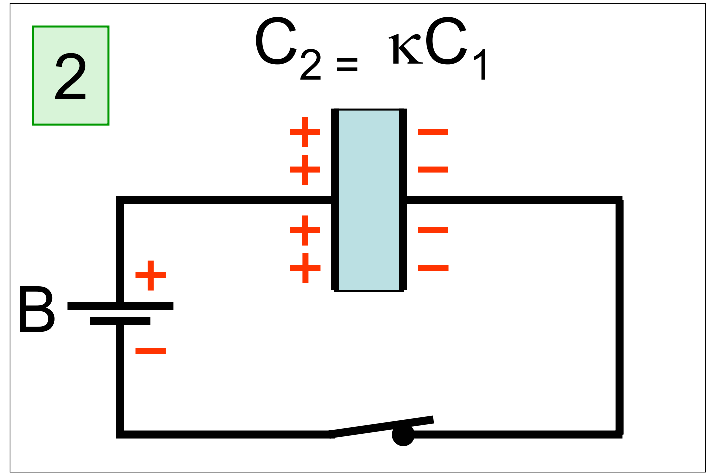


Dielectric strength of a dielectric is the maximum electric field it can tolerate without breakdown

Dielectric Strength	E_{max} (kV/mm)
Air	3
Paper	16
Strontium titanate	8

26-6 Capacitor with dielectric



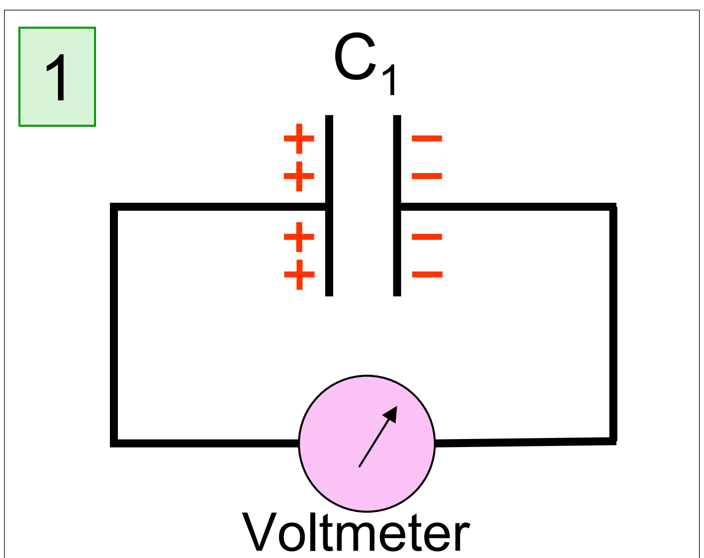


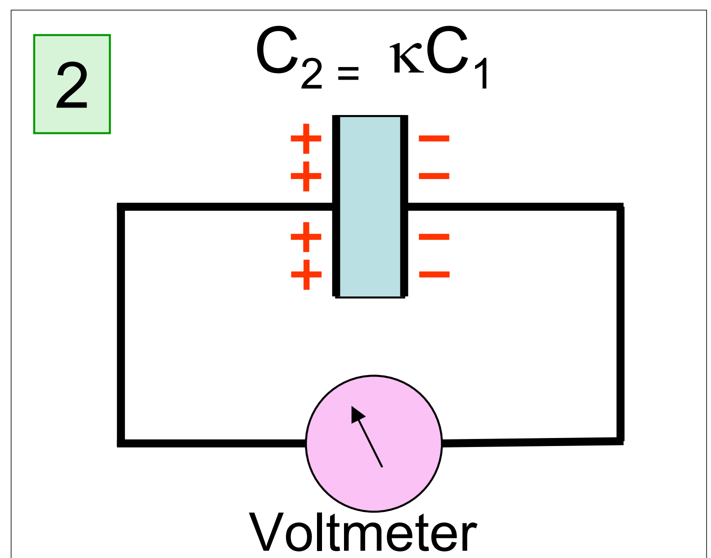
$$q = C\,V$$

$$q_2 = \kappa C_1\,V$$

$$q_2 = \kappa q_1$$

Voltage does not change (battery)





$$V = \frac{q}{C}$$

$$V_2 = \frac{q}{C_2}$$

$$V_2 = \frac{q}{\kappa C_1}$$

$$V_2 = \frac{V_1}{\kappa}$$

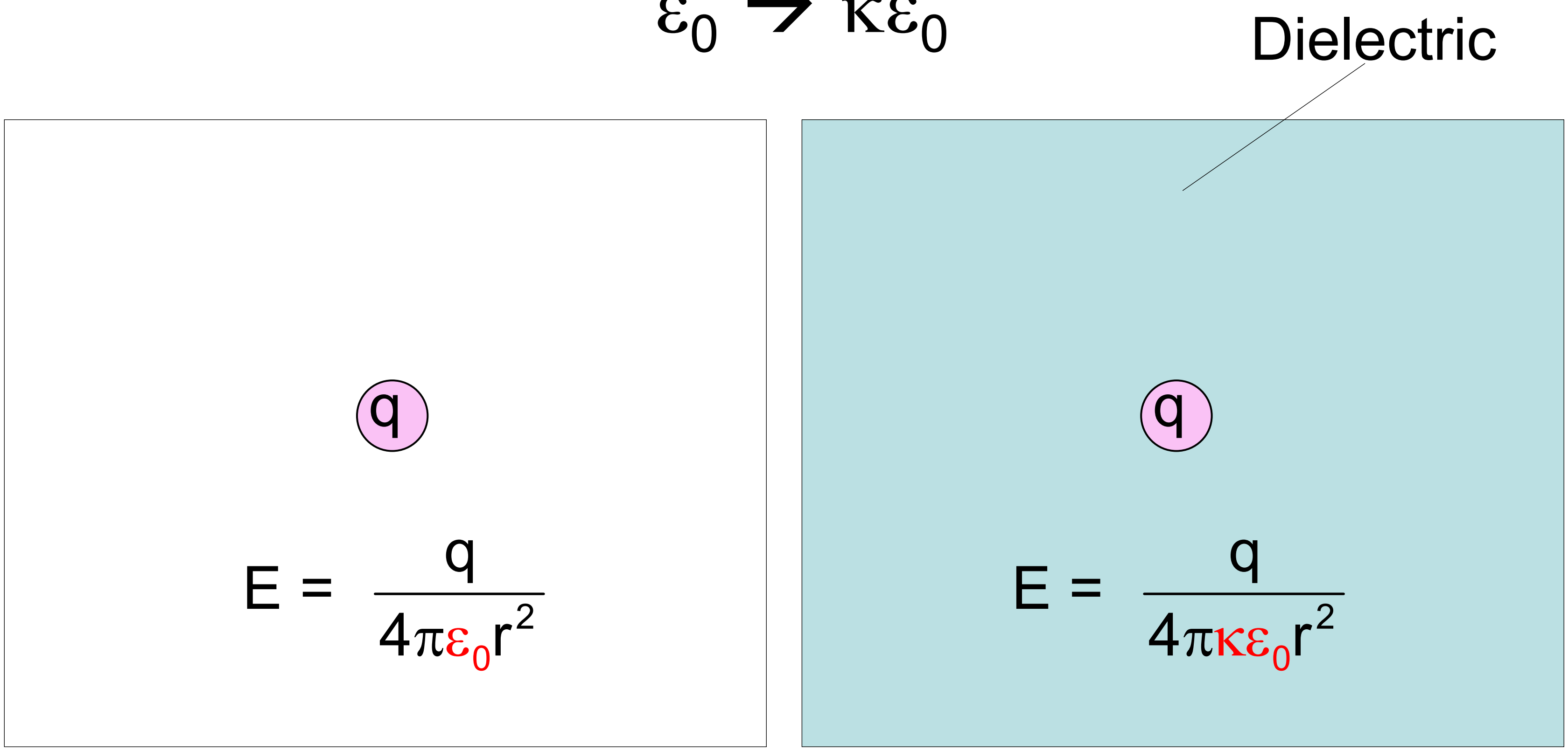
Charge does not change (no battery)

26-6 Capacitor with dielectric

In a region completely filled by a dielectric material of dialectic constant κ

In any equation,

$$\varepsilon_0 \rightarrow \kappa\varepsilon_0$$



$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{q}{4\pi\kappa\varepsilon_0 r^2}$$

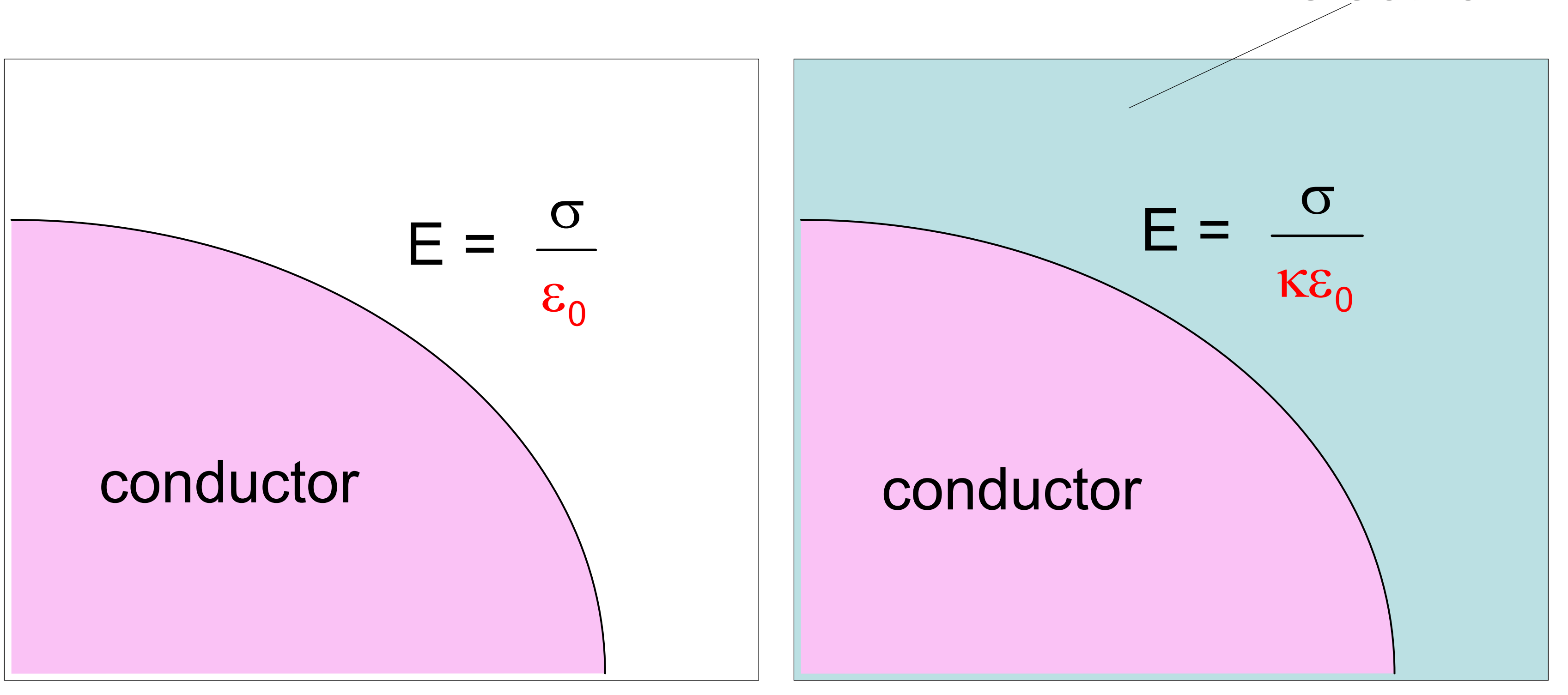
Electric field from a point charge

26-6 Capacitor with dielectric

In a region completely filled by a dielectric material of dialectic constant κ

In any equation,

$$\varepsilon_0 \rightarrow \kappa\varepsilon_0$$



Electric field near the surface of a conductor

26-6 Capacitor with dielectric

For a fixed distribution of charges

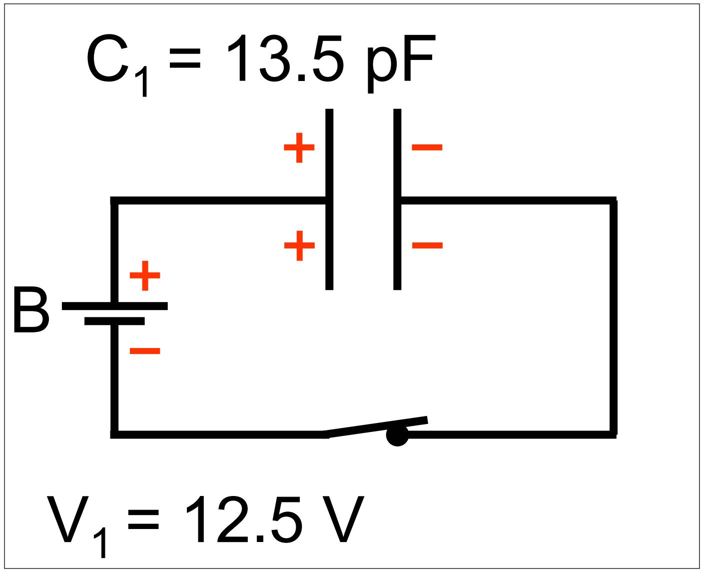
$$E \rightarrow \frac{E}{\kappa}$$

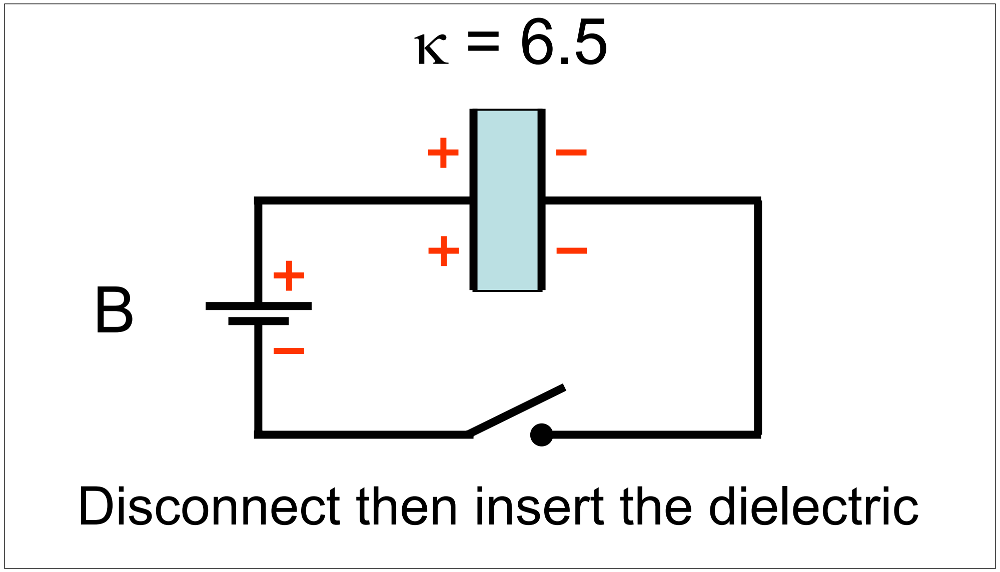
Without dielectric

With dielectric
weaker

26-6 Capacitor with dielectric

Sample Problem 26- 5





What is the potential energy before and after insertion?

$$U_{before} = \frac{1}{2} C_1 V_1^2 = 1055 \text{ pJ}$$

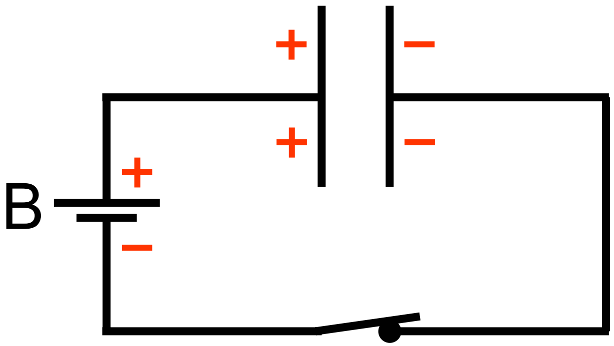
$$U_{after} = \frac{1}{2} C_2 V_2^2 = \frac{q_2^2}{2C_2} = \frac{q^2}{2\kappa C_1} = \frac{U_{before}}{\kappa} = 162 \text{ pJ}$$

$$q_1 = q_2 = q$$

26-6 Capacitor with dielectric

Sample Problem 26- 5

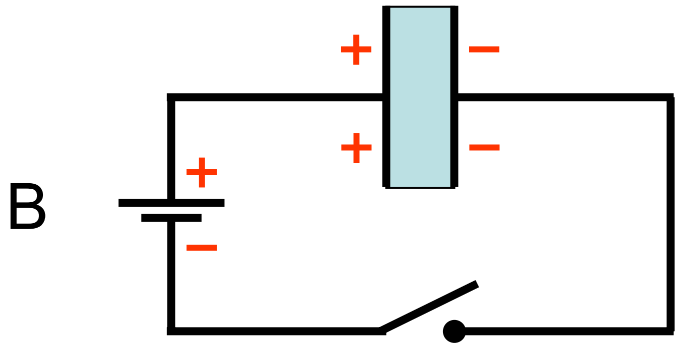
$C_1 = 13.5$ pF



$V = 12.5$ V

$U_{before} = 1055$ pJ

$\kappa = 6.5$



Disconnect then insert the dielectric

$U_{after} = 162$ pJ

What is the work done by the person who inserts the dielectric slab?

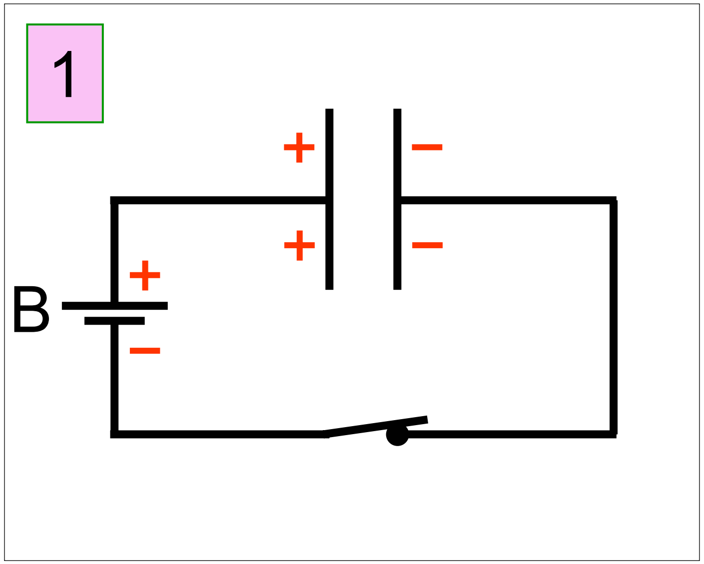
$$W_{appl} = U_{after} - U_{before} = -893 \text{ pJ}$$

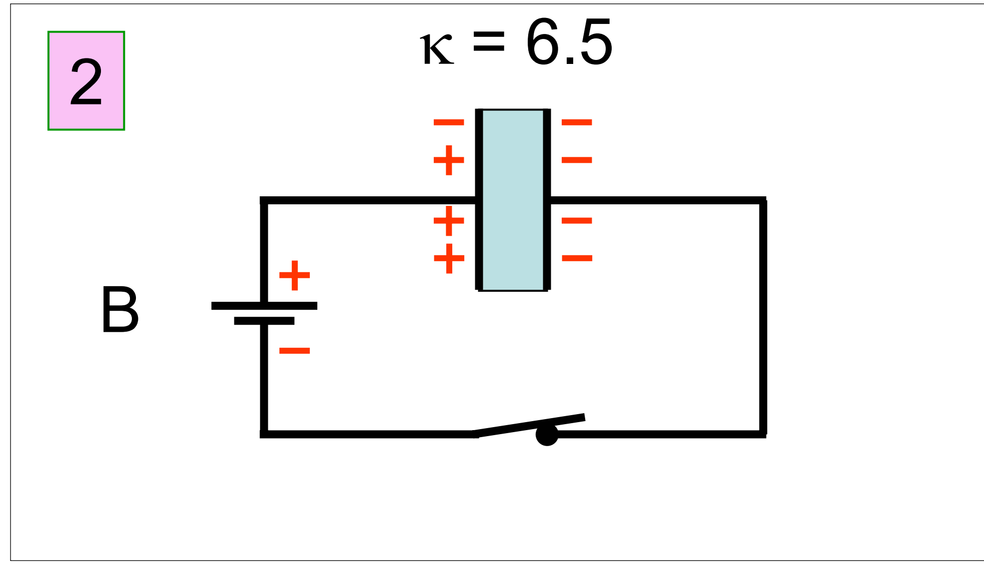
$$W_{cap} = -893 \text{ pJ}$$

The capacitor will pull the dielectric slab in

26-6 Capacitor with dielectric

Checkpoint 5





What happens to …

Potential difference ?	Same	Battery $V_2 = V_1$
Capacitance ?	Increase	$C_2 = \kappa C_1$
Charge on the capacitor ?	Increase	$q_2 = C_2 V_2 = \kappa\, q_1$
The potential energy ?	Increase	$U_2 = 0.5\, C_2\, V_2^2 = \kappa U_1$
The electric field between plated ?	Same	$E_2 = V_2/d = V_1/d = E_1$