Chapter 25

Electric Potential

25-1 Electric Potential Energy

Electrostatic Force is a conservative force

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We can define potential energy to a system of charged particles

$$\Delta U = - W$$

Change in electric potential energy

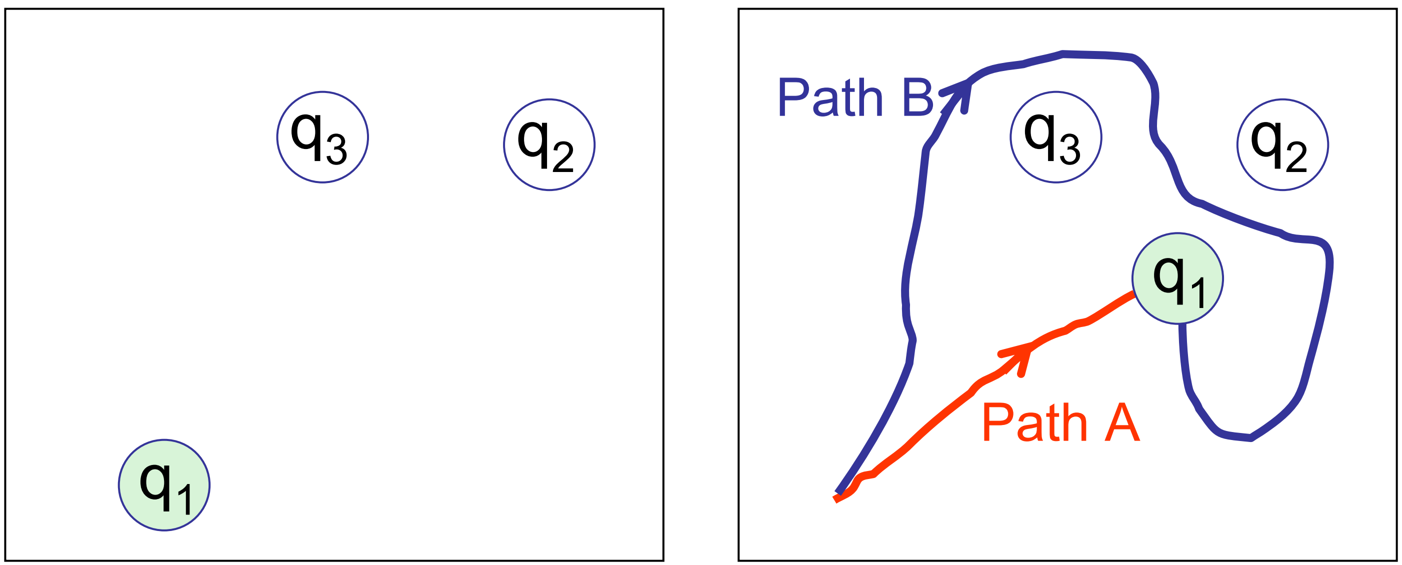
work done by the electrostatic force on the particles

25-1 Electric Potential Energy

Electrostatic Force is a conservative force

$$\Delta U = - W$$

Work done by the electrostatic force on charged particles is independent of path



Initial state

Final state

Work done by the electrostatic force on charged particles depends only on the state of the system

25-1 Electric Potential Energy

Only differences in potential energy have physical meaning

$$\Delta U = U_f - U_i = -W$$

We are free to define zero potential energy configuration

We will choose our reference configuration when the particles are all infinitely separated from each other

$U_\infty = 0$ when all particles at infinity

$$U = -W_\infty$$

Electric potential energy of any charged particles configuration

Work done by the electrostatic force to bring the charged particles from infinity

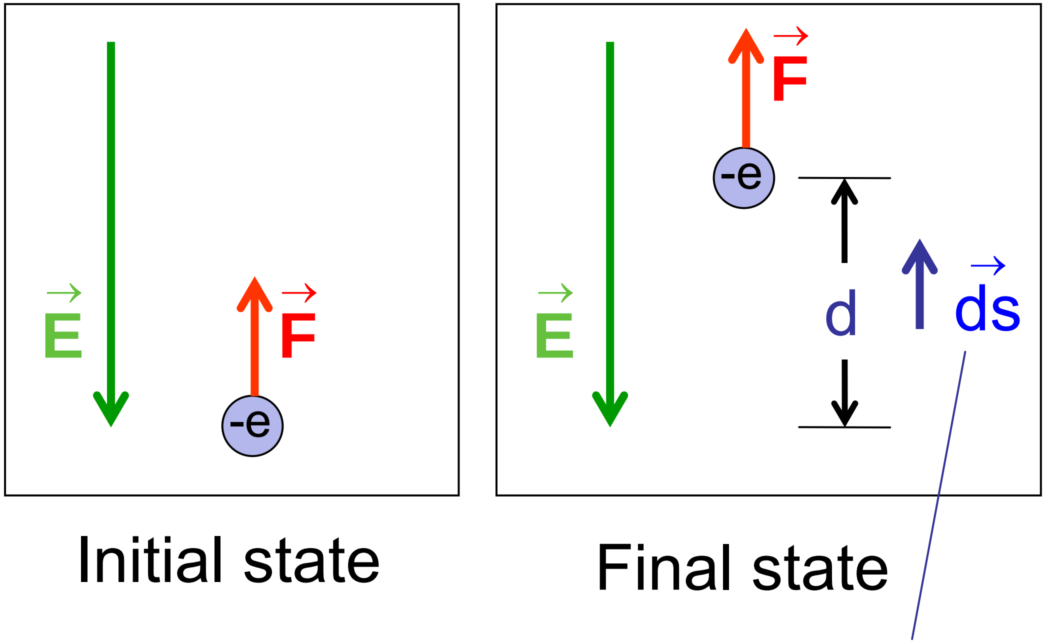
25-1 Electric Potential Energy

Sample Problem 25-1

E = 150 N/m uniform

d = 520 m

What is the change in the electric potential energy of the electron?



small displacement vector points towards the final position

$$\Delta U = -W = -\int_{s=0}^{s=d} \vec{F}\bullet d\vec{s} = -\int_{s=0}^{s=d} (-e)\,\vec{E}\bullet d\vec{s}$$

$$= e\int_{s=0}^{s=d} \vec{E}\bullet d\vec{s} = e\int_{s=0}^{s=d} E\,ds\cos(180) = -e\,E\,d$$

$$= (1.6\times10^{-19}C)(150N/C)(520m) = -1.2\times10^{-14}\ J$$

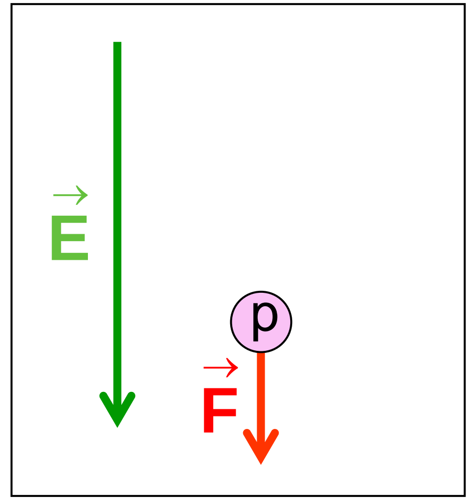
The electric potential energy of the electron decreases

25-1 Electric Potential Energy

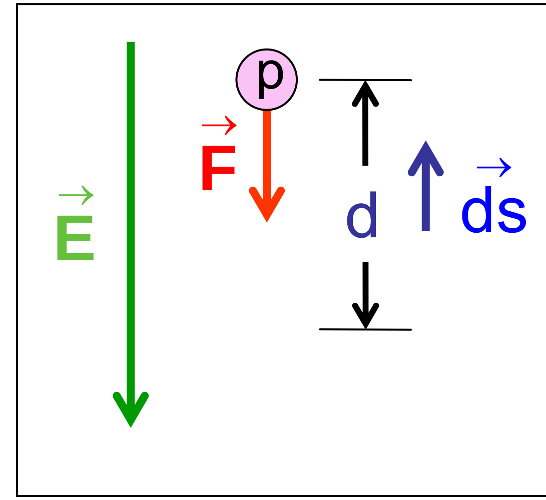
Checkpoint 1

E uniform

Does the electric field do positive or negative work on the proton?



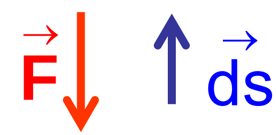
Initial state



Final state

Negative work

$$W = \int_{s=0}^{s=d} \vec{F} \bullet d\vec{s}$$

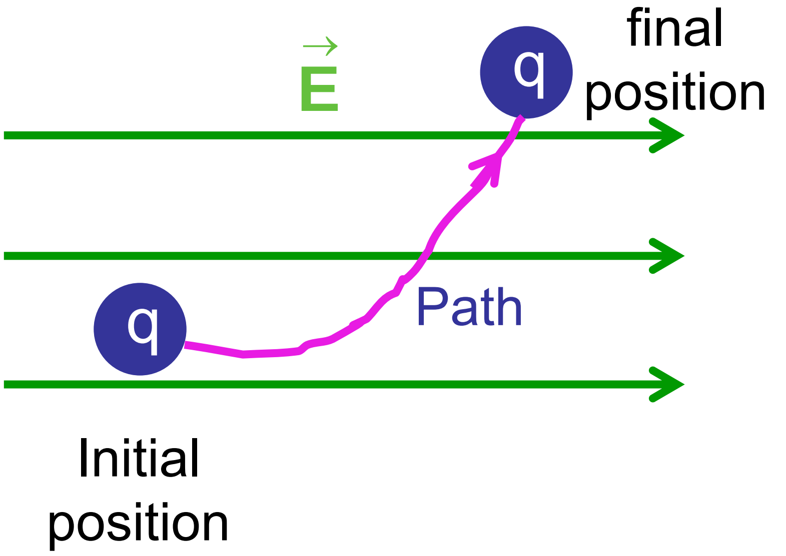


Does the electric potential energy of the proton increase or decrease?

$$\Delta U = - W$$

The electric potential energy of the proton increases

25-1 Electric Potential Energy



$$\Delta U = - W$$

$$\Delta U = - \int_i^f \vec{F} \cdot d\vec{s}$$

$$\Delta U = -q \int_i^f \vec{E} \cdot d\vec{s}$$

The change in the electric potential energy depends on the charge as well as on the electric field

25-2 Electric Potential

Electric potential — Electric potential Energy

$$V = \frac{U}{q}$$

Electric potential difference

$$\Delta V = \frac{\Delta U}{q}$$

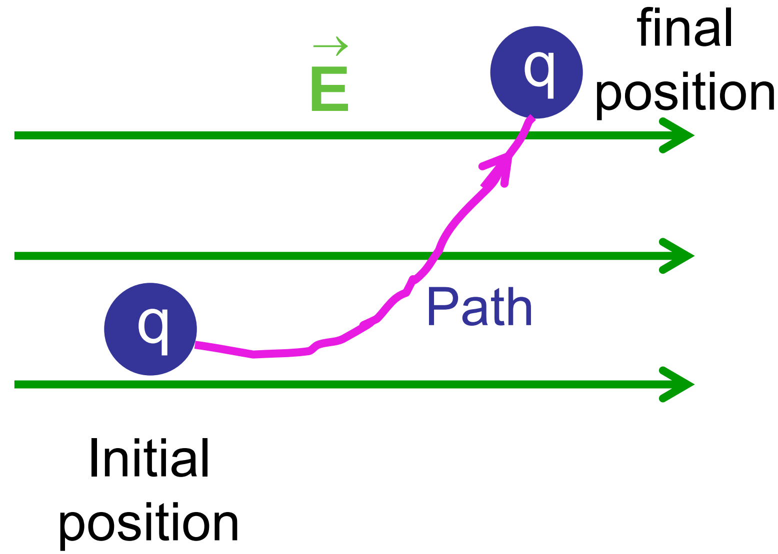
The electric potential difference ΔV is the difference in the electric potential energy ΔU per unit charge

25-2 Electric Potential

$$\Delta V = \frac{\Delta U}{q}$$

$$\Delta V = \frac{-W}{q}$$

$$\Delta V = \frac{-q \int_i^f \vec{E} \bullet d\vec{s}}{q}$$

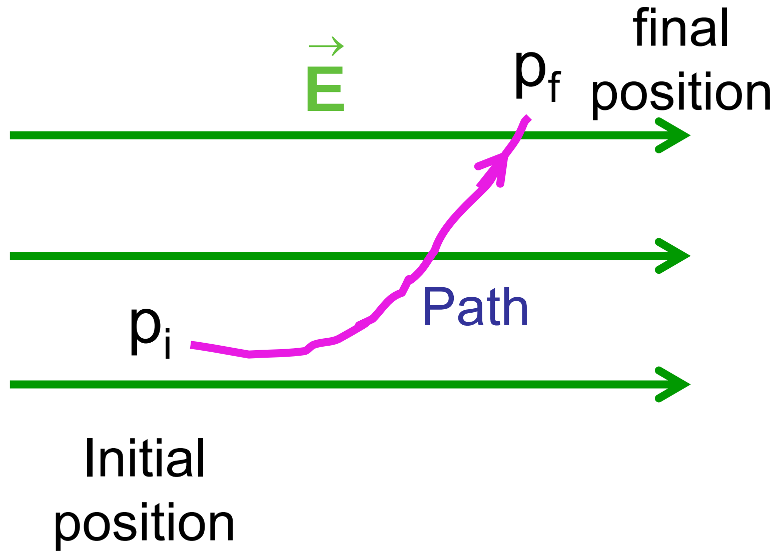


$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{s}$$

Electric potential difference depends only on the electric field

Electric potential is property of an electric field

25-2 Electric Potential



$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$$

Electric potential difference depends only on the electric field

Electric potential is property of an electric field

25-2 Electric Potential

$$\Delta V = \frac{\Delta U}{q}$$

$$\Delta V = \frac{-W}{q}$$

If we choose $U_\infty = 0$ when all particles at infinity

The electric potential at any point in an electric field

$$V = \frac{-W_\infty}{q}$$

Work done by the electrostatic field on a charged particle as that particle moves in form infinity to the point

25-2 Electric Potential

Electric potential is measured in volts

$$\text{Volt} = \frac{\text{Joule}}{\text{Coulomb}}$$

$$\Delta V = \frac{-W}{q}$$

25-2 Electric Potential

Electric field is measured in

Newton/Coulomb
N/C

or

Volt/meter
V/m

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\Delta V = -\int_i^f \vec{E} \bullet \vec{ds}$$

$$\frac{\text{Newton}}{\text{Coulomb}} = \frac{\text{Volt}}{\text{meter}}$$

25-2 Electric Potential

Energy is measured in

Joule	or	Electron-volt
J		eV

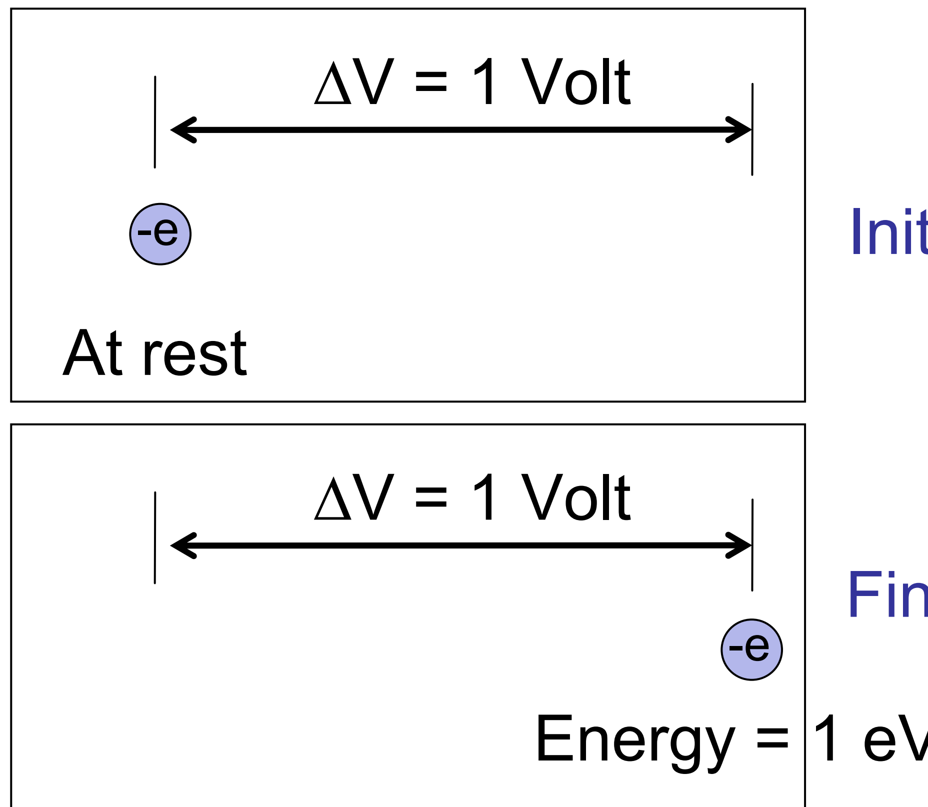
$$\Delta V = \frac{\Delta U}{q}$$

$$\Delta U = q\,\Delta V$$

$$1\ \text{eV} = e\ \frac{\text{J}}{\text{C}}$$

$$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{C}\ \frac{\text{J}}{\text{C}}$$

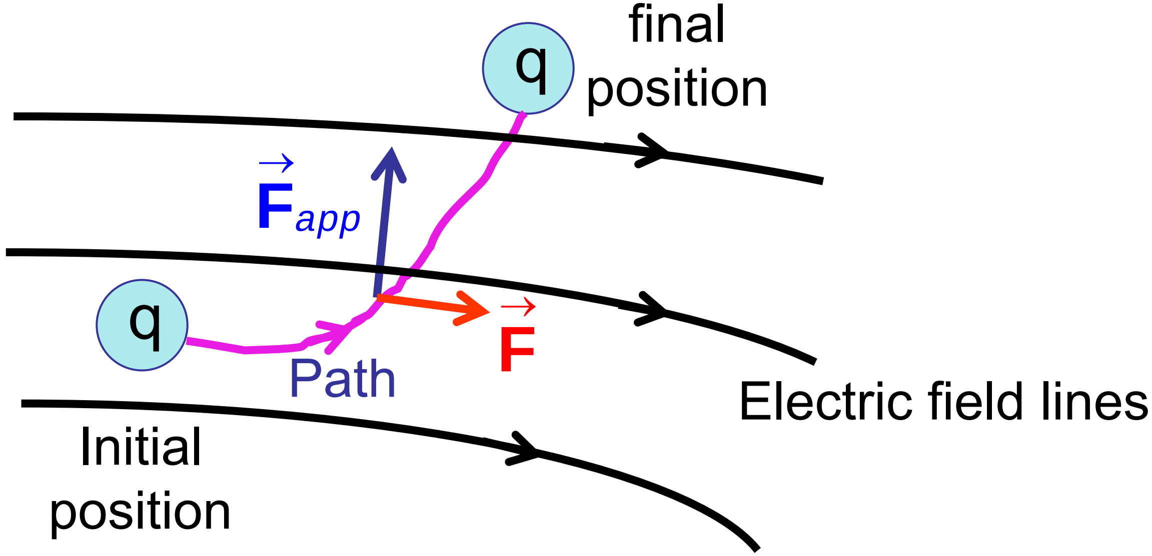
$$\boxed{1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}}$$



Initial

Final

25-2 Electric Potential



Work-kinetic energy theorem

$$\Delta K = W_{app} + W$$

Change in the kinetic energy of the particle $K_f - K_i$

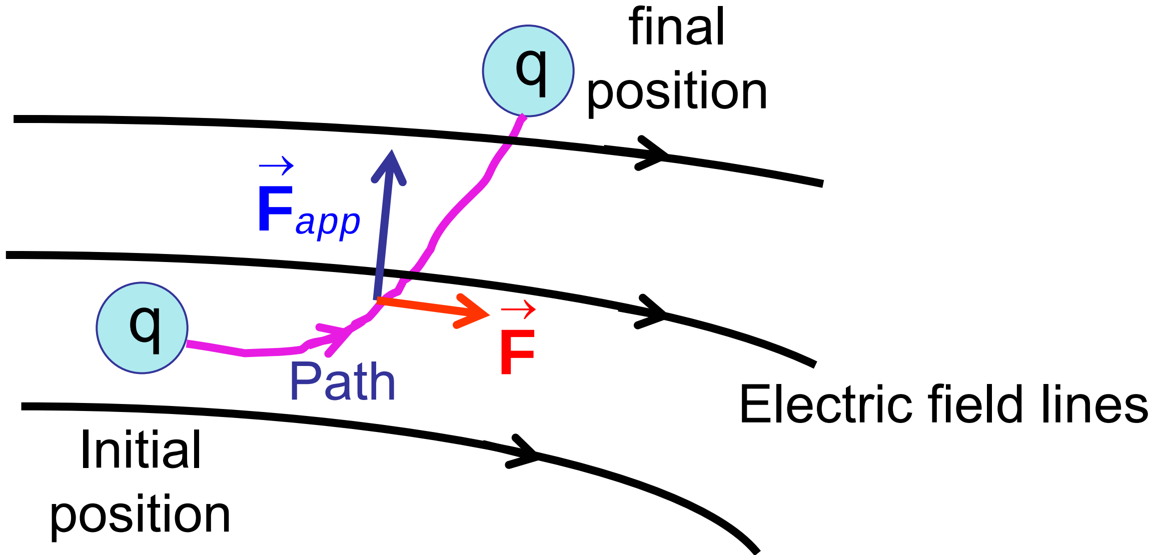
Work done by the applied forces

Work done by the field $-\Delta U$

25-2 Electric Potential

$$\Delta K = W_{app} + W$$

$$W = -\Delta U = -q\,\Delta V$$



If the kinetic energy of the particle does not change
$K_f = K_i$

$$0 = W_{app} + W$$

$$W_{app} = -W = -(-\Delta U)$$

$$W_{app} = \Delta U$$

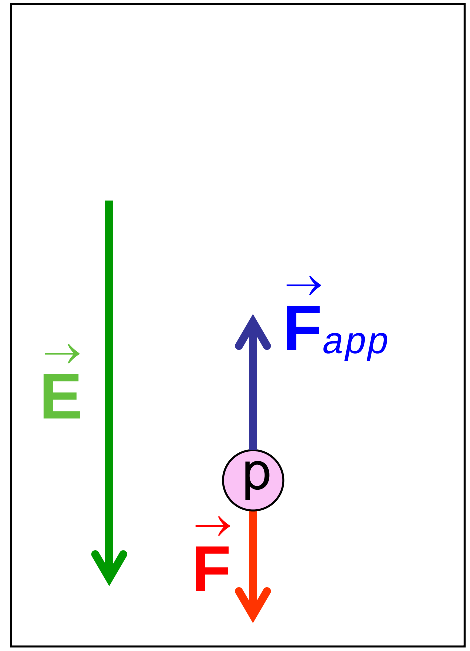
$$W_{app} = q\,\Delta V$$

25-2 Electric Potential

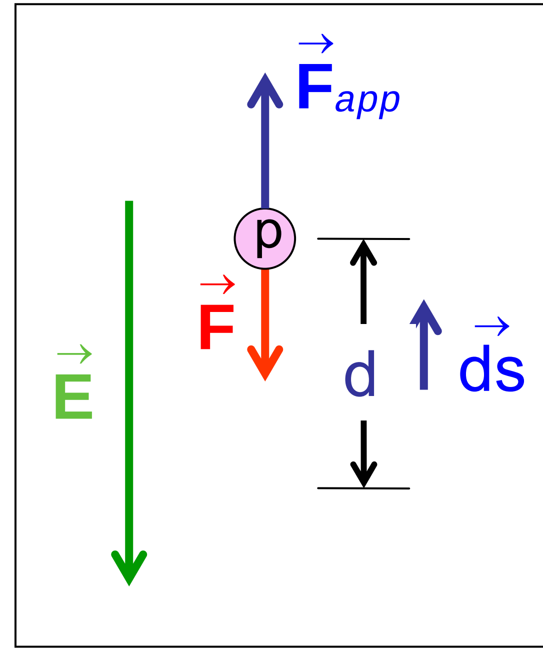
Checkpoint 2

E uniform

Does our force do positive or negative work on the proton?



Initial state



Final state

Positive work

$$W_{app} = \int_{s=0}^{s=d} \vec{F}_{app} \bullet \vec{ds}$$

Does the proton move to a point of higher or lower electric potential?

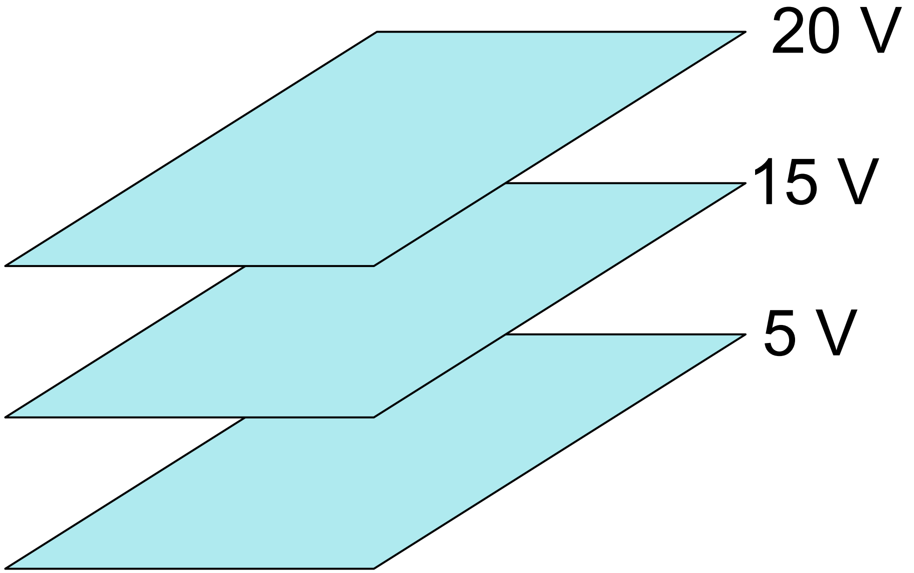
$$\Delta U = W_{app}$$

Higher potential

$$\Delta V = \frac{W_{app}}{q}$$

25-3 Equipotential surfaces

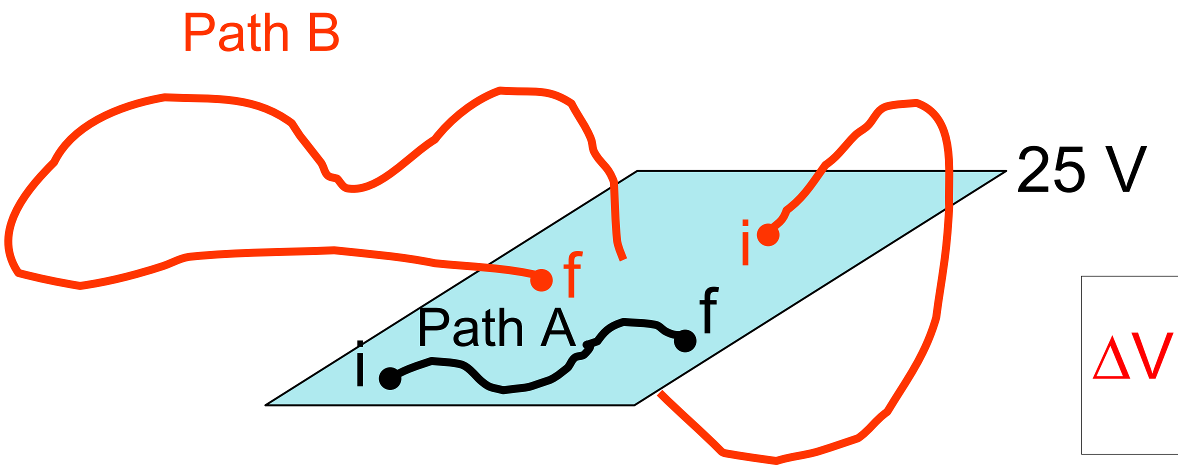
All points on an equipotential surface have the same electric potential



equipotential surfaces

25-3 Equipotential surfaces

When a charge particle moves between two points on the same equipotential surface, no net work is done on it.

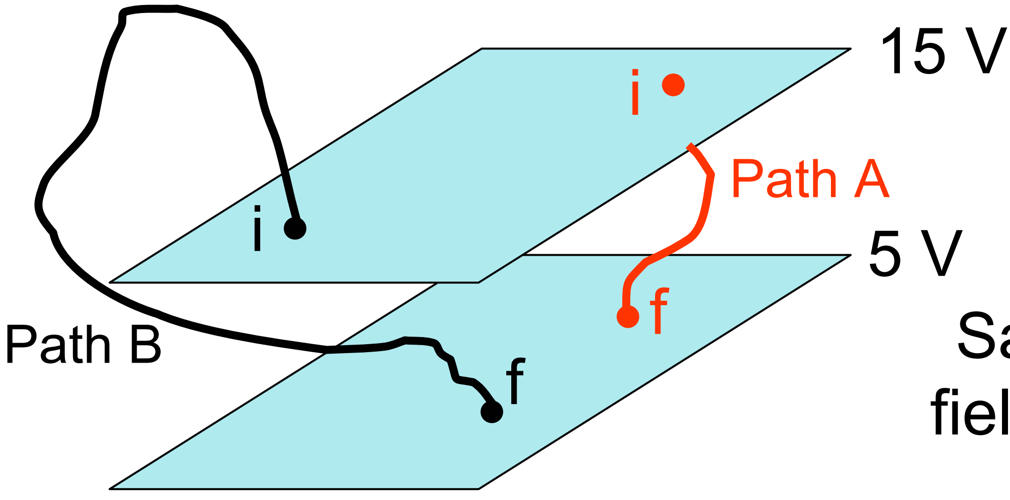


$$\Delta V = \frac{-W}{q}$$

equipotential surface $\Delta V = 0$

$W = 0$

25-3 Equipotential surfaces



equipotential surfaces

$$\Delta V = \frac{-W}{q}$$

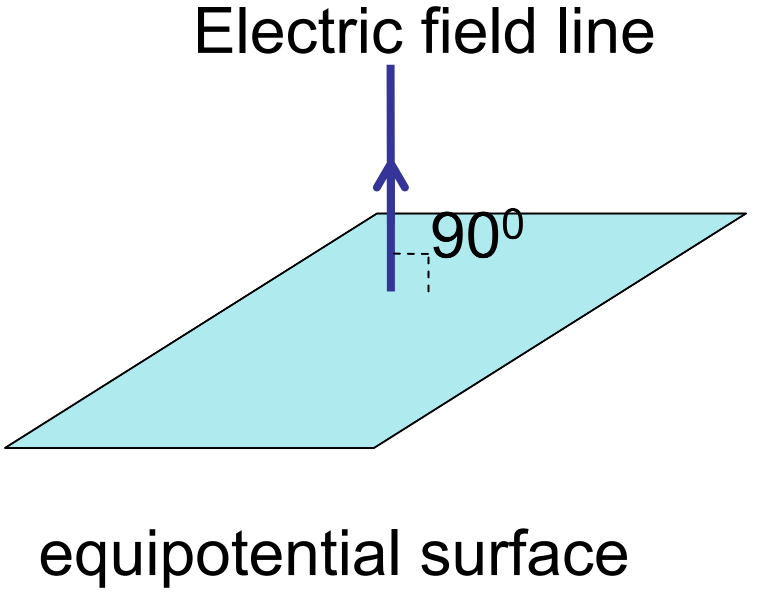
Same work is done by the field on the charged particle

$$W = -q\,\Delta V$$

$$W = -q\,(5 - 15) = 10\,q$$

25-3 Equipotential surfaces

Equipotential surfaces are always perpendicular to the electric field lines



25-3 Equipotential surfaces

Why are equipotential surfaces always perpendicular to the electric field lines?

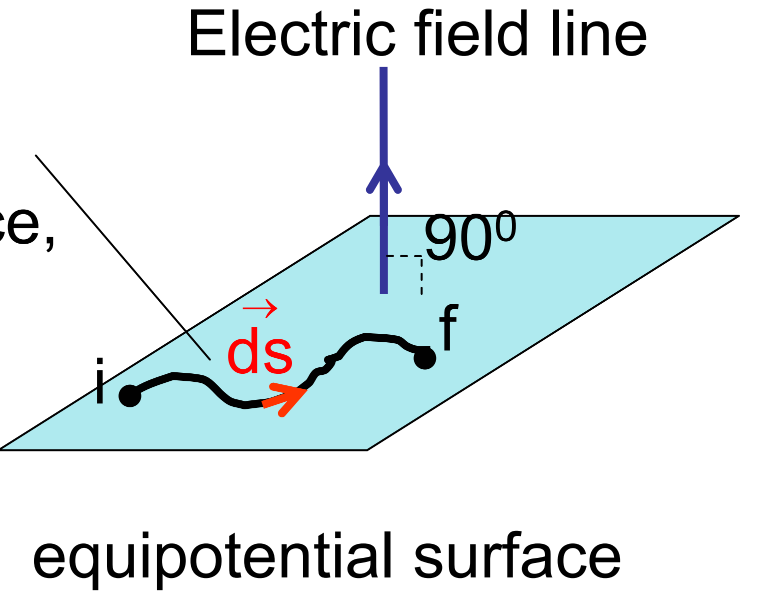
$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{s}$$

Equipotential surface → $\Delta V = 0$

$$0 = -\int_i^f \vec{E} \bullet d\vec{s}$$

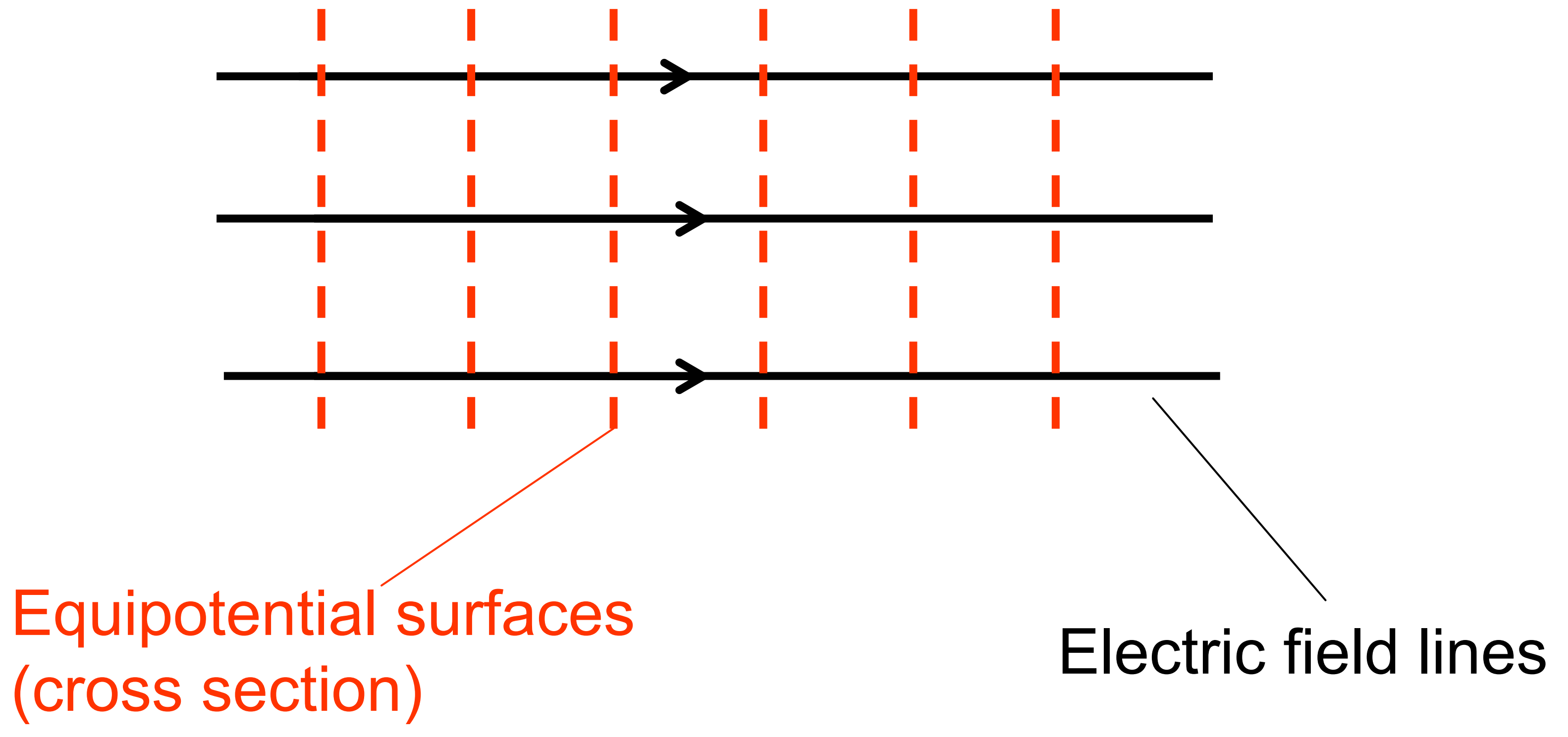
To satisfy this on any path on the equipotential surface,

$$\vec{E} \perp d\vec{s}$$



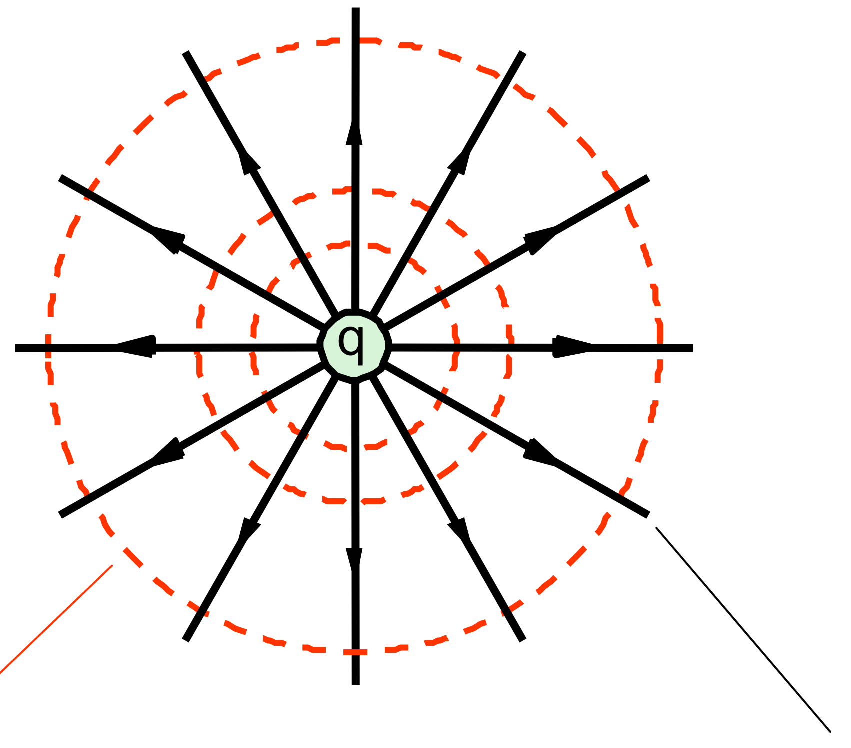
25-3 Equipotential surfaces

Equipotential surfaces for a uniform electric field



25-3 Equipotential surfaces

Equipotential surfaces for a point charge

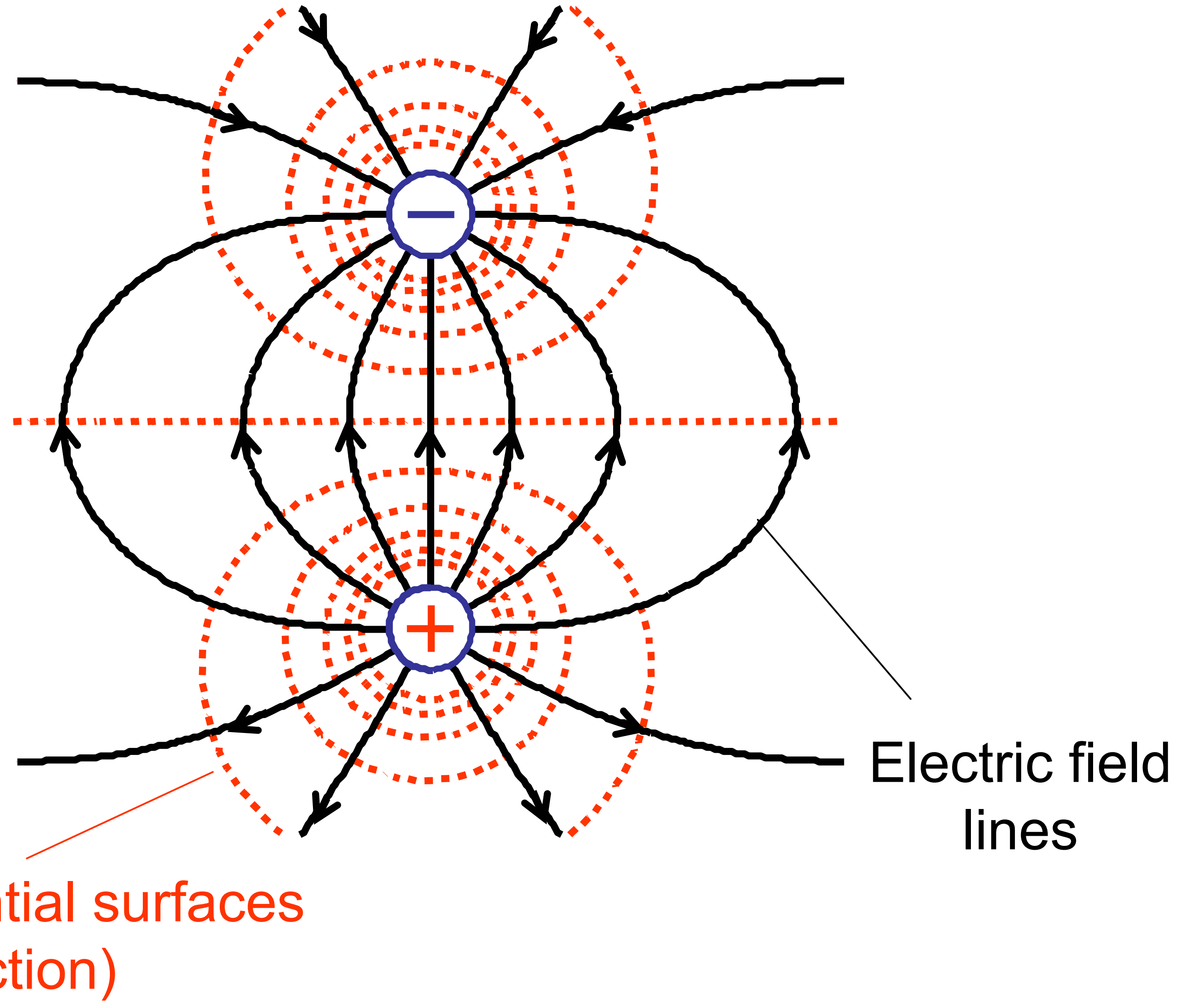


Equipotential surfaces
(cross section)

Electric field lines

25-3 Equipotential surfaces

Equipotential surfaces for an electric dipole



25-4 Calculating the potential from the field

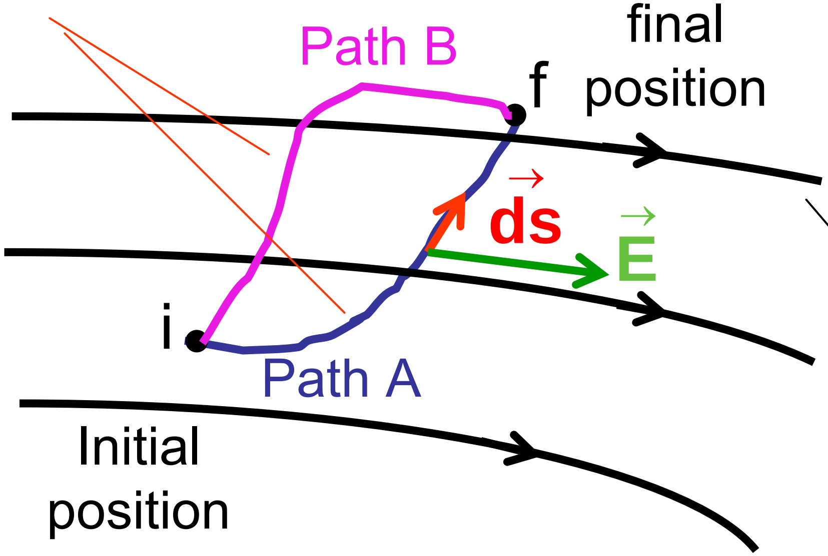
If the electric field is known, the electric potential difference can be found from

$$\Delta V = -\int_{i}^{f} \vec{E}\cdot d\vec{s}$$

Differential displacement points toward the final position

Line integral (integral over particular path)

Any path will produce the same result



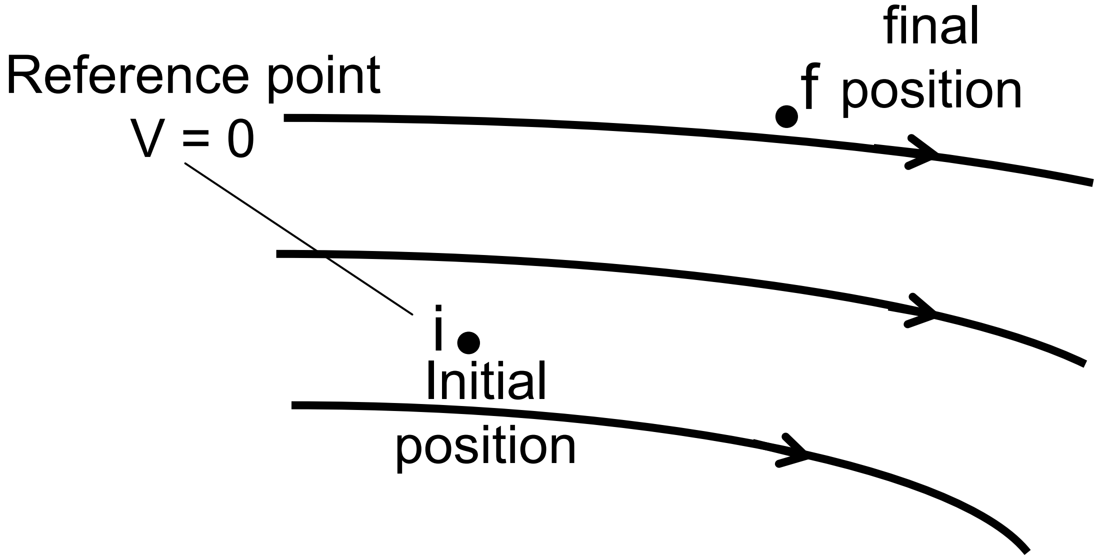
Electric field lines

25-4 Calculating the potential from the field

$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{s}$$

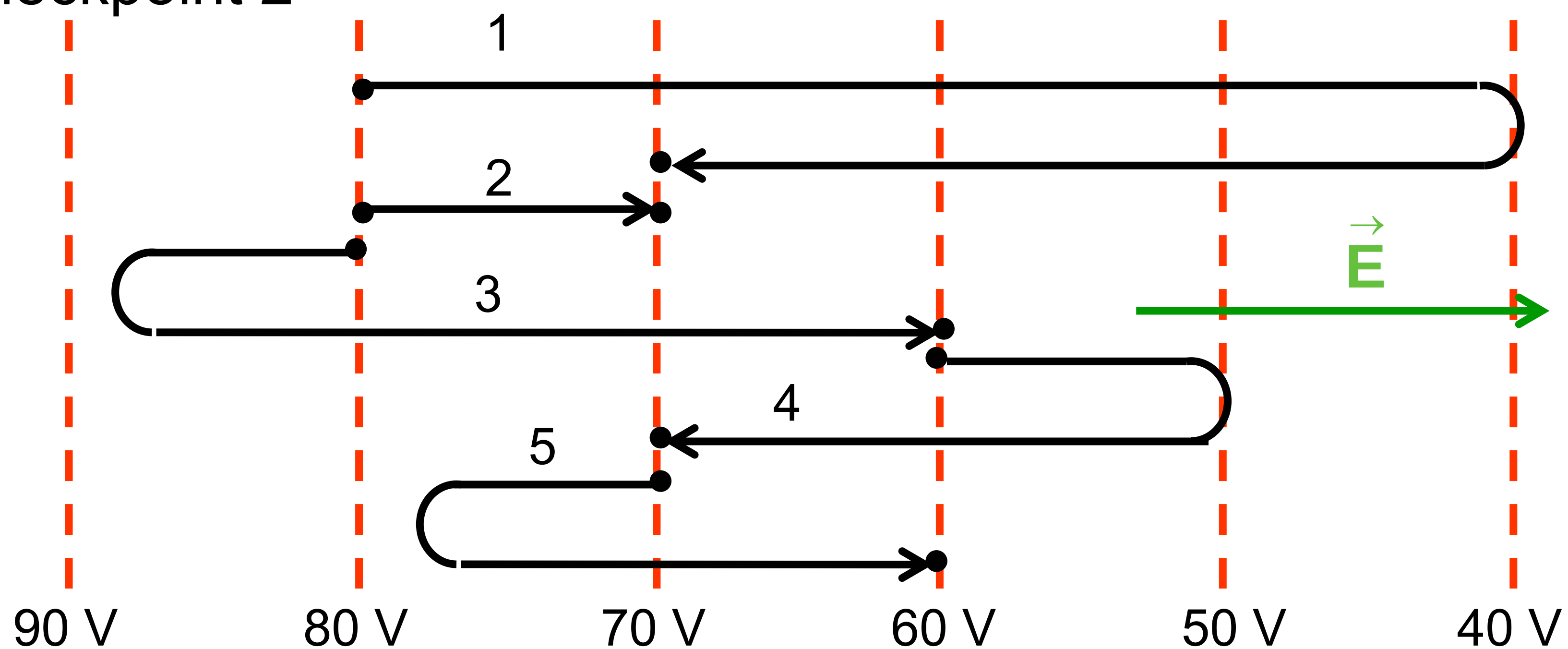
If we choose $V_i = 0$ at point i , we can write the electric potential at any point relative to zero potential at point i

$$V = -\int_i^f \vec{E} \bullet d\vec{s}$$



25-4 Calculating the potential from the field

Checkpoint 2



We will move an electron along these lines

What is the direction of the electric field?

What is the work we do for each path?

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{s}$$

$$W_{app} = q\,\Delta V$$

1 → $W_{app} = (-e)(-10) = 10e$

2 → $W_{app} = (-e)(-10) = 10e$

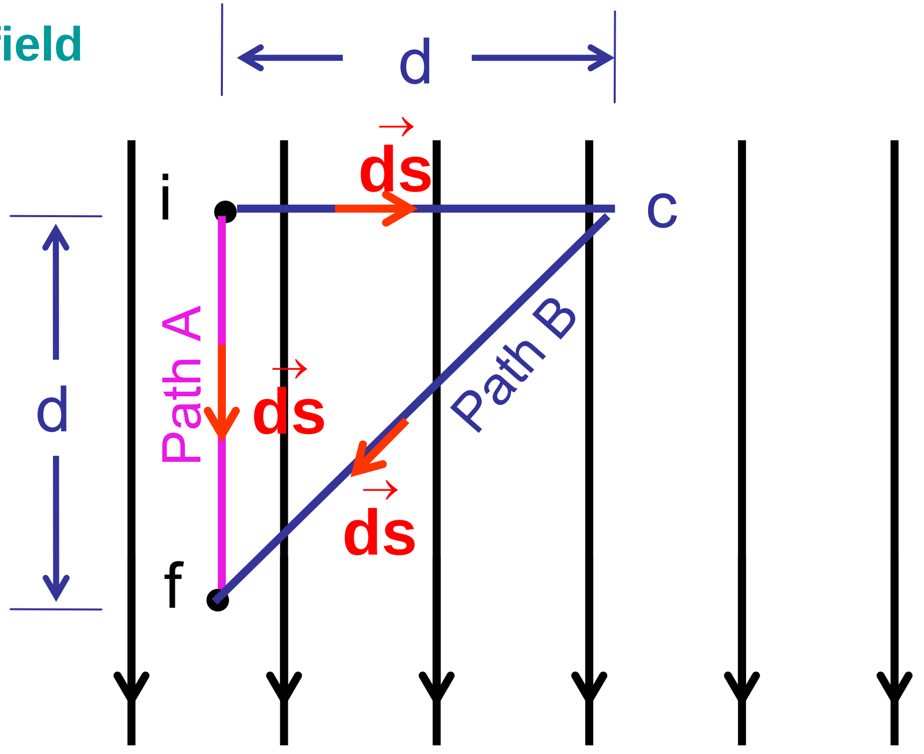
3 → $W_{app} = (-e)(-20) = 20e$

4 → $W_{app} = (-e)(10) = -10e$

5 → $W_{app} = (-e)(-10) = 10e$

25-4 Calculating the potential from the field

Sample Problem 25-2



Find the electric potential difference between i and f by using path A and B

Path A

$$\Delta V = -\int_i^f \vec{E}\bullet d\vec{s} = -\int_i^f E\,ds$$

$$= -E\int_i^f ds = -Ed$$

Path B

From i to c $\quad V_c - V_i = -\int_i^c \vec{E}\bullet d\vec{s} = -\int_i^c E\,ds\cos(90^0) = 0$

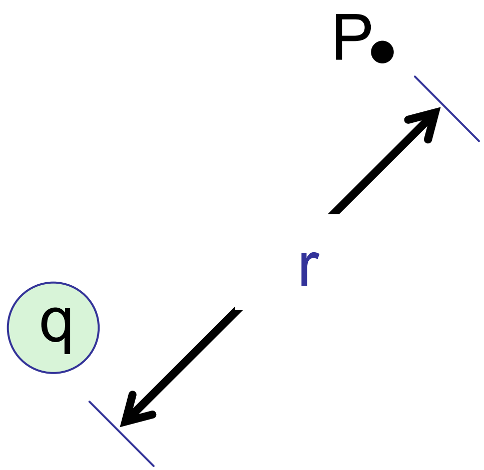
From c to f $\quad V_f - V_c = -\int_c^f \vec{E}\bullet d\vec{s} = -\int_c^f E\,ds\cos(45^0)$

$$= -\frac{E}{\sqrt{2}}\int_i^f ds = -\frac{E}{\sqrt{2}}\sqrt{2}d = -E\,d$$

25-5 Potential Due to a Point Charge

The electric potential due to a particle of charge q at distance r form the particle

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



$V = 0$ when $r = \infty$

25-5 Potential Due to a Point Charge

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

A positively charged particle produces
a positive electric potential

A negatively charged particle produces
a negative electric potential

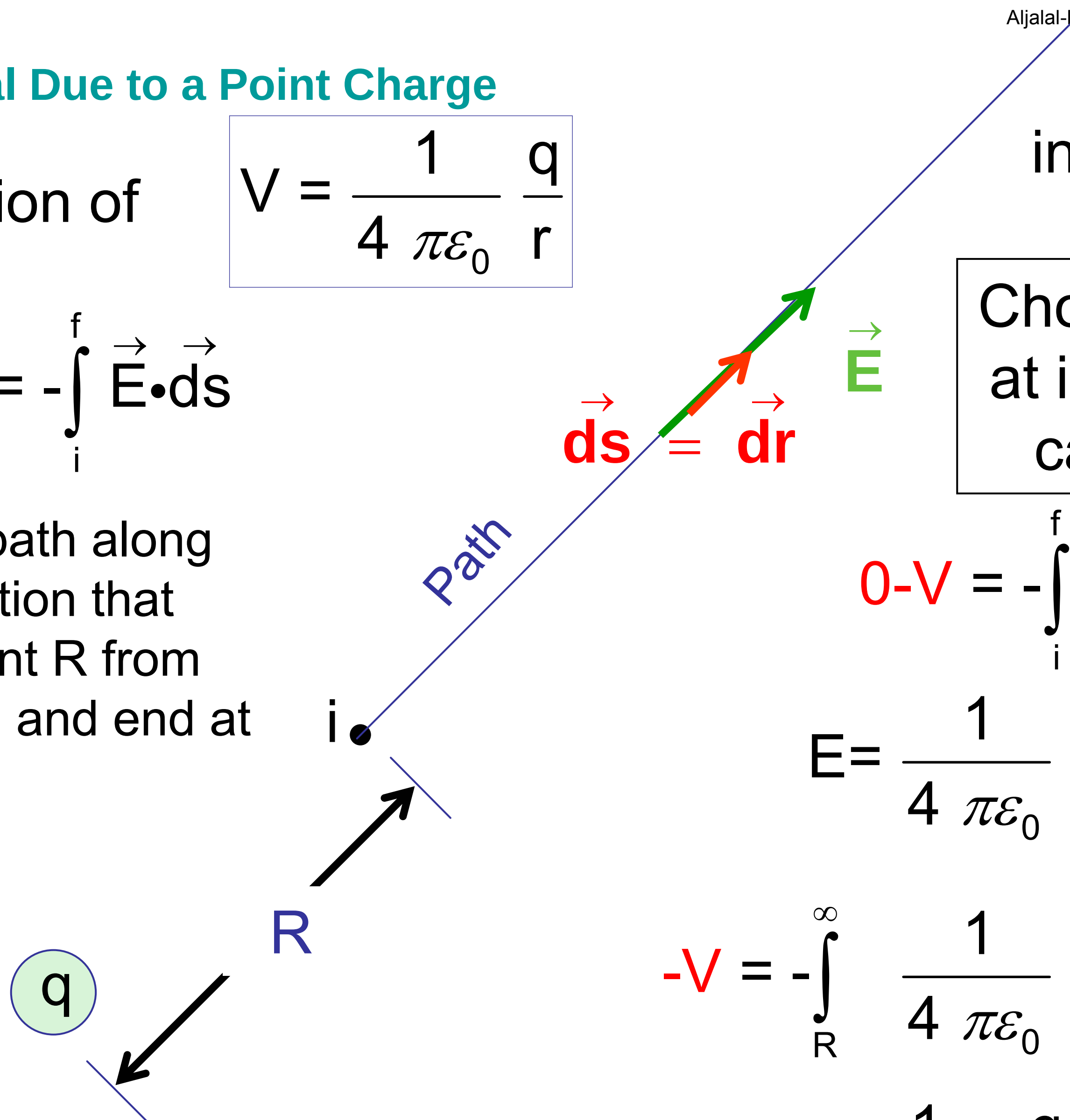
25-5 Potential Due to a Point Charge

Derivation of

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

Choose a path along radial direction that starts a point R from the charge and end at infinity



Choose $V_f = 0$ at infinity and call $V_i = V$

$$0 - V = -\int_i^f E\, dr$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$-V = -\int_R^\infty \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\, dr$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

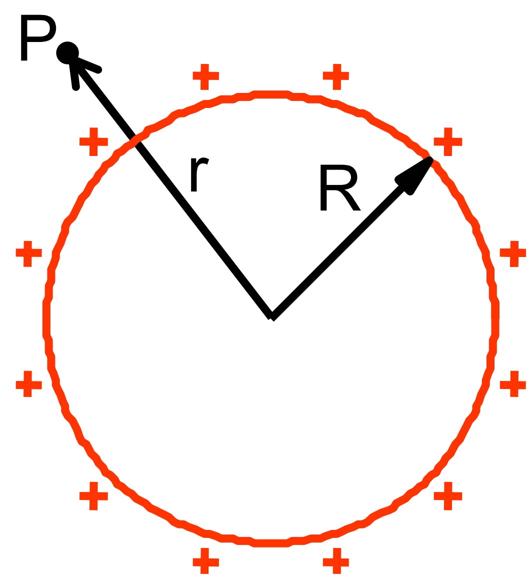
25-5 Potential Due to a Point Charge

Electric potential outside a spherically symmetric charge distribution of radius R and with total charge q

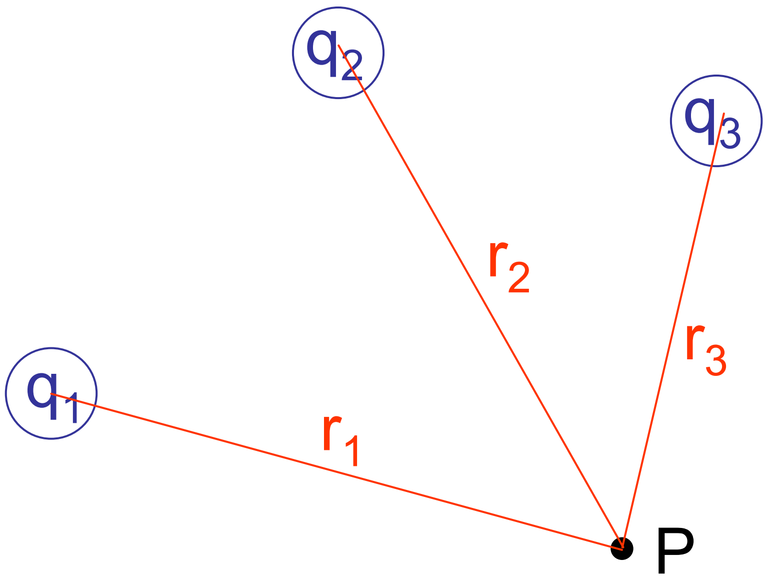
r > R

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Similar to the electric potential due to a point charge q placed at the center of the sphere



25-6 Potential Due to a group of Point Charges

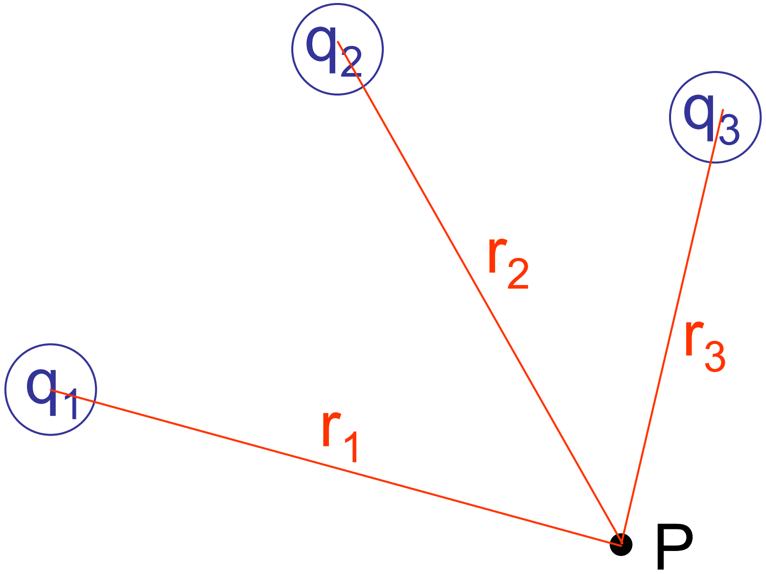


Superposition

Total electric potential V at point P is the algebraic sum of the electric potential at point P from each point charge

$$V = V_1 + V_2 + V_3$$

25-6 Potential Due to a group of Point Charges



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

Vector

Vector sum

Difficult

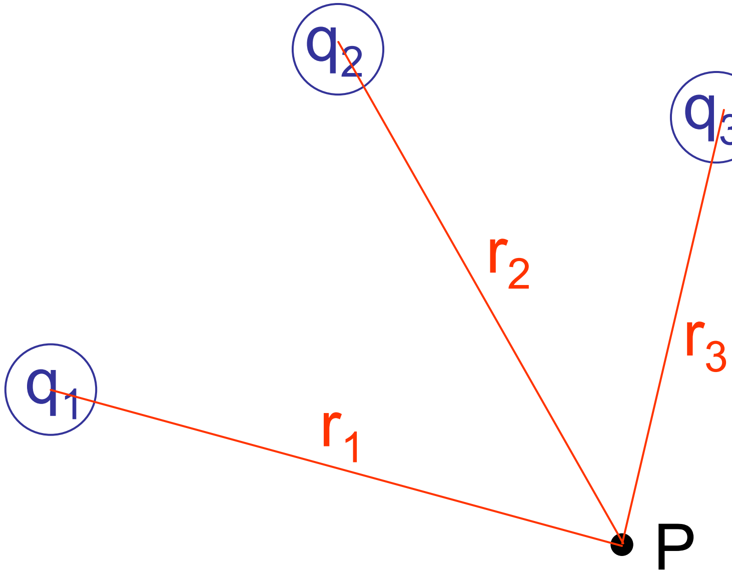
$$V = V_1 + V_2 + V_3$$

Scalar

Algebraic sum

Easy

25-6 Potential Due to a group of Point Charges



$$V = V_1 + V_2 + V_3$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

Distance from q_3

25-6 Potential Due to a group of Point Charges

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{3}\frac{q_i}{r_i}$$

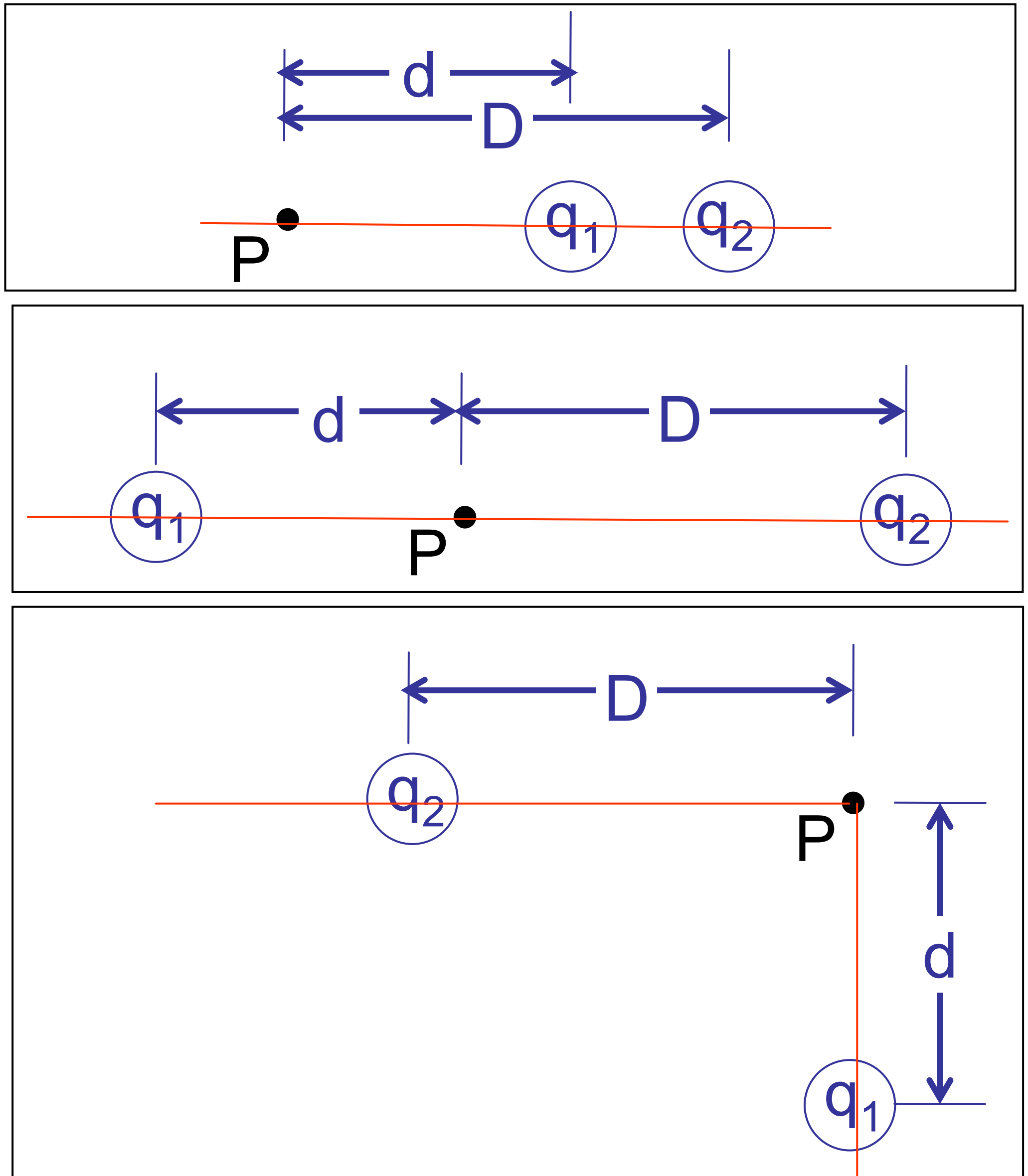
For 3 charged particles

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

For n charged particles

25-6 Potential Due to a group of Point Charges

Checkpoint 4



Rank the net electric potential at P, greatest first

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{d} + \frac{q_2}{D}\right)$$

All tie

25-6 Potential Due to a group of Point Charges

Sample Problem 25-3

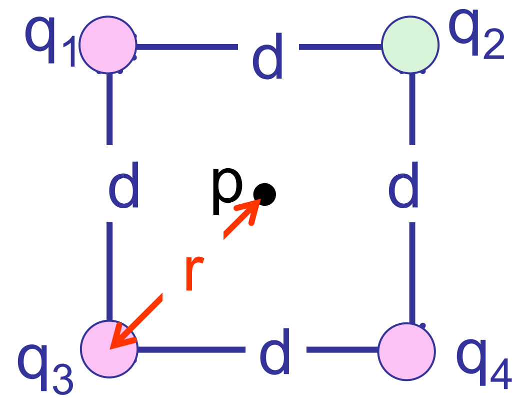
d =1.3 m

q_1= +12 nC

q_2= -24 nC

q_3= +31 nC

q_4= +17 nC



$$r = \frac{d}{\sqrt{2}}$$

Electric potential at point P?

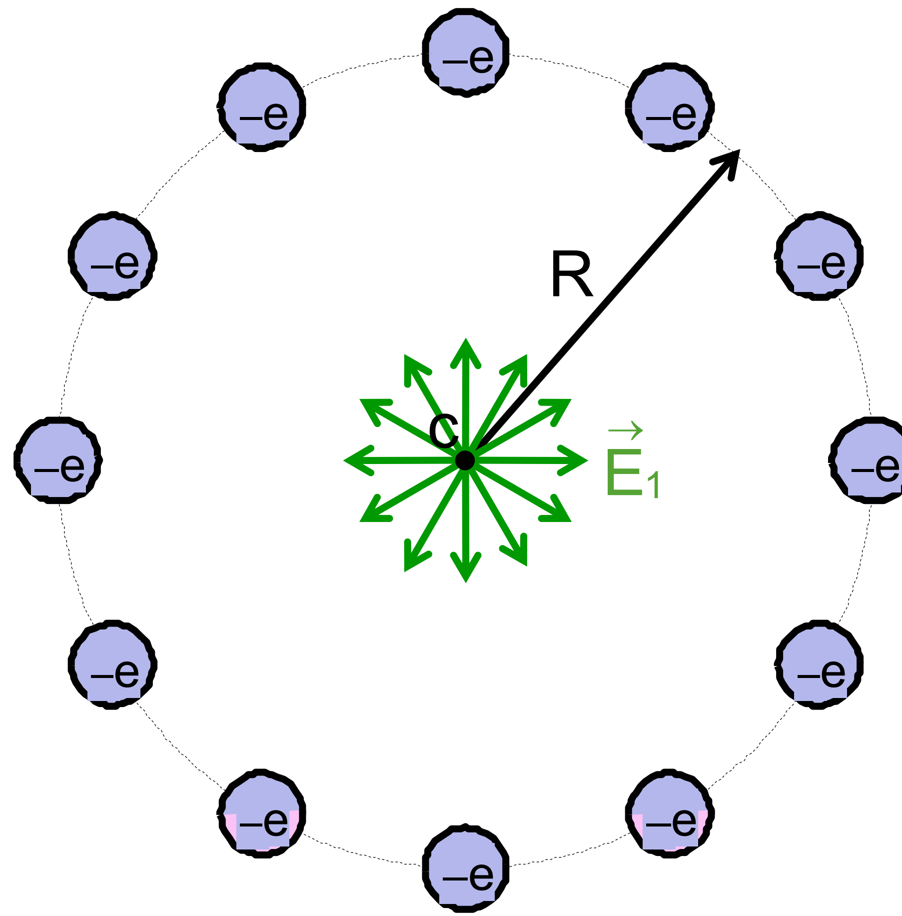
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right)$$

V = 350 V

25-6 Potential Due to a group of Point Charges

Sample Problem 25-4

12 electron are equally spaced on a circle of radius R



What is electric potential at the center C?

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{12}\frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{12(-e)}{R}$$

What is electric Field at the center C?

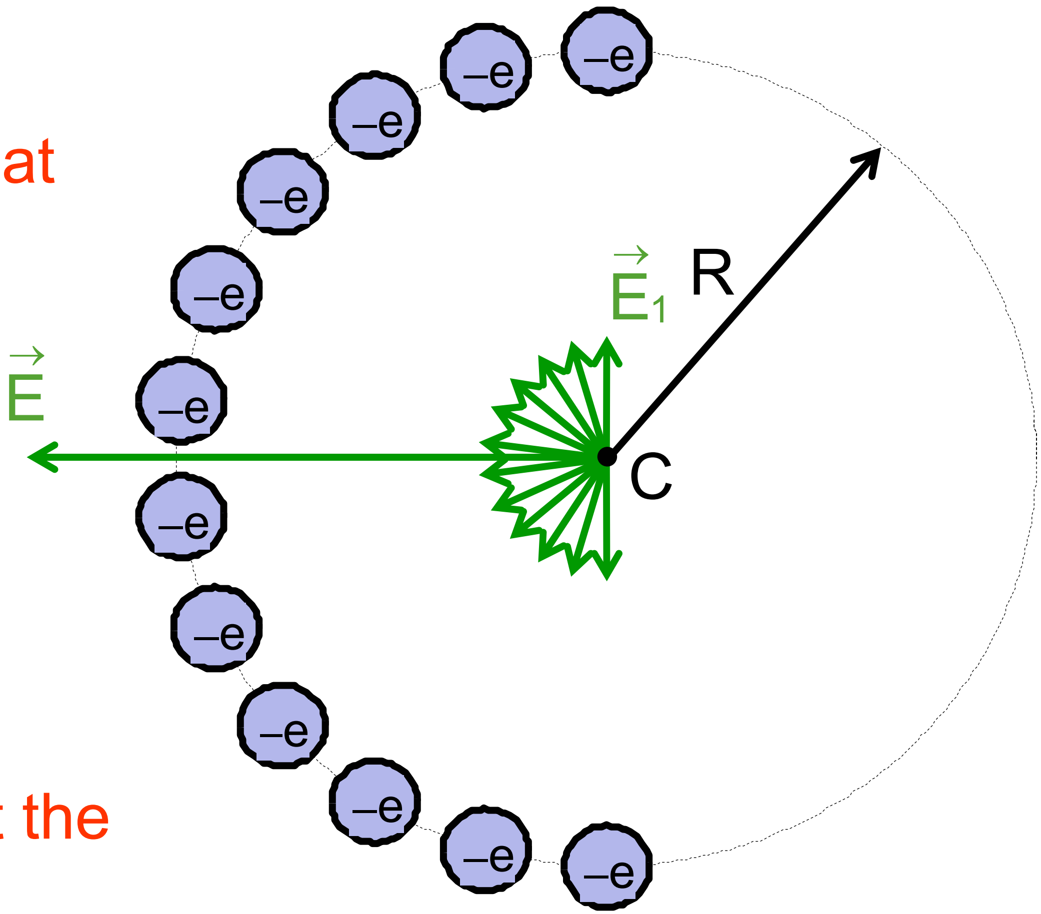
$$\vec{E} = \sum_{i=1}^{12}\vec{E}_i$$

$E = 0$

25-6 Potential Due to a group of Point Charges

Sample Problem 25-4

12 electron on a circle of radius R



What is electric potential at the center C?

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{12}\frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{12(-e)}{R}$$

What is electric Field at the center C?

$$\vec{E} = \sum_{i=1}^{12}\vec{E}_i \neq 0$$

25-9 Calculating the field form the potential

If you know the electric potential V,
You can calculate the component of the electric field along direction s, form

$$E_s = -\frac{\partial V}{\partial s}$$

Partial derivative,
Treat all variables except s variable as constants

The component of the electric field in any direction is the negative of the rate of change of the electric potential with distance in that direction

25-9 Calculating the field form the potential

The component of the electric field along x-axis

$$E_x = -\frac{\partial V}{\partial x}$$

Treat y and z variables as constants

The component of the electric field along y-axis

$$E_y = -\frac{\partial V}{\partial y}$$

Treat x and z variables as constants

The component of the electric field along z-axis

$$E_z = -\frac{\partial V}{\partial z}$$

Treat x and y variables as constants

25-9 Calculating the field form the potential

Sample Problem 25-5

Find the electric field associated with the following electric potential

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

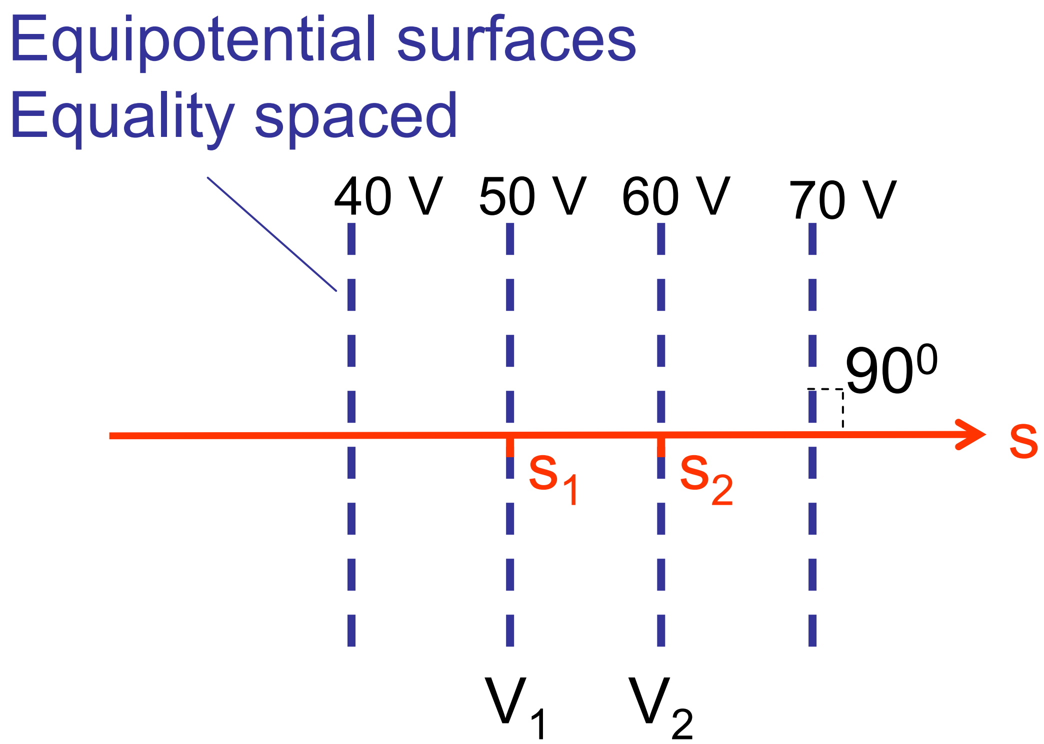
$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right) = 0$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right) = 0$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

25-9 Calculating the field form the potential

Uniform electric field



$$E_s = -\frac{\partial V}{\partial s}$$

$$E_s = -\frac{V_2 - V_1}{s_2 - s_1}$$

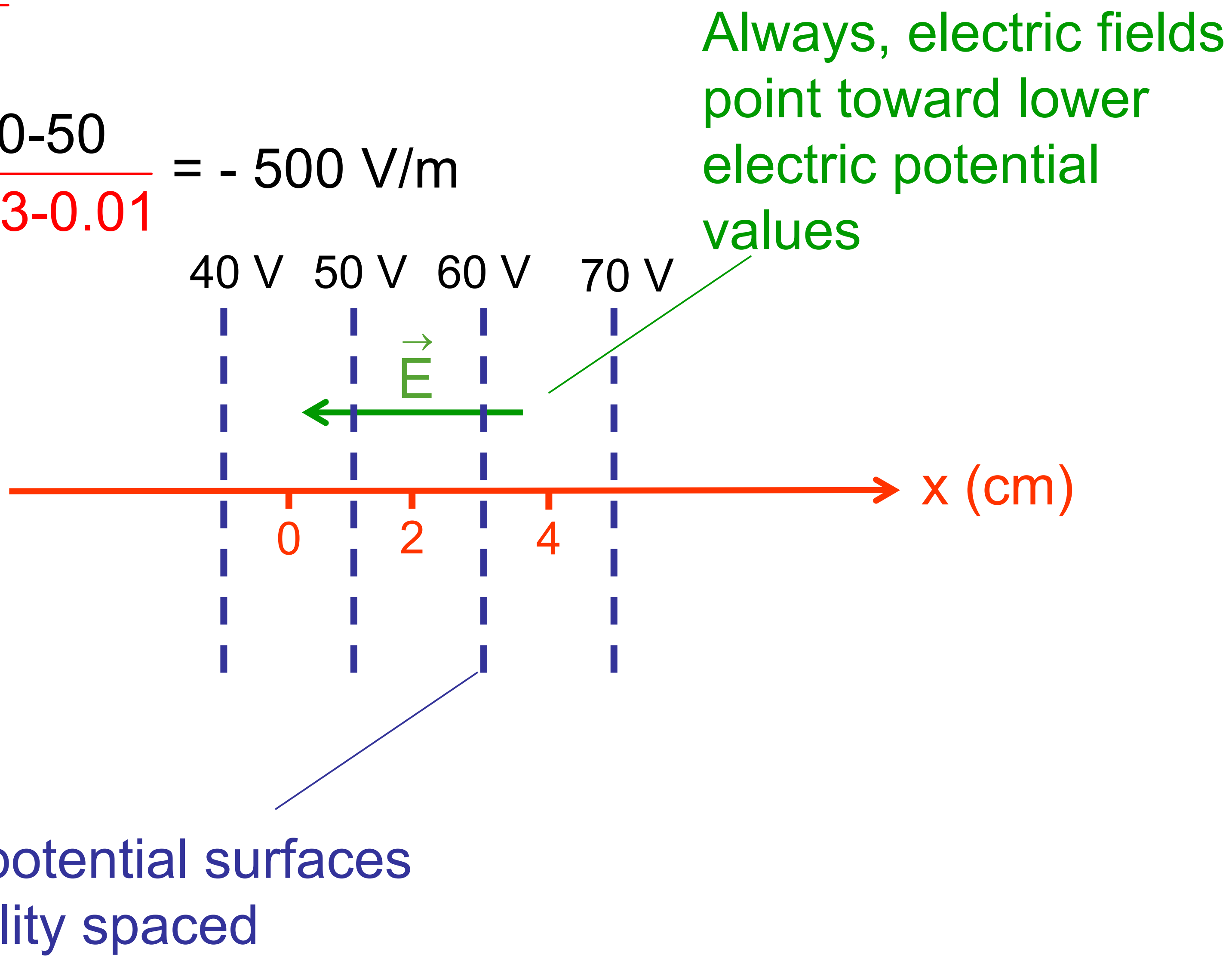
$$E_s = -\frac{\Delta V}{\Delta s}$$

25-9 Calculating the field form the potential

Uniform electric field

$$E_x = -\frac{\Delta V}{\Delta x}$$

$$E_x = -\frac{60-50}{0.03-0.01} = -500\ \text{V/m}$$



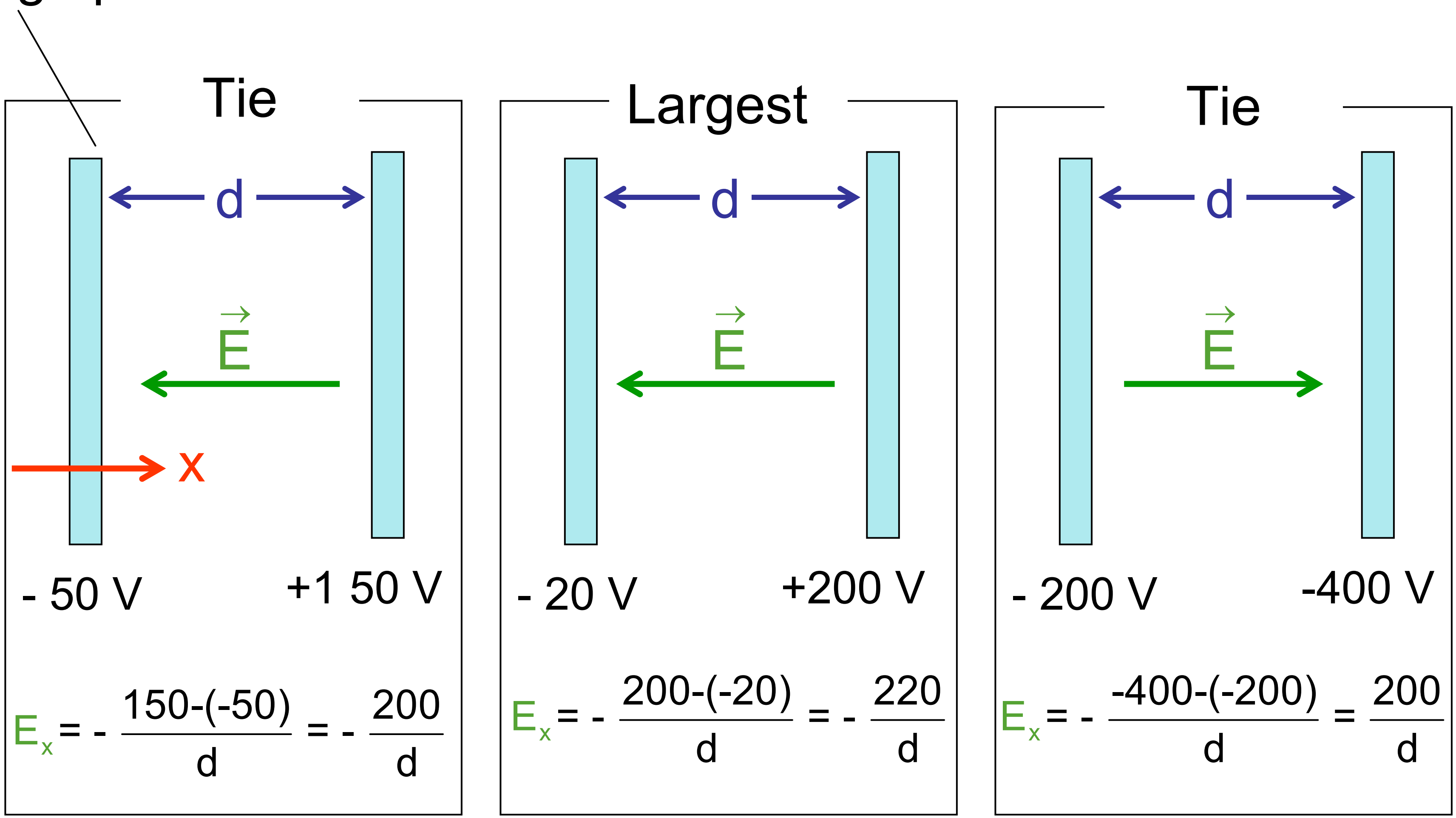
25-9 Calculating the field form the potential

Checkpoint 6

Rank the electric field between the plates

$$E_s = -\frac{\Delta V}{\Delta s}$$

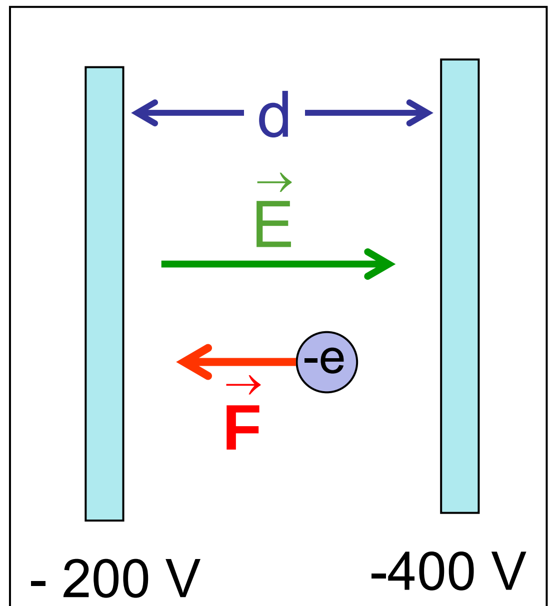
Large plates



25-9 Calculating the field form the potential

Checkpoint 6

What happens to an electron between the plates?



$$E_x = -\frac{-400-(-200)}{d} = \frac{200}{d}$$

The electron accelerates to the left

25-9 Calculating the field form the potential

Deviation of $E_s = -\dfrac{\partial V}{\partial s}$

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{s}$$

If Δs is very small such the electric field does not change much over Δs

$$\Delta V \approx -\vec{E} \bullet \Delta\vec{s}$$

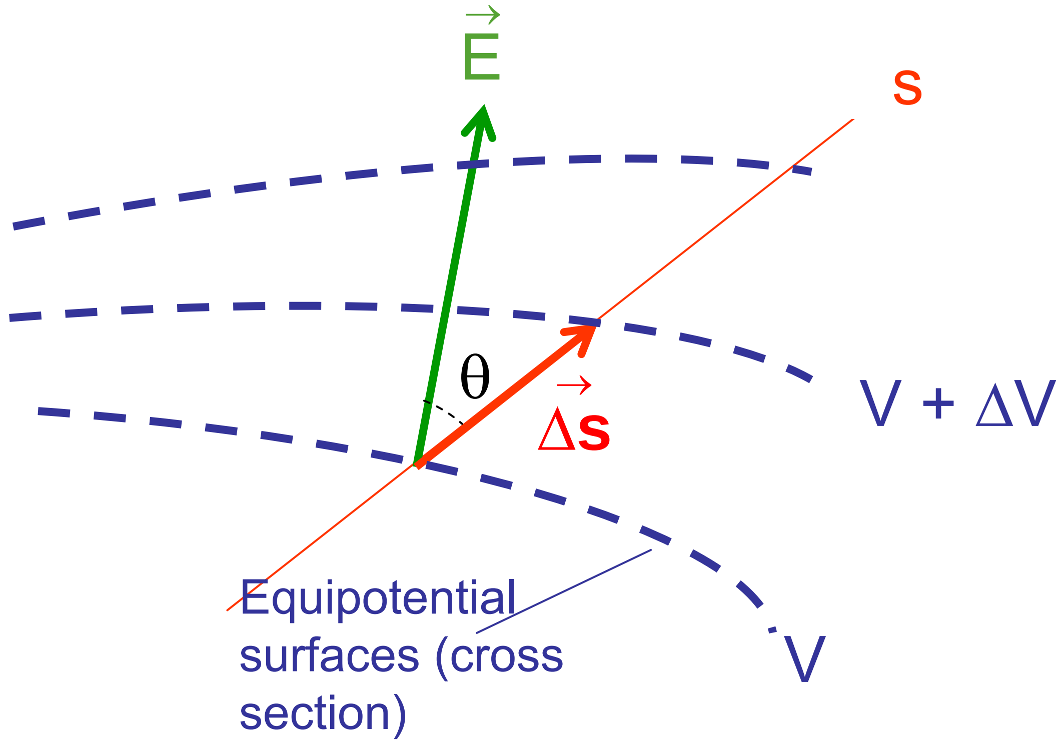
$$\Delta V \approx -E\,\Delta S \cos\theta$$

$$E\cos\theta \approx -\frac{\Delta V}{\Delta S}$$

$$E_s \approx -\frac{\Delta V}{\Delta S}$$

$$E_s = -\lim_{\Delta S \to 0} \frac{\Delta V}{\Delta S}$$

$$E_s = -\frac{\partial V}{\partial s}$$



25-9 Calculating the field form the potential

Know E
Want ΔV

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$$

Uniform E

$$\Delta V = -E\cos\theta\,\Delta s$$

Know V
Want E

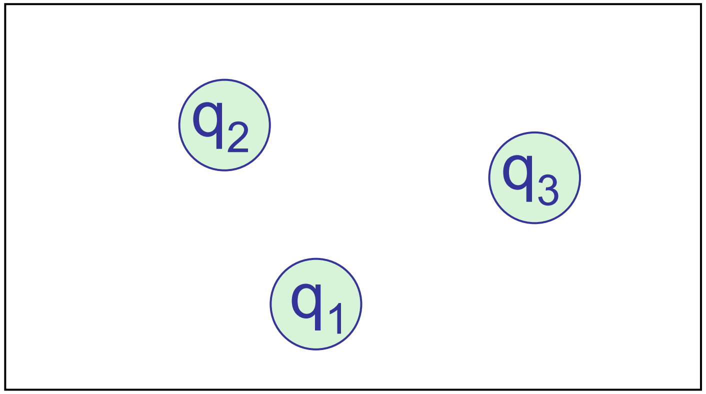
$$E_s = -\frac{\partial V}{\partial s}$$

Uniform E

$$E\cos\theta = -\frac{\Delta V}{\Delta s}$$

$$E\cos\theta = E_s$$

25-10 Electric Potential Energy of a System of Point Charges



What is the electric potential energy of these fixed point charges?

$$\Delta K = W_{app} + W$$

Assume these point charges were stationary at infinity and we brought them to this fixed configuration

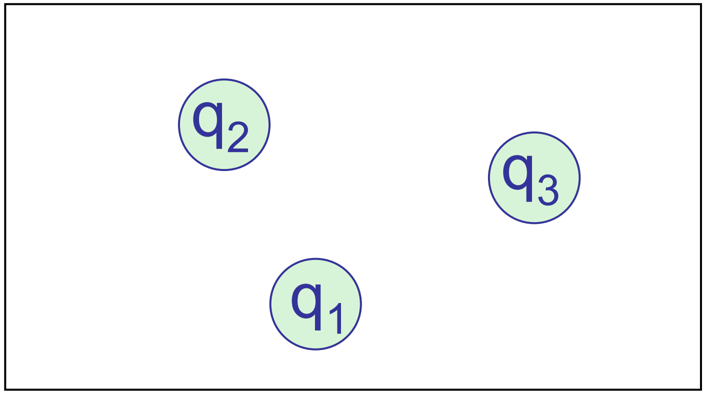
$$0 = W_{app,\infty} + W_{\infty}$$

$$0 = W_{app,\infty} - (U - U_{\infty})$$

We will choose our reference $U_{\infty} = 0$

$$U = W_{app,\infty}$$

25-10 Electric Potential Energy of a System of Point Charges

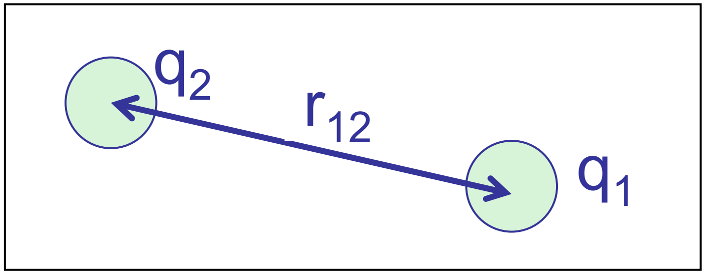


What is the electric potential energy of these fixed point charges?

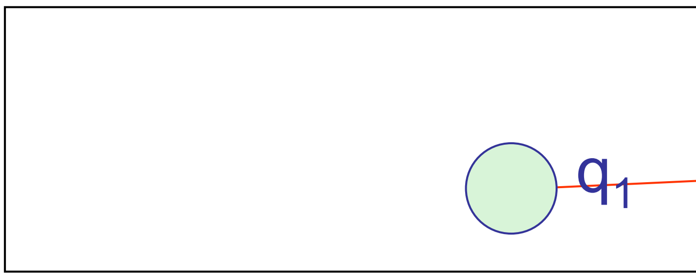
$$U = W_{app,\infty}$$

The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system by bringing each point form an infinite distance.

25-10 Electric Potential Energy of a System of Point Charges



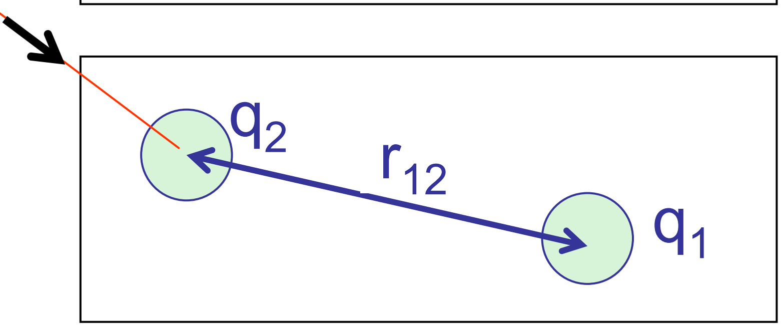
What is the electric potential energy of these fixed point charges?



First, we will bring q_1 form infinity

Since there is no external field

$$W_{1\text{-app},\infty} = 0$$



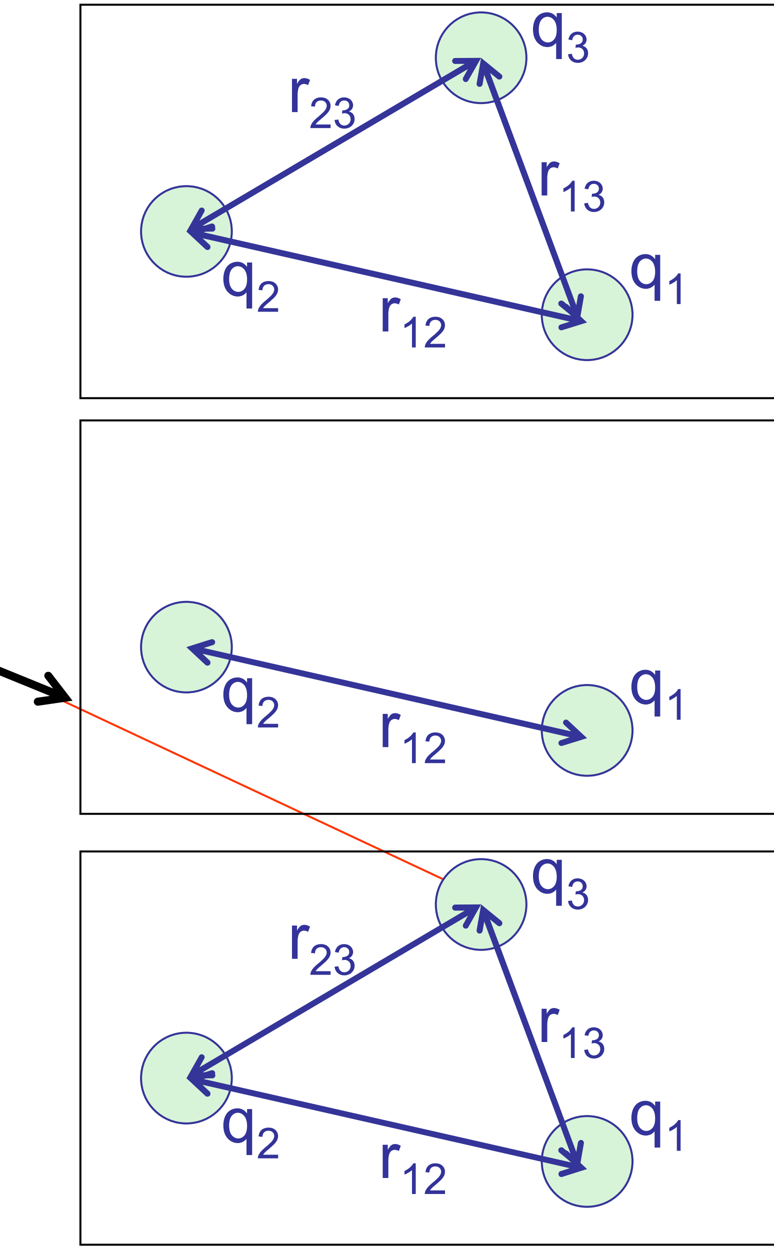
Second, we will bring q_2 form infinity

It will move in the electric field of q_1

$$W_{1\text{-app},\infty} = q_2\ V\ = q_2\ \frac{1}{4\ \pi\varepsilon_0}\frac{q_1}{r_{12}}$$

$$U = W_{app,\infty} = \frac{1}{4\ \pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} = U_{12}$$

25-10 Electric Potential Energy of a System of Point Charges



What is the electric potential energy of these fixed point charges?

We know that we need to do work of U_{12} to bring q_1 and q_2 at this position

Next, we will bring q_3 form infinity
It will move in the electric field of q_1 and q_2

$$W_{3\text{-app},\infty} = q_3 V$$

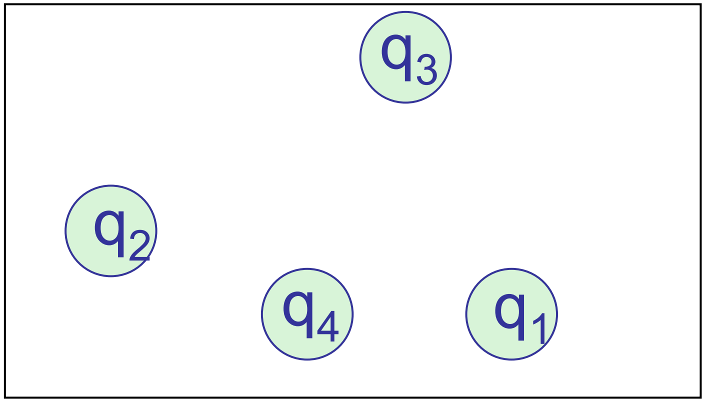
$$= q_3 \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

$$= U_{13} + U_{23}$$

$$U = W_{app,\infty} = U_{12} + U_{13} + U_{23}$$

25-10 Electric Potential Energy of a System of Point Charges



What is the electric potential energy of these fixed point charges?

$U = \quad +0$	Applied work to bring 1st charge form ∞
$+U_{12}$	Applied work to bring 2nd charge form ∞
$+U_{13} + U_{23}$	Applied work to bring 3rd charge form ∞
$+U_{14} + U_{24} + U_{23}$	Applied work to bring 4th charge form ∞

$$U_{ij} = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}$$

r_{ij}: Distance between q_i and q_j

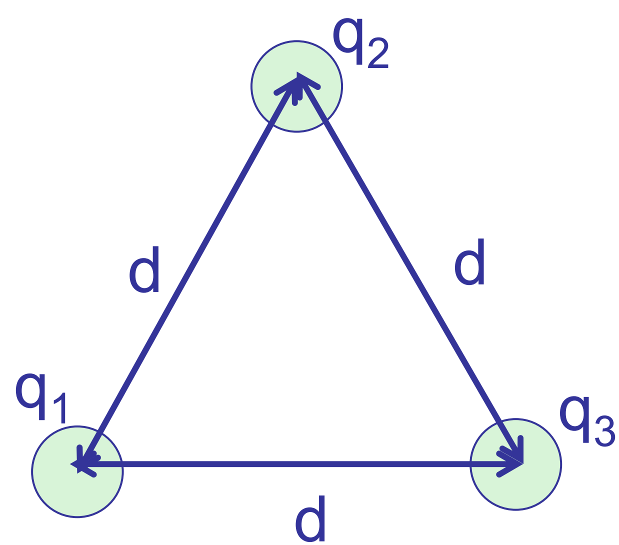
25-10 Electric Potential Energy of a System of Point Charges

Sample Problem 25-6

d = 12 cm   q =150 nC

q_1 =+q q_2 = -4q q_3 = +2 q

What is the electric potential energy of the system?



$$U = U_{12} + U_{13} + U_{23}$$

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{d} + \frac{q_1 q_3}{d} + \frac{q_2 q_3}{d}\right) = -17 \text{ mJ}$$

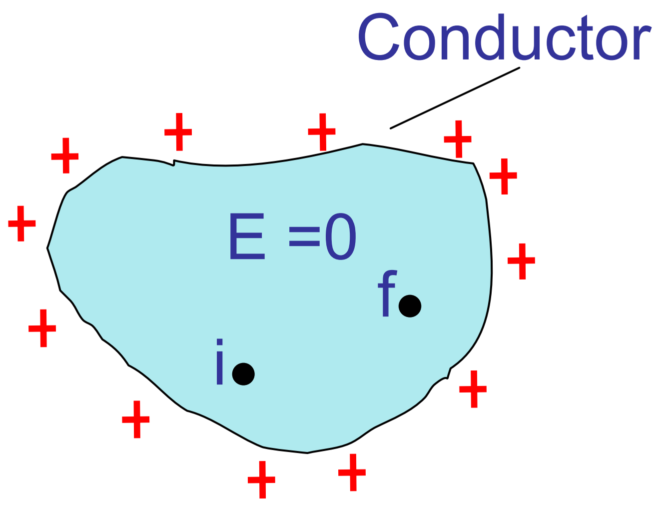
You have to do a work of – 17 mJ to assemble this structure from points at infinity

Or

You have to do a work of + 17 mJ to dissemble this structure to points at infinity

25-11 Potential of a Charged Isolated Conductor

In electrostatic equilibrium,
All points within a conductor have the same potential

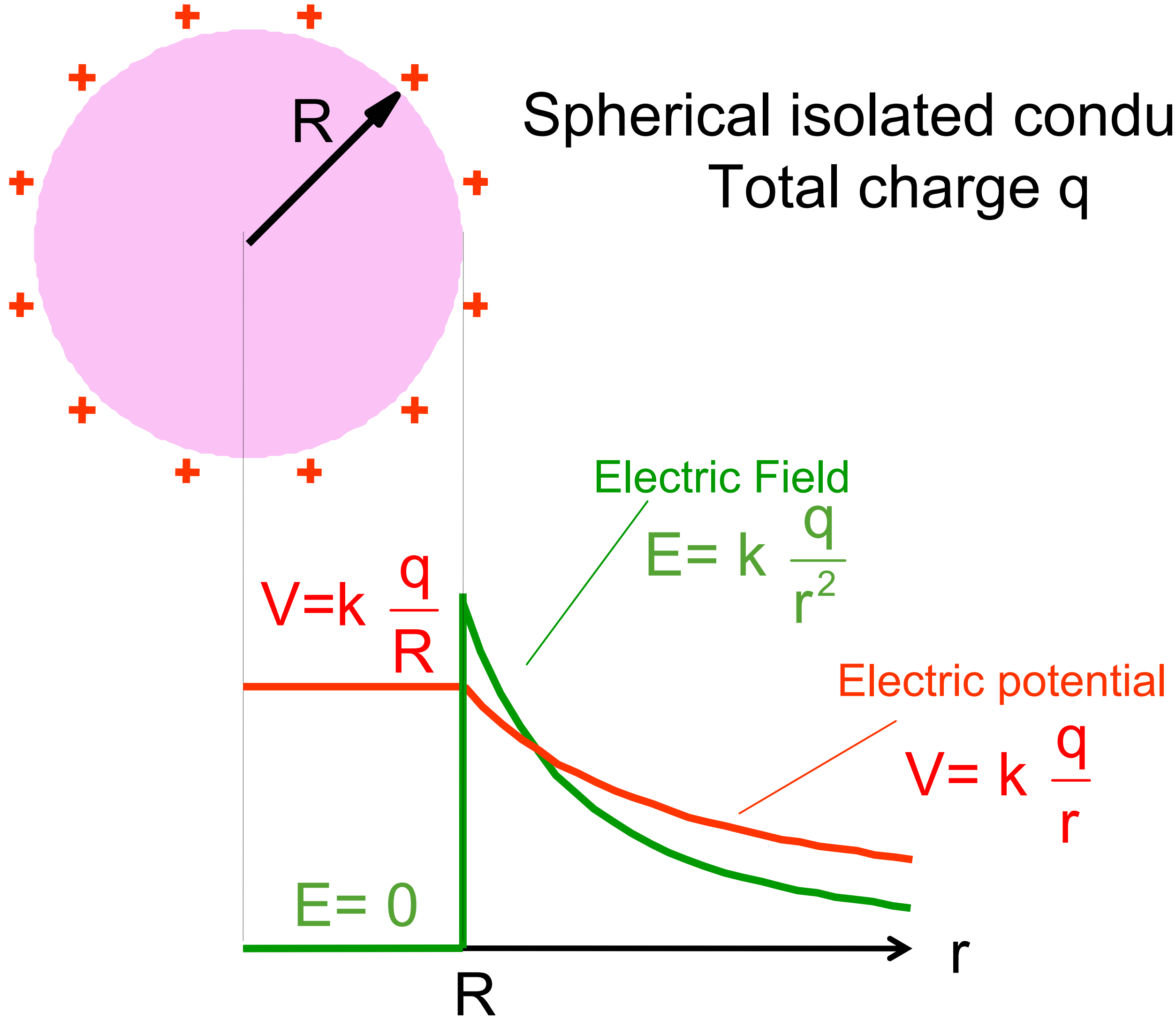


$$\Delta V = -\int_i^f \vec{E} \bullet \vec{ds} = -\int_i^f 0 \bullet \vec{ds} = 0$$

$$\Delta V = V_f - V_i = 0$$

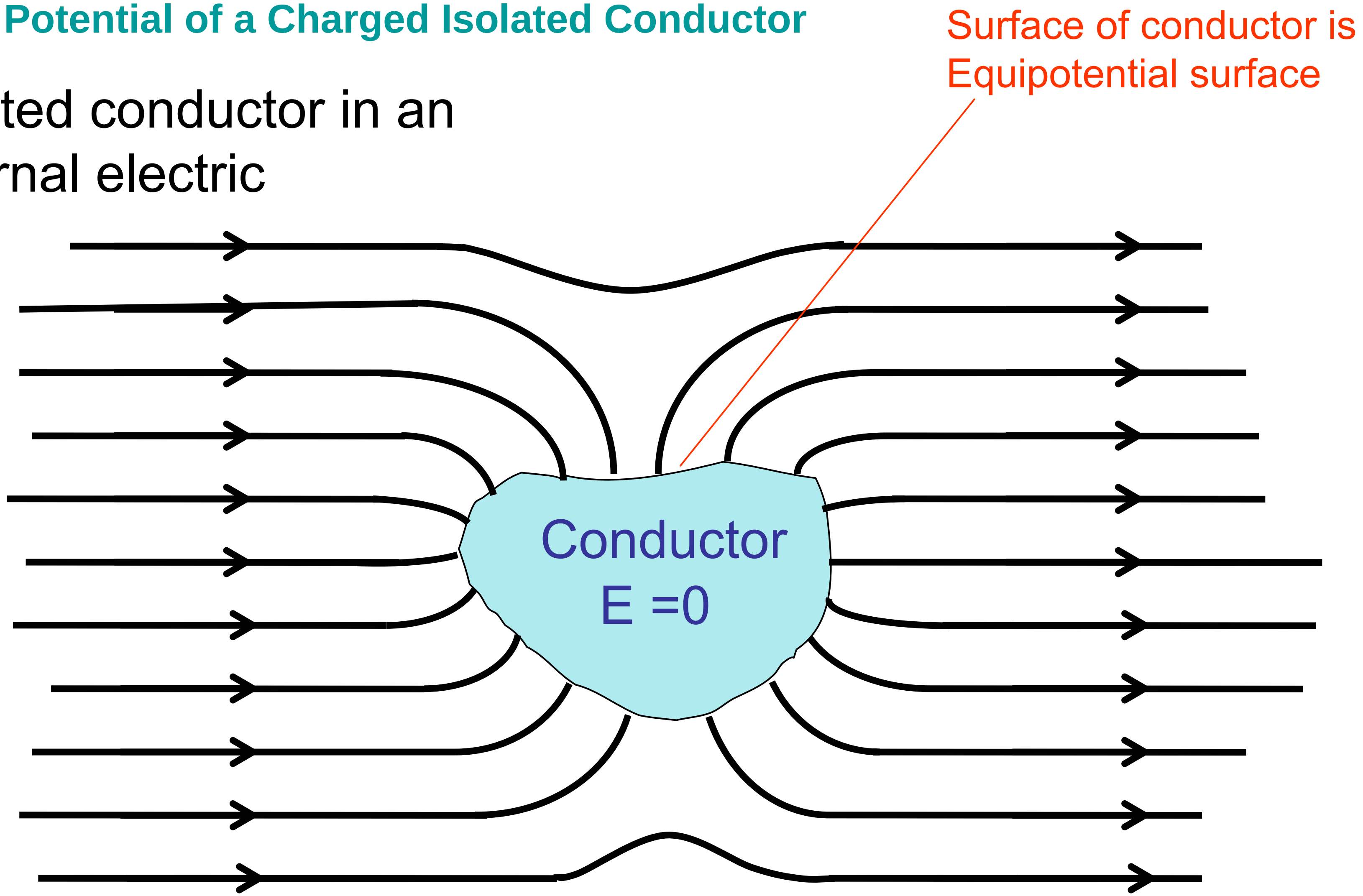
$$V_f = V_i$$

25-11 Potential of a Charged Isolated Conductor



25-11 Potential of a Charged Isolated Conductor

isolated conductor in an external electric



Electrons distribute themselves on the surface such that E= 0 inside the conductor and electric field is perpendicular to the surface