Chapter 24

Gauss' Law

24-1 A New Look at Coulomb's Law

Gauss' Law ⟺ Same as ⟺ Coulomb' Law

Very useful in calculating the eclectic field from symmetrical shapes

Spheres, cylinders, …

24-3 Flux of an Electric Field

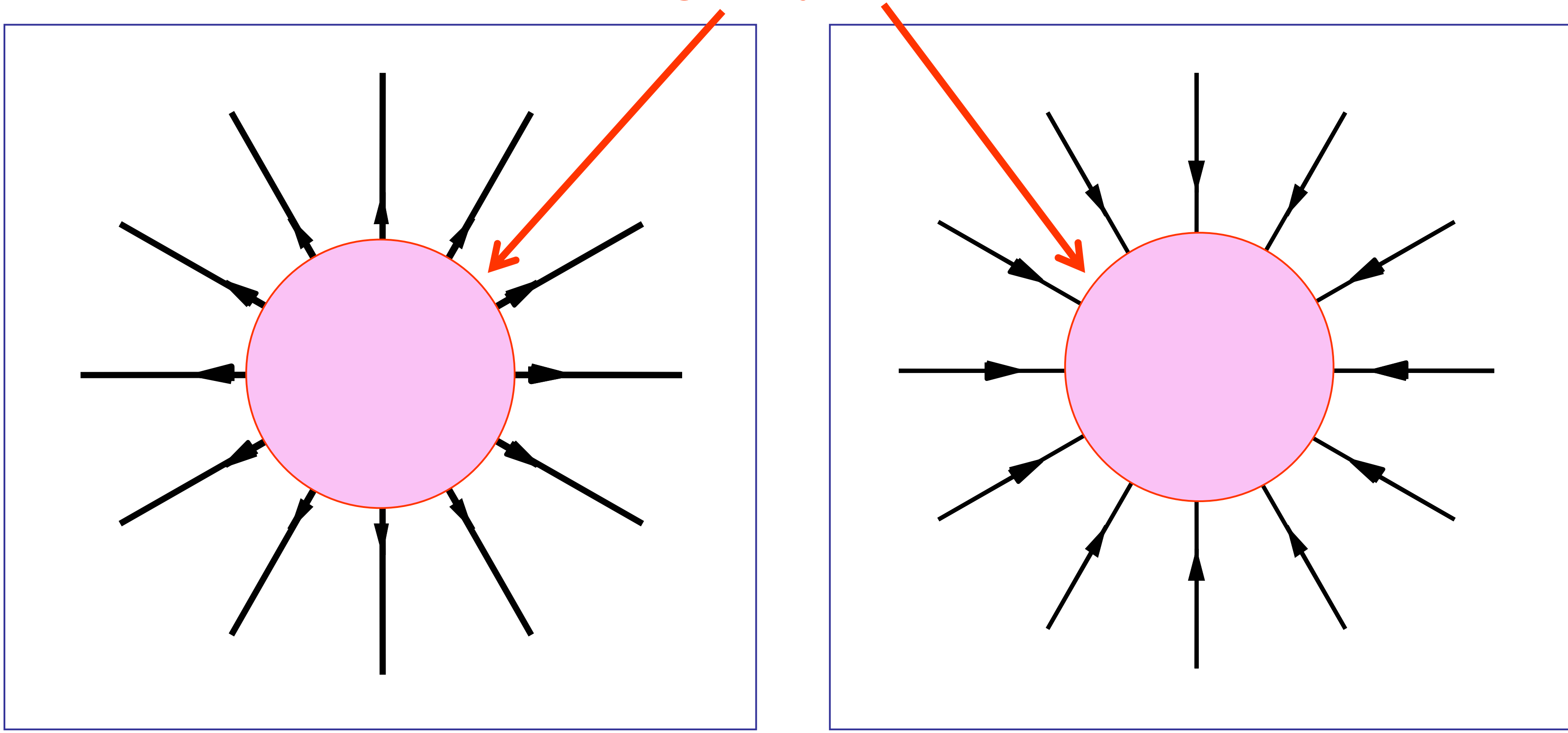
Gauss' Law

Gaussian surface

If you know the electric field over
any imaginary closed surface,
you can calculate
the net electric charge inside this closed surface

24-3 Flux of an Electric Field

Gaussian surface
imaginary sphere



You can guess that the net charge inside the Gaussian surface is positive charge

You can guess that the net charge inside the Gaussian surface is negative charge

24-3 Flux of an Electric Field

Gauss' Law

ε_0 (Electric flux through any closed surface)
= charge inside the surface

$$\varepsilon_0 \Phi = q_{enc}$$

permittivity constant

ε_0 **= 8.85 × 10^{-12}C^2/N•m^2**

24-3 Flux of an Electric Field

Electric flux through a Gaussian surface

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

Integration over a closed surface

Area vector

24-3 Flux of an Electric Field

Gauss' Law

ε_0 (Electric flux through any closed surface)
= charge inside the surface

$$\varepsilon_0 \Phi = q_{enc}$$

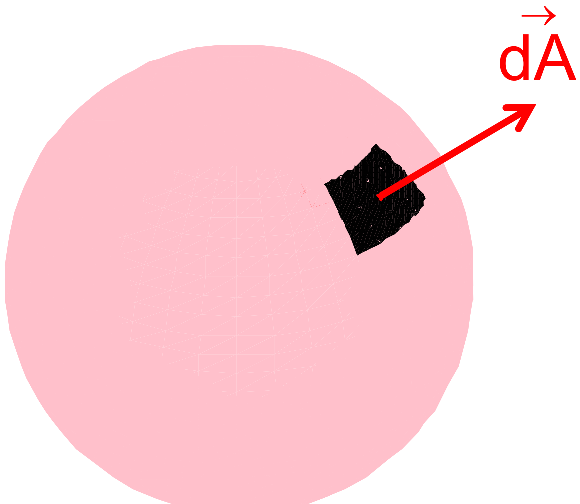
$$\varepsilon_0 \oint \vec{E} \bullet d\vec{A} = q_{enc}$$

24-3 Flux of an Electric Field

Vector area $d\vec{A}$

Magnitude : Area of the small surface

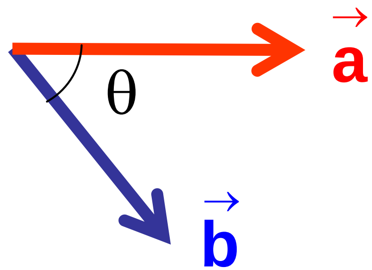
Direction : Normal to the surface
And
pointing to the outer side of the closed surface



Gaussian surface
imaginary closed surface

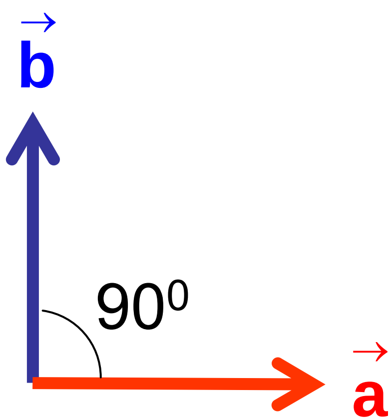
24-3 Flux of an Electric Field

Dot product: review

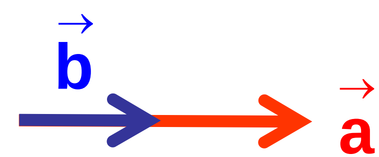


$$\vec{a}\cdot\vec{b} = a\,b\cos\theta$$

Scalar quantity

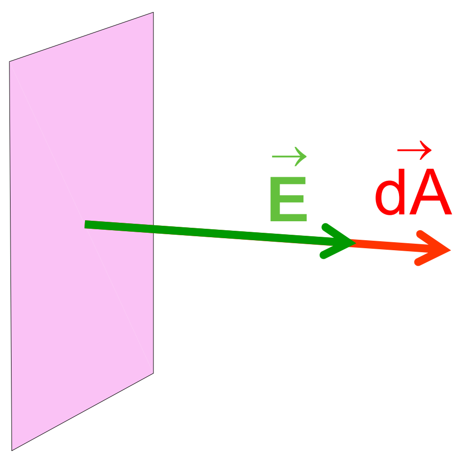


$$\vec{a}\cdot\vec{b} = 0$$



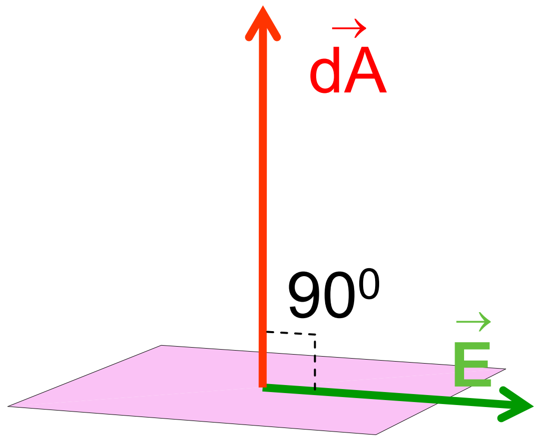
$$\vec{a}\cdot\vec{b} = a\,b$$

24-3 Flux of an Electric Field



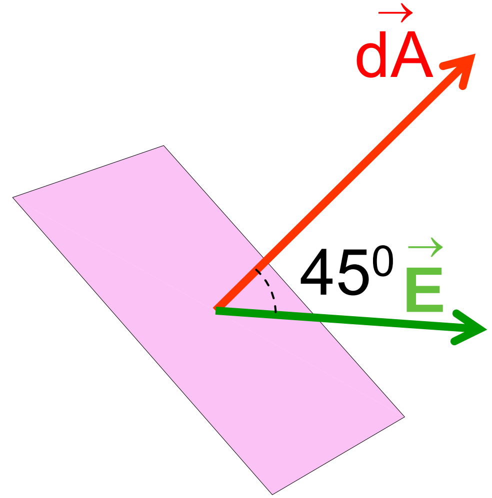
$$d\Phi = \vec{E}\bullet d\vec{A}$$

$$d\Phi = E\ dA$$



$$d\Phi = \vec{E}\bullet d\vec{A}$$

$$d\Phi = 0$$



$$d\Phi = \vec{E}\bullet d\vec{A}$$

$$d\Phi = E\ dA\ \cos 45^0$$

Flux through area $d\vec{A}$

24-3 Flux of an Electric Field

The magnitude of the electric field is proportional to the electric field lines per unit area perpendicular to the lines

⇩

$\vec{E} \bullet d\vec{A}$ is proportional to the electric field lines passing though area $d\vec{A}$

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

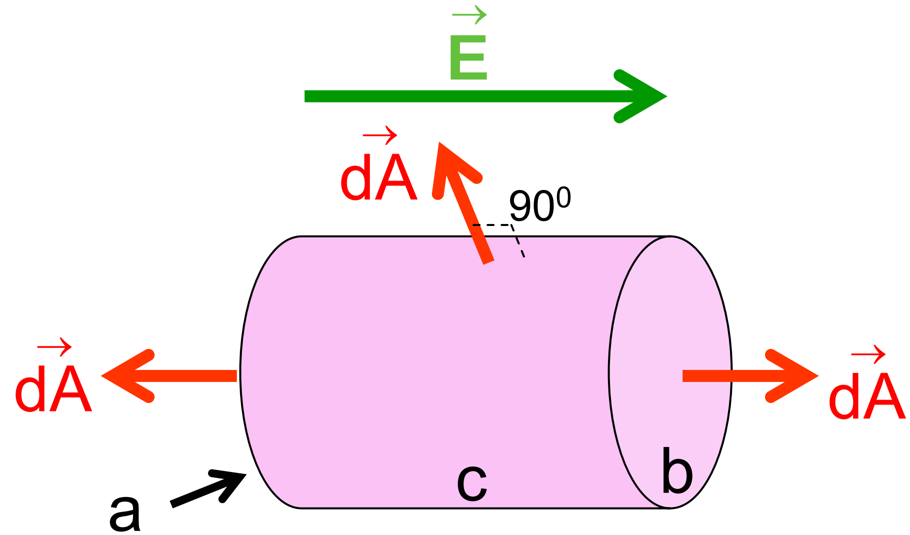
⇩

The electric flux Φ though a Gaussian surface is proportional to the net number of electric field lines passing through that surface

24-3 Flux of an Electric Field

Sample Problem 24-1

Find the flux of the electric field though the closed surface of the cylinder



total flux =

flux through surface a

+ flux though surface b

+ flux though surface c

$$\Phi = \int_a \vec{E}\bullet d\vec{A} + \int_b \vec{E}\bullet d\vec{A} + \int_c \vec{E}\bullet d\vec{A}$$

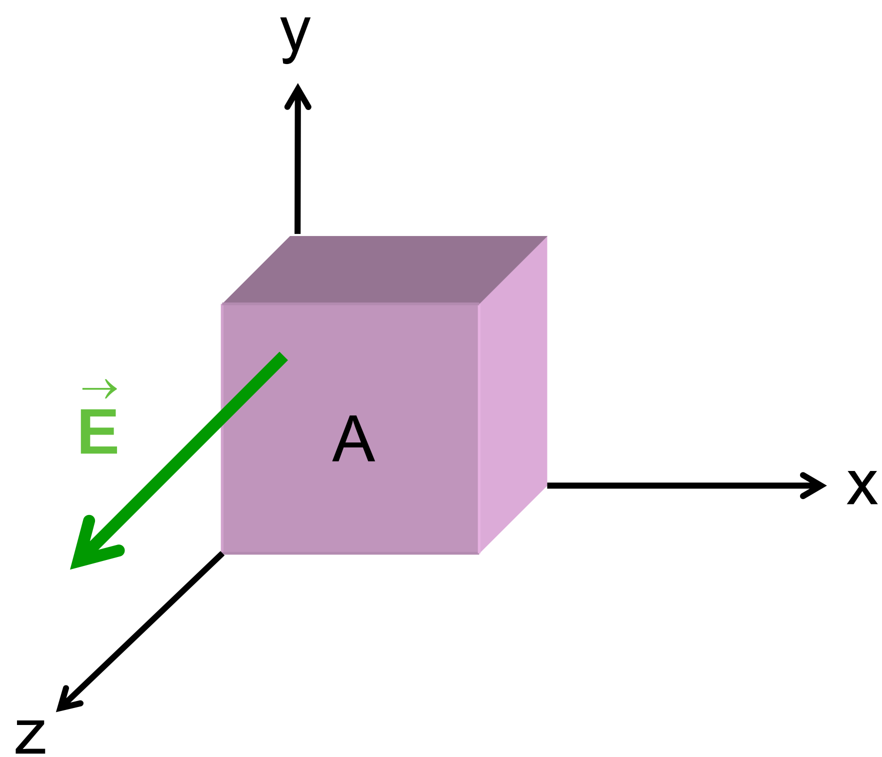
$$\Phi = \int_a E\, dA \cos 180^0 + \int_b E\, dA \cos 0^0 + \int_c E\, dA \cos 90^0$$

$$\Phi = -E\int_a dA + E\int_b dA + \int_c 0\, dA$$

$$\Phi = -E\,(\text{cap''s area}) + E\,(\text{cap''s area}) + 0 = 0$$

24-3 Flux of an Electric Field

Checkpoint 1



Flux though the front face = E A

Flux though the rear face = - E A

Flux though the top face = 0

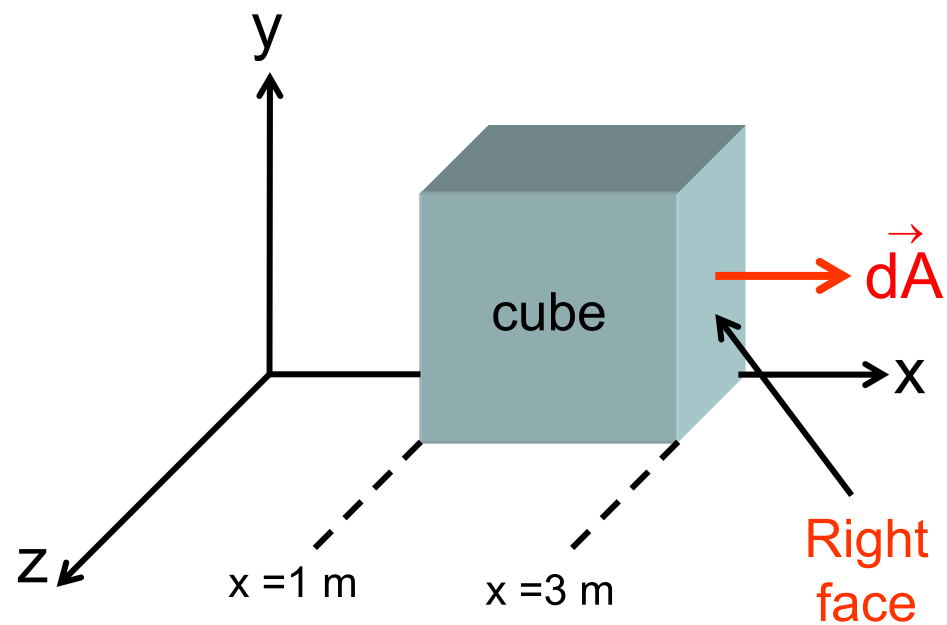
Flux though the whole cube = 0

24-3 Flux of an Electric Field

Sample Problem 24-2

$\vec{E} = 3x\,\hat{i} + 4\,\hat{j}$

What is the electric flux though the right face?



$$\Phi = \int_{right} \vec{E} \bullet d\vec{A}$$

$$d\vec{A} = dA\,\hat{i} = dy\,dz\,\hat{i}$$

$$\vec{E} \bullet d\vec{A} = (3x\,\hat{i} + 4\,\hat{j}) \bullet (dy\,dz\,\hat{i}) = (3x\,\hat{i}) \bullet (dy\,dz\,\hat{i}) = 3x\,dy\,dz$$

$$\Phi = \int_{y=0}^{y=2}\int_{z=0}^{z=2} 3x\,dy\,dz = \int_{y=0}^{y=2}\int_{z=0}^{z=2} 3(3)\,dy\,dz = 9\int_{y=0}^{y=2}\int_{z=0}^{z=2} dy\,dz$$

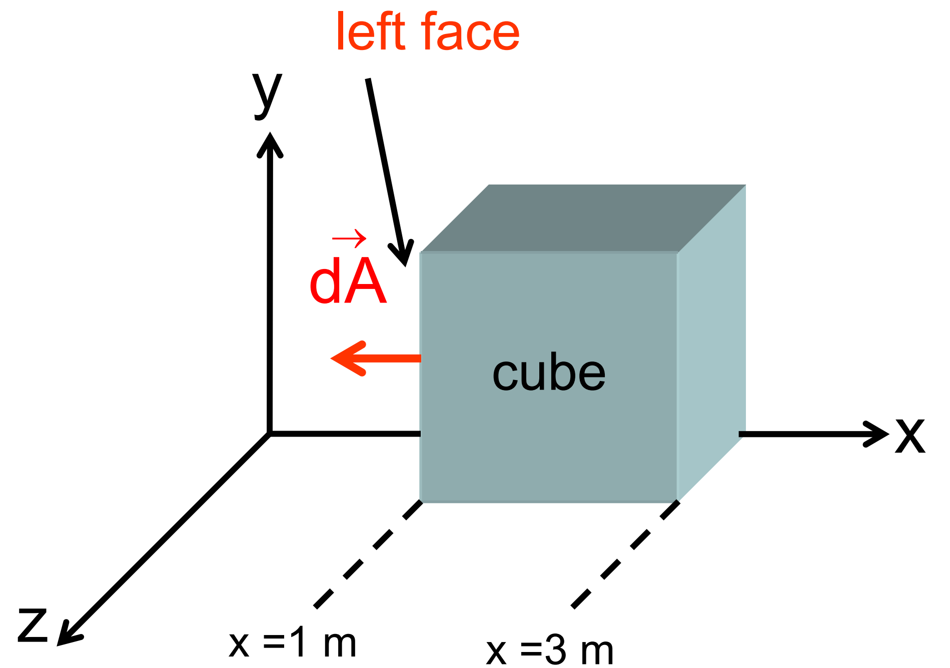
$$= 9 \times 4 = 36\ \mathrm{N\,m^2/C}$$

24-3 Flux of an Electric Field

Sample Problem 24-2

$\vec{E} = 3x\,\hat{i} + 4\,\hat{j}$

What is the electric flux though the left face?



$$\Phi = \int_{\text{left}} \vec{E} \bullet d\vec{A}$$

$$d\vec{A} = -\,dA\,\hat{i} = -\,dy\,dz\,\hat{i}$$

$$\vec{E} \bullet d\vec{A} = (3x\,\hat{i} + 4\,\hat{j}) \bullet (-\,dy\,dz\,\hat{i}) = (3x\,\hat{i}) \bullet (-\,dy\,dz\,\hat{i}) = -\,3x\,dy\,dz$$

$$\Phi = -\int_{y=0}^{y=2}\int_{z=0}^{z=2} 3x\,dy\,dz = -\int_{y=0}^{y=2}\int_{z=0}^{z=2} 3(1)\,dy\,dz = -3\int_{y=0}^{y=2}\int_{z=0}^{z=2} dy\,dz$$

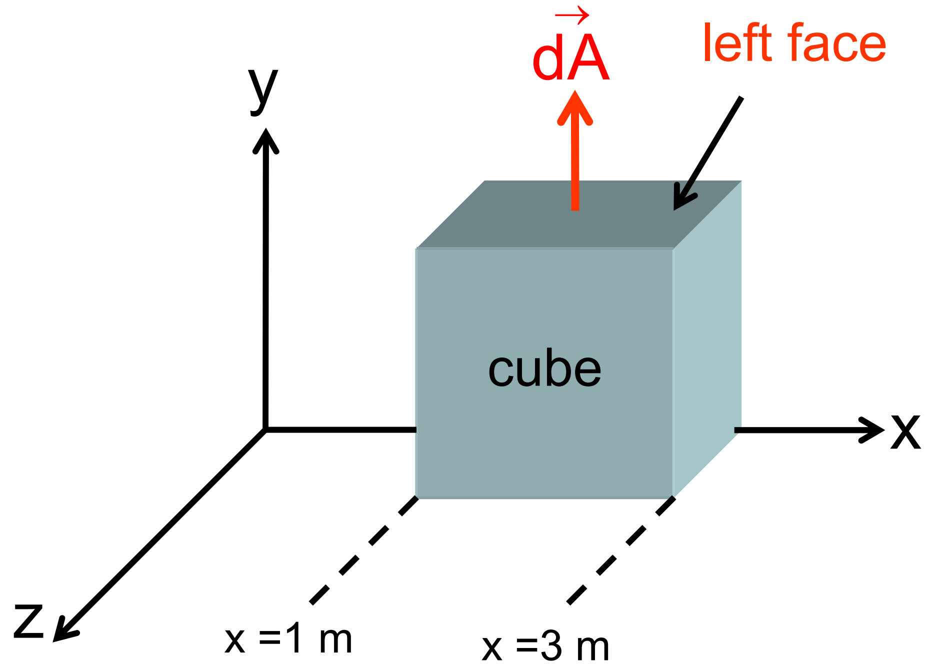
$$= -3 \times 4 = -12\ \text{N m}^2/\text{C}$$

24-3 Flux of an Electric Field

Sample Problem 24-2

$\vec{E} = 3x\,\hat{i} + 4\,\hat{j}$

What is the electric flux though the top face?



$$\Phi = \int_{top} \vec{E} \bullet d\vec{A}$$

$$d\vec{A} = dA\,\hat{j} = dx\,dz\,\hat{j}$$

$$\vec{E} \bullet d\vec{A} = (3x\,\hat{i} + 4\,\hat{j}) \bullet (dx\,dz\,\hat{j}) = (4\hat{j}) \bullet (dx\,dz\,\hat{j}) = 4\,dx\,dz$$

$$\Phi = \int_{x=1}^{x=3} \int_{z=0}^{z=2} 4\,dx\,dz = 4 \int_{x=1}^{x=3} \int_{z=0}^{z=2} dx\,dz = 4 \times 4 = 16\ \mathrm{N\,m^2/C}$$

24-4 Gauss' Law

Gauss' Law

ε_0 (Electric flux through any closed surface) = charge inside the surface

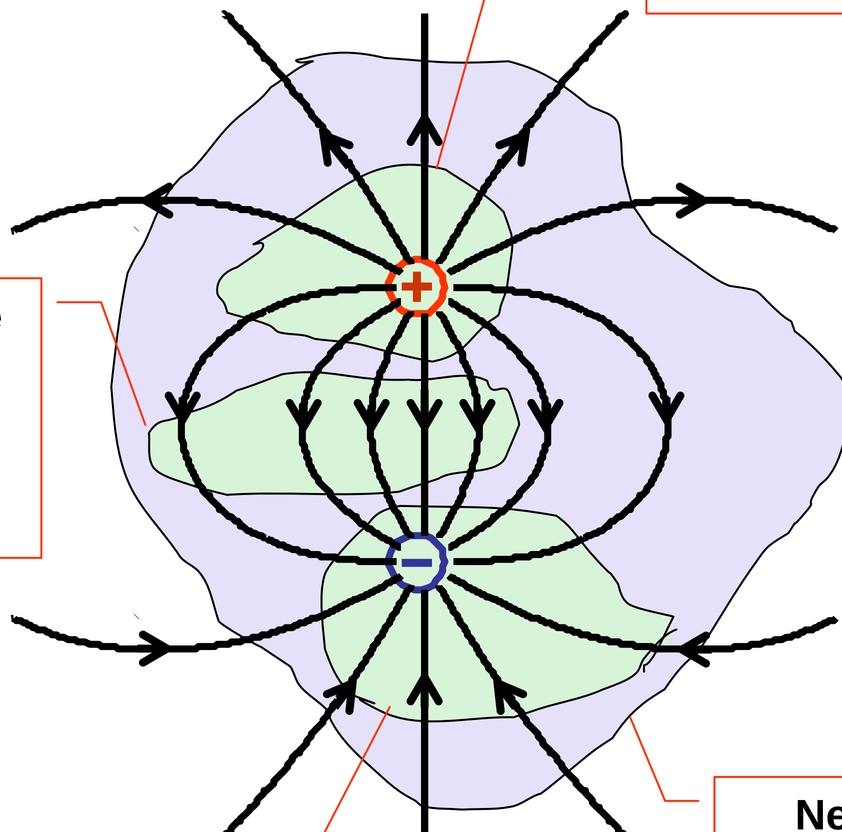
$$\varepsilon_0 \Phi = q_{enc}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

24-4 Gauss' Law

Electric field is outward for all points on this closed surface. Flux and charge inside are both positive.

No Charge inside the surface. Electric Field lines entering the surface also leaving it



Electric field is intward for all points on the surface. Flux and charge inside are both positive

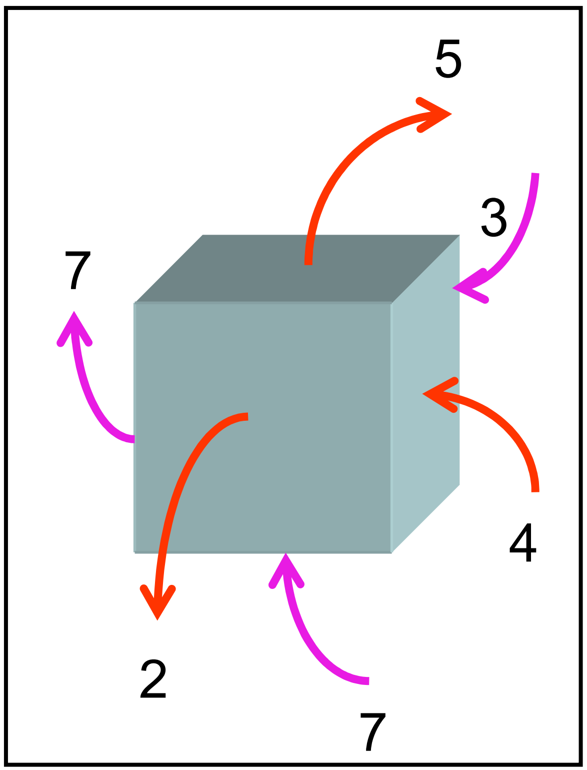
Net charge inside this closed surface is zero. Number of the electric field lines entering the surface = number of lines leaving it.

24-4 Gauss' Law

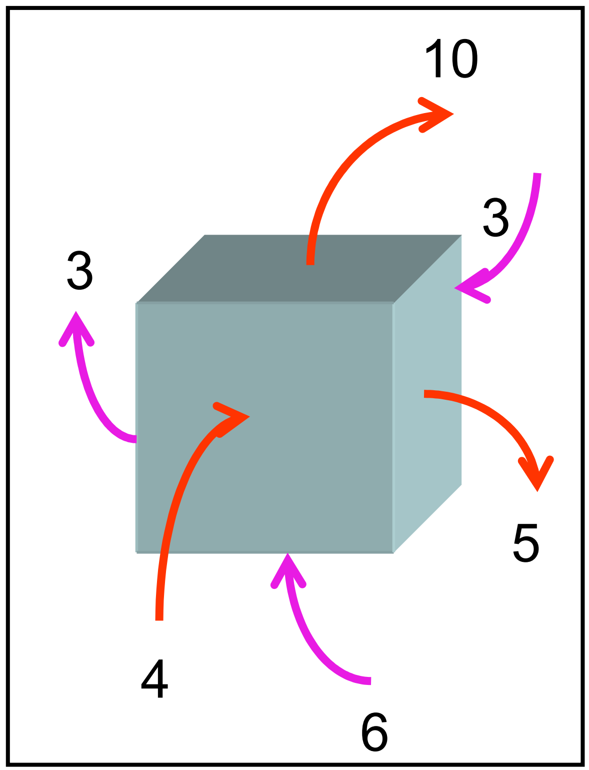
Checkpoint 2

Numbers indicate the flux on each face and arrows indicate the direction of the electric lines

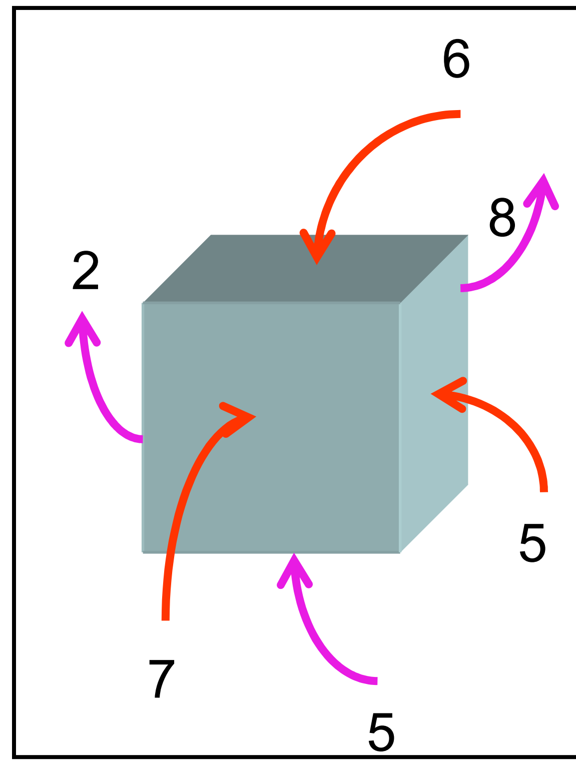
What kind of charge enclosed by these Gaussian cubes?



No net charge



Positive net charge



Negative net charge

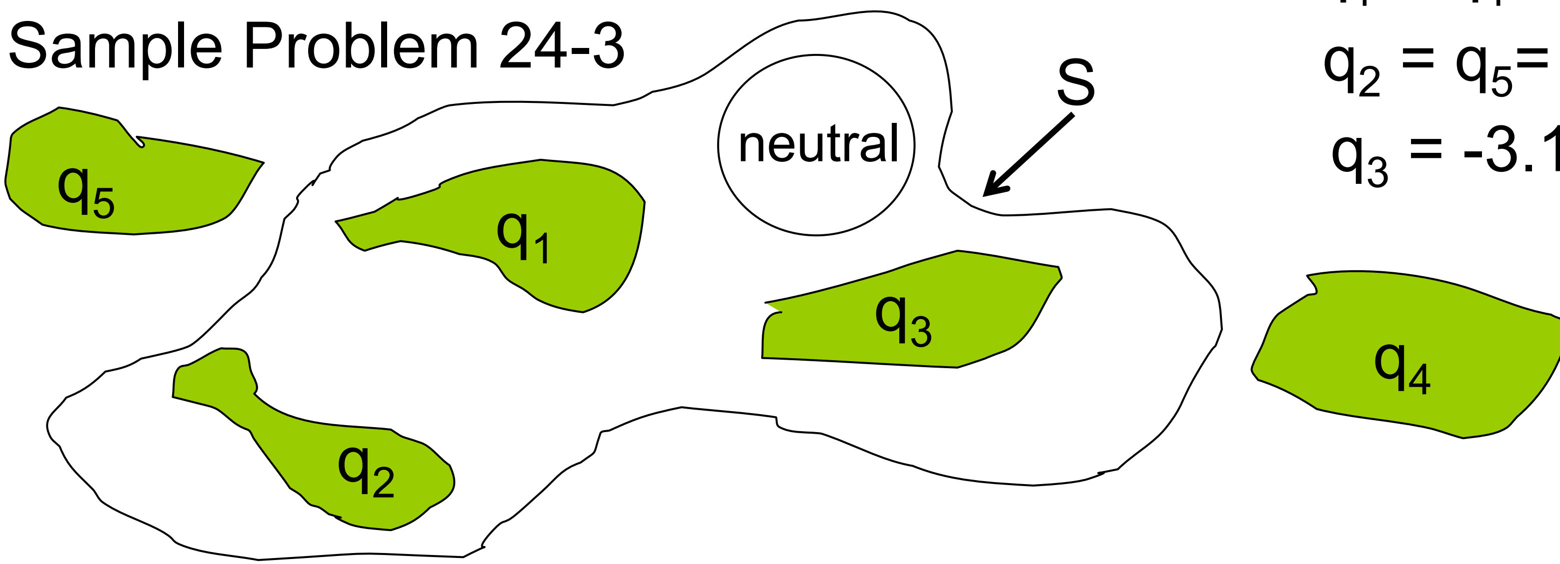
24-4 Gauss' Law

Sample Problem 24-3

$q_1 = q_4 = 3.1$ nC

$q_2 = q_5 = -5.9$ nC

$q_3 = -3.1$ nC



What is the net electric flux though the Gaussian surface S?

$$\varepsilon_0 \Phi = q_{enc}$$

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{q_1+q_2+q_3}{\varepsilon_0} = \frac{3.1 \times 10^{-9}\text{C} - 5.9 \times 10^{-9}\text{C} - 3.1 \times 10^{-9}\text{C}}{8.85 \times 10^{-12}\text{C}^2/\text{N m}^2}$$

$$= -670 \text{ C/N m}^2$$

24-5 Gauss' Law and Coulomb's Law

Gauss' Law ⟺ Same as ⟺ Coulomb' Law

Gauss' Law

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

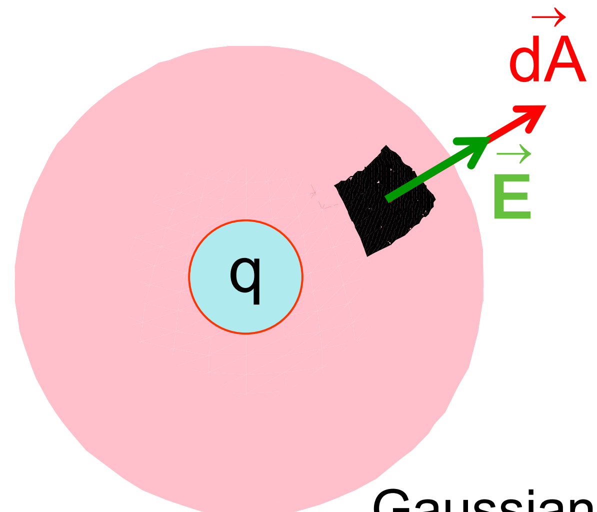
$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{enc}$$

$$\varepsilon_0 E \oint dA = q$$

$$\varepsilon_0 E\, (4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

Coulomb's Law

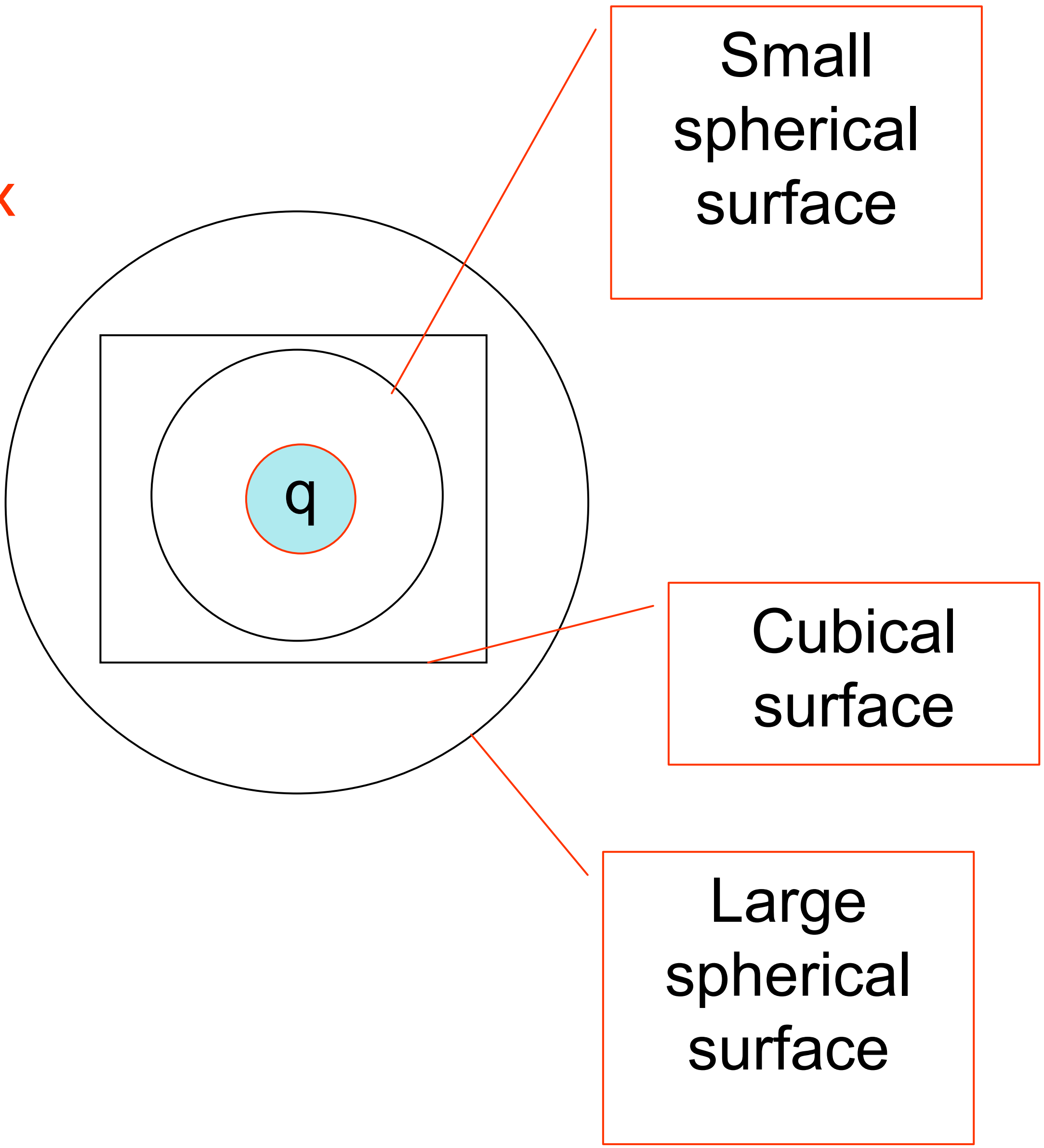


Gaussian spherical surface of radius r

24-5 Gauss' Law and Coulomb's Law

Checkpoint 3

What is the relation between the electric flux though the three Gaussian surfaces?

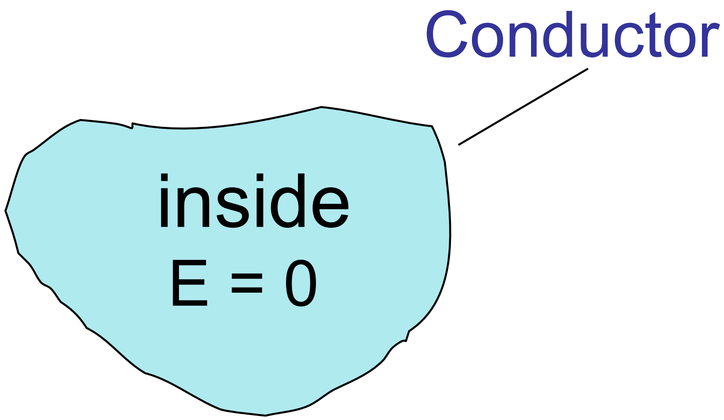


All the same

24-6 A Charged Isolated Conductor

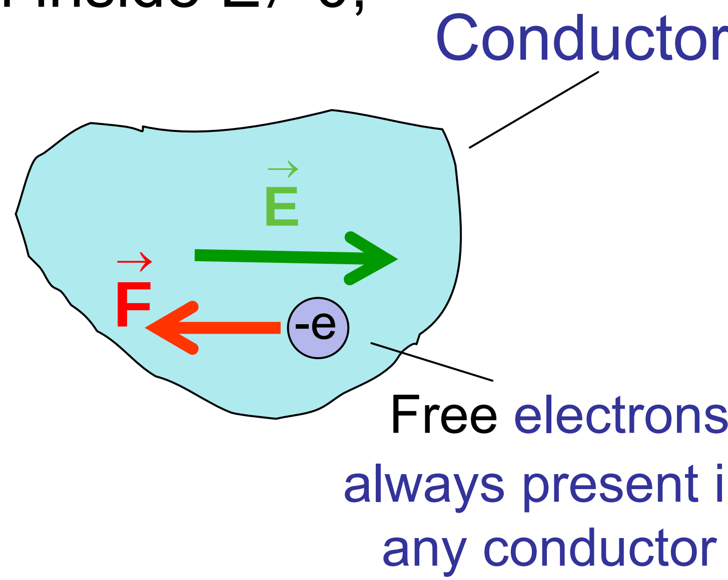
In electrostatic equilibrium,
all excess charge on a conductor
is entirely on the conductor’s outer surface

24-6 A Charged Isolated Conductor



Electrostatic equilibrium

If inside $E \neq 0$,

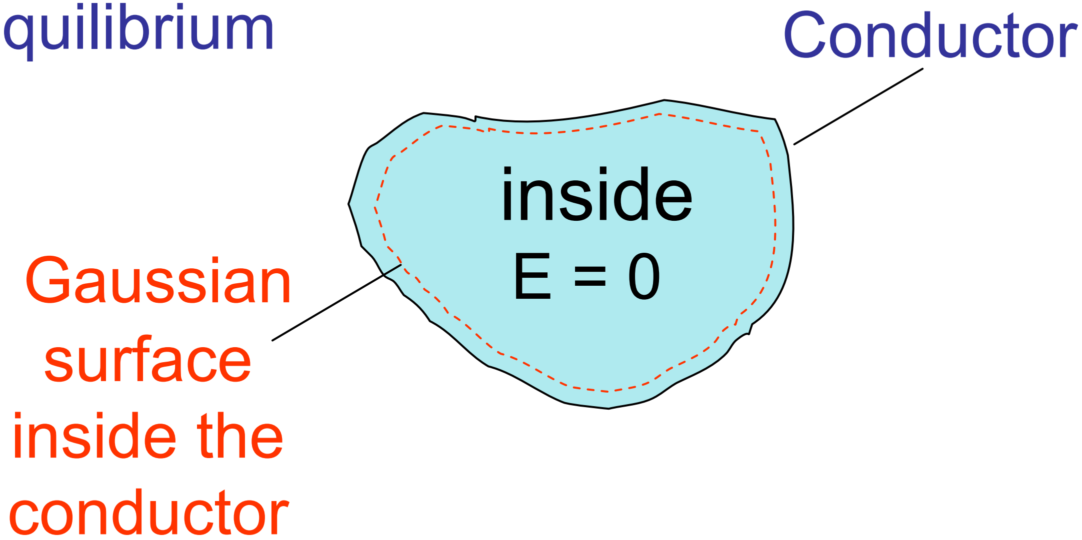


free electrons move

Not electrostatic equilibrium.

24-6 A Charged Isolated Conductor

electrostatic equilibrium



Since the electric field inside the conductor = 0 ⇒ Flux through any Gaussian surface inside the conductor = 0

Net charge inside the conductor = 0

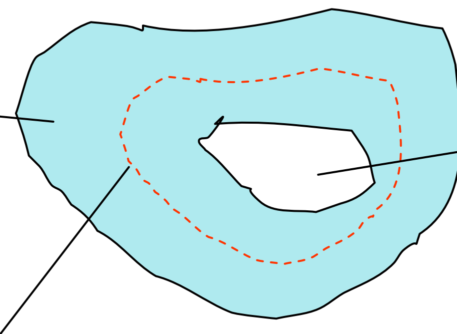
All excess charge is on the outer surface of the conductor

24-6 A Charged Isolated Conductor

electrostatic equilibrium

Conductor

inside
E = 0

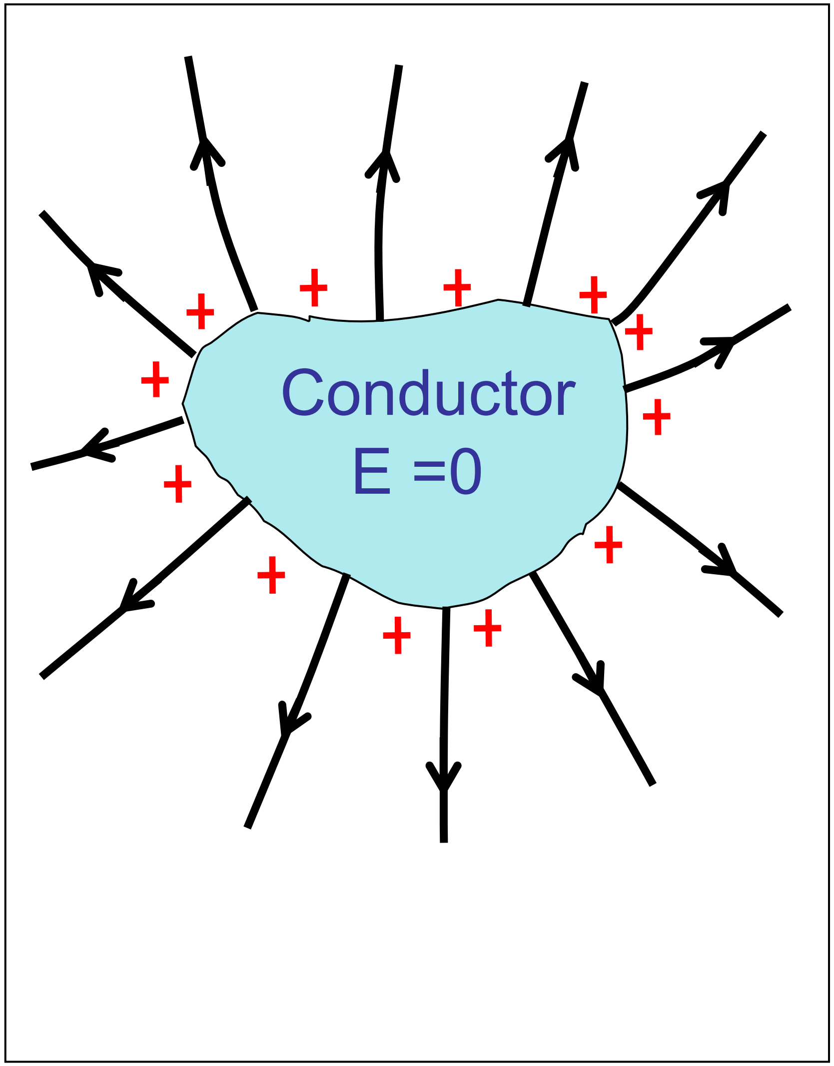


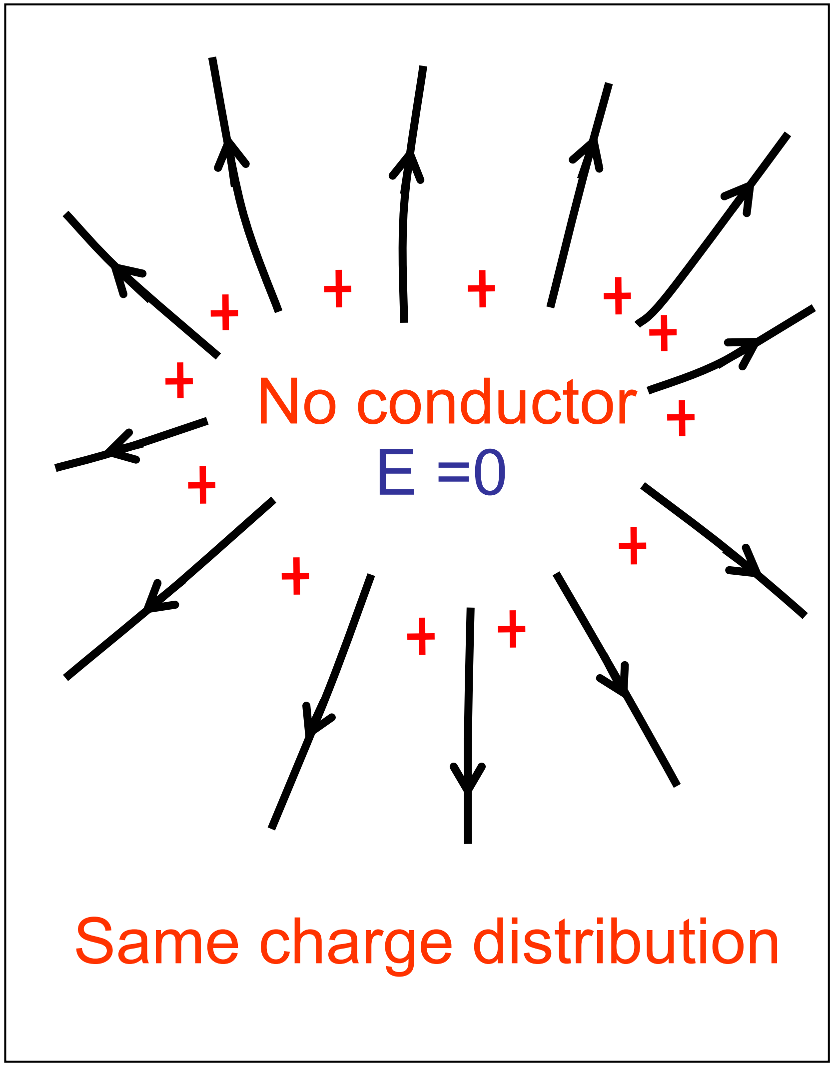
cavity totally within the conductor

Flux though the closed surface = 0 ⇒ No net charge inside the surface

No Net charge on the cavity wall

24-6 A Charged Isolated Conductor

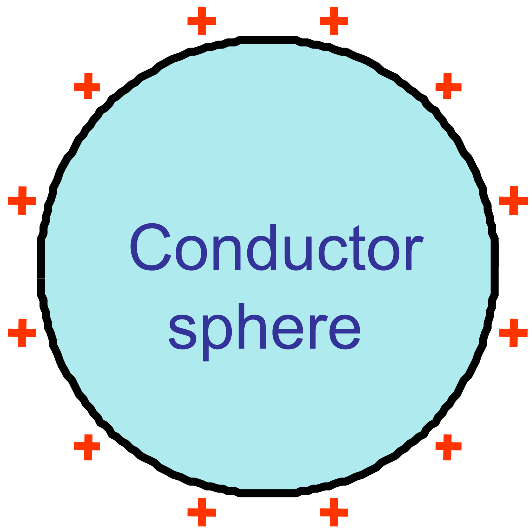




Same electric field

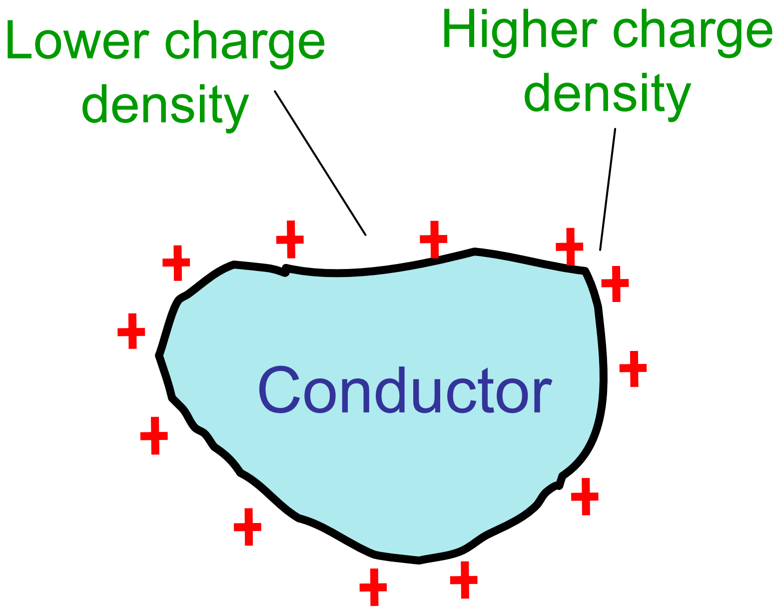
Electric field is produced by the charge distribution

24-6 A Charged Isolated Conductor



Uniform charge distribution

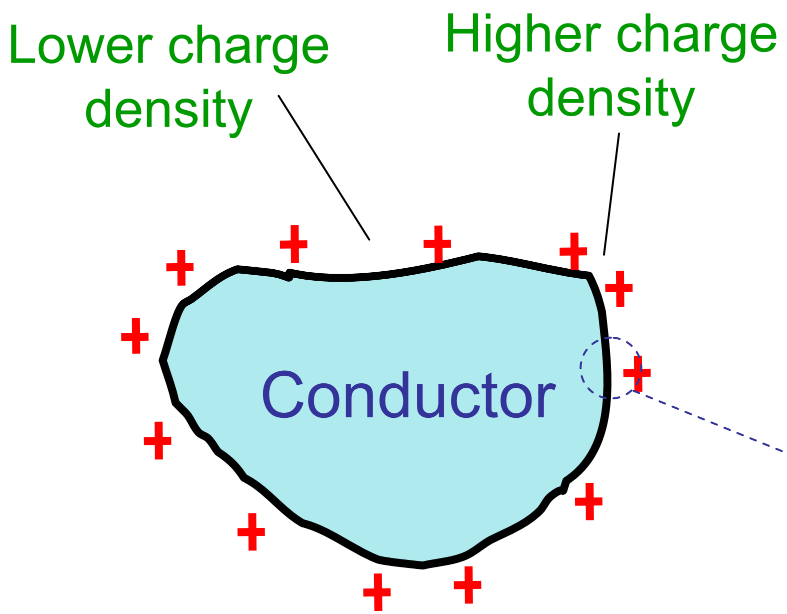
Electric field is very easy to calculate



charge distribution is not uniform

In general, electric field is very difficult to calculate

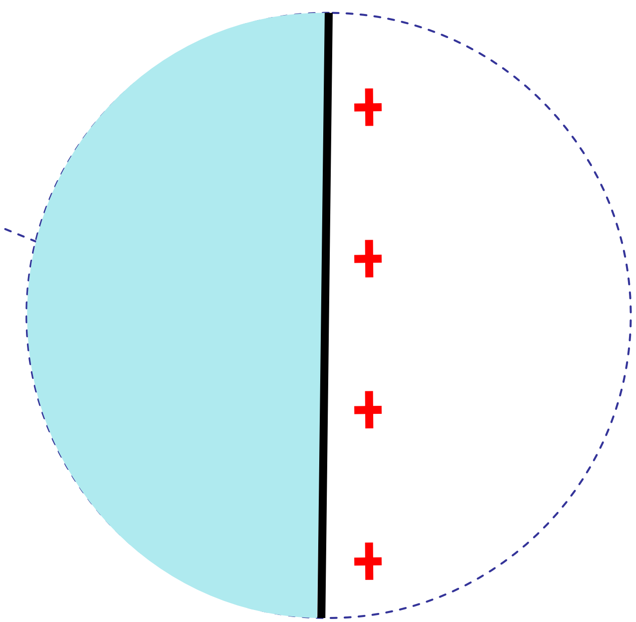
24-6 A Charged Isolated Conductor



charge distribution is not uniform

In general, electric field is very difficult to calculate

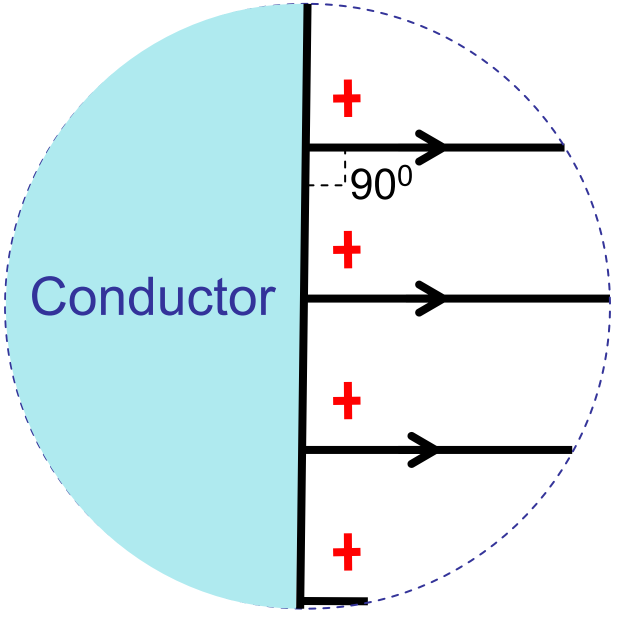
Choose very small area on the conductor's surface such that the surface is almost flat and the charge distribution is almost uniform



But we can find the electric field just neat the surface by using Gauss' law

24-6 A Charged Isolated Conductor

Electrostatic equilibrium

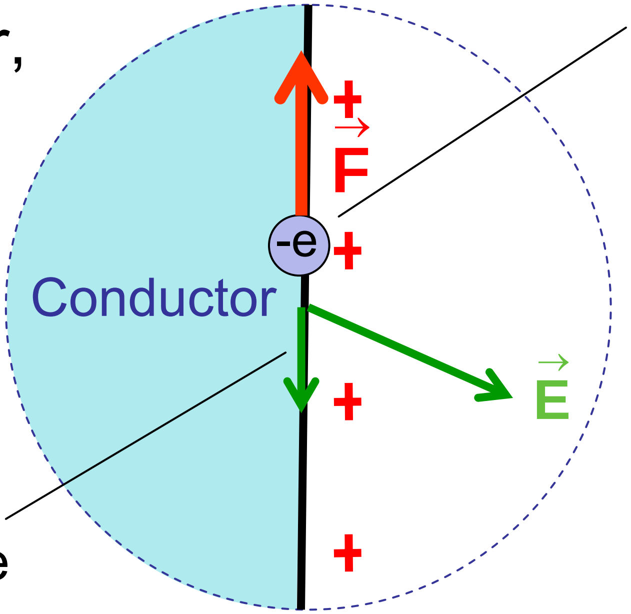


Electric field is perpendicular
to the surface of conductors

24-6 A Charged Isolated Conductor

If electric field is not perpendicular to the surface of conductor,

Free electrons always present on surface of conductor



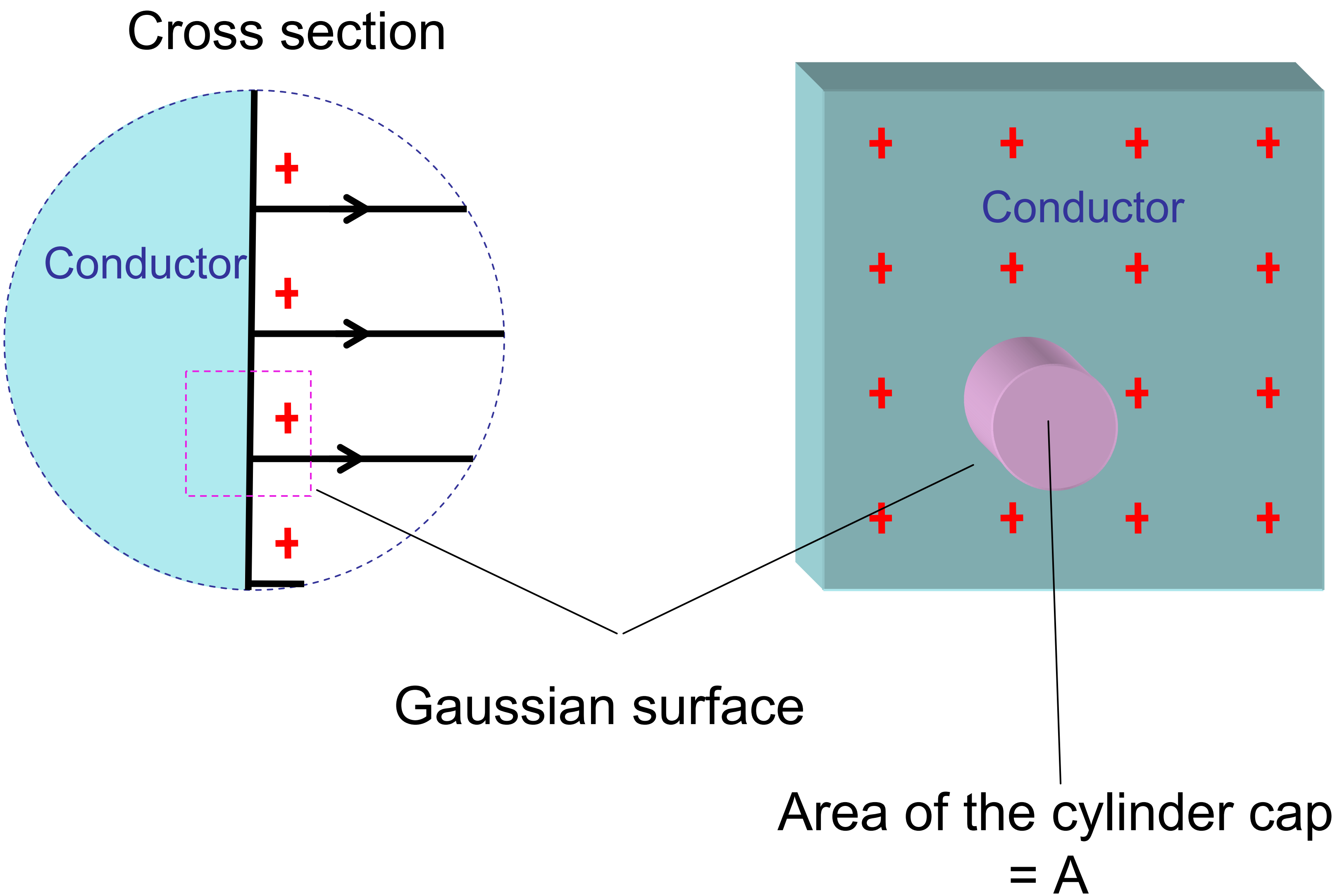
component of E along the surface

free electrons move

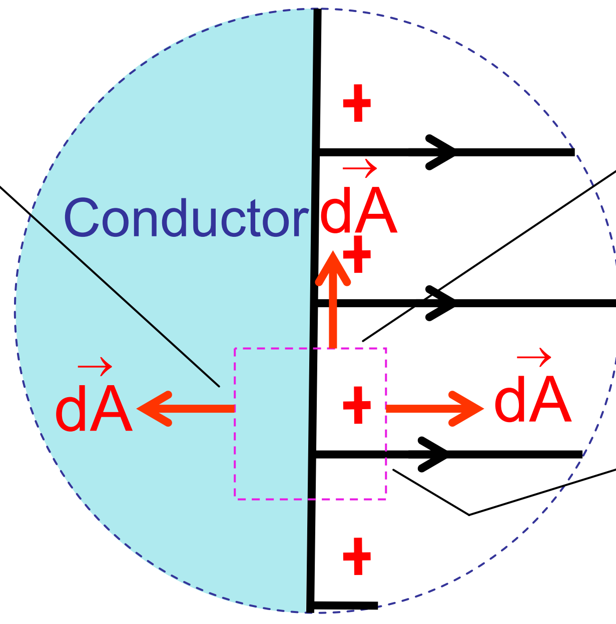
Not electrostatic equilibrium.

In electrostatic equilibrium, electric field is perpendicular to the surface of conductors

24-6 A Charged Isolated Conductor



24-6 A Charged Isolated Conductor



Flux through the rear cap = 0
Since inside conductor
$\vec{E} = 0$

Flux through the cylinder surface = 0
Since
$\vec{E} \perp \vec{dA}$

Flux through the front cap = E A
Since inside conductor
$\vec{E}$ parallel to $\vec{dA}$

Total electric flux through the cylinder = E A

$$\varepsilon_0 \Phi = q_{enc}$$

$$\varepsilon_0 E A = q_{enc}$$

$$E = \frac{q_{enc}}{\varepsilon_0 A} = \frac{\sigma}{\varepsilon_0}$$

surface charge density

$$= \frac{\text{Charge}}{\text{Area}}$$

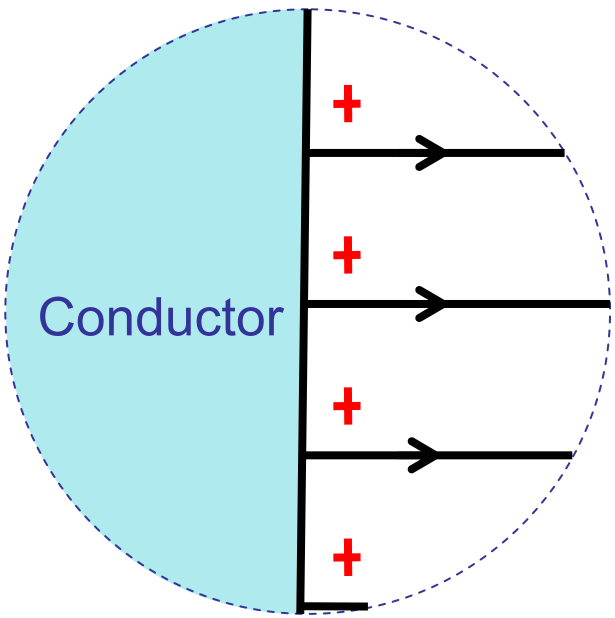
24-6 A Charged Isolated Conductor

Electric field just near the surface of a conductor

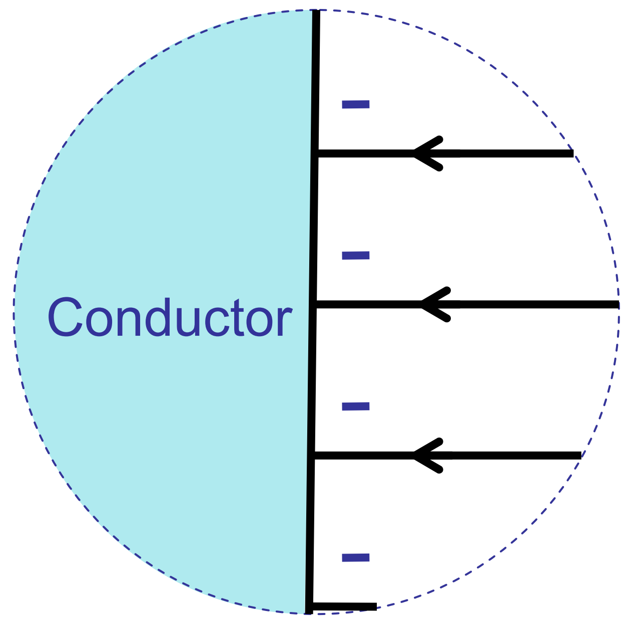
surface charge density

Magnitude: $E = \frac{\sigma}{\varepsilon_0}$ $= \frac{\text{Charge}}{\text{Area}}$

Direction: Perpendicular to the conductor's surface



Positively charged

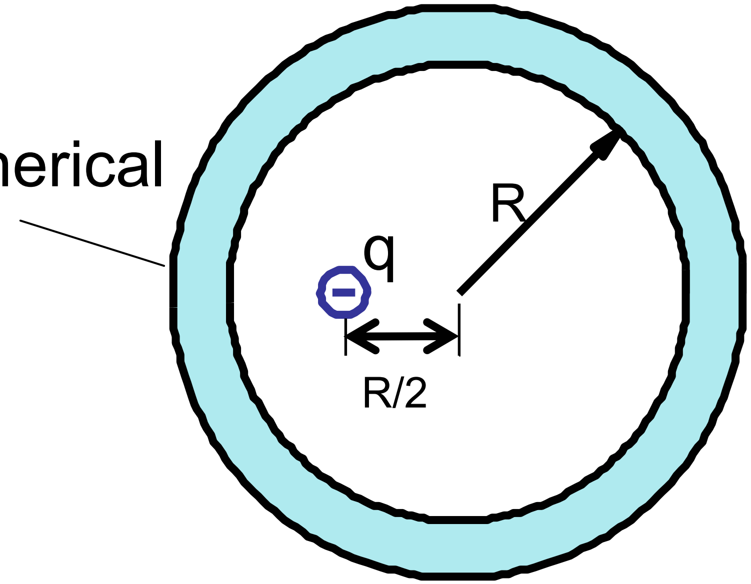


Negatively charged

24-6 A Charged Isolated Conductor

Sample Problem 24-4

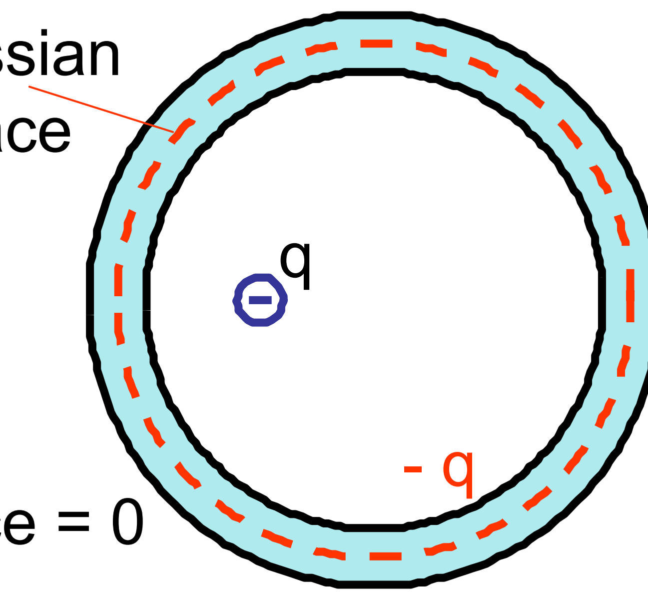
$q = -5\ \mu C$



What is the induced charges on its inner and outer surfaces?

metal = conductor

Inside the conductor, E = 0



Electric flux through the Gaussian surface = 0

$\varepsilon_0\, \Phi = q_{enc} = 0$

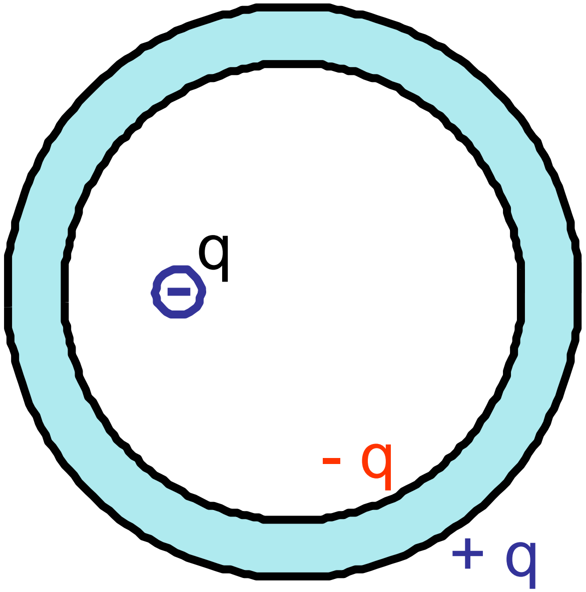
Net charge inside the Gaussian surface = 0

q + charge on the inner surface of the shell= 0

Charge on the inner surface of the shell= - q = + 5 μC

24-6 A Charged Isolated Conductor

Sample Problem 24-4



The metal shell is neutral

Net charge of the shell = 0

charge on the outer surface + charge on the inner surface = 0

charge on the outer surface = - charge on the inner surface

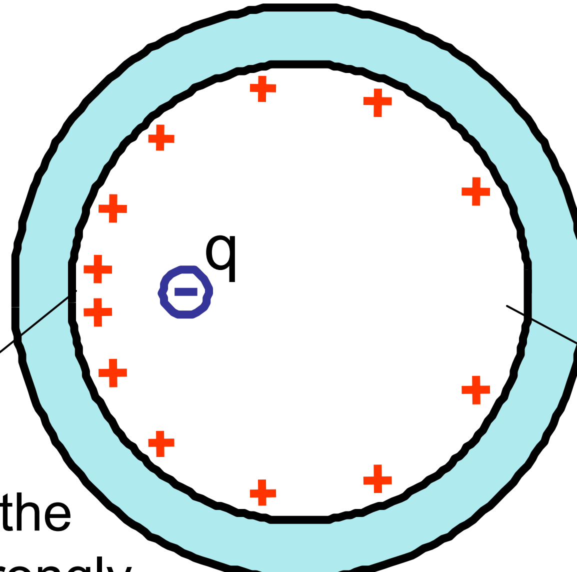
charge on the outer surface = - (-q) = q

charge on the outer surface = - 5 μC

24-6 A Charged Isolated Conductor

Sample Problem 24-4

Are those charges uniformly distributed?



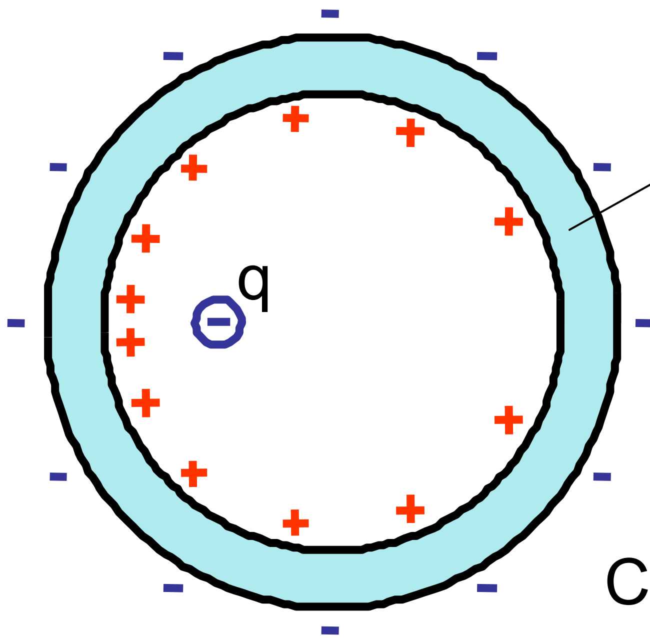
The point charge inside the shell repels electrons strongly from this area.
More positive

The point charge inside the shell repels electrons weakly from this area.
less positive

On the inner surface, charge are not uniformly distributed

24-6 A Charged Isolated Conductor

Sample Problem 24-4



Inside E = 0

Inside charges can not affect (applying force) on the outer charges

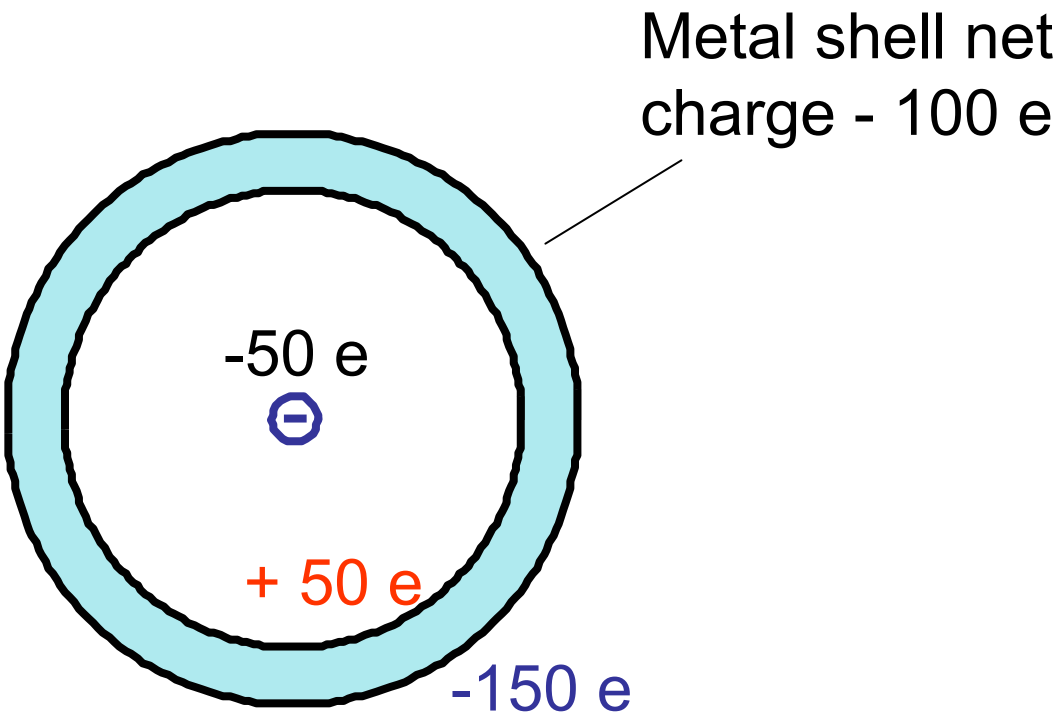
Charges on the outer spherical surface distribute themselves to minimize the forces among themselves

On the outer surface, charge is uniformly distributed

24-6 A Charged Isolated Conductor

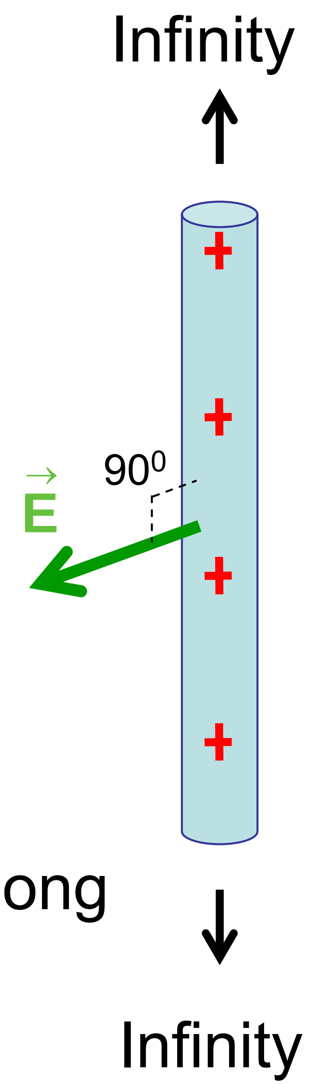
Checkpoint 4

What is the charge on the shell's inner and outer surfaces?



24-7 Applying Gauss' Law: Cylindrical Symmetry

What is the electric felid at a distance r from the axis?

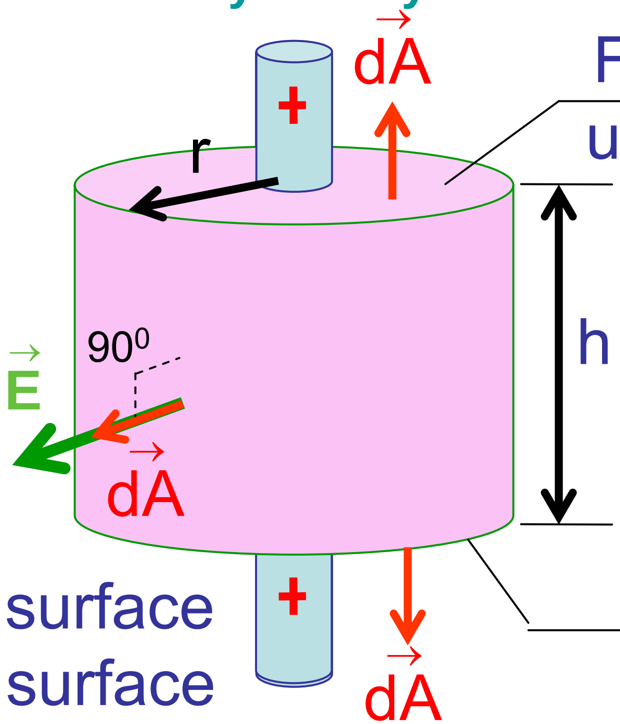


Infinitely long cylindrical rod with a uniform linear charge density λ

From symmetry, the electric field points along the radial direction

24-7 Applying Gauss' Law: Cylindrical Symmetry

Cylindrical Gaussian cylinder of radius r and coaxial with the rod



Flux through the upper and lower caps = 0

Since

$\vec{E} \perp \vec{dA}$

Since

$\vec{E}$ parallel to $\vec{dA}$

flux through the cylinder surface

= E(area of the cylinder surface

= E (2 π r h)

Total electric flux through the Gaussian surface = E (2 π r h)

$$\varepsilon_0 \Phi = q_{enc}$$

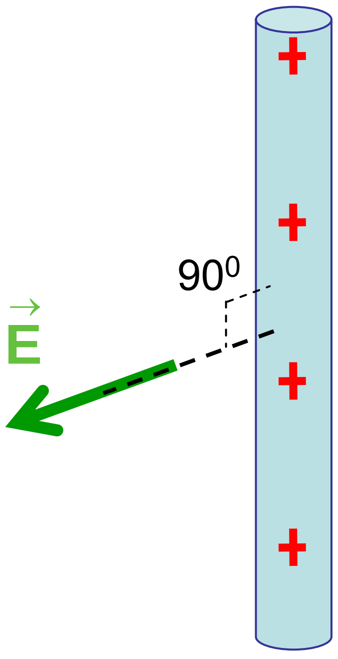
$$\varepsilon_0 E (2 \pi r h) = q_{enc}$$

$$E = \frac{q_{enc}}{2 \pi r h \varepsilon_0} = \frac{\lambda}{2 \pi \varepsilon_0 r}$$

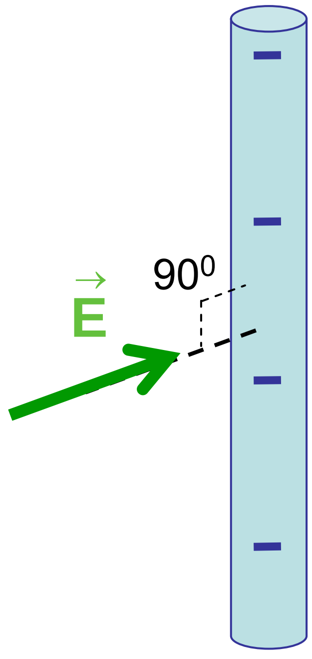
Linear charge density $= \frac{q_{enc}}{h}$

24-7 Applying Gauss' Law: Cylindrical Symmetry

Magnitude: $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$



outward in radial direction

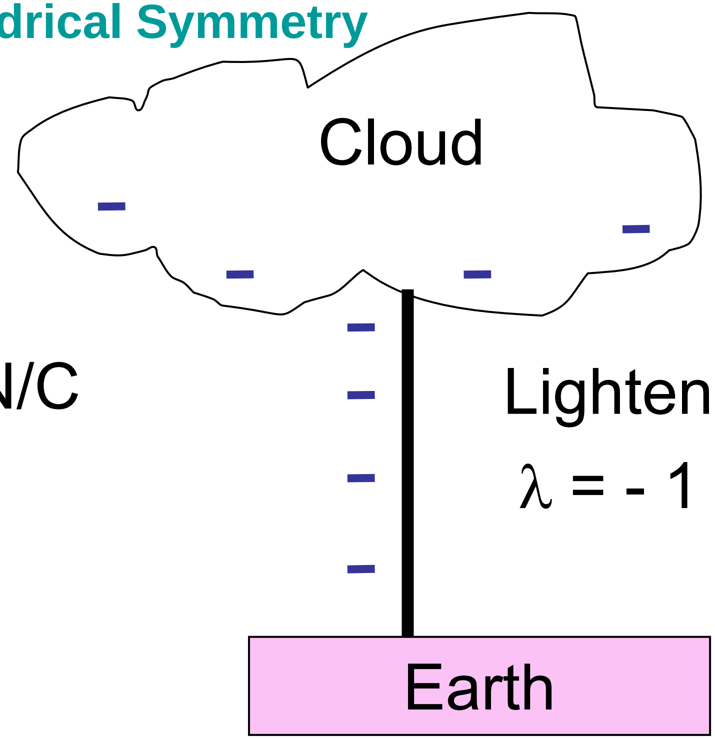


inward in radial direction

24-7 Applying Gauss' Law: Cylindrical Symmetry

Sample Problem 24-5

What is the radius of the visible lightening Column ?



Lightening

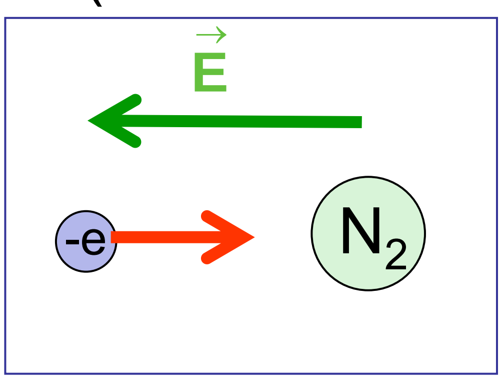
$\lambda = -1 \times 10^{-3}$ C/m

To see light E > 3 x 10^6 N/C

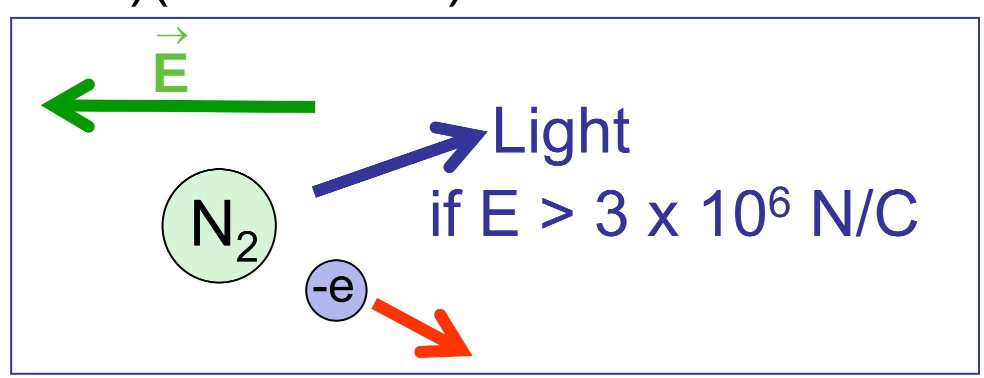
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$r = \frac{\lambda}{2\pi\varepsilon_0 E}$$

$$r = \frac{1\times10^{-3}\,\mathrm{C/m}}{2\pi\,(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(3\times10^{6}\,\mathrm{N/C})} = 6\ \mathrm{m}$$



before collision



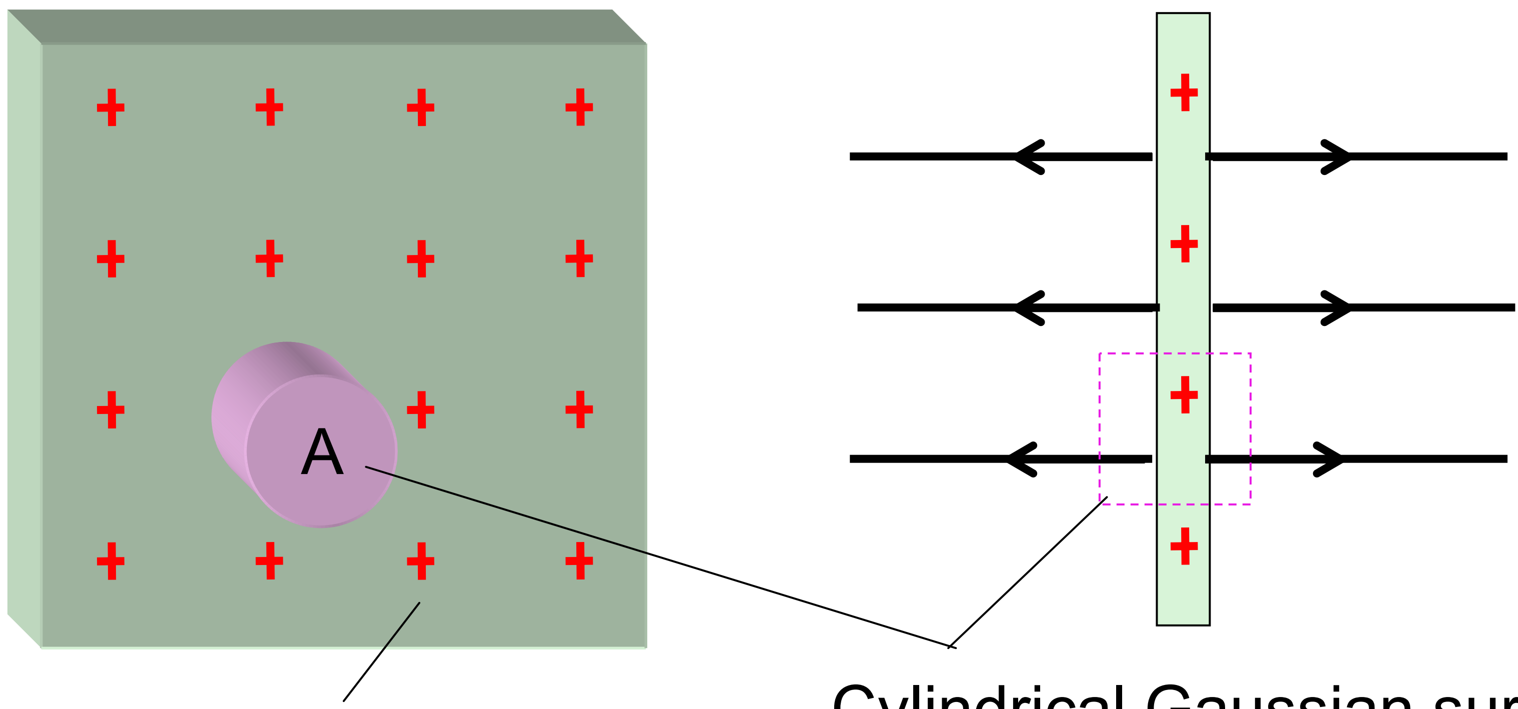
After collision

24-8 Applying Gauss' Law: Planar Symmetry

What is the electric felid from thin infinite nonconducting sheet?

From symmetry, the electric field is perpendicular to the sheet

Cross section

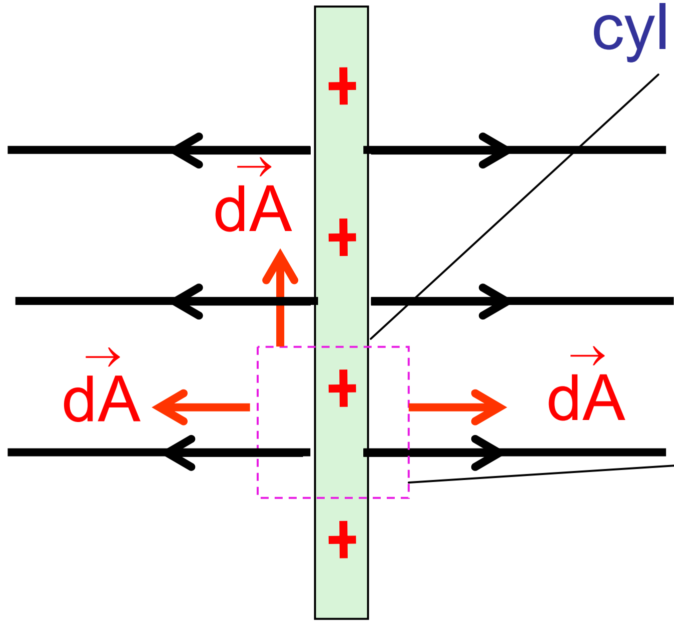


Thin infinite nonconducting sheet

Cylindrical Gaussian surface with cap area of A

24-8 Applying Gauss' Law: Planar Symmetry

Cross section



Flux through the cylinder surface = 0

Since

$\vec{E} \perp d\vec{A}$

Flux through the right cap

= Flux through the left cap

= E A

$\vec{E}$ parallel to $d\vec{A}$

Total electric flux through the cylinder = E A + E A = 2 E A

$$\varepsilon_0 \Phi = q_{enc}$$

$$\varepsilon_0 (2 E A) = q_{enc}$$

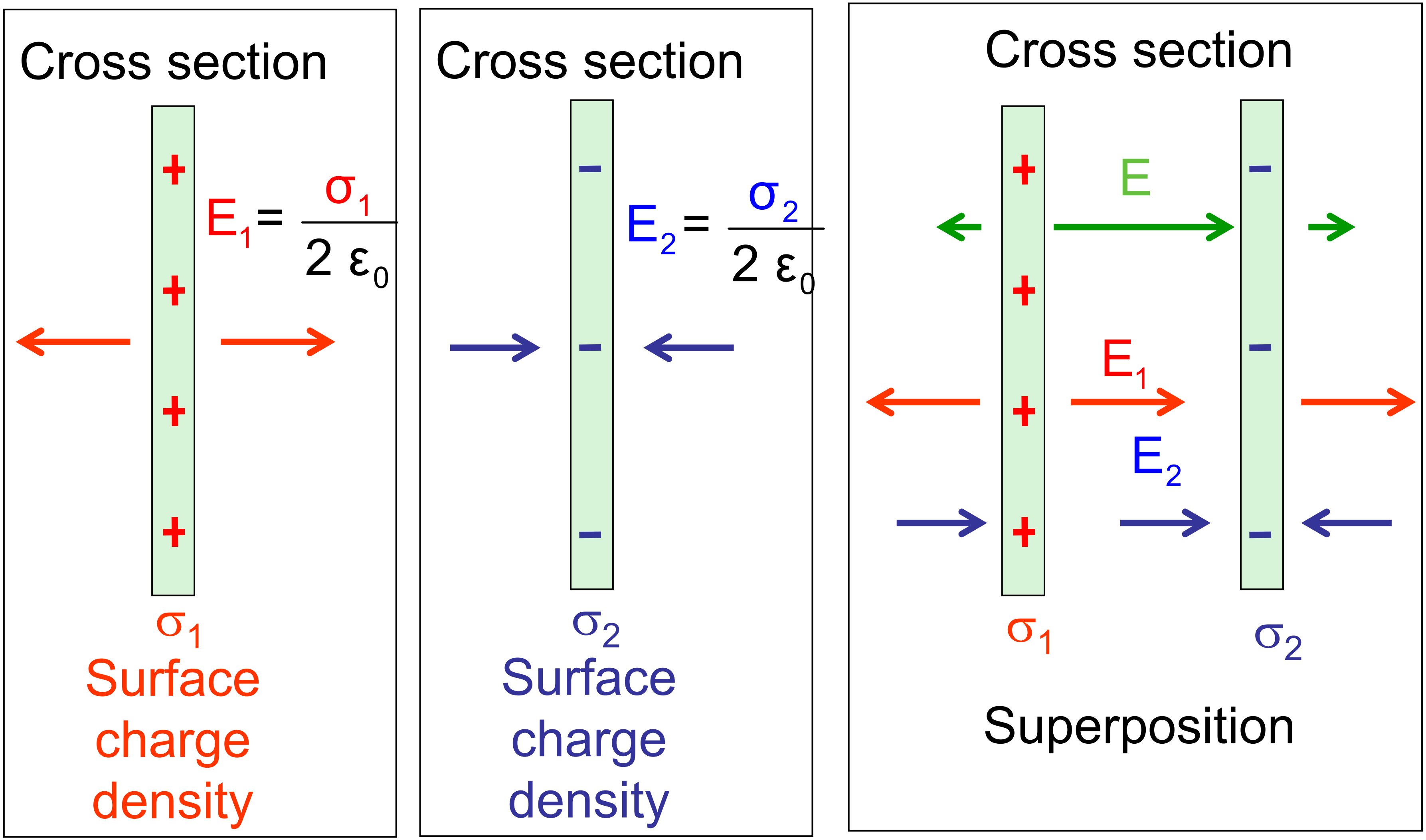
$$E = \frac{q_{enc}}{2 \varepsilon_0 A} = \frac{\sigma}{2 \varepsilon_0}$$

surface charge density

$$= \frac{\text{Charge}}{\text{Area}}$$

24-8 Applying Gauss' Law: Planar Symmetry

Nonconducting sheets

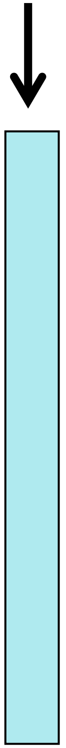


24-8 Applying Gauss' Law: Planar Symmetry

Conducting sheets

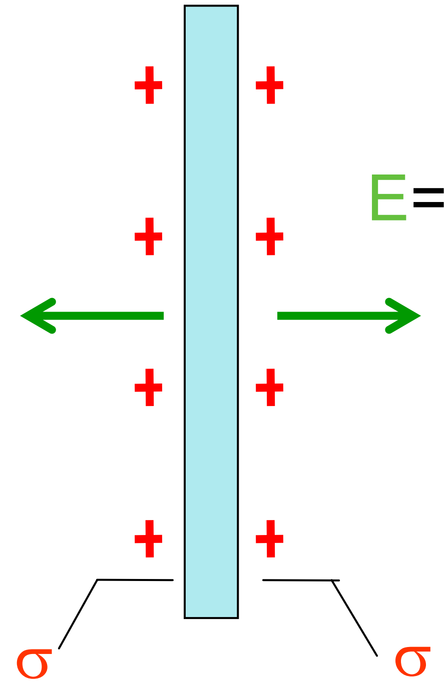
Cross section

put charge



No charge inside the conductor.

All charge on the surface



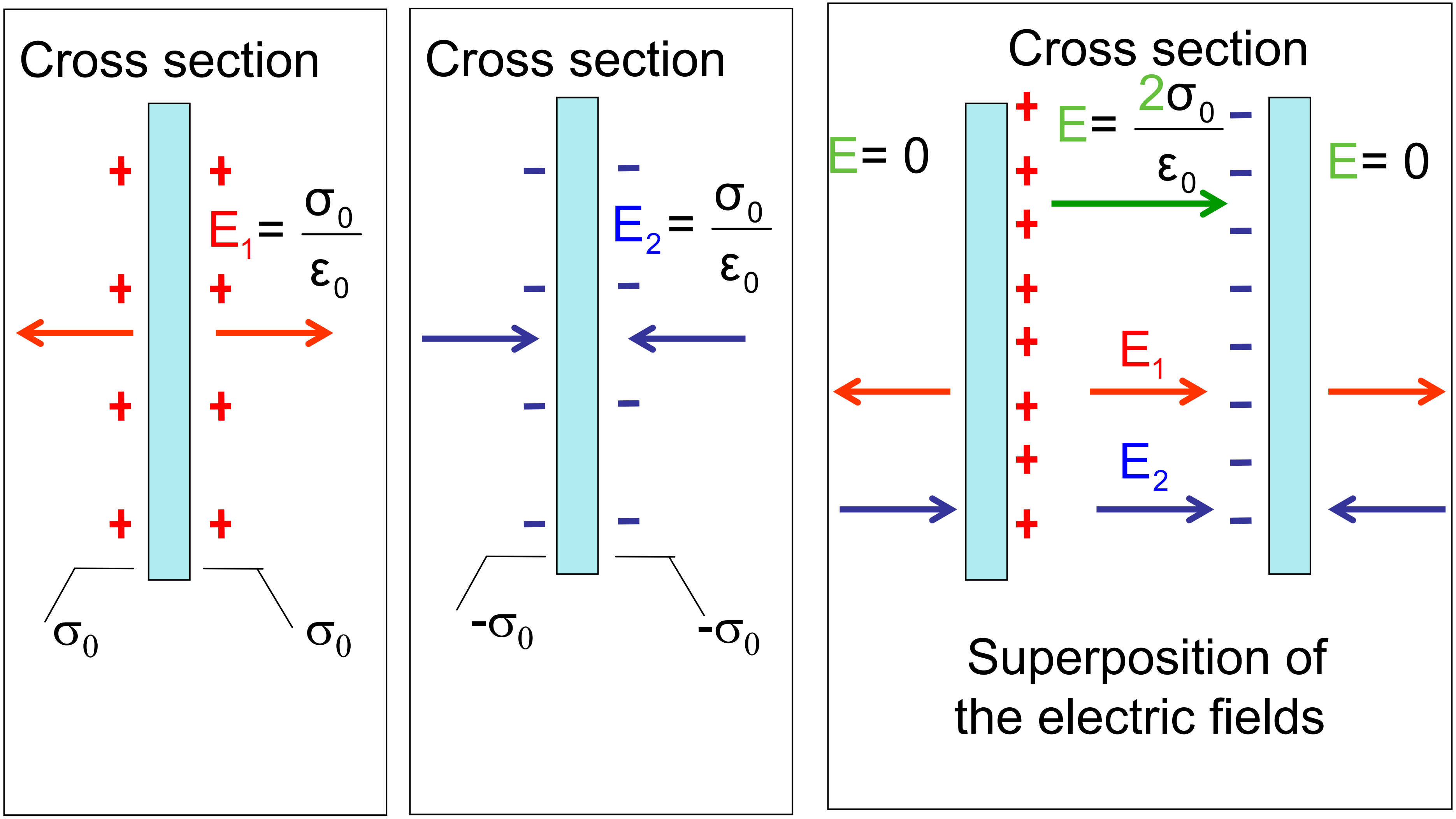
$$E = \frac{\sigma}{\varepsilon_0}$$

for conductors.

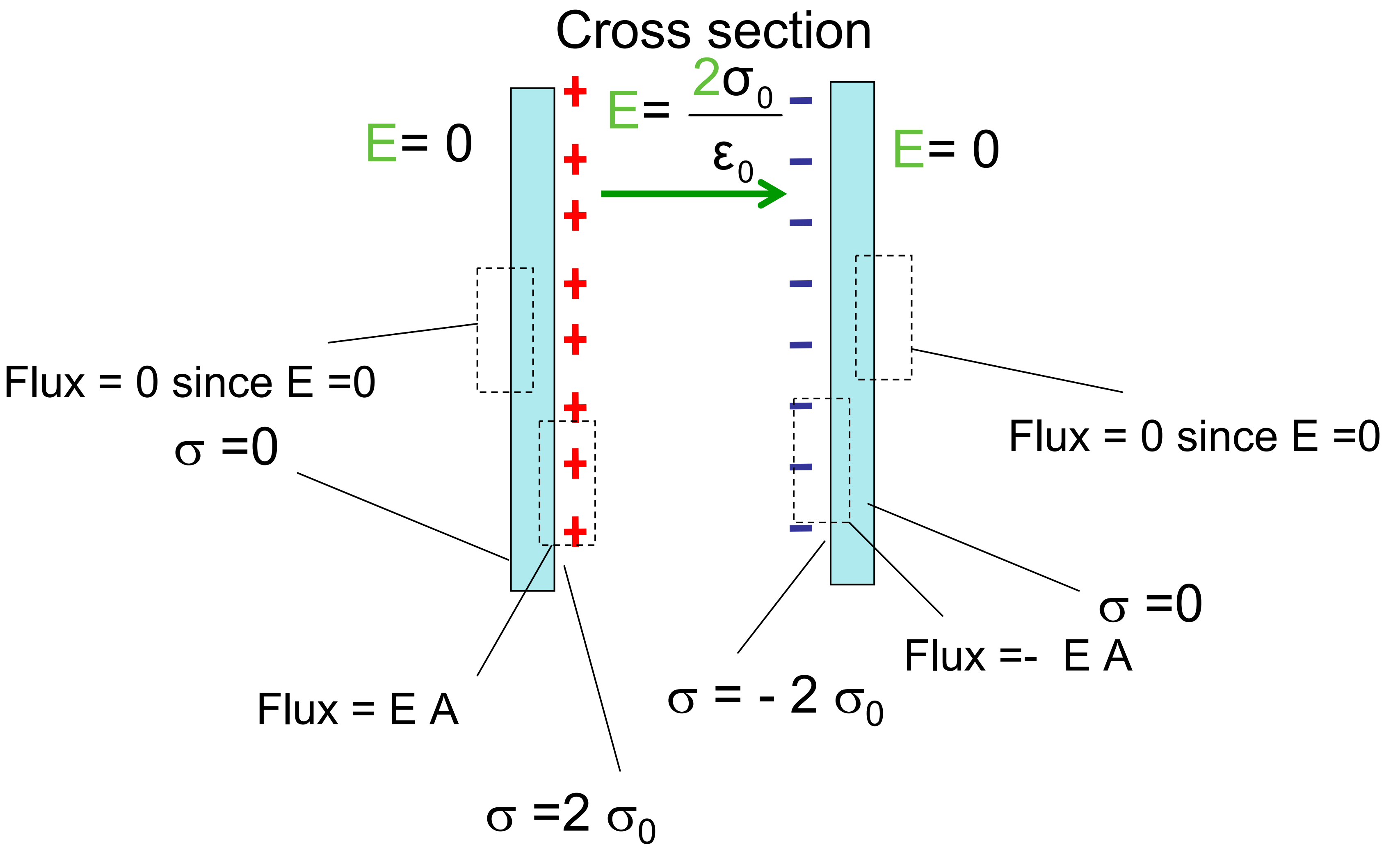
we assign charge density for each side

24-8 Applying Gauss' Law: Planar Symmetry

Conducting sheets



24-8 Applying Gauss' Law: Planar Symmetry



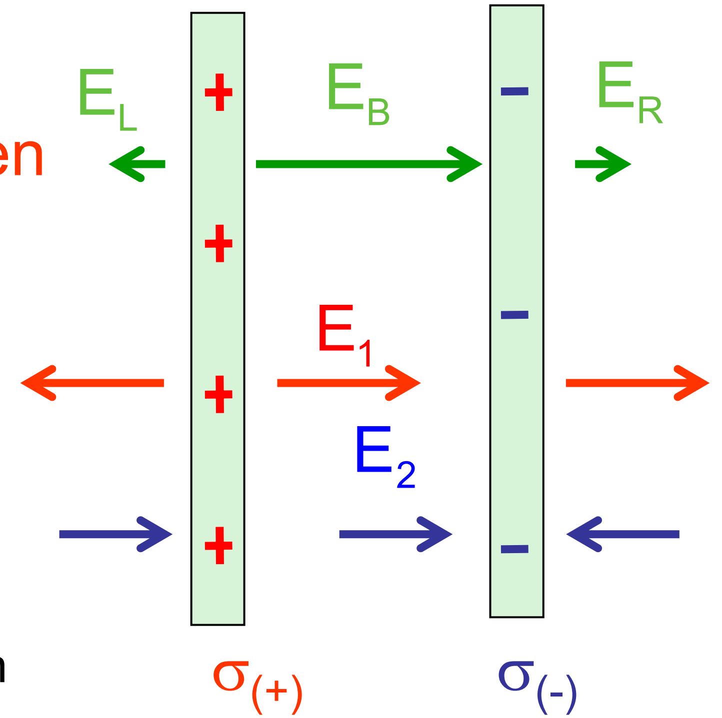
Conducting sheets

24-8 Applying Gauss' Law: Planar Symmetry

Sample Problem 24-6

Two nonconducting large sheets

Cross section



$\sigma_{(+)} = 6.8\ \mu C/m^2$
$\sigma_{(-)} = 4.3\ \mu C/m^2$

Find the electric field to the right, between and to the left of the two plates

$$E_1 = \frac{\sigma_{(+)}}{2\varepsilon_0} \qquad E_2 = \frac{\sigma_{(-)}}{2\ \varepsilon_0}$$

$$E_L = E_1 - E_2 = \frac{\sigma_{(-)}}{2\ \varepsilon_0} - \frac{\sigma_{(-)}}{2\ \varepsilon_0}$$

$$= \frac{(6.8-4.3)\times10^{-6} C/m^2}{2(8.85\times10^{-12} C^2/Ngm^2)} = 1.4\times10^5\ N/m$$

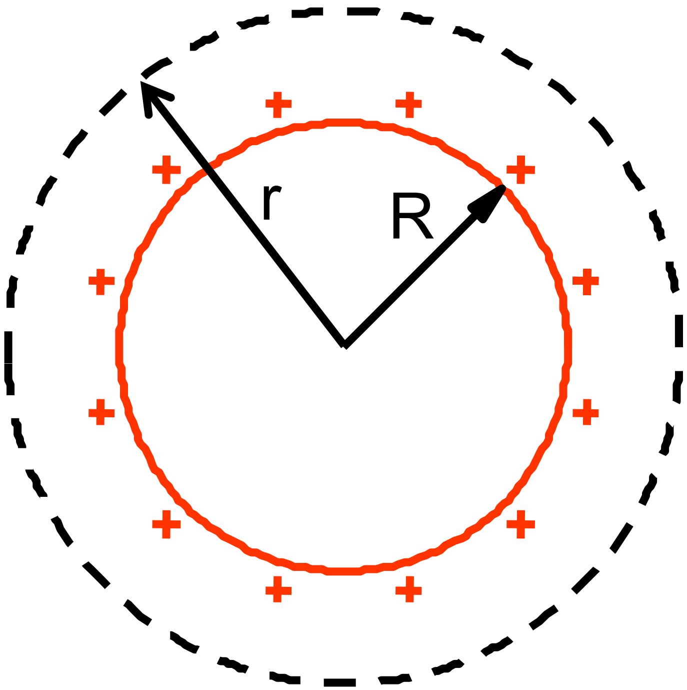
$E_L = 1.4\times10^5\ N/m$ To the left

$E_R = E_L = 1.4\times10^5\ N/m$ To the right

$E_{BL} = E_1 + E_2 = \quad = 6.3\times10^5\ N/m$ To the right

24-9 Applying Gauss' Law: spherical Symmetry

A thin, uniformly charged spherical shell with total charge q and of radius R



$r > R$

$$\varepsilon_0 \Phi = q_{enc}$$

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

$$\Phi = \oint E\, dA = E \oint dA = E(4\pi r^2)$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{q}{4\pi \varepsilon_0 r^2}$$

Similar to the electric field from a point charge placed at the center of the sell

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center

24-9 Applying Gauss' Law: spherical Symmetry

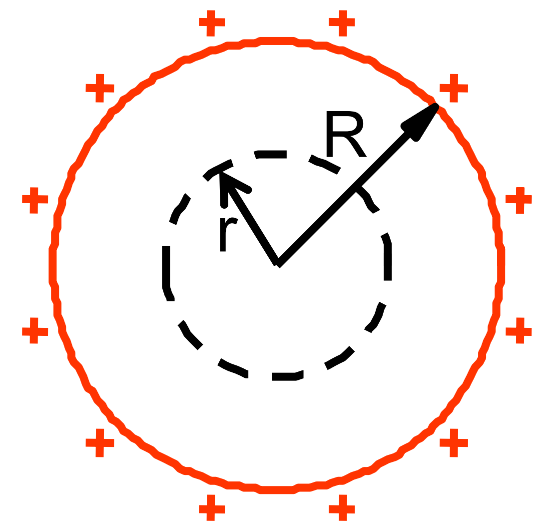
A thin, uniformly charged spherical shell with total charge q and of radius R

r < R

$$\varepsilon_0 \Phi = q_{enc} = 0$$

$$\Phi = 0$$

$$E = 0$$



If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

24-9 Applying Gauss' Law: spherical Symmetry

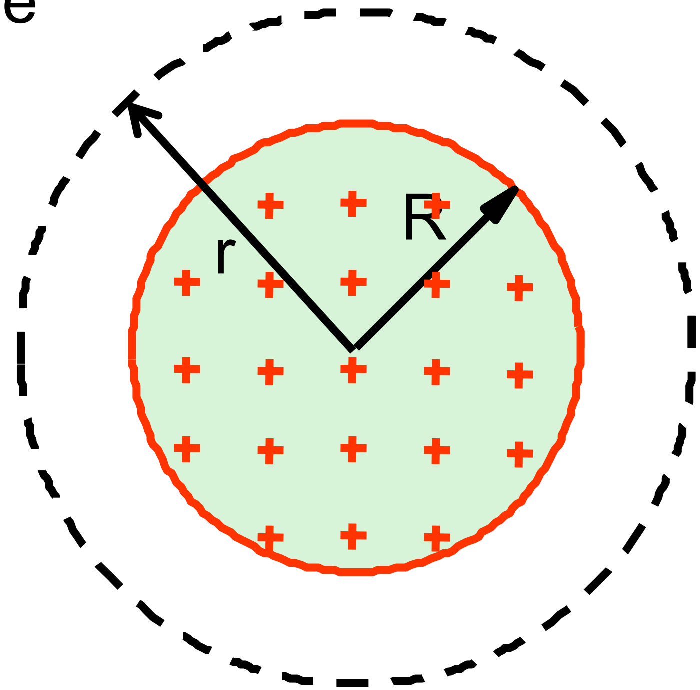
A sphere of radius R and with total charge q uniformly charged throughout its volume

r > R

$$\varepsilon_0 \Phi = q_{enc}$$

$$\Phi = E(4\pi r^2)$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$



24-9 Applying Gauss' Law: spherical Symmetry

A sphere of radius R and with total charge q uniformly charged throughout its volume

r < R

$$\varepsilon_0 \Phi = q_{enc}$$

$$\Phi = E(4\pi r^2)$$

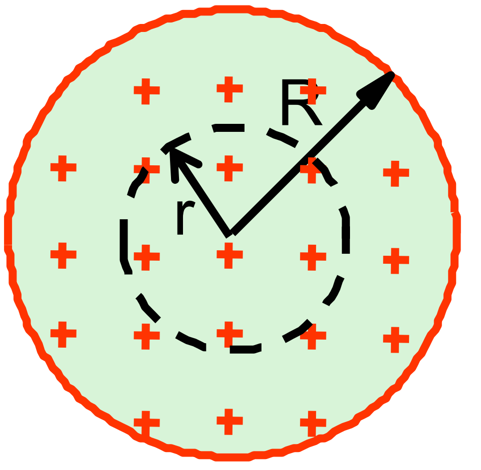
$$E = \frac{q_{enc}}{4\pi\varepsilon_0 r^2}$$

$$\frac{\text{Charge enclosed by r}}{\text{Volume enclosed by r}} = \frac{\text{total charge}}{\text{Totla volume}}$$

$$\frac{q_{enc}}{\frac{4\pi r^3}{3}} = \frac{q}{\frac{4\pi R^3}{3}}$$

$$q_{enc} = \frac{r^3}{R^3} q$$

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r$$



24-9 Applying Gauss' Law: spherical Symmetry

Checkpoint 5

Rank electric field at points, greatest first

3 and 4 tie
then 2
then 1

Electric field due the sheets is zero.
You need only to consider the field from the sphere