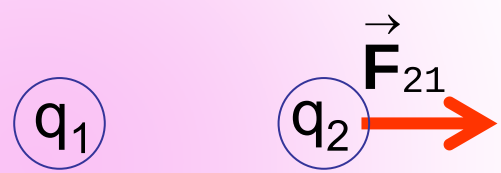
# Chapter 23

# Electric Fields

## 23-1 Charges and Forces: A Closer Look

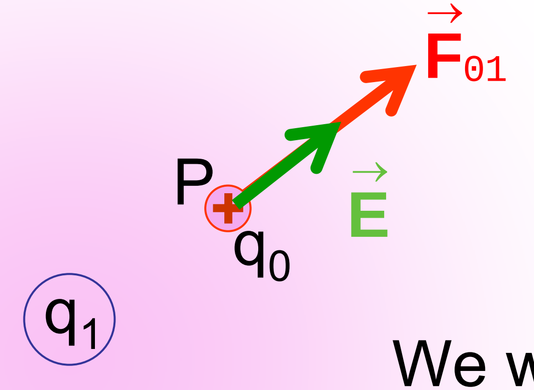
Charged particle $q_1$ sets up an electric field in all space




How does the point charge $q_2$ know about the presence of point charge $q_1$?

## 23-2 The Electric Field

To feel the electric field of a charged particle $q_1$ at a point P, we bring a positive charged particle $q_0$ at point P and measure the force $F_{01}$ on $q_0$ from $q_1$.

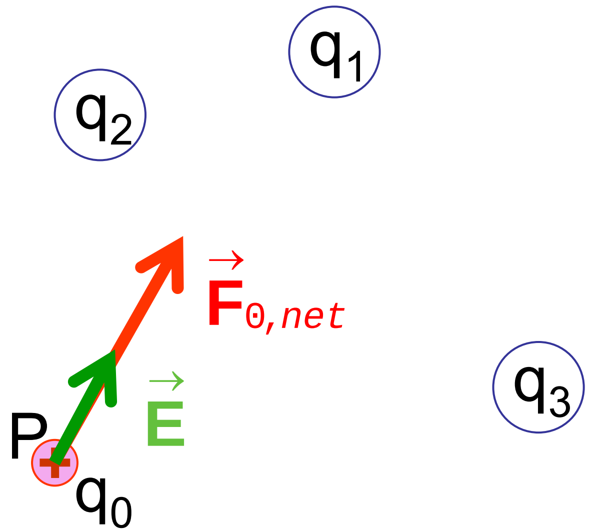



We will define the electric field of charged particle $q_1$ at point P as

$$\vec{E} = \frac{\vec{F}_{01}}{q_0}$$

We call $q_0$ a test charge

## 23-2 The Electric Field




The electric field from the charged particles $q_1$, $q_2$, $q_3$ at point P is

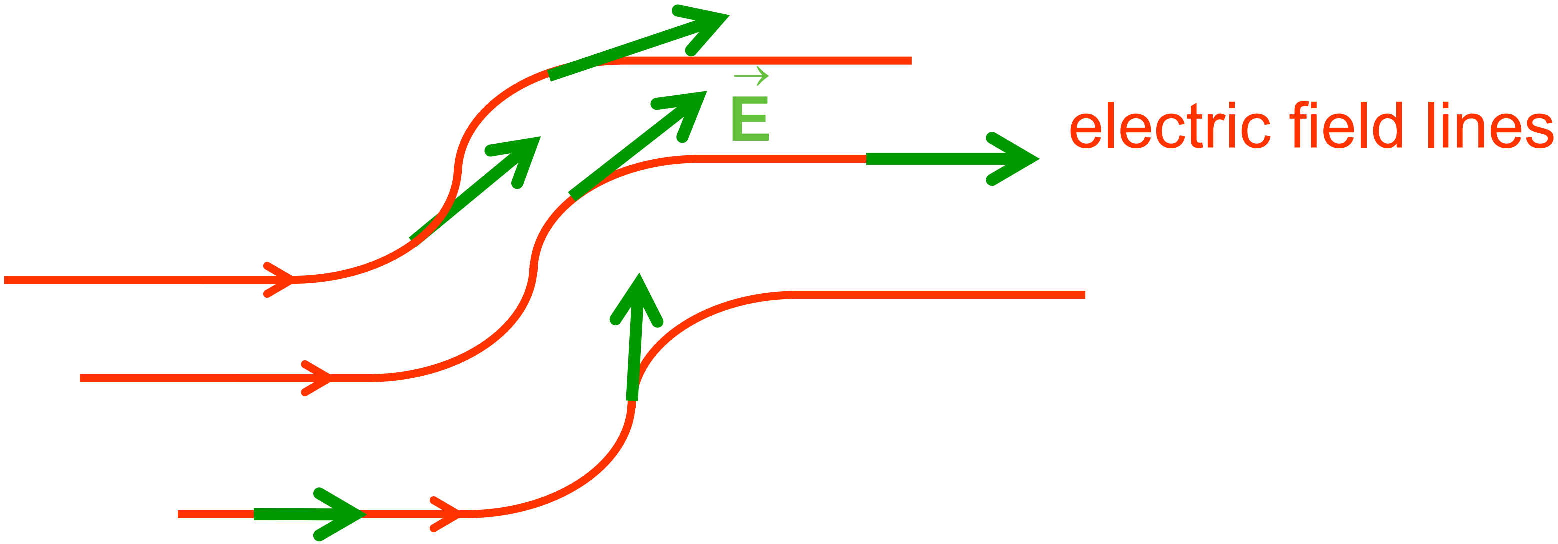
$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_{0,net}}{q_0}$$

At each point in space, you can find the electric field by the same method.

$q_0$ is a positive test charge

## 23-2 The Electric Field Lines

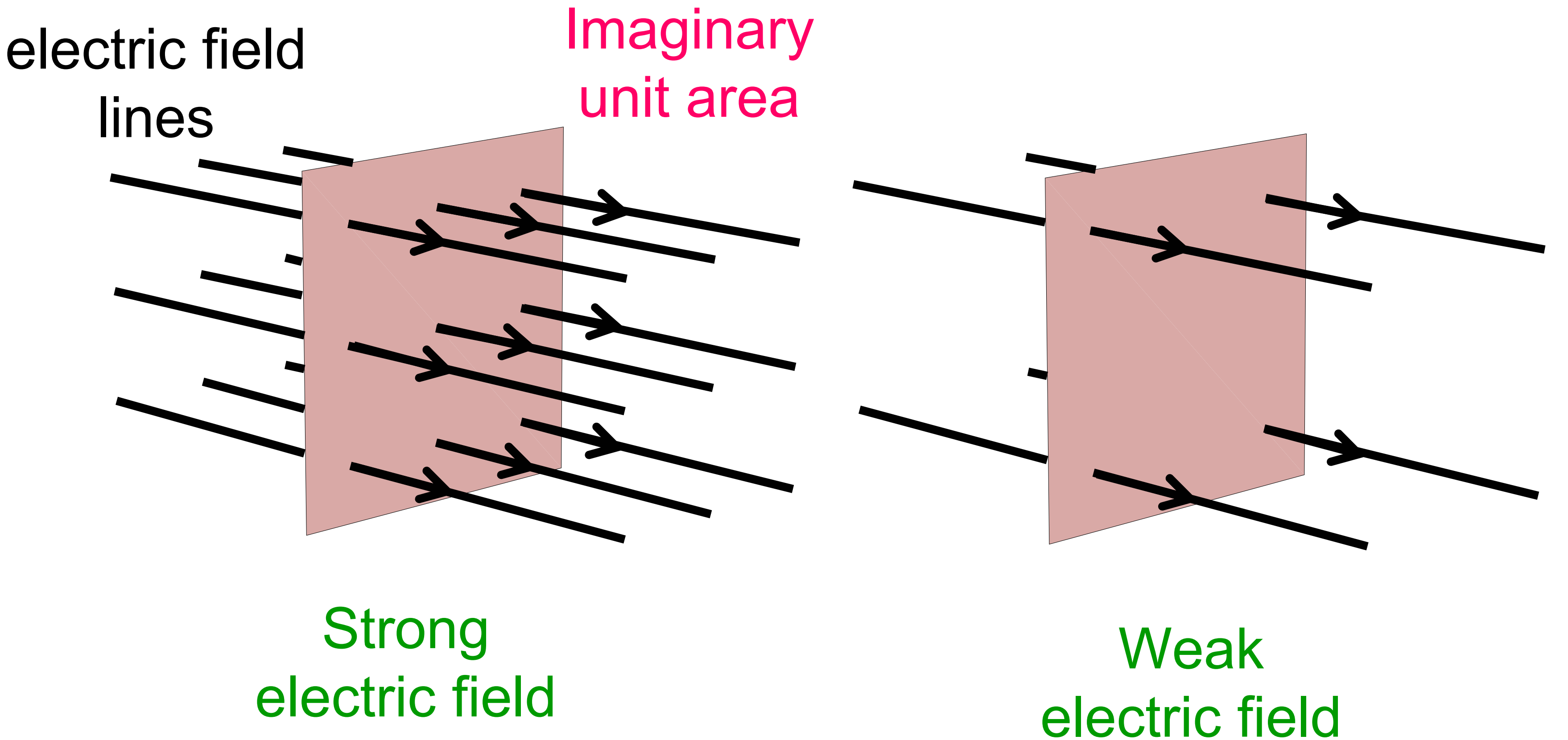
We use electric field lines to help us representing the electric field




At any point, the tangent of the electric field lines gives the direction of the electric field

## 23-2 The Electric Field Lines

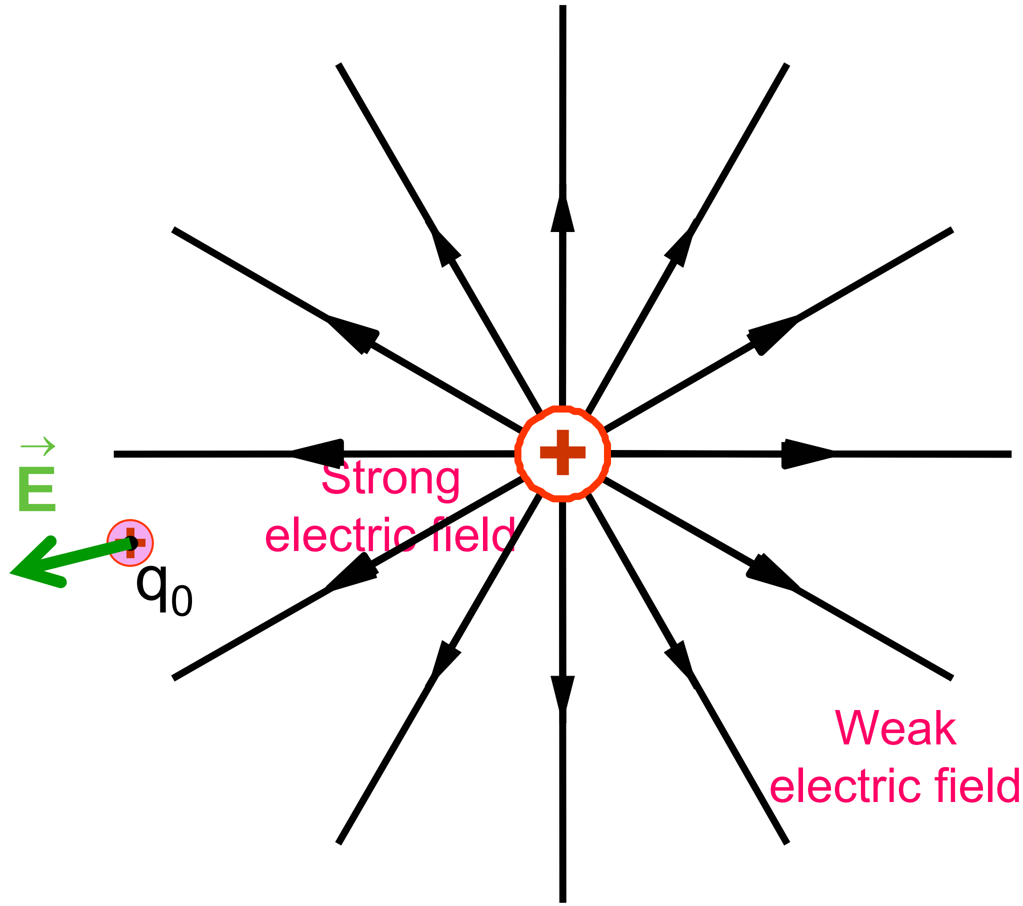
We use electric field lines to help us representing the electric field




Number of lines per unit area in a plane perpendicular to the electric field lines is proportional the magnitude of the electric field

## 23-2 The Electric Field Lines

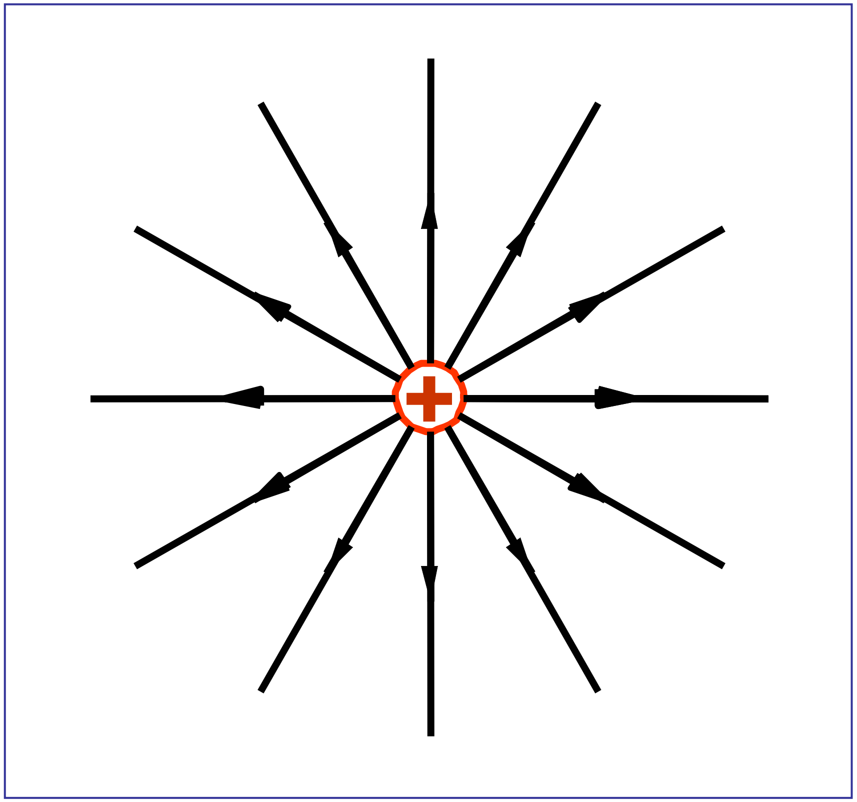
# Electric field lines from a positive point charge

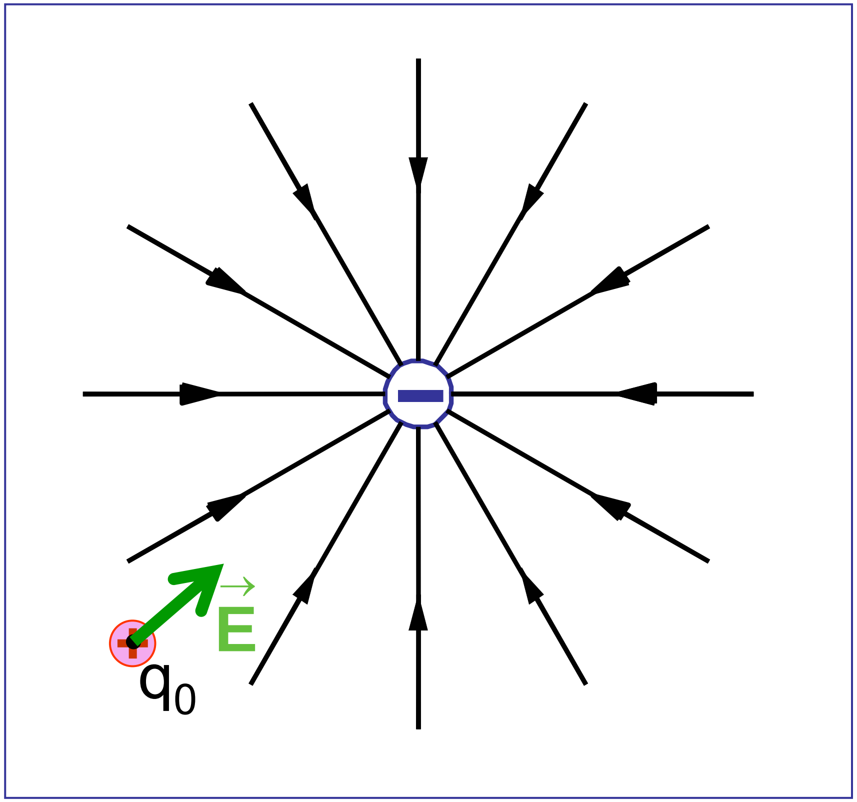



$q_0$ is a positive test charge

## 23-2 The Electric Field Lines

# Electric field lines from a point charge

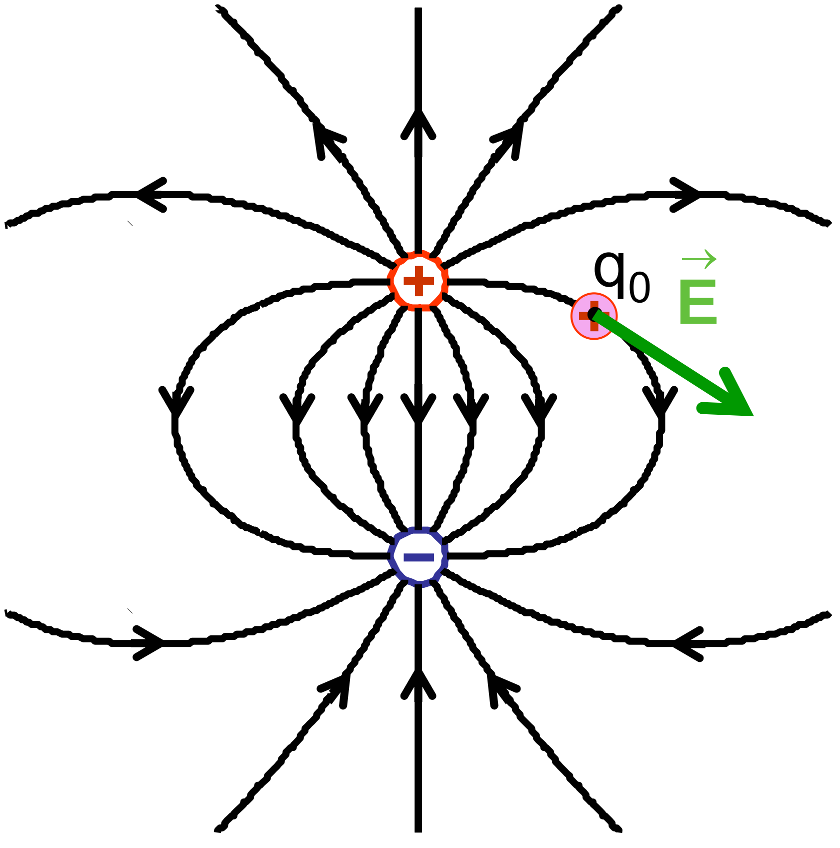





Electric field lines extend away form positive charge and toward negative charge

## 23-2 The Electric Field Lines

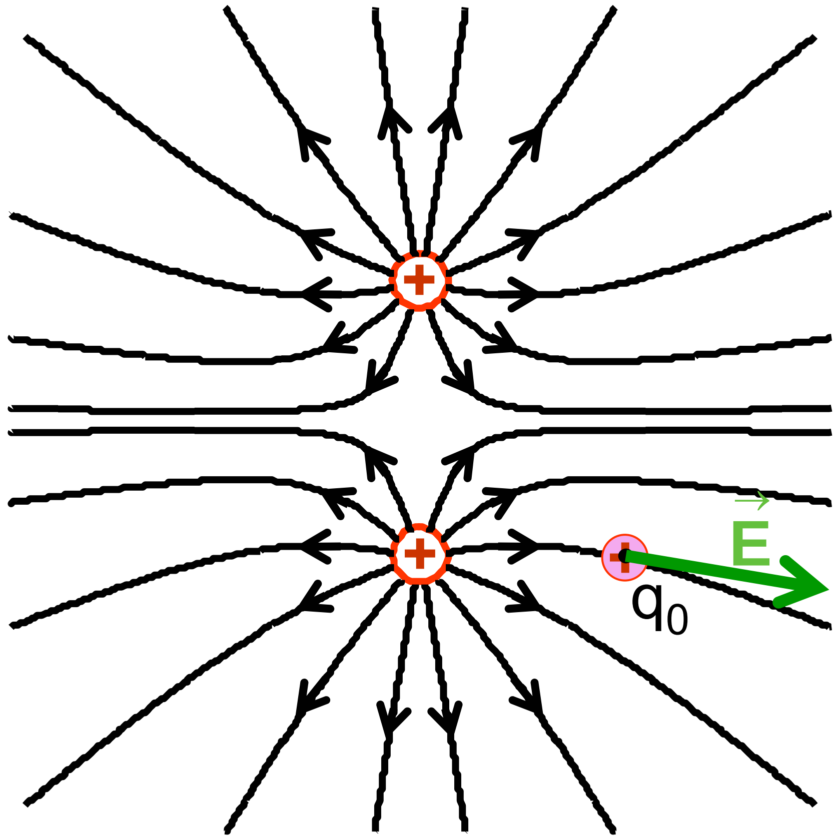
Electric field lines from a positive and a nearby negative point charge




Electric field lines extend away form positive charge and toward negative charge

## 23-2 The Electric Field Lines

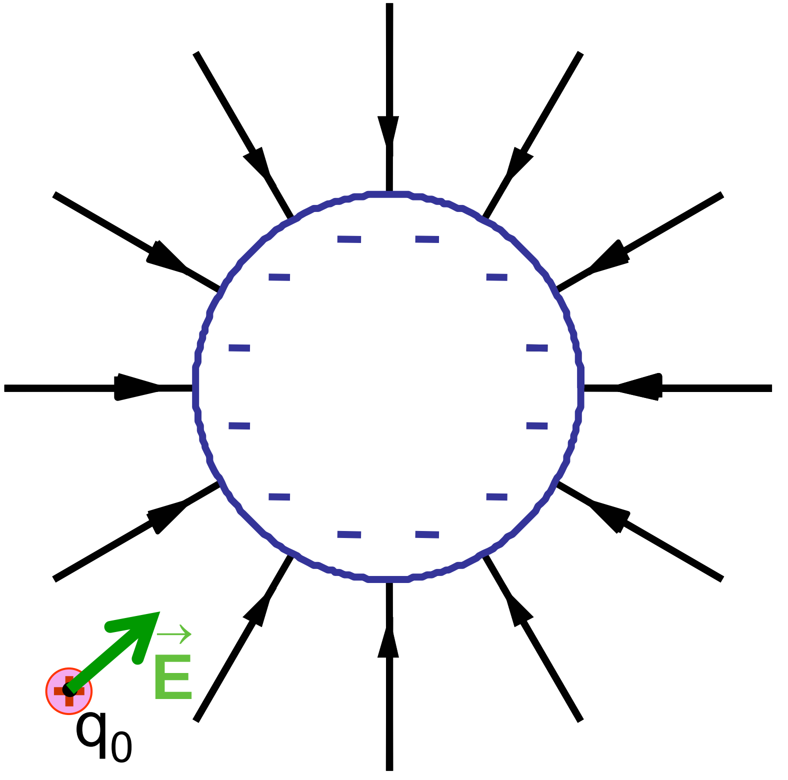
Electric field lines from two equal and opposite point charges




Electric field lines extend away form positive charge and toward negative charge

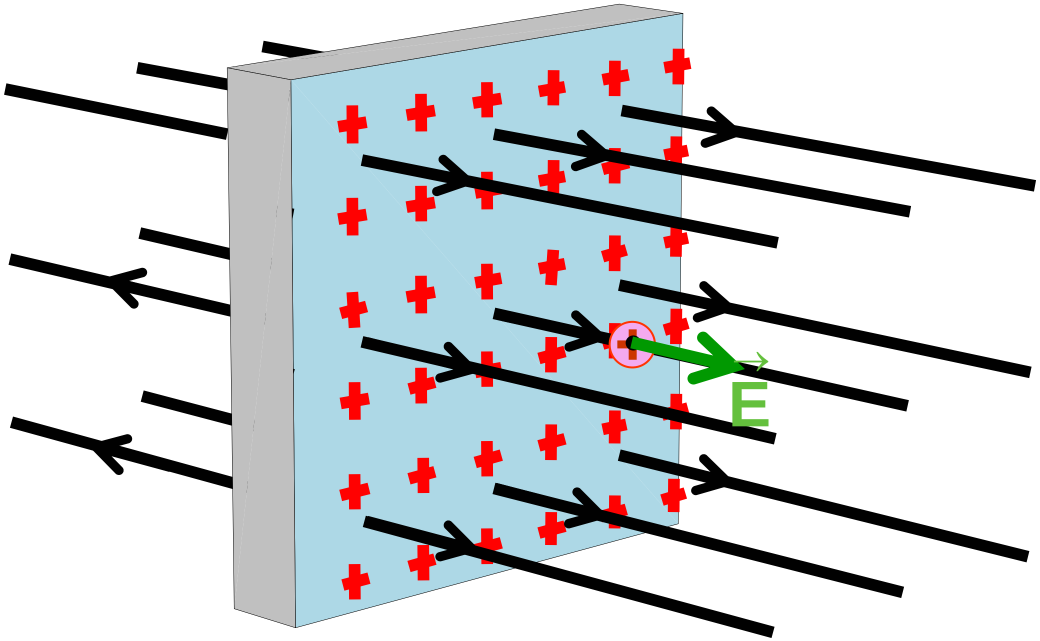
## 23-2 The Electric Field Lines

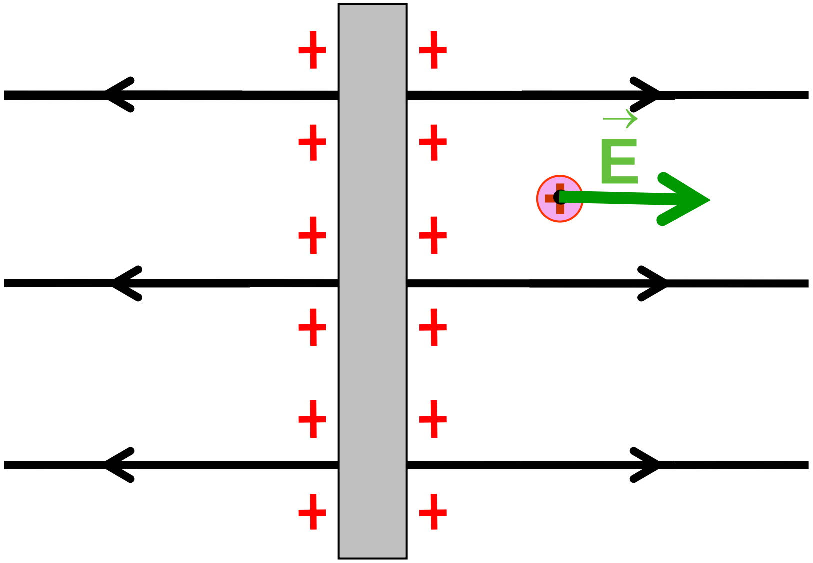
# Electric field lines from a sphere of uniform negative charge



## 23-2 The Electric Field Lines

Electric field lines from a very large nonconducting sheet



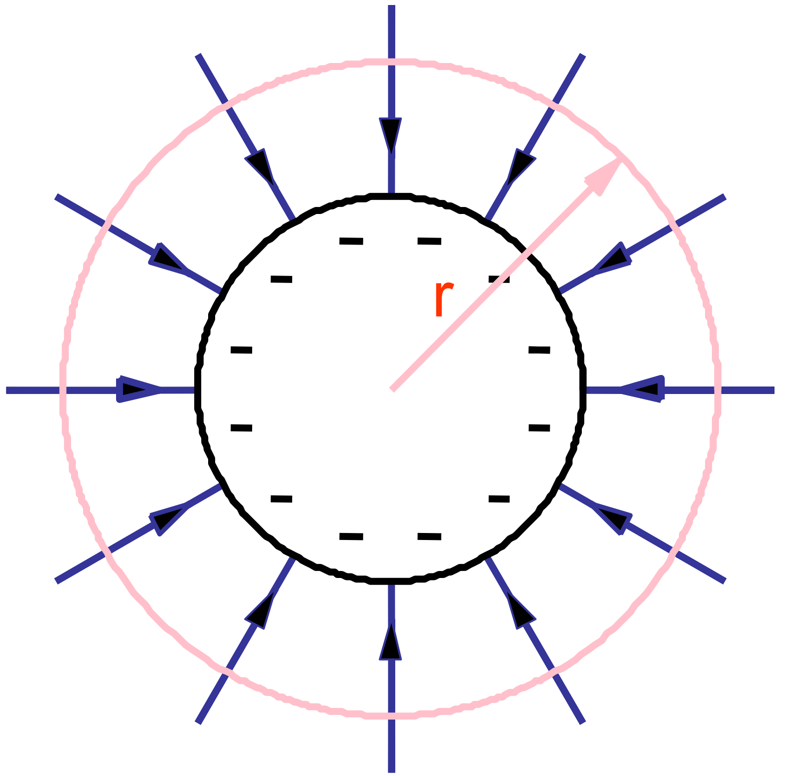




Side view

## 23-2 The Electric Field Lines

# Sample Problem 23-1

Use an argument based on electric field lines to describe how the electric field varies with distance from the center of a uniformly charged sphere




The magnitude of the electric field is proportional to the number of lines per unit area in a plane perpendicular to the lines.

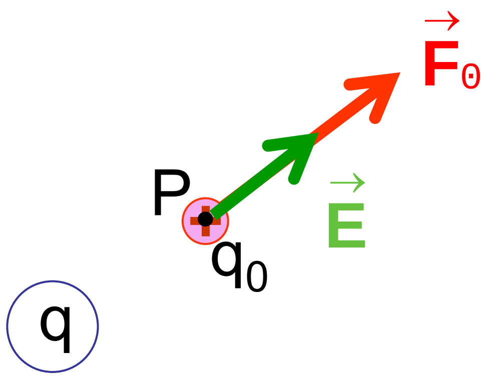
Suppose we have a number of N lines extend toward the sphere.

For any imaginary sphere with radius r and concentric with the charged sphere, the number of lines per unit perpendicular to the imaginary sphere is

$$\frac{N}{4\pi r^2} \propto E$$

The magnitude of the electric field varies as the inverse square of the distance form the center of the sphere

# 23-2 The Electric Field Due to a Point Charge

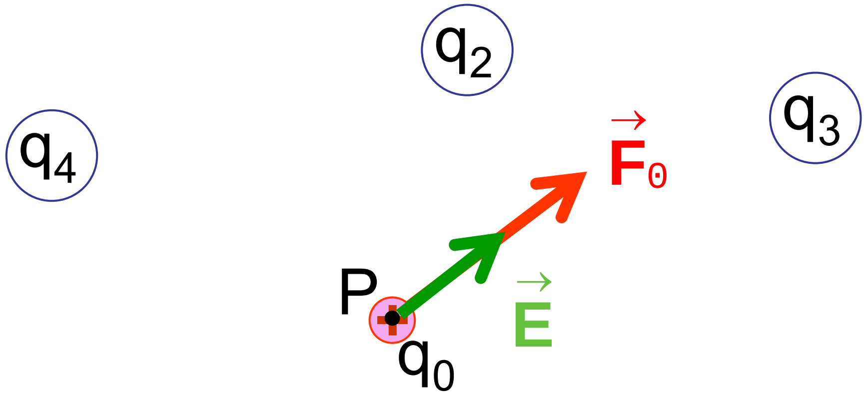



$$E = \frac{F_0}{q_0} = \frac{k\,\frac{|q_0||q|}{r^2}}{q_0} = k\,\frac{|q|}{r^2}$$

$$\boxed{E = k\,\frac{|q|}{r^2} = \frac{1}{4\pi\varepsilon_0}\,\frac{|q|}{r^2}}$$

Point charge

## 23-2 The Electric Field Due to a Point Charge




$$\vec{F}_{0,net} = \vec{F}_{02} + \vec{F}_{03} + \vec{F}_{04}$$

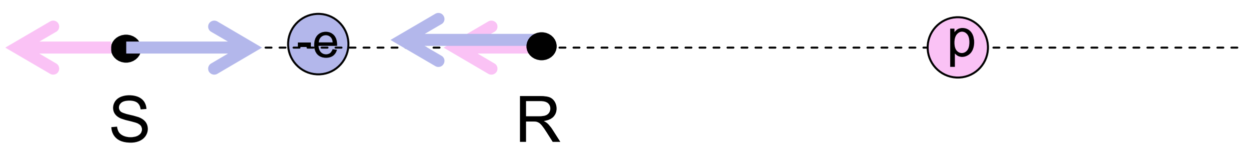
$$\vec{E} = \frac{\vec{F}_{0,net}}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \frac{\vec{F}_{03}}{q_0}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

Superposition principle

# 23-2 The Electric Field Due to a Point Charge

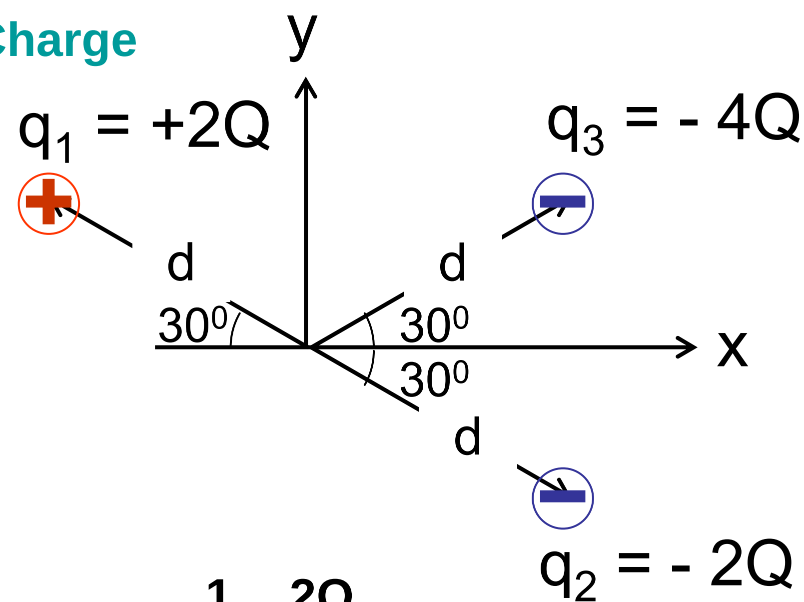
## Checkpoint 1

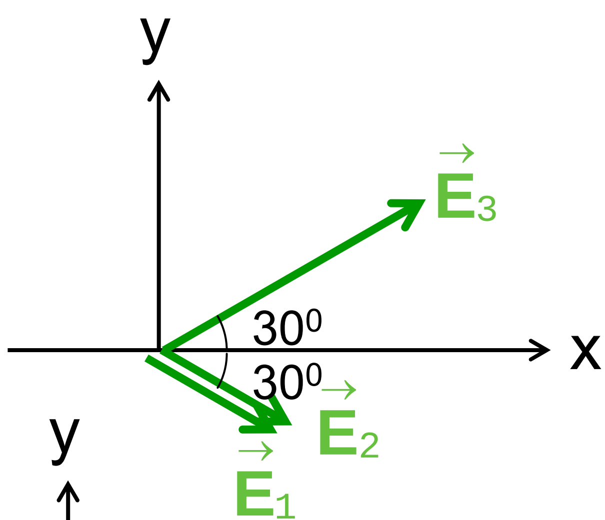


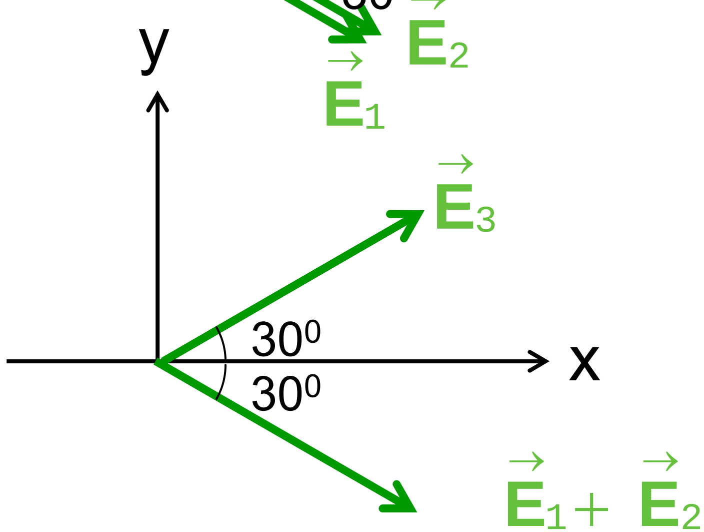
## 23-2 The Electric Field Due to a Point Charge

### Sample Problem 23-2

What net electric field is produced at the origin?




$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}$$

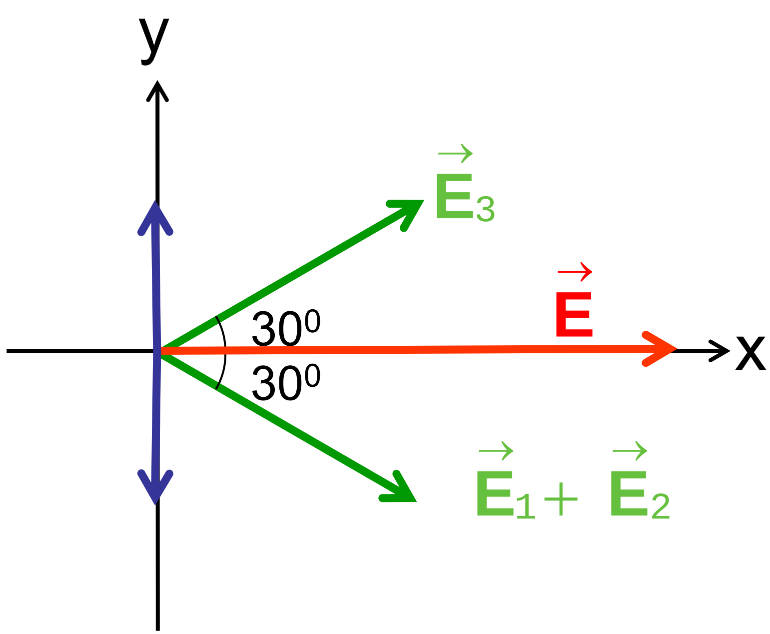
$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}$$

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}$$

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}$$

## 23-2 The Electric Field Due to a Point Charge

# Sample Problem 23-2



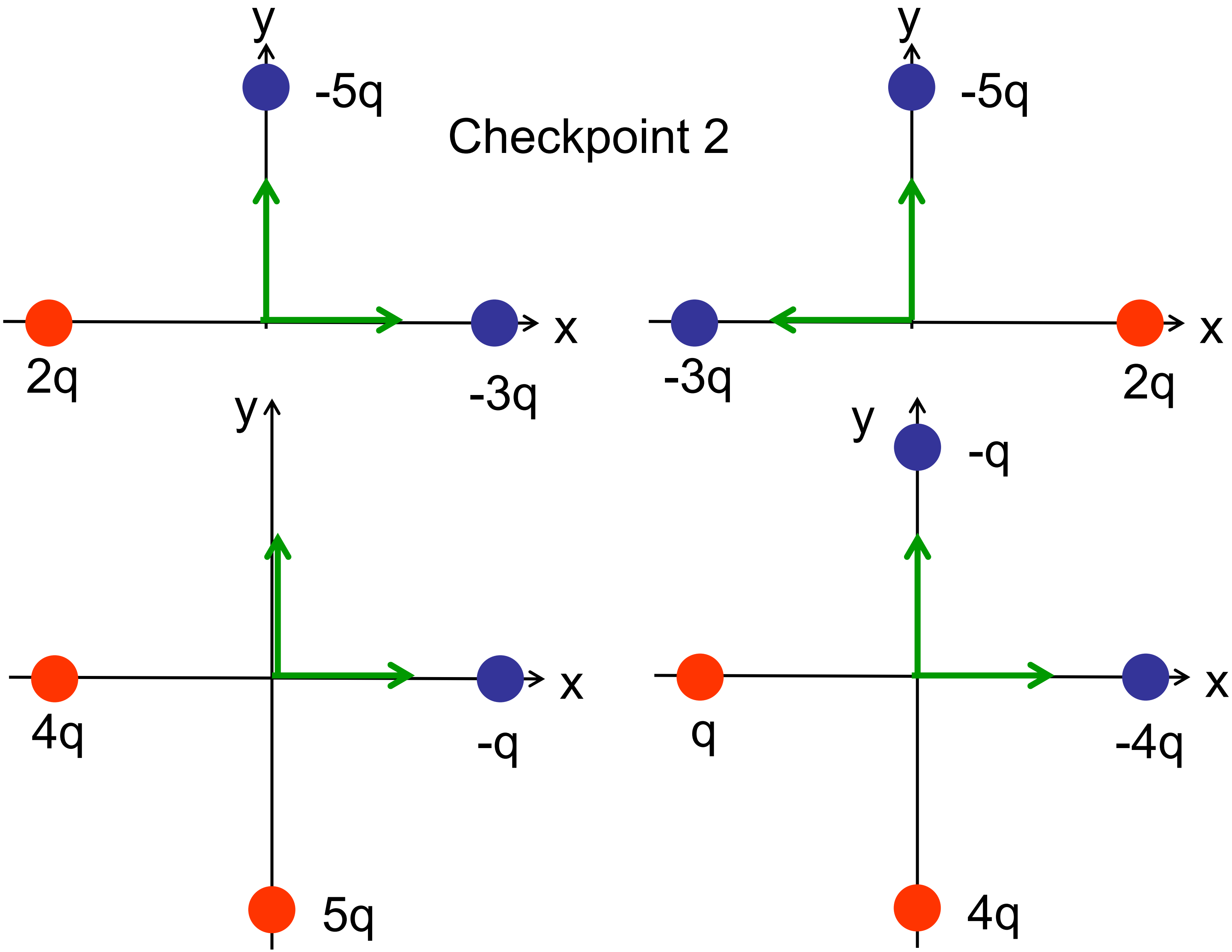

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}$$

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}$$

$$E = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}\right)\cos(30^0)$$

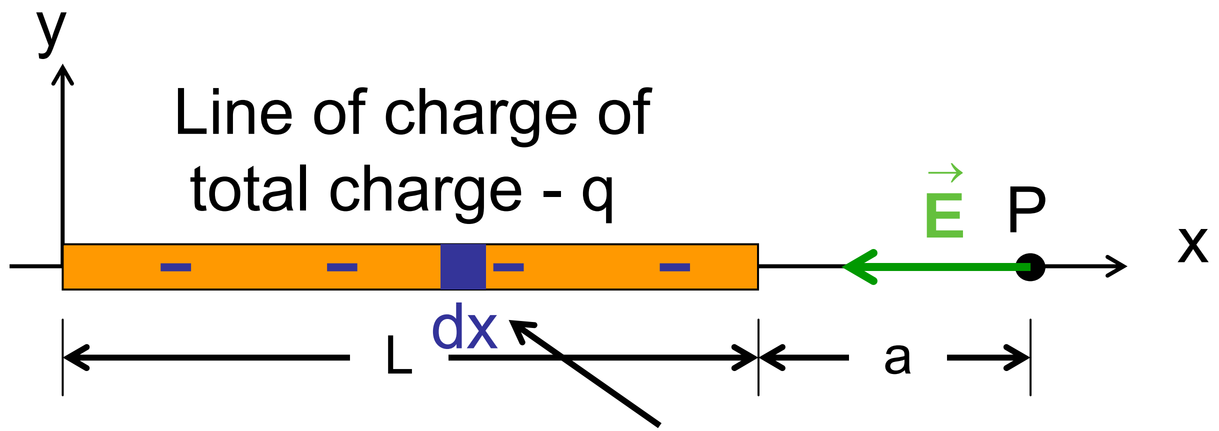
## 23-2 The Electric Field Due to a Point Charge

Checkpoint 2



## 23-6 The Electric Field Due to a Line of Charge

Find the electric field at point P




$$\frac{dq}{-q} = \frac{dx}{L}$$

$$dq = \frac{-q}{L} dx$$

$$dq = \lambda\, dx \qquad \lambda = \frac{\text{Charge}}{\text{Length}} = \text{linear charge density}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(L+a-x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(L+a-x)^2}$$

$$E = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(L+a-x)^2} = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{-1}{L+a-x}\right]_{x=0}^{x=L} = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{L+a} - \frac{1}{a}\right)$$

## 23-6 The Electric Field Due to a Line of Charge

$$E=\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{L+a}-\frac{1}{a}\right)=\frac{1}{4\pi\varepsilon_0}\frac{-q}{L}\left(\frac{1}{L+a}-\frac{1}{a}\right)$$

What is the electric field for a >> L ?

We can write the electric field as

$$E=\frac{1}{4\pi\varepsilon_0}\frac{-q}{L}\left(\frac{1}{a}\frac{1}{1+\frac{L}{a}}-\frac{1}{a}\right)=\frac{1}{4\pi\varepsilon_0}\frac{-q}{L}\frac{1}{a}\left(\frac{1}{1+\frac{L}{a}}-1\right)$$

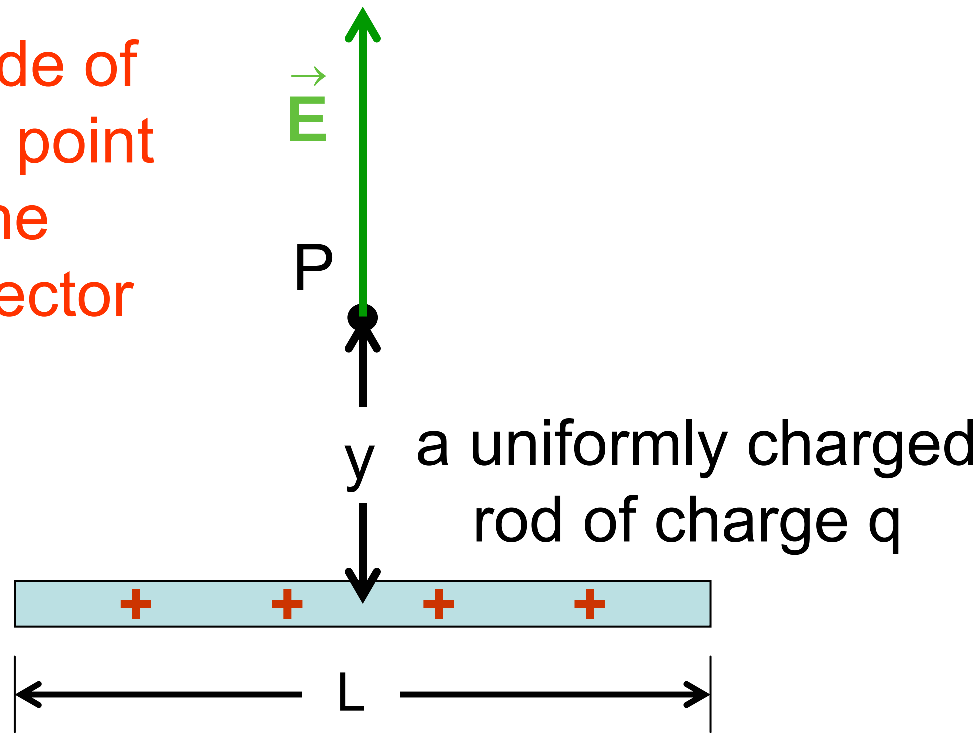
For a >> L, $\frac{1}{1+\frac{L}{a}}\approx 1-\frac{L}{a}$

$$E\approx\frac{1}{4\pi\varepsilon_0}\frac{-q}{L}\frac{1}{a}\left(1-\frac{L}{a}-1\right)=\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$$

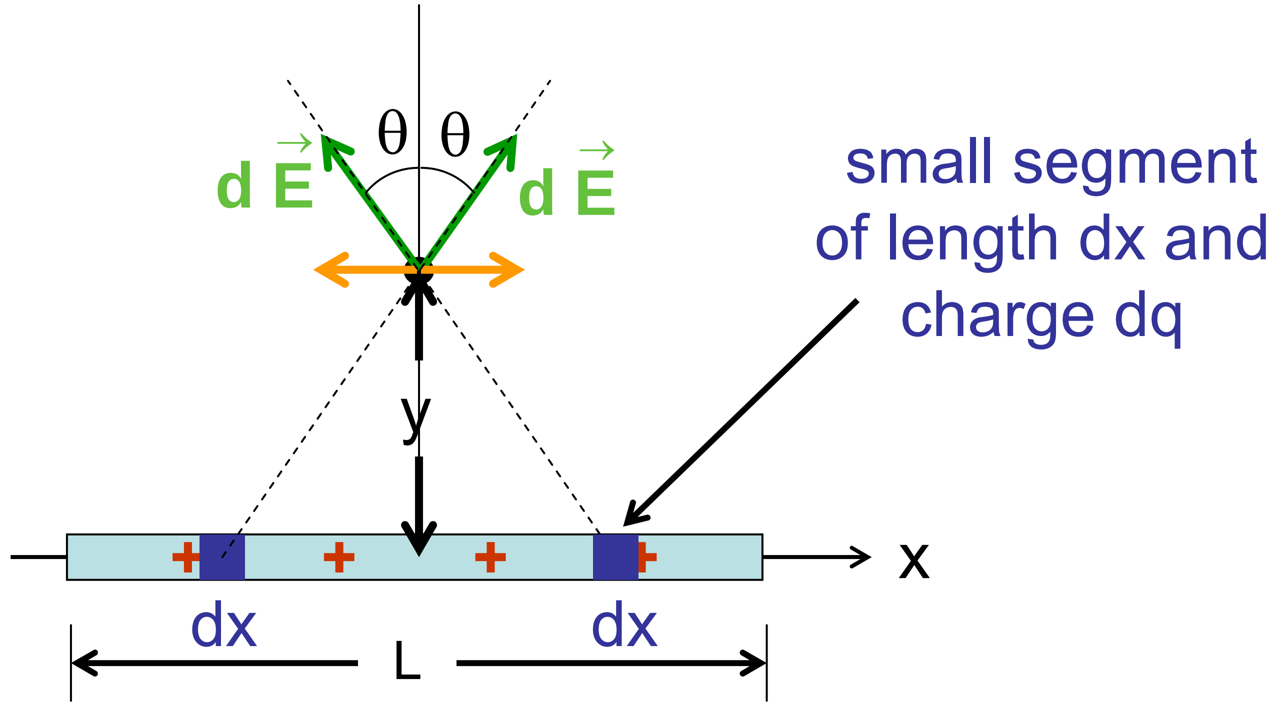
For a >> L, the electric field is similar to that from a point charge

## 23-6 The Electric Field Due to a Line of Charge

Find the magnitude of the electric field at point P located on the perpendicular bisector of the rod.



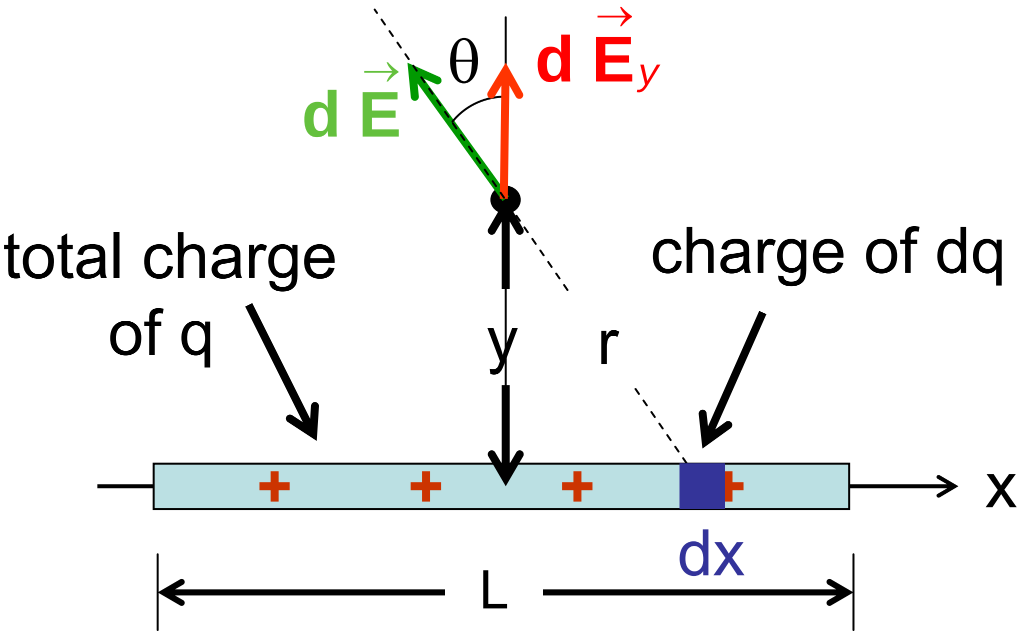
## 23-6 The Electric Field Due to a Line of Charge




From symmetry,
components along x-axis cancel each others.
We need only to consider components along y-axis

## 23-6 The Electric Field Due to a Line of Charge

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$




$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2+y^2}$$

$$dq = \frac{q}{L}dx$$

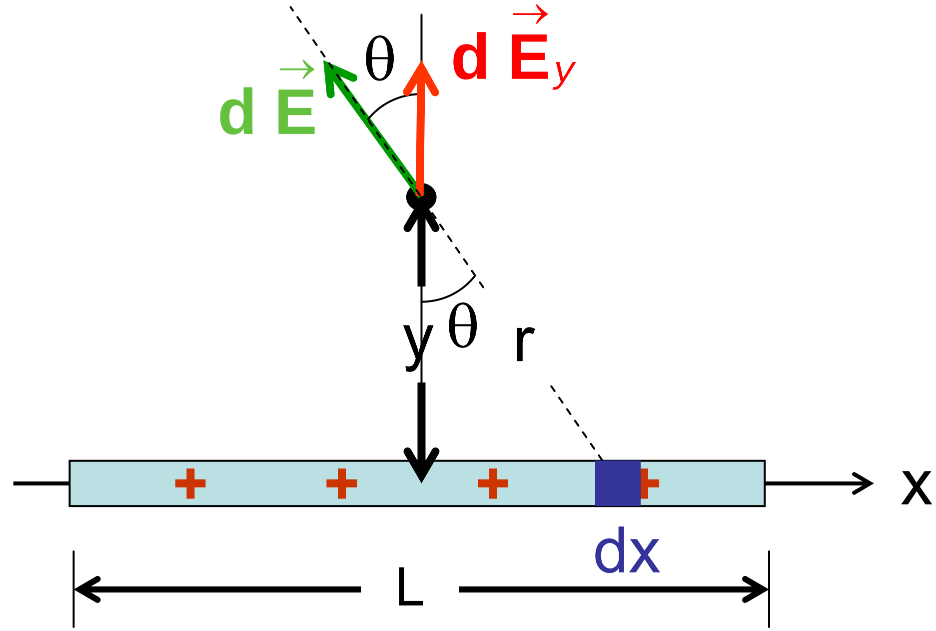
$$dq = \lambda\, dx$$

$$\lambda = \frac{\text{Charge}}{\text{Length}} = \text{linear charge density}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{x^2+y^2}$$

$$dE_y = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{x^2+y^2}\cos\theta$$

## 23-6 The Electric Field Due to a Line of Charge




$$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{x^2+y^2}\cos\theta$$

$$\cos\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

$$dE_y = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{x^2+y^2}\frac{y}{\sqrt{x^2+y^2}}$$

$$E = \int dE_y = \int_{-L/2}^{L/2} \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, y\, dx}{(x^2+y^2)^{3/2}}$$

From Appendix E

$$E = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{y\, dx}{(x^2+y^2)^{3/2}} = \frac{\lambda y}{4\pi\varepsilon_0}\left[\frac{x}{y^2(x^2+y^2)^{1/2}}\right]_{x=-L/2}^{x=L/2}$$

## 23-6 The Electric Field Due to a Line of Charge

$$E=\frac{\lambda y}{4\pi\varepsilon_0}\left(\frac{L/2}{y^2((L/2)^2+y^2)^{1/2}}-\frac{-L/2}{y^2((-L/2)^2+y^2)^{1/2}}\right)$$

$$E=\frac{\lambda}{2\pi\varepsilon_0}\frac{L}{y(L^2+4y^2)^{1/2}}$$

$$E=\frac{1}{2\pi\varepsilon_0}\frac{q}{y(L^2+4y^2)^{1/2}}$$

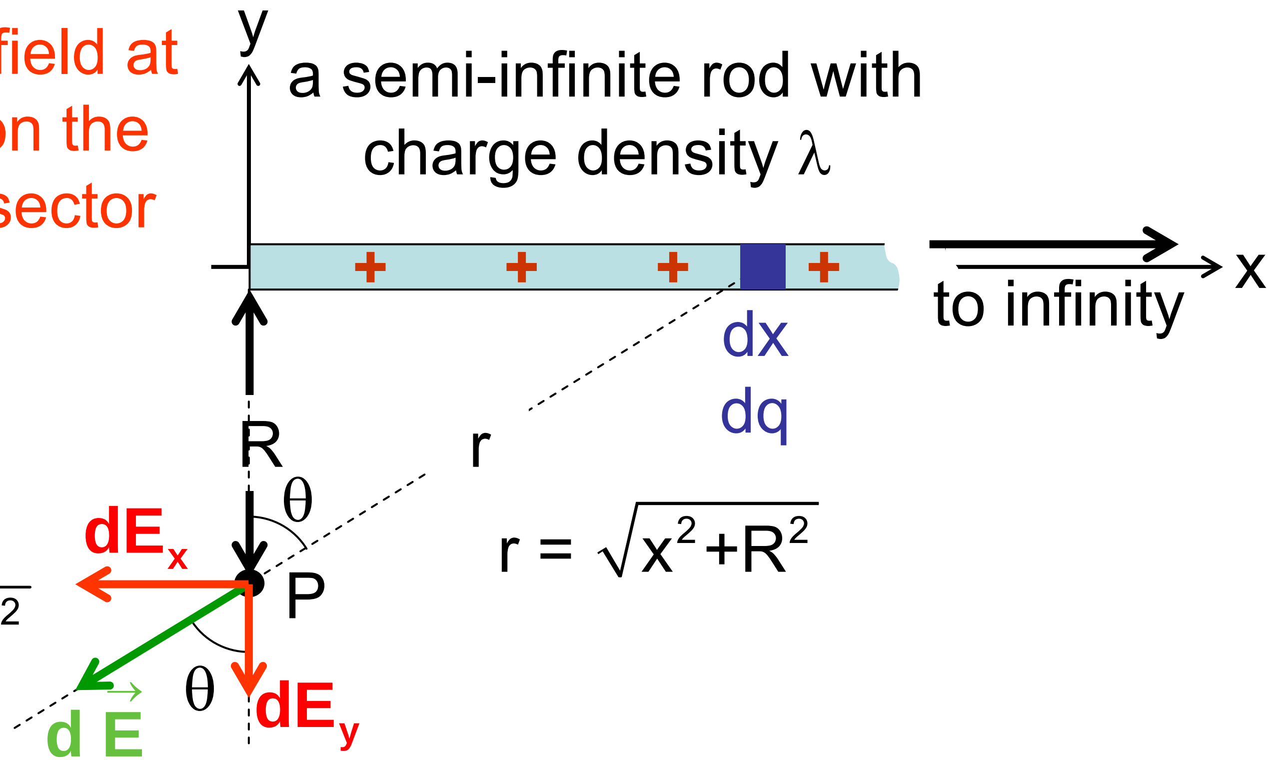
What is the magnitude of the electric field for y >> L ?

$$E=\frac{1}{2\pi\varepsilon_0}\frac{q}{y(L^2+4y^2)^{1/2}}\approx\frac{1}{2\pi\varepsilon_0}\frac{q}{y(4y^2)^{1/2}}=\frac{1}{4\pi\varepsilon_0}\frac{q}{y^2}$$

For y >> L, The electric field is similar to that form a point charge

## 23-6 The Electric Field Due to a Line of Charge

Find the electric field at point P located on the perpendicular bisector of the rod.




$$r = \sqrt{x^2+R^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2+R^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{x^2+R^2}$$

$$dE_x = dE \sin\theta = dE\,\frac{x}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{x^2+R^2}\frac{x}{\sqrt{x^2+R^2}}$$

$$dE_y = dE \cos\theta = dE\,\frac{R}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{x^2+R^2}\frac{R}{\sqrt{x^2+R^2}}$$

## 23-6 The Electric Field Due to a Line of Charge

$$E_x = \int dE_x = \int_0^\infty \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dx}{x^2+R^2} \frac{x}{\sqrt{x^2+R^2}} = \frac{\lambda}{4\pi\varepsilon_0} \int_0^\infty \frac{x\, dx}{(x^2+R^2)^{3/2}}$$

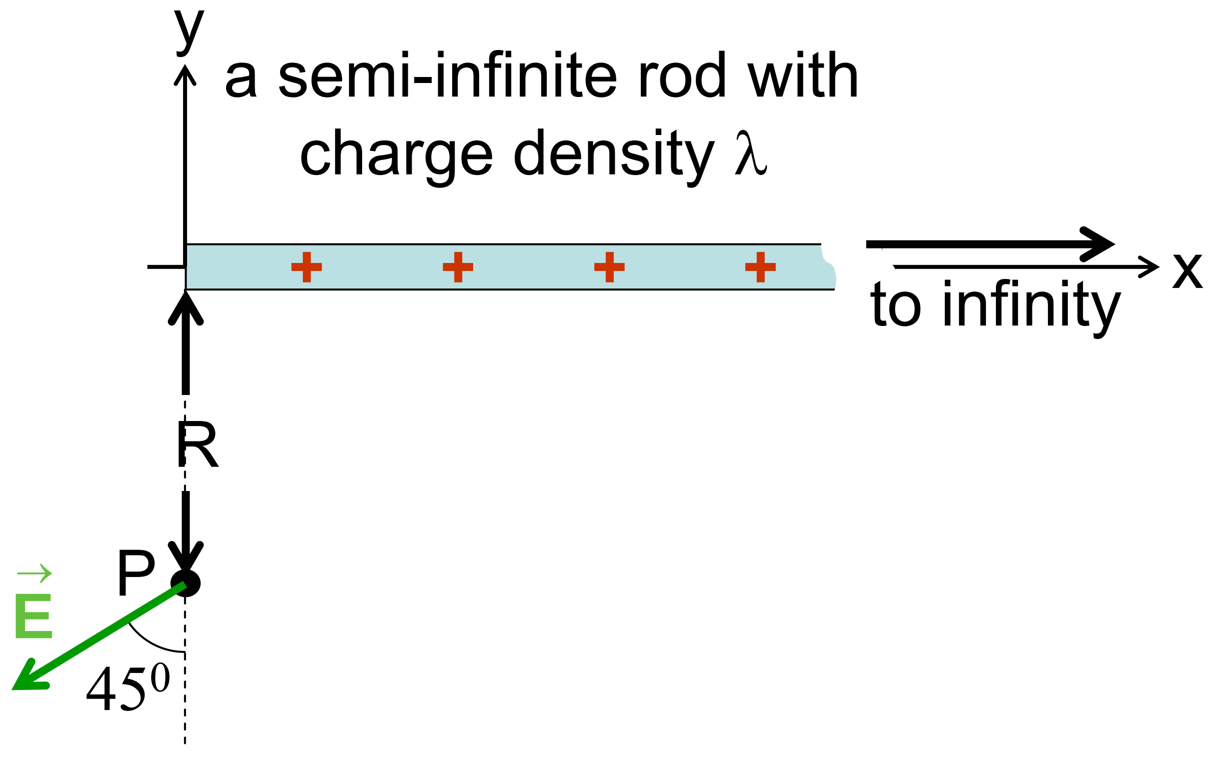
$$= \frac{\lambda}{4\pi\varepsilon_0} \left[ -\frac{1}{(x^2+R^2)^{1/2}} \right]_0^\infty = \frac{\lambda}{4\pi\varepsilon_0 R}$$

$$E_y = \int dE_y = \int_0^\infty \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dx}{x^2+R^2} \frac{R}{\sqrt{x^2+R^2}} = \frac{\lambda R}{4\pi\varepsilon_0} \int_0^\infty \frac{dx}{(x^2+R^2)^{3/2}}$$

$$= \frac{\lambda R}{4\pi\varepsilon_0} \left[ \frac{x}{R^2(x^2+R^2)^{1/2}} \right]_0^\infty = \frac{\lambda}{4\pi\varepsilon_0 R}$$

$$E_x = E_y = \frac{\lambda}{4\pi\varepsilon_0 R}$$

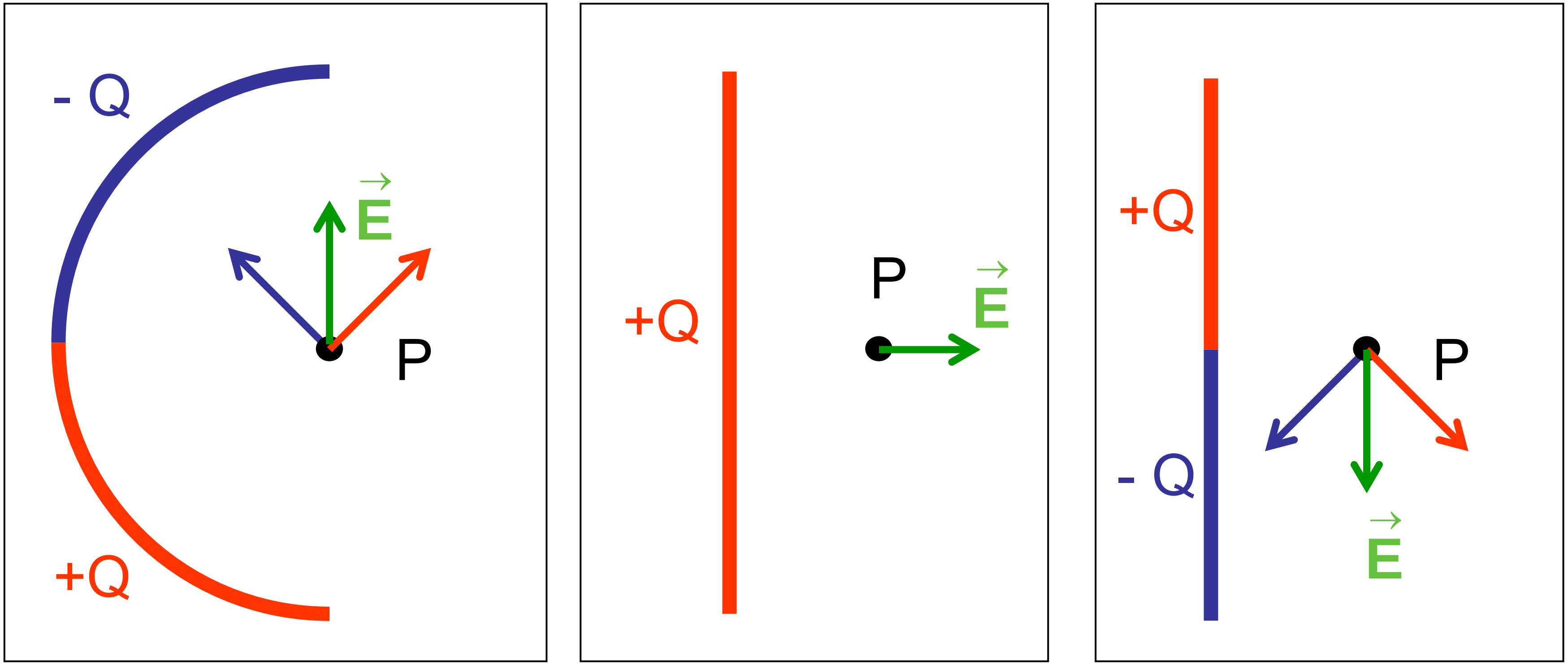
## 23-6 The Electric Field Due to a Line of Charge




$$\vec{E} = -\frac{\lambda}{4\pi\varepsilon_0 R}(\hat{i}+\hat{j})$$

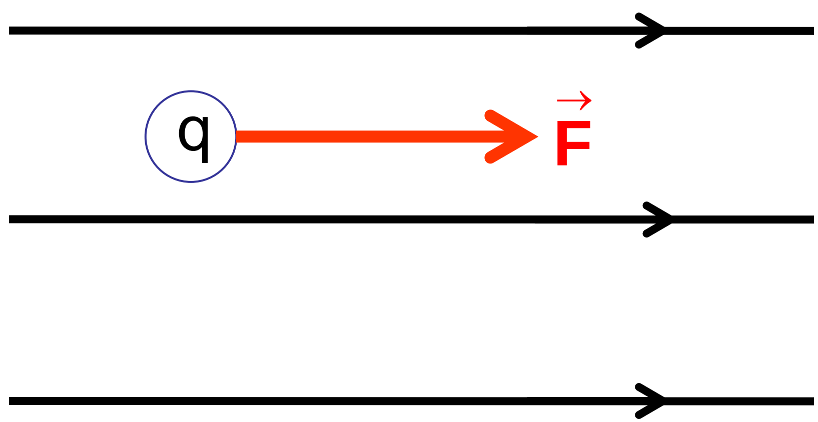
# 23-6 The Electric Field Due to a Line of Charge

## Checkpoint 3



## 23-8 A Point Charge in an Electric Field

What happens when a charged particle is placed in an external electric field?




External electric field (not produced by the point charge)

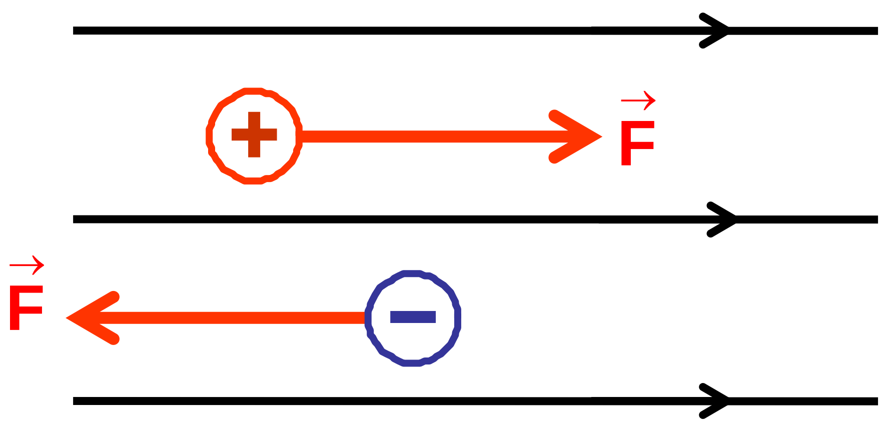
An electrostatic force acts on the particle

$$\vec{F} = q\,\vec{E}$$

External electric field

## 23-8 A Point Charge in an Electric Field

$$\vec{F} = q\,\vec{E}$$

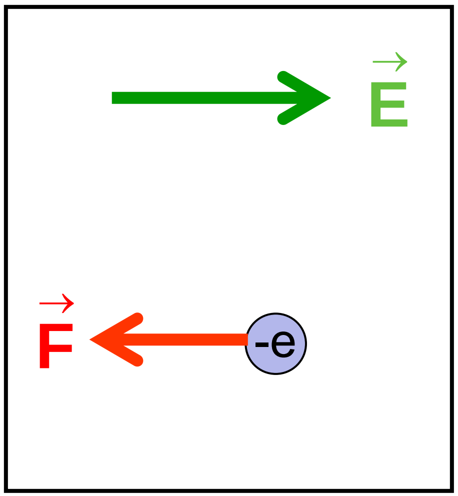



External electric field (not produced by the point charge)

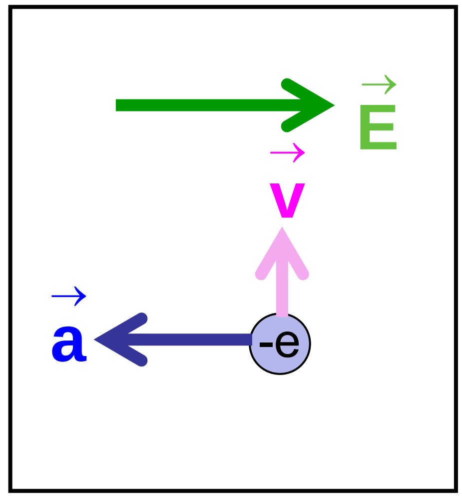
If the charge point is positive, the force has the same direction of electric field.
If the charge point is negative, the force has the opposite direction of the electric field.

## 23-8 A Point Charge in an Electric Field

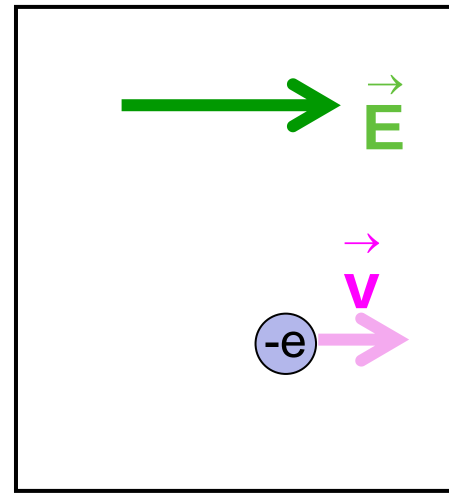
# Checkpoint 4




Direction of the force?.



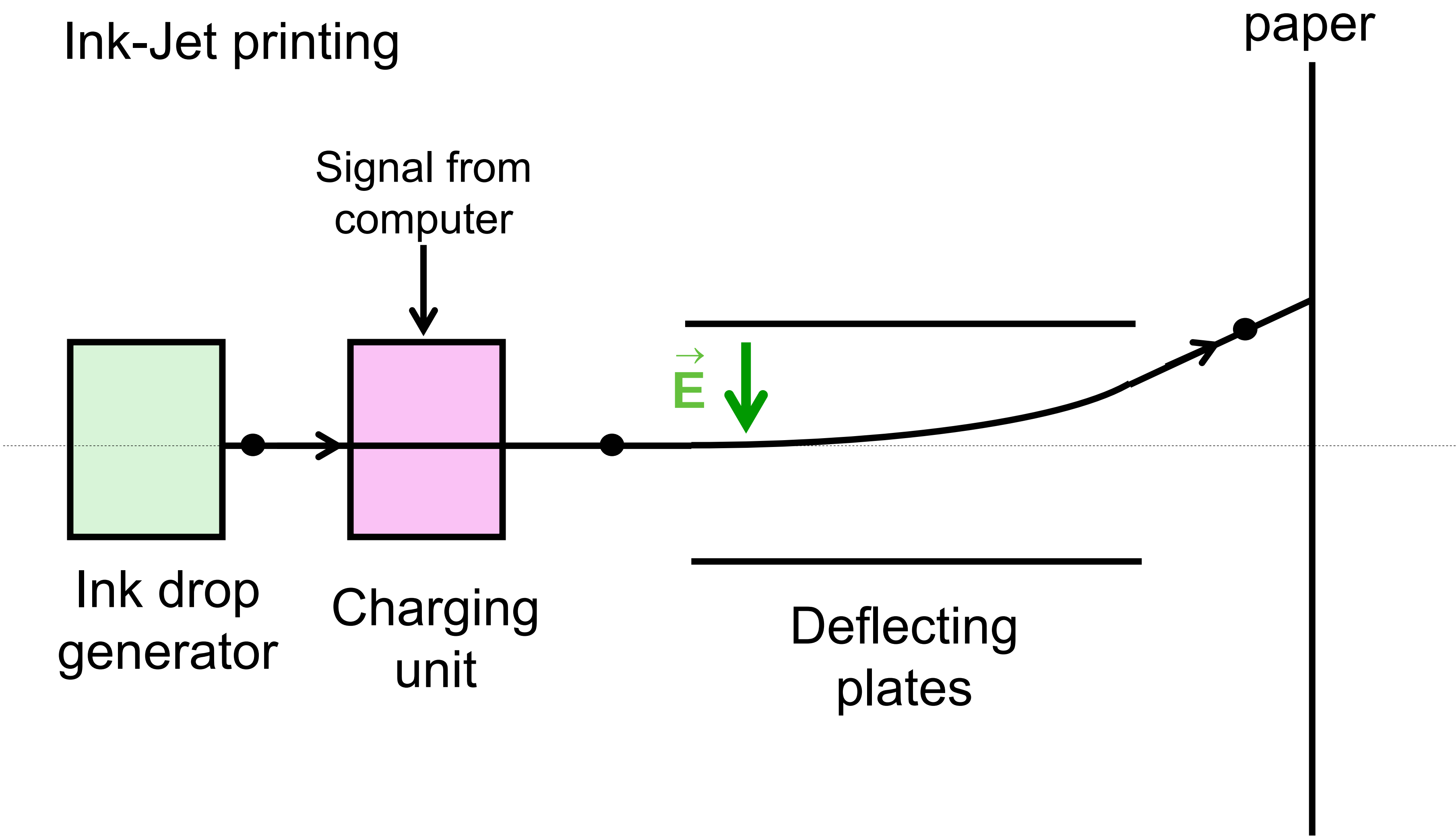

Direction of the acceleration?.




How does velocity change?.

decreases

## 23-8 A Point Charge in an Electric Field



## 23-8 A Point Charge in an Electric Field

# Sample Problem 23-4

Ink drop

Mass $m = 1.3 \times 10^{-10}$ kg

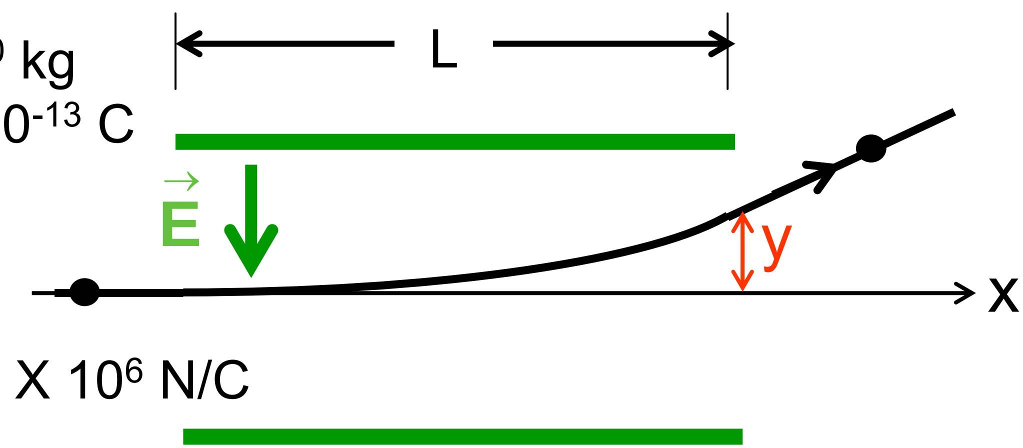
Charge $Q = -1.5 \times 10^{-13}$ C

initial $v_x = 18$ m/s

Deflecting plates

Length $L = 1.6$ cm

Electric Field $E = 1.4 \times 10^{6}$ N/C




Deflecting plates

What is the vertical deflection at the far edge of the plates ?

$$y = \frac{1}{2} a_y \, t^2$$

$$a_y = \frac{F}{m} = \frac{Q\,E}{m}$$

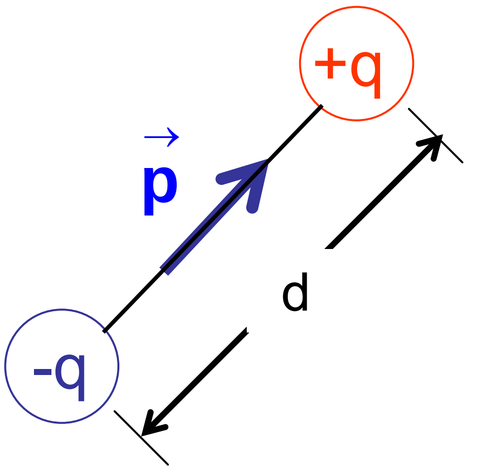
$$t = \frac{L}{v_x}$$

$$y = \frac{1}{2} \frac{Q\,E}{m} \left( \frac{L}{v_x} \right)^2 = 0.64 \text{ mm}$$

## 23-9 A Dipole in an Electric Field

Electric Dipole

Is two charges
which are equal in magnitude
but of opposite sign
and separated by distance d.




Dipole moment $\vec{p}$

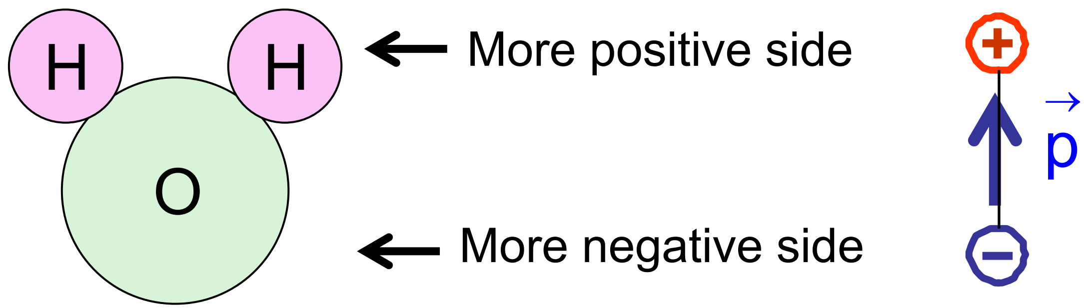
has a magnitude of $p = q\,d$
and
points towards the positive charge
along the line joining the two charges

## 23-9 A Dipole in an Electric Field

### Examples

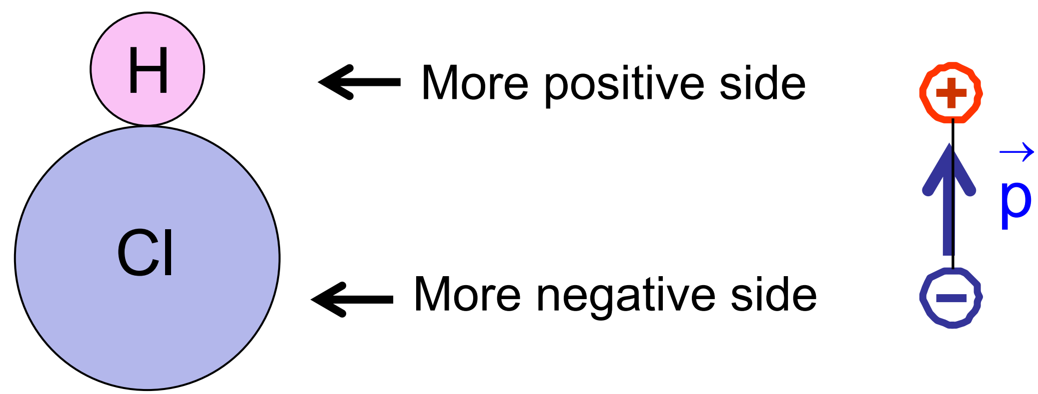
Water molecule, $H_2O$, is an electric dipole

The oxygen atom attracts the electrons of the the hydrogen atoms

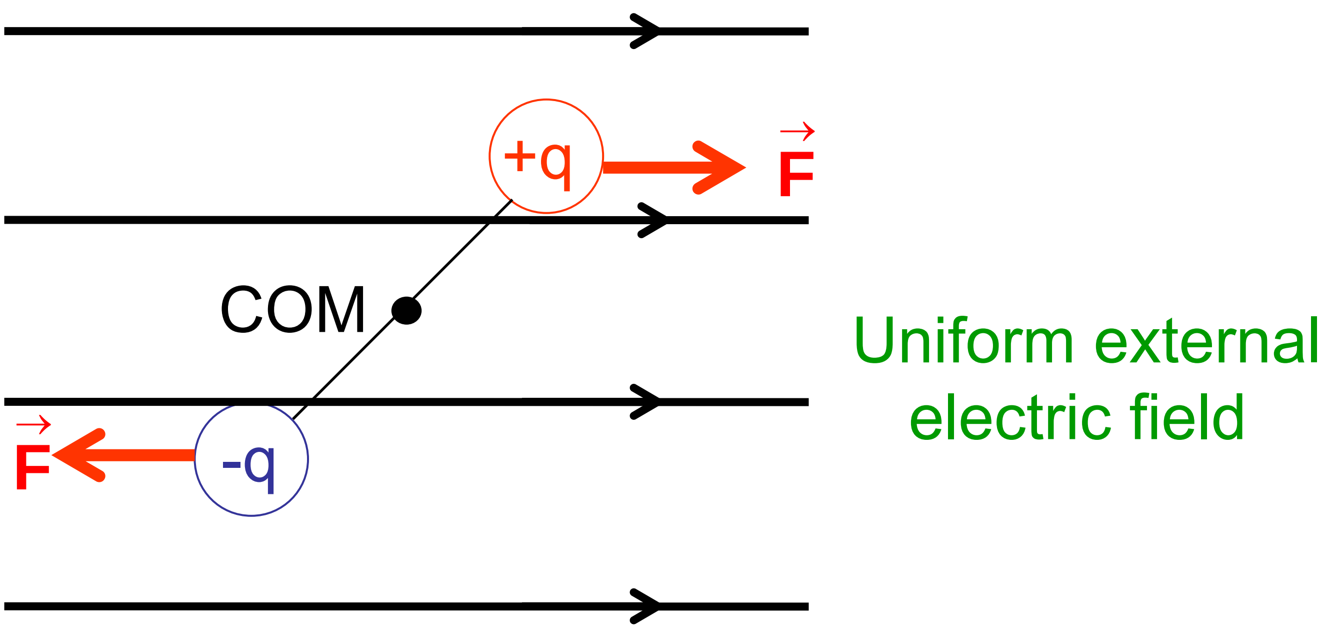



Hydrogen chloride molecule, HCl, is an electric dipole

The chlorine atom attracts the electrons of the the hydrogen atom



## 23-9 A Dipole in an Electric Field

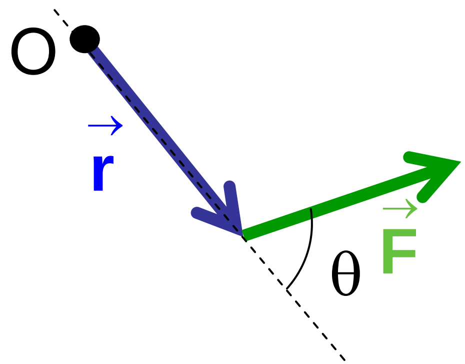


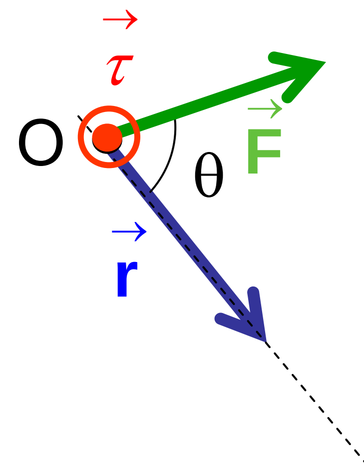

net force on the dipole = 0

The dipole will rotate about its center of mass (COM) because of the torque on the dipole

## 23-9 A Dipole in an Electric Field

### Torque: review

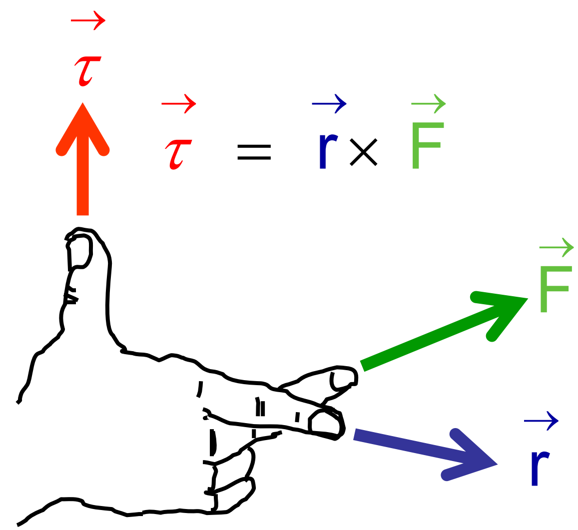



$$\vec{\tau} = \vec{r} \times \vec{F}$$

Magnitude: $\tau = r F \sin \theta$

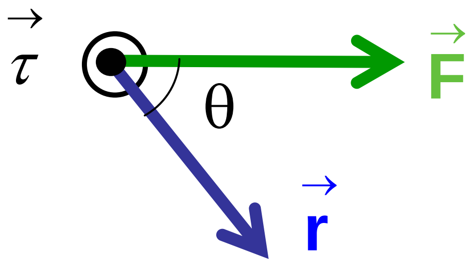
Direction: right-hand rule




$$\vec{\tau} = \vec{r} \times \vec{F}$$

right hand

## 23-9 A Dipole in an Electric Field

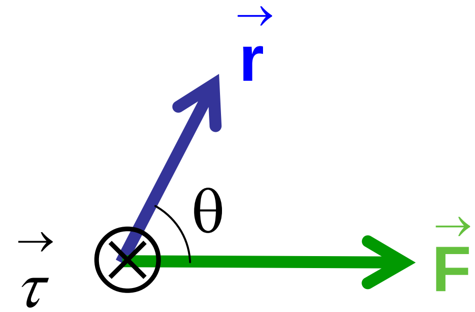



$\tau = r F \sin\theta$

torque points out of the page

Another way

$\tau = + r F \sin\theta$



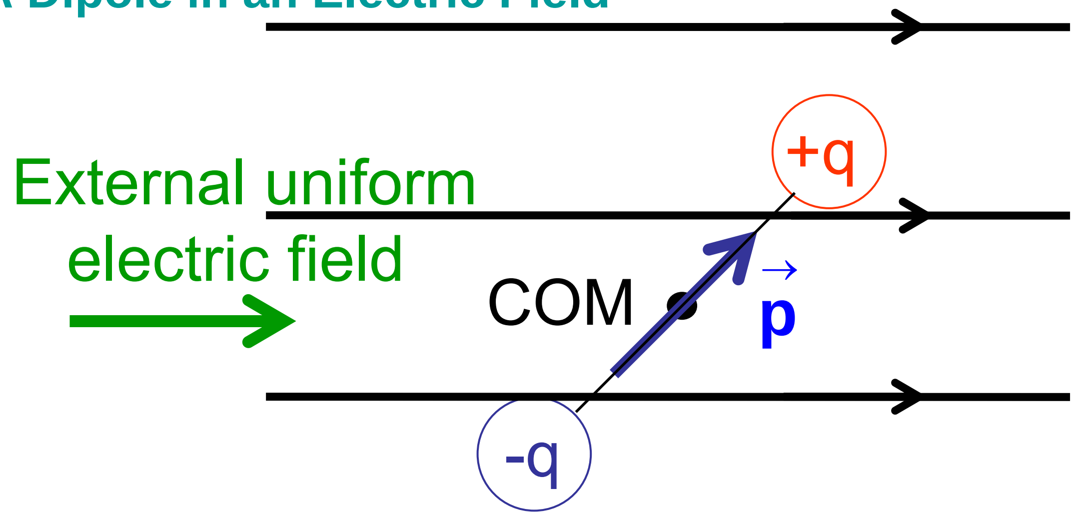

$\tau = r F \sin\theta$

torque points into the page

Another way

$\tau = - r F \sin\theta$

## 23-9 A Dipole in an Electric Field

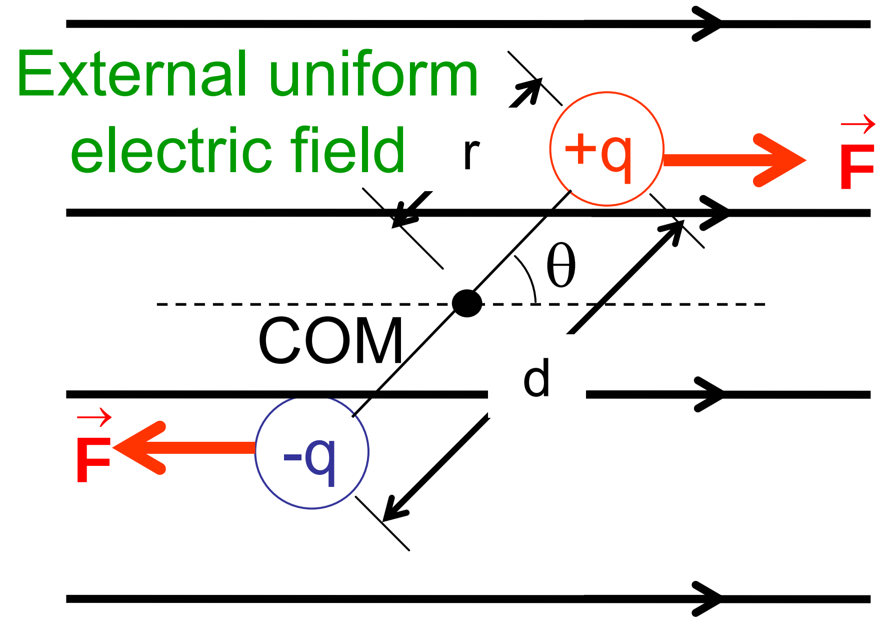



$$\vec{\tau} = \vec{p} \times \vec{E}$$

## 23-9 A Dipole in an Electric Field

Derivation of

$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$




Torque $\tau = - F\, r \sin\theta - F\,(d - r)\sin\theta$

$$\tau = - F\, d \sin\theta$$

$$\tau = - q\, E\, d \sin\theta$$

$$\tau = - q\, d\, E \sin\theta$$

$$\tau = - p\, E \sin\theta$$

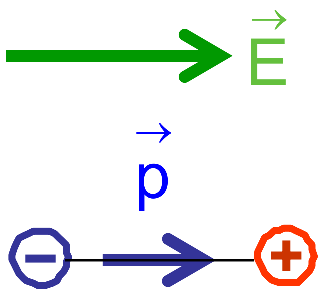
$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$$

## 23-9 A Dipole in an Electric Field

# Potential energy of an electric dipole

$$U = -\vec{p} \cdot \vec{E}$$

$$U = -p\,E\cos\theta$$



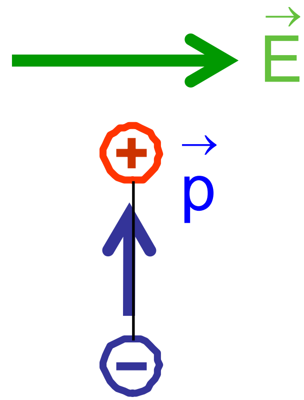

$\theta = 0^0$

$U = -p\,E$

minimum potential energy

$\tau = 0$

Stable equilibrium

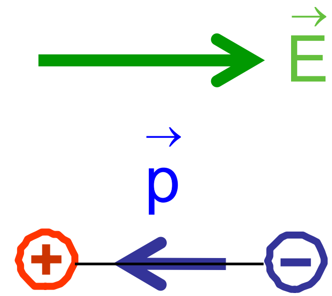



$\theta = 90^0$

$U = 0$

maximum torque

$\tau = -\ p\,E$




$\theta = 180^0$

$U = p\,E$

maximum potential energy

$\tau = 0$

unstable equilibrium

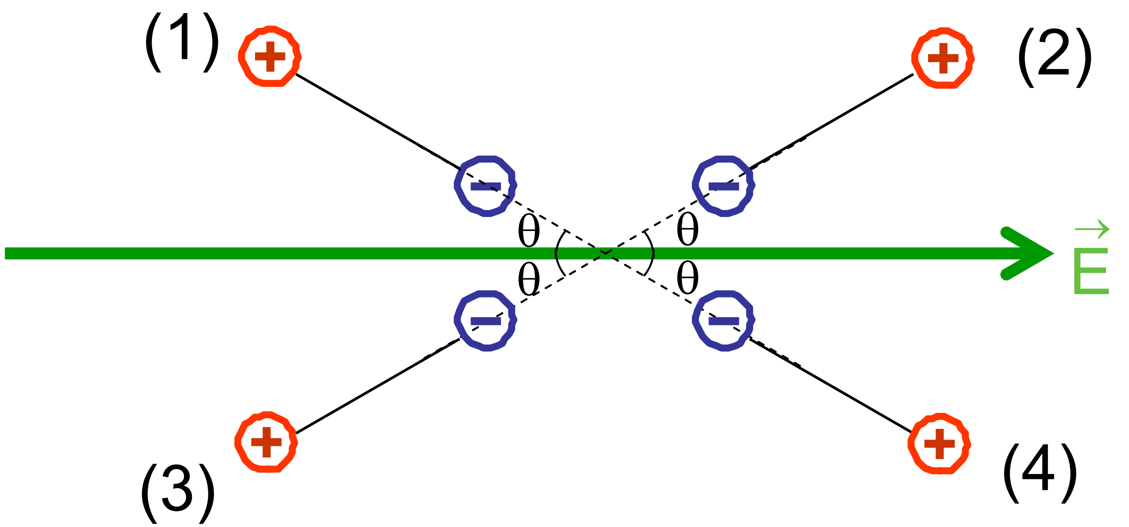
## 23-9 A Dipole in an Electric Field

# Potential energy of an electric dipole

| | |
|---|---|
| Work done by an electric field on a dipole when the dipole rotates from $\theta_i$ to $\theta_f$ | Work done by an applied force on a dipole when the dipole rotates from $\theta_i$ to $\theta_f$ |
| $W = -(U_f - U_i)$ | $W = (U_f - U_i)$ |

## 23-9 A Dipole in an Electric Field

### Checkpoint 5




Rank magnitude of the torque $\tau = | p E \sin \theta |$ All tie

Rank Potential energy $U = -p E \cos \theta$ 1 and 3 tie then 2 and 4 tie

## 23-9 A Dipole in an Electric Field

### Sample Problem 23-5

Water molecule, $H_2O$, has an electric dipole of $6.2 \times 10^{-30}$ C.m

How far apart the molecule's centers of positive and negative charges

$p = q\,d$

Oxygen atom has 8 protons and each hydrogen atom has one proton

$q = 10\,e$

$$d = \frac{p}{q} = \frac{6.2\times10^{-30}\,\text{C.m}}{10\times1.6\times10^{-19}\,\text{m}} = 3.9\times10^{-12}\,\text{m}$$

$H_2O$ molecule is placed in an external field of $1.5\text{X}10^{4}$ N/C. What is the maximum torque the field applied on the molecule

$\tau = |\,p\,E\sin\theta\,|$

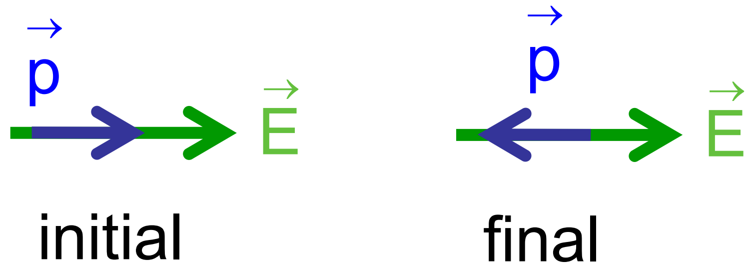
maximum $\tau = p\,E$

maximum $\tau = (6.2 \text{ X } 10^{-30}\text{ C.m})(1.5\text{X}10^{4}\text{ N/C}) = 9.3\text{X}10^{-26}$ N.m

## 23-9 A Dipole in an Electric Field

# Sample Problem 23-5

How much work must an external agent do to turn water molecule end for end in this field starting from its fully aligned position for which $\theta = 0^0$




$$W = (U_f - U_i)$$

$$W = U(\theta_f) - U(\theta_i)$$

$$W = -pE \cos 180^0 - (-pE \cos 0^0)$$

$$W = 2\ pE$$

$$W = 1.9 \times 10^{-25}\ J$$