

# Chapter 20

## The Kinetic Theory of Gases

## 20-1 A New Way to Look at Gases

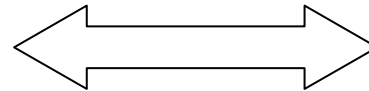
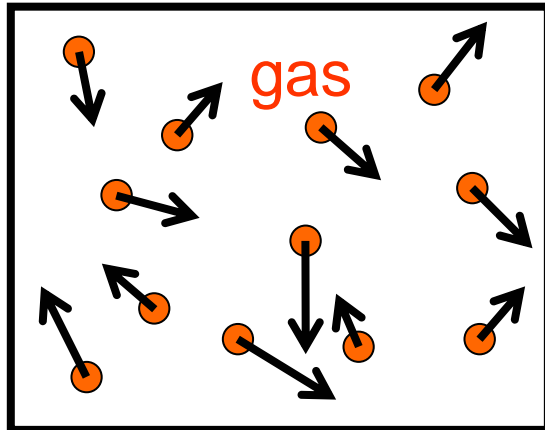
# Kinetic theory of gases

relates

the macroscopic property of gases  
(pressure – temperature – volume - internal energy)

to

the motion of atoms or molecules



pressure  
temperature  
volume  
internal energy

## 20-2 Avogadro's Number

### Avogadro's Number

The number of atoms in a 12 g sample of carbon 12 (1 mole)

From experiments

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

Number of moles  $\longrightarrow$   $n = \frac{N}{N_A}$

Number of molecules or atoms  $\longleftarrow$   $N$

Avogadro's number  $\longleftarrow$   $N_A$

mass of the sample  $\longleftarrow$   $M_{\text{Sam}}$

$n = \frac{M_{\text{Sam}}}{M} = \frac{M_{\text{Sam}}}{mN_A}$

Molar mass (mass of one mole)  $\longleftarrow$   $M$

Molecular mass (mass of one molecule)  $\longleftarrow$   $mN_A$

## 20-3 Ideal Gases

At low enough densities, all gases tend to obey ideal gas law

Gas Constant  
= 8.31 J/mole·K

Ideal gas Law

$$pV = nRT$$

Absolute pressure → Volume → Number of moles → Temperature In Kelvins

Although, there is no such thing in nature as a truly ideal gas, all real gases approach the ideal state at low enough densities

## 20-3 Ideal Gases

Number of  
molesGas Constant  
= 8.31 J/mole·K

Ideal gas Law

$$pV = nRT$$

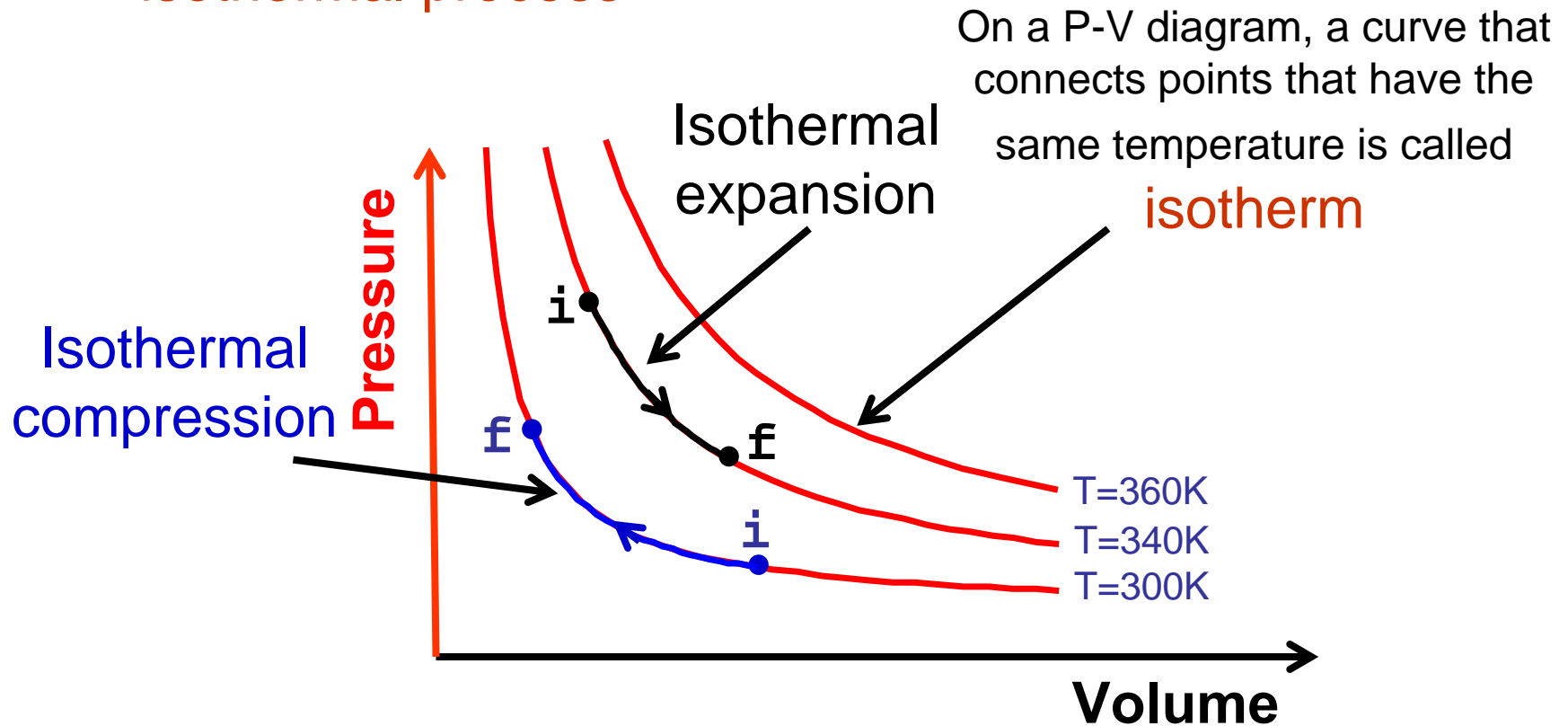
$$pV = NkT$$

Number of  
moleculesBoltzmann  
constant  
=  $1.38 \times 10^{-23}$  J/K

$$k = \frac{nR}{N} = \frac{R}{N_A} = \frac{8.31 \text{ J/mole}\cdot\text{K}}{6.02 \times 10^{23} \text{ mol}^{-1}} = 1.38 \times 10^{-23} \text{ J/K}$$


## 20-3 Ideal Gases

A process in which the temperature is kept constant is called **isothermal process**



## 20-3 Ideal Gases

Work done by an ideal gas at constant temperature


$$W = \int_{V_i}^{V_f} p dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV$$

$$W = nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

Ideal gas  
isothermal process

## 20-3 Ideal Gases

Work done by an ideal gas **at constant volume**

$$W = \int_{V_i}^{V_f} p dV = 0$$

Constant-volume process  
**isochoric process**

Work done by an ideal gas **at constant pressure**

$$W = \int_{V_i}^{V_f} p dV = p(V_f - V_i)$$

Constant-pressure process  
**isobaric process**

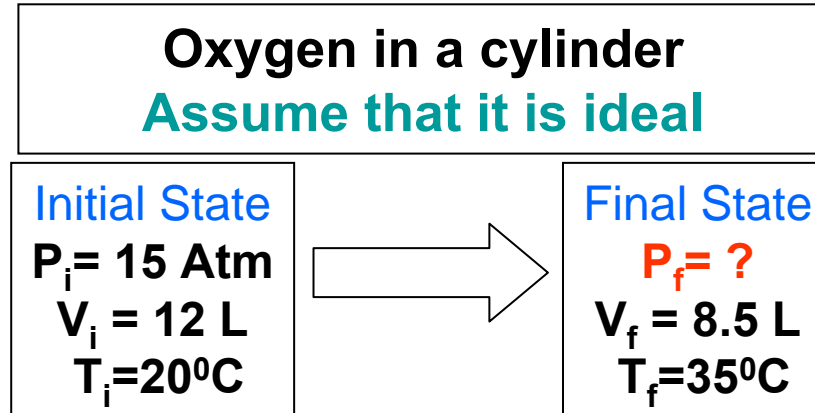


## 20-3 Ideal Gases

### Checkpoint 1

## 20-3 Ideal Gases

## Sample Problem 20-1



$$p_i V_i = nRT_i$$

$$p_f V_f = nRT_f$$

$$\frac{p_f V_f}{p_i V_i} = \frac{\cancel{nRT_f}}{\cancel{nRT_i}}$$

$$p_f = p_i \frac{V_i T_f}{V_f T_i}$$

$$p_f = (15 \text{ atm}) \frac{(12\text{L})((35+273)\text{K})}{(8.5\text{L})((20+273)\text{K})} = 22\text{atm}$$

Need to convert to

**Kelvins**



## 20-3 Ideal Gases

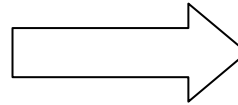
### Sample Problem 20-2

1 mole of oxygen  
expands at a constant  
temperature  
Assume that it is ideal

Initial State

$$V_i = 12 \text{ L}$$

$$T_i = 310 \text{ K}$$



Final State

$$V_f = 19 \text{ L}$$

$$T_f = 310 \text{ K}$$

Work done by the gas?

$$W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

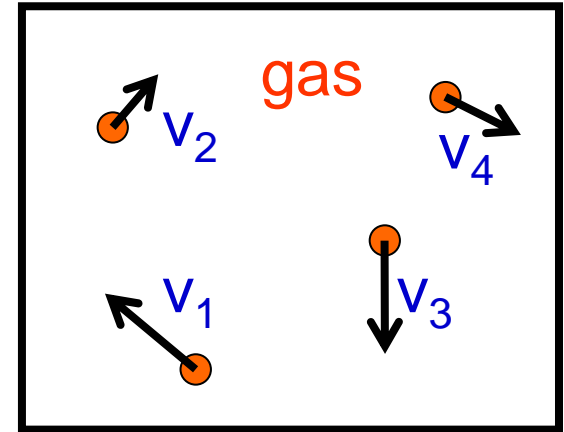
$$W = (1 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(310 \text{ K}) \ln\left(\frac{19 \text{ L}}{12 \text{ L}}\right) = 1190 \text{ J}$$

## 20-4 Pressure, Temperature, and RMS speed

### Root Mean Square (RMS) speed

For 4 atoms

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2}{4}}$$



For N atoms

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_N^2}{N}}$$

## 20-4 Pressure, Temperature, and RMS speed

Root Mean Square  
(RMS) speed

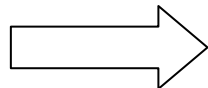
Gas constant =  
8.31 J/mol·K

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Temperature  
In Kelvins

Molar mass  
(mass of one mole)

Using ideal gas law



$$v_{\text{rms}} = \sqrt{\frac{3pV}{nM}}$$

## 20-4 Pressure, Temperature, and RMS speed

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

At room Temperature (300K)		
Gas	Molar Mass (g/mol)	$v_{\text{rms}}$ (m/s)
Hydrogen	2	1920
Nitrogen	28	517
Oxygen	32	483

## 20-4 Pressure, Temperature, and RMS speed

Derivation of

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

## 20-4 Pressure, Temperature, and RMS speed

### Sample Problem 20-3

You have five numbers  
5, 11, 32, 67, and 89

Find the average values of these numbers

$$\text{average value} = \frac{5+11+32+67+89}{5} = 50.8$$

Find the rms values of these numbers

$$\text{rms value} = \sqrt{\frac{5^2+11^2+32^2+67^2+89^2}{5}} = 52.1$$



## 20-5 Translational Kinetic Energy

The average translational kinetic energy  
per molecule of an ideal gas

$$K_{\text{avg}} = \frac{3}{2} kT$$

Boltzmann constant  
 $= 1.38 \times 10^{-23} \text{ J/K}$

Temperature  
in Kelvins

At a given temperature,  
all ideal gas molecules  
– no matter what their masses –  
have the same average translational kinetic energy.

## 20-5 Translational Kinetic Energy

Checkpoint 2

## 20-5 Translational Kinetic Energy

Derivation of

$$K_{\text{avg}} = \frac{3}{2} kT$$

$$K_{\text{avg}} \equiv \frac{1}{2} m \frac{v_1^2 + v_2^2 + \dots + v_N^2}{N}$$

$$K_{\text{avg}} = \frac{1}{2} m v_{\text{rms}}^2 \xrightarrow{v_{\text{rms}} = \sqrt{\frac{3RT}{M}}} K_{\text{avg}} = \frac{1}{2} m \left( \frac{3RT}{M} \right)$$

$$K_{\text{avg}} = \frac{3}{2} \left( \frac{m}{M} \right) RT \xrightarrow{\frac{m}{M} = \frac{1}{N_A}} K_{\text{avg}} = \frac{3}{2} \left( \frac{1}{N_A} \right) RT$$

$$\xrightarrow{\frac{R}{N_A} = k} K_{\text{avg}} = \frac{3}{2} kT$$

## 20-8 The Molar Specific Heat of an Ideal Gas

For a monatomic gas (which has individual atoms rather than molecules), the internal energy  $E_{\text{int}}$  is the sum of the translational kinetic energies of the atoms

$$E_{\text{int}} = N K_{\text{avg}}$$

$$E_{\text{int}} = N \left( \frac{3}{2} kT \right) = \frac{3}{2} N k T = \frac{3}{2} n R T$$

Internal energy  $E_{\text{int}}$   
of a monatomic gas

Gas constant =  
8.31 J/mole·K

$$E_{\text{int}} = \frac{3}{2} n R T$$

Temperature  
in Kelvins

Number of  
moles

## 20-8 The Molar Specific Heat of an Ideal Gas

$$E_{\text{int}} = \frac{3}{2} nRT$$

The internal energy  $E_{\text{int}}$  of an ideal gas is a function of **the gas temperature only**, it does not depend on any other variable.

## 20-8 The Molar Specific Heat of an Ideal Gas

Review from Ch. 19

Heat capacity

Heat

Change in temperature (K)

$$Q = C \Delta T$$

$$Q = C_{\text{molar}} n \Delta T$$

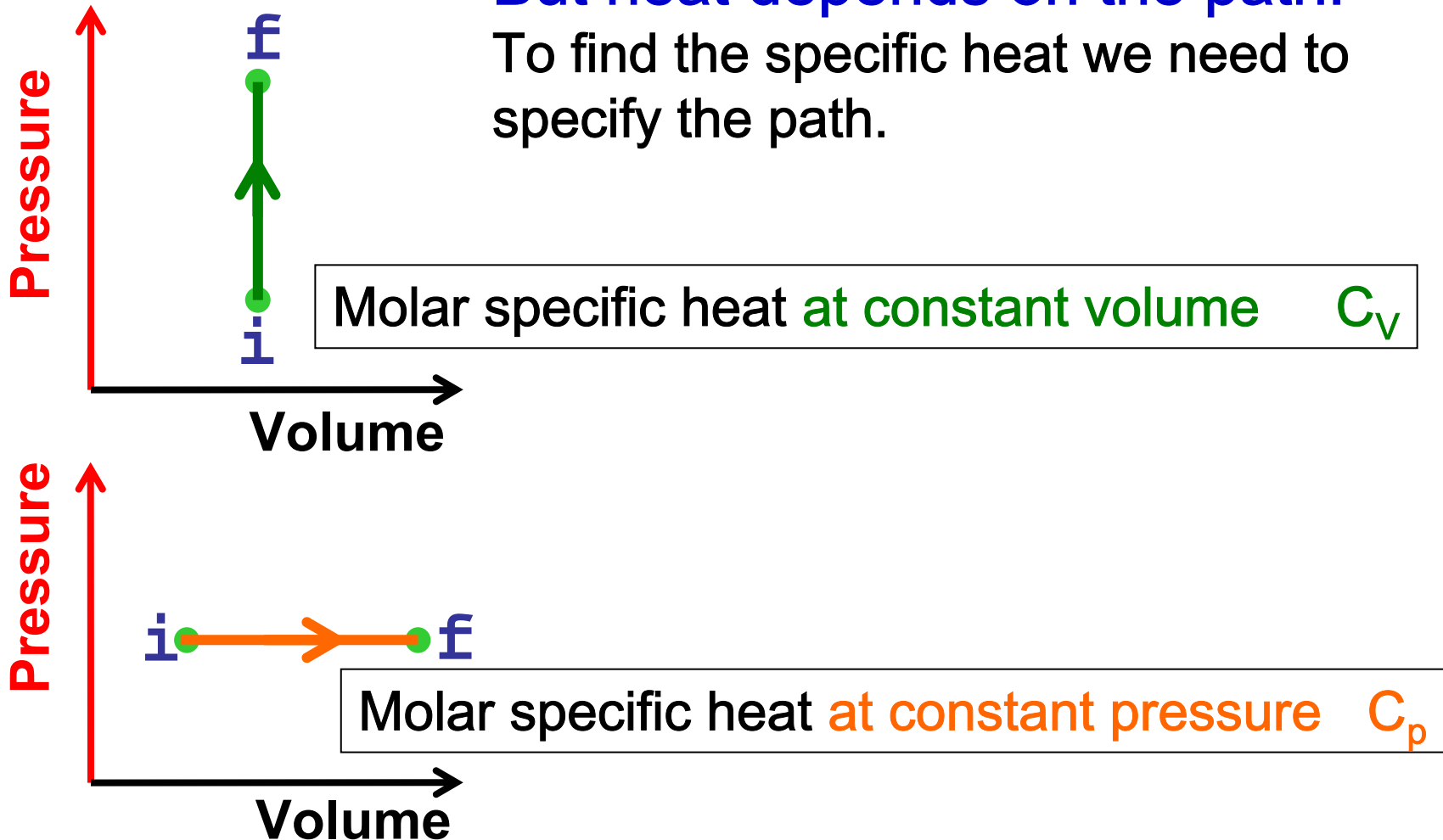
Heat capacity = (Molar specific heat) (Number of moles)

**Molar specific heat** is the heat needed to raise one mole of a substance by one unit temperature

## 20-8 The Molar Specific Heat of an Ideal Gas

**Molar specific heat** is the heat needed to raise one mole of a substance by one unit temperature

**But heat depends on the path!**  
To find the specific heat we need to specify the path.



## 20-8 The Molar Specific Heat of an Ideal Gas

For **solids** and **liquids**,  
the difference between  $C_V$  and  $C_p$  is **small**.

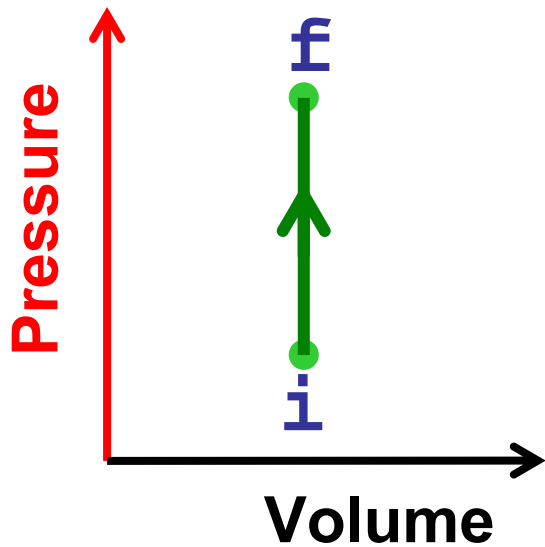
**Usually we do not need to distinguish  $C_V$  from  $C_p$ .**

**We just use one value for specific heat or molar specific heat**

For **gases**,  
the difference between  $C_V$  and  $C_p$  is **big**.  
**We need to distinguish  $C_V$  from  $C_p$ .**



## 20-8 The Molar Specific Heat of an Ideal Gas

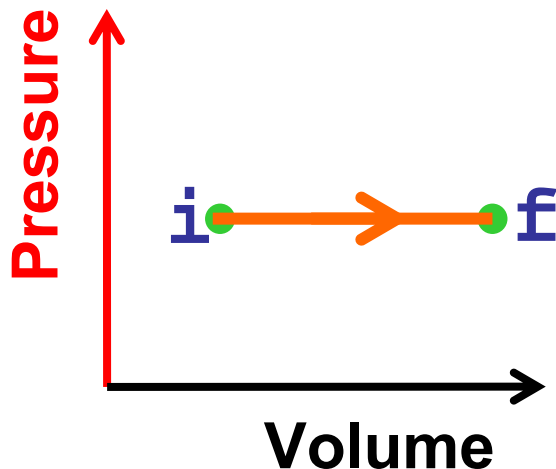


Molar specific heat at constant volume

$$C_V = \frac{3}{2} R$$

monatomic ideal gas  
(Gas consists of single atoms)

Gas constant =  
8.31 J/mol·K



Molar specific heat at constant pressure

$$C_P = \frac{5}{2} R$$

monatomic ideal gas

## 20-8 The Molar Specific Heat of an Ideal Gas

### Derivation of $C_V$

First law of thermodynamics

monatomic gas

$$\Delta E_{\text{int}} = \frac{3}{2} nR\Delta T$$

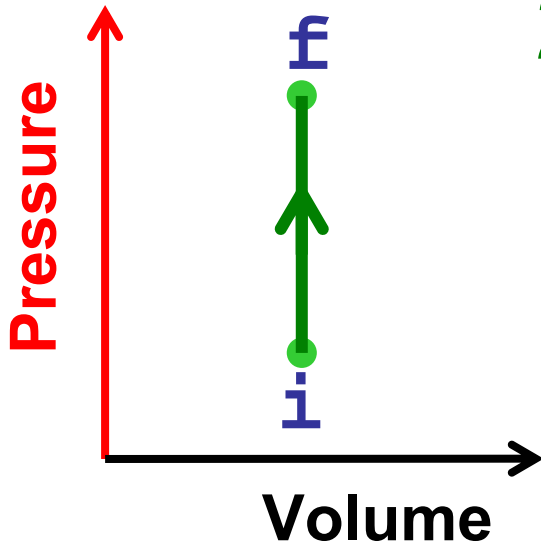
$$\Delta E_{\text{int}} = Q - W$$

$$W = 0$$

$$Q = nC_V\Delta T$$

~~$$\frac{3}{2} nR\Delta T = nC_V\Delta T$$~~

$$\frac{3}{2} R = C_V$$



## 20-8 The Molar Specific Heat of an Ideal Gas

### Derivation of $C_p$

First law of thermodynamics

monatomic gas

$$\Delta E_{\text{int}} = \frac{3}{2} nR\Delta T$$

$$\Delta E_{\text{int}} = Q - W$$

$$W = P\Delta V$$

$$PV = nRT$$

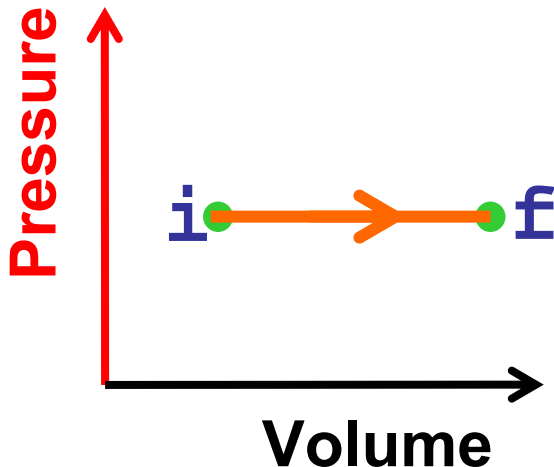
$$P\Delta V = nR\Delta T$$

$$Q = nC_p\Delta T$$

$$\cancel{\frac{3}{2} nR\Delta T} = \cancel{nC_p\Delta T} - \cancel{nR\Delta T}$$

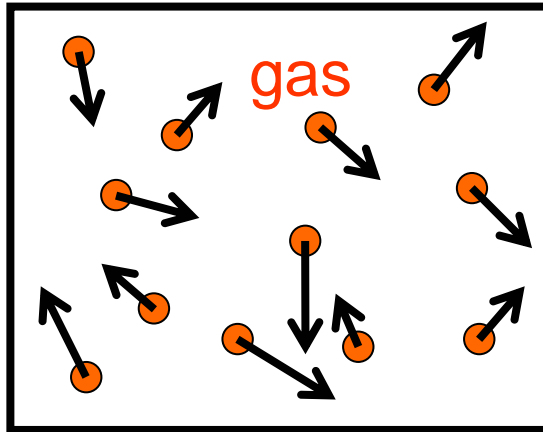
$$\frac{3}{2} R = C_p - R$$

$$\frac{5}{2} R = C_p$$

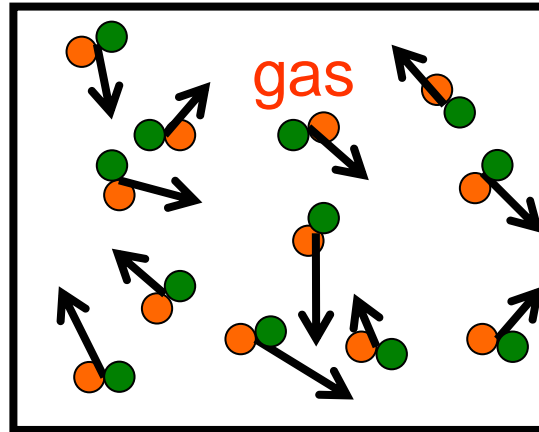


## 20-8 The Molar Specific Heat of an Ideal Gas

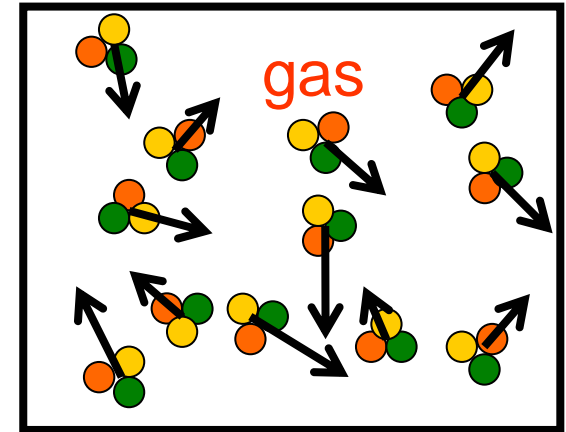
### Extra: Not for Phys102



**Monatomic gas**  
Gas consists of  
single atoms



**Diatomic gas**  
Each molecule  
has two atoms



**Polyatomic gas**  
Each molecule  
has more than  
two atoms

## 20-8 The Molar Specific Heat of an Ideal Gas

## Extra: Not for Phys102

	$C_v$	$E_{\text{int}}=nC_vT$	$C_p=C_v+R$	Degrees of freedom
Monatomic gas Gas consists of single atoms	$\frac{3}{2} R$	$\frac{3}{2} nRT$	$\frac{5}{2} R$	3
Diatomic gas Each molecule has two atoms	$\frac{5}{2} R$	$\frac{5}{2} nRT$	$\frac{7}{2} R$	5
Polyatomic gas Each molecule has more than two atoms	$\frac{6}{2} R$	$\frac{6}{2} nRT$	$\frac{8}{2} R$	6
$E_{\text{int}}=nC_vT$				

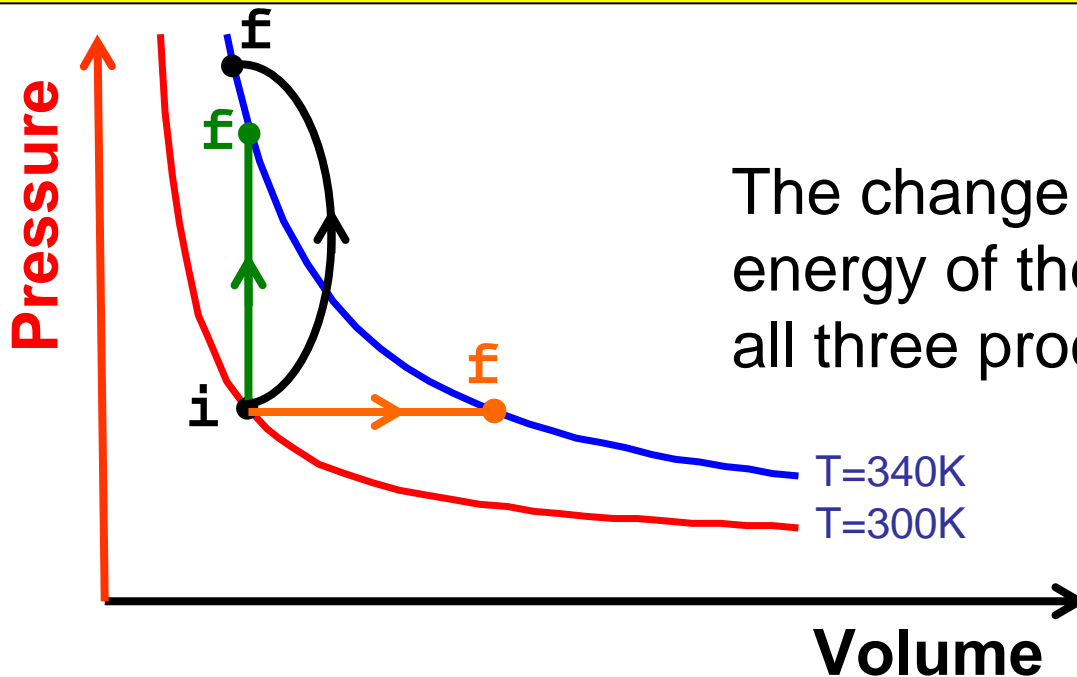
## 20-8 The Molar Specific Heat of an Ideal Gas

$$\Delta E_{\text{int}} = nC_V\Delta T$$

$n = \text{constant}$



A change in the internal energy  $E_{\text{int}}$  of a **confined** ideal gas depends on the change of the gas temperature only; it does not depend on what type of process produces the change in the temperature



The change in the internal energy of the gas is the same for all three processes

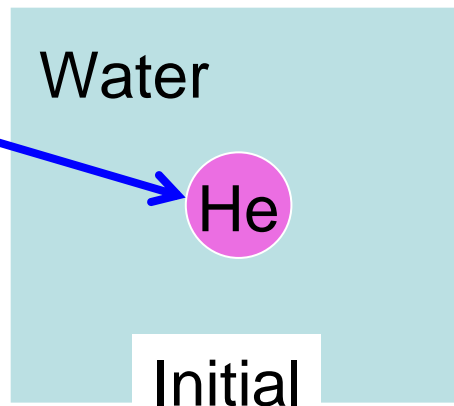
## 20-8 The Molar Specific Heat of an Ideal Gas

Checkpoint 4

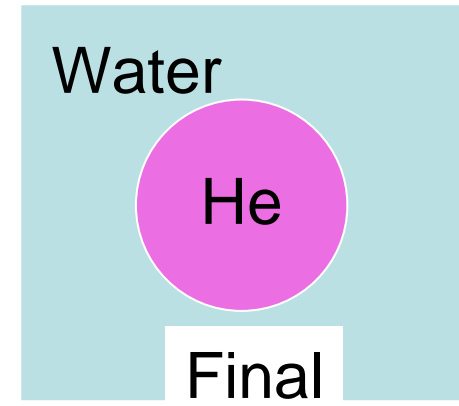
## 20-8 The Molar Specific Heat of an Ideal Gas

### Sample Problem 20-8

Helium  
bubble  
5 moles



Expansion  
Same pressure  
 $\Delta T = 20\text{ C}^\circ$



Heat added to the helium?

$$\begin{aligned}
 Q &= n C_p \Delta T \\
 &= n \left( \frac{5}{2} R \right) \Delta T \\
 &= (5 \text{ mol}) \left( \frac{5}{2} 8.13 \frac{\text{J}}{\text{mol K}} \right) (20\text{C}^\circ) \\
 &= 2077.5 \text{ J}
 \end{aligned}$$

Change in the internal energy  
of the helium?

$$\begin{aligned}
 \Delta E_{\text{int}} &= n C_v \Delta T \\
 &= n \left( \frac{3}{2} R \right) \Delta T \\
 &= (5 \text{ mol}) \left( \frac{3}{2} 8.13 \frac{\text{J}}{\text{mol K}} \right) (20\text{C}^\circ) \\
 &= 1246.5 \text{ J}
 \end{aligned}$$

Work done by the helium?

$$\Delta E_{\text{int}} = Q - W \quad \rightarrow \quad W = Q - \Delta E_{\text{int}} = 2077.5 \text{ J} - 1246.5 \text{ J} = 831 \text{ J}$$



## 20-11 Adiabatic Expansion

Review form Ch. 19

### Adiabatic processes

No heat transfer to the system

$$Q=0$$

Either

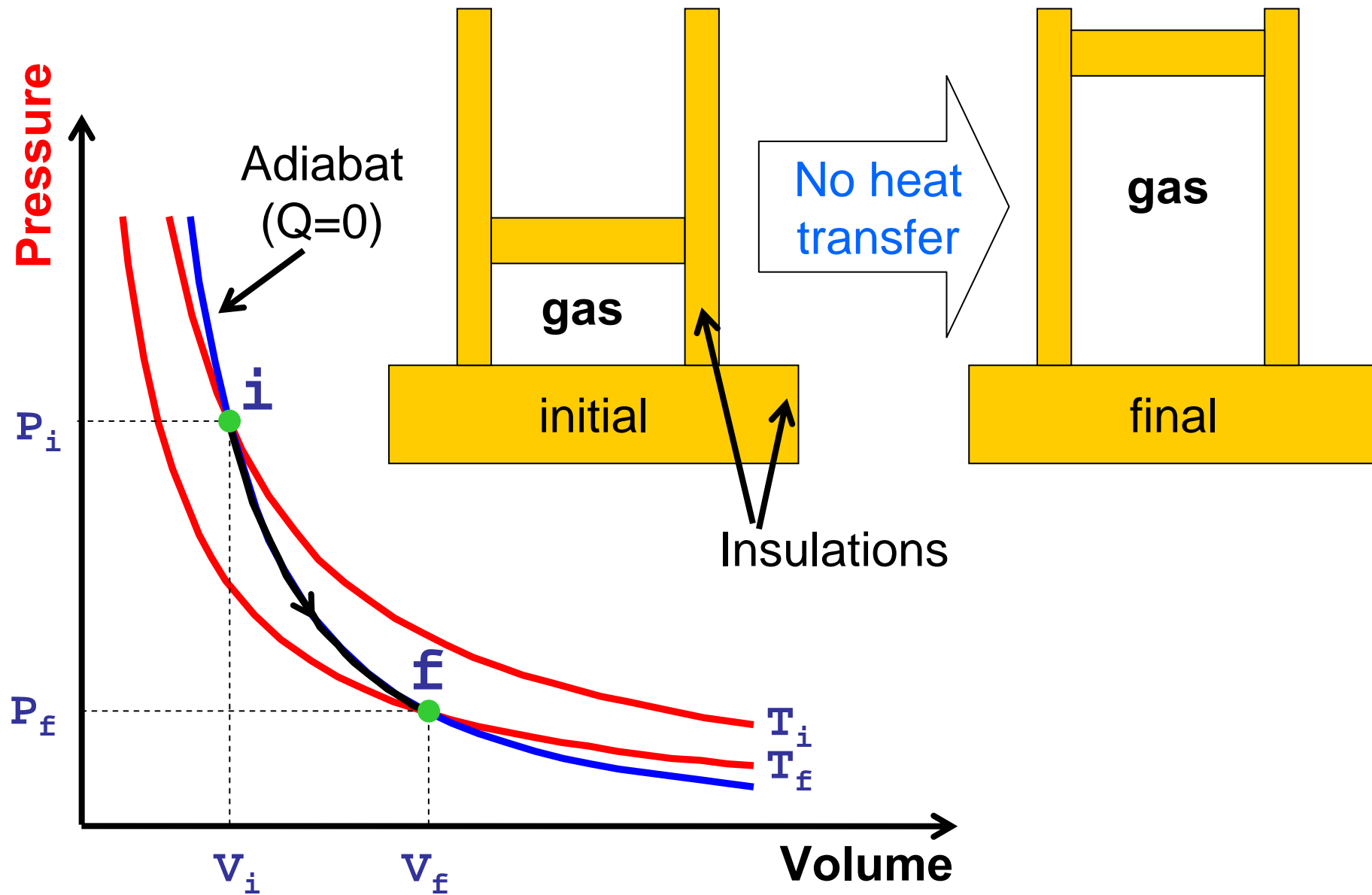
**System is well insulated**

Or

**Process occurs so rapidly that  
no heat transferred to the  
system (sound waves)**

## 20-11 Adiabatic Expansion

## Adiabatic Expansion

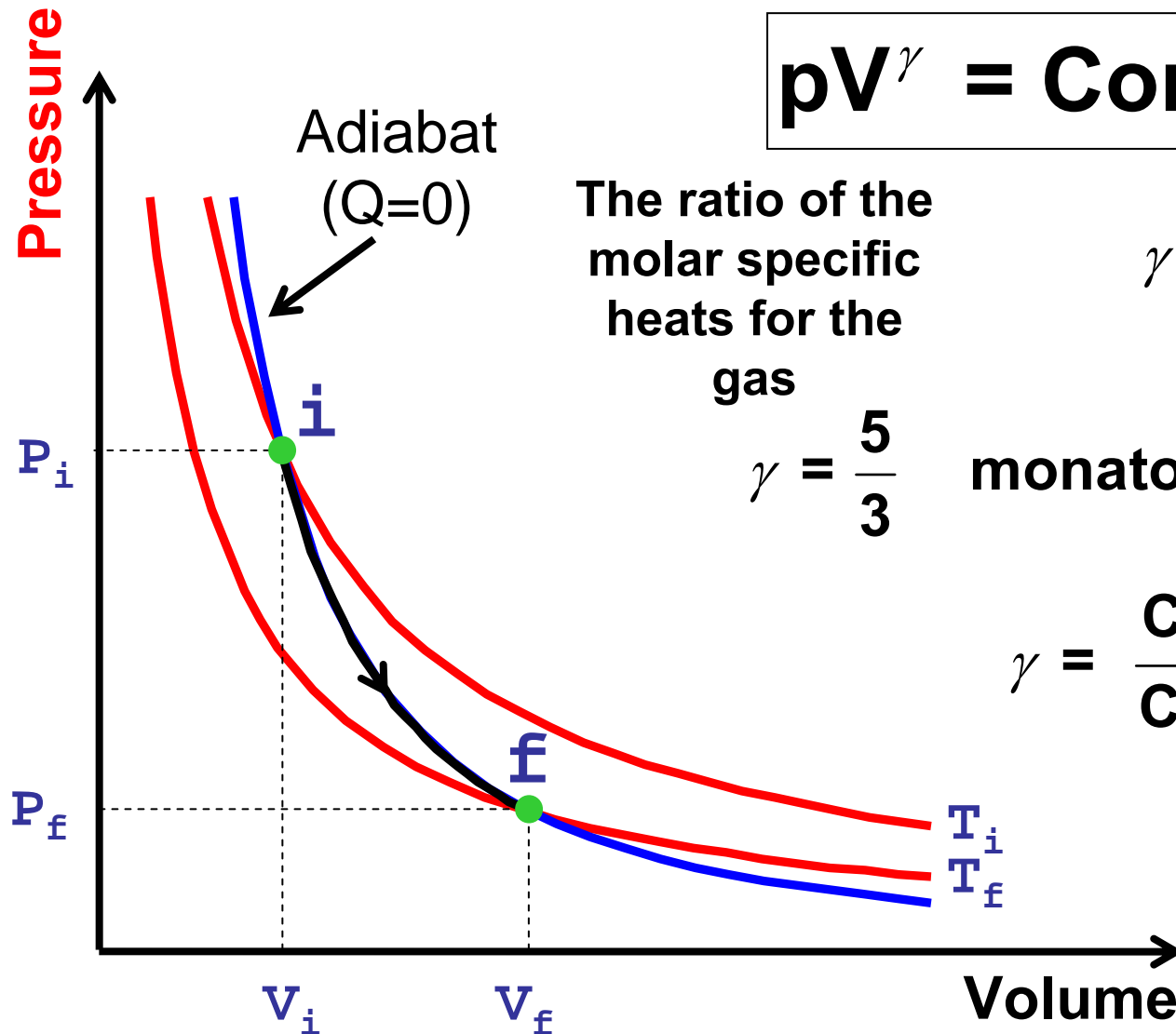


## 20-11 Adiabatic Expansion

## Adiabatic Expansion

$$p_i V_i^\gamma = p_f V_f^\gamma$$

$$pV^\gamma = \text{Constant}$$



$$\gamma = \frac{C_p}{C_v}$$

monatomic gas

$$\gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

## 20-11 Adiabatic Expansion

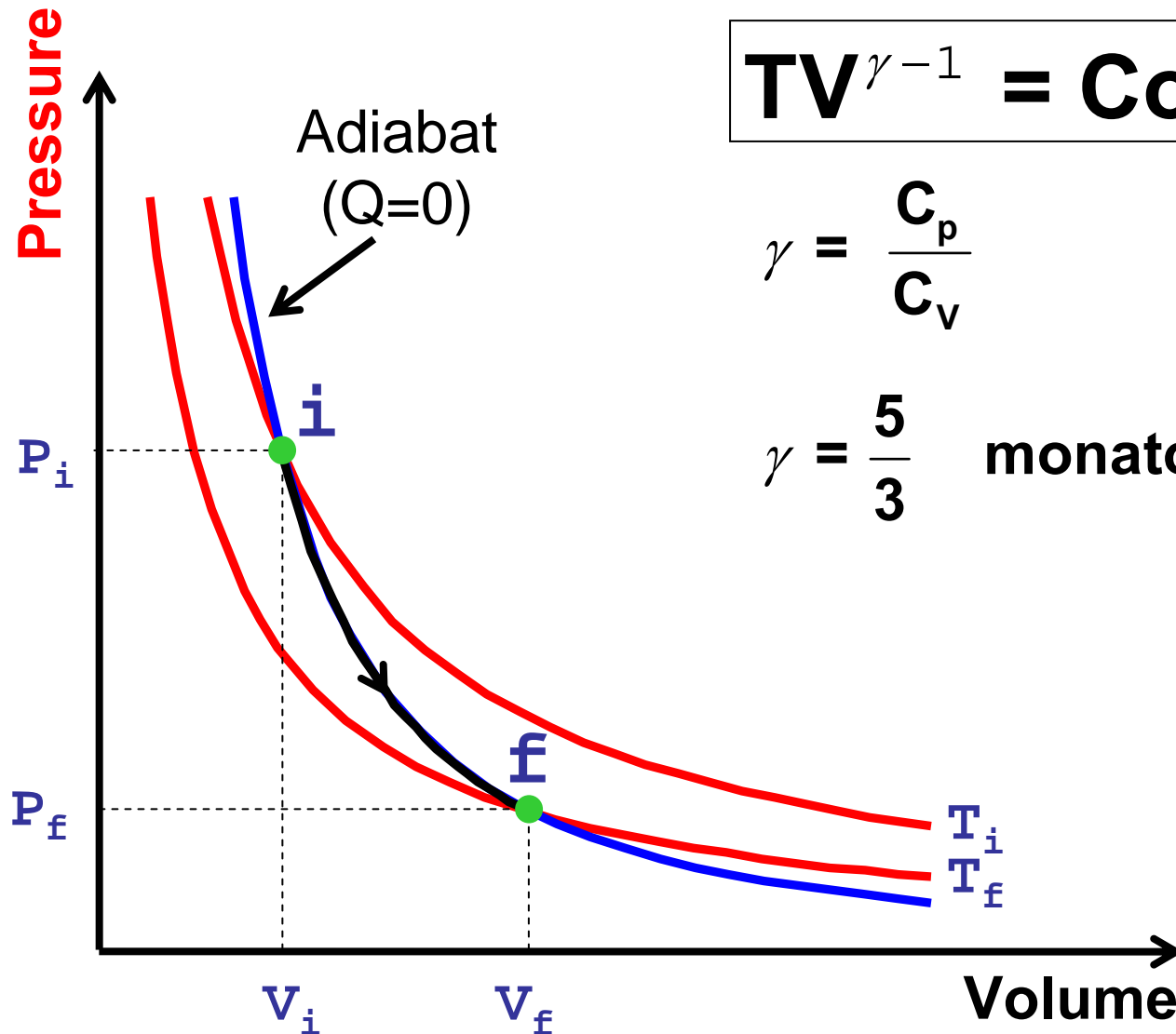
## Adiabatic Expansion

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$TV^{\gamma-1} = \text{Constant}$$

$$\gamma = \frac{C_p}{C_v}$$

$$\gamma = \frac{5}{3} \quad \text{monatomic gas}$$



## 20-11 Adiabatic Expansion

# Adiabatic Expansion

$$pV^\gamma = \text{Constant}$$

$$TV^{\gamma-1} = \text{Constant}$$

$$\gamma = \frac{C_p}{C_v} \quad \text{general}$$

$$\gamma = \frac{5}{3} \quad \text{monatomic gas}$$

## 20-11 Adiabatic Expansion

Adiabatic Expansion Derivation of  $PV^\gamma = \text{Constant}$ 

$$\Delta E_{\text{int}} = Q - W \xrightarrow{\text{Small change}} \begin{aligned} dE_{\text{int}} &= -dW \\ dE_{\text{int}} &= -pdV \end{aligned}$$

$$\Delta E_{\text{int}} = nC_v\Delta T \xrightarrow{\text{Small change}} dE_{\text{int}} = nC_v dT$$

$$dT = \frac{-pdV}{nC_v}$$

$$pV = nRT \xrightarrow{\text{Small change}} pdV + Vdp = nRdT$$

$$pdV + Vdp = nR \frac{-pdV}{nC_v}$$

$$pdV + Vdp = -\frac{R}{C_v} pdV \longrightarrow \left(1 + \frac{R}{C_v}\right) pdV + Vdp = 0 \longrightarrow$$

## 20-11 Adiabatic Expansion

$$\left(1 + \frac{R}{C_V}\right)pdV + Vdp = 0 \longrightarrow \left(\frac{C_V + R}{C_V}\right)pdV + Vdp = 0$$

$$\xrightarrow{C_V + R = C_P} \left(\frac{C_P}{C_V}\right)pdV + Vdp = 0 \xrightarrow{\frac{C_P}{C_V} = \gamma} \gamma pdV + Vdp = 0$$

$$\longrightarrow V^{\gamma-1}(\gamma pdV + Vdp = 0) \longrightarrow \gamma V^{\gamma-1}pdV + V^\gamma dp = 0$$

$$\longrightarrow d(pV^\gamma) = 0 \longrightarrow$$

$$\boxed{pV^\gamma = \text{Constant}}$$

## 20-11 Adiabatic Expansion

**Adiabatic Expansion** Derivation of  $TV^{\gamma-1} = \text{Constant}$ 

$$pV = nRT \longrightarrow p = \frac{nRT}{V}$$

$$pV^{\gamma} = \text{Constant} \longrightarrow \frac{nRT}{V} V^{\gamma} = \text{Constant}$$

$$nRTV^{\gamma-1} = \text{Constant}$$

$$TV^{\gamma-1} = \frac{\text{Constant}}{nR} = \text{Another Constant}$$

$$\boxed{TV^{\gamma-1} = \text{Constant}}$$

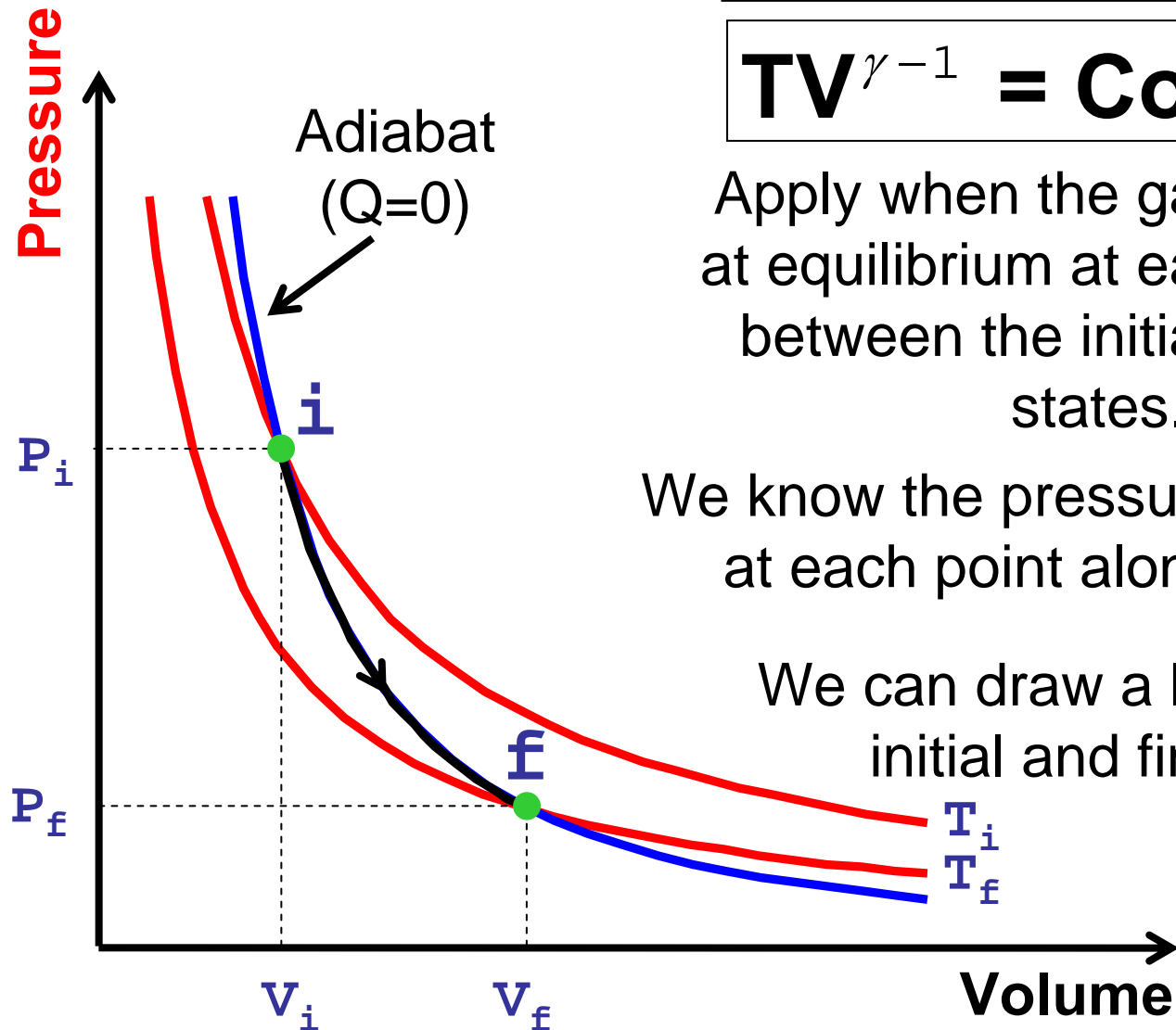


## 20-11 Adiabatic Expansion

## Adiabatic Expansion

$$pV^\gamma = \text{Constant}$$

$$TV^{\gamma-1} = \text{Constant}$$



Apply when the gas is almost at equilibrium at each moment between the initial and final states.

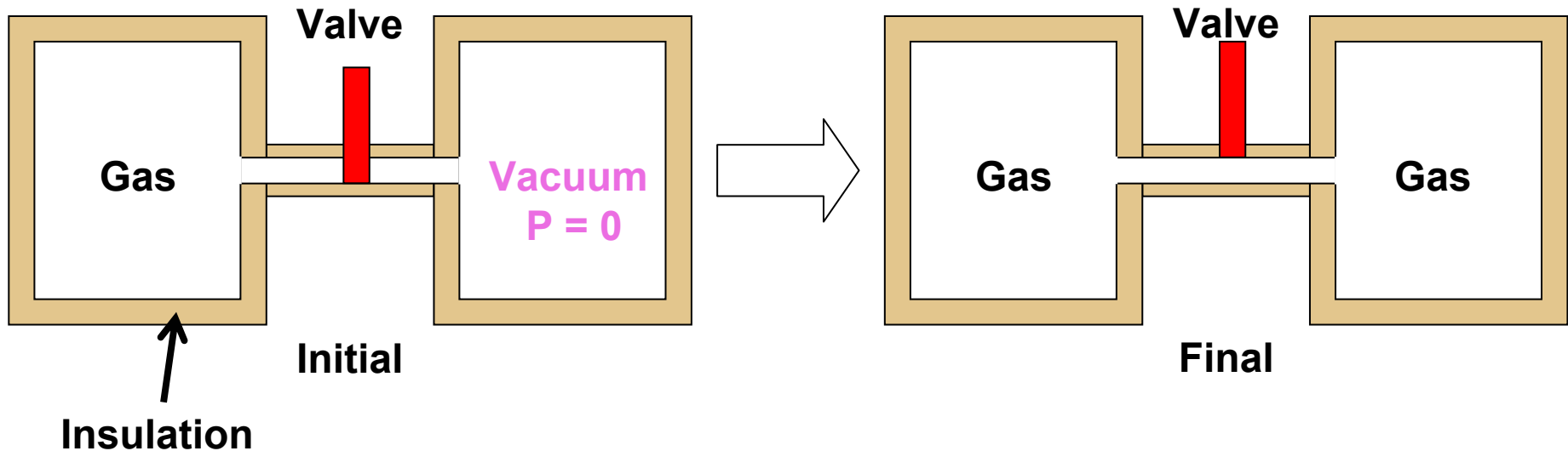
We know the pressure and volume at each point along the path.

We can draw a line between initial and final point

## 20-11 Adiabatic Expansion

### Review form Ch. 19

## Free expansions



System is insulated  $\rightarrow Q = 0$

$$W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} 0 dV = 0$$

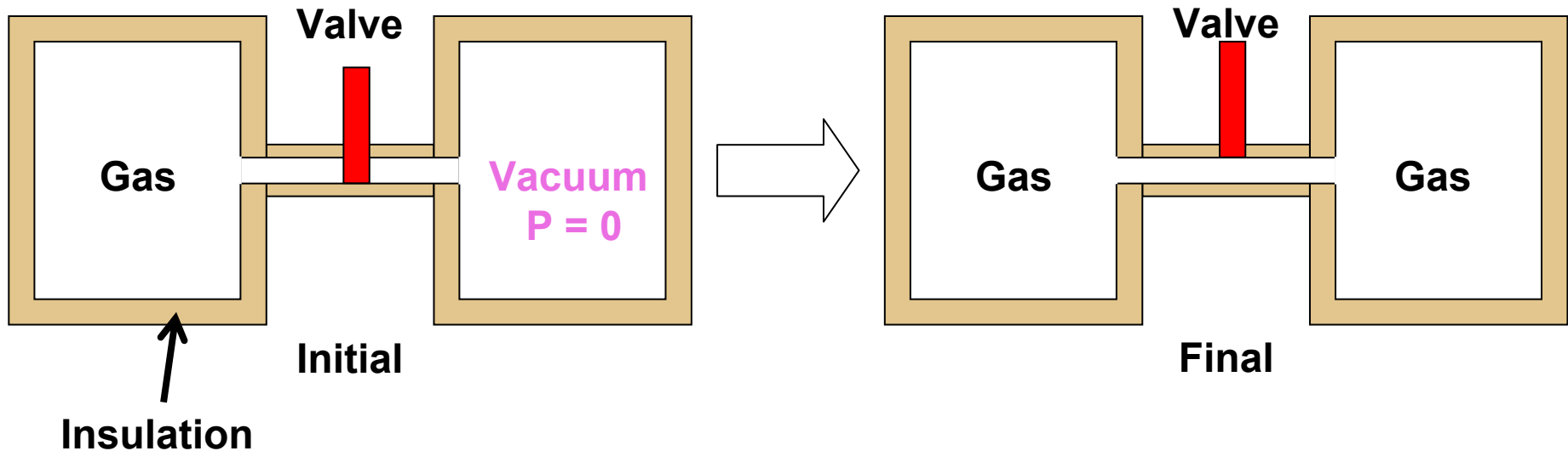
$$\Delta E_{\text{int}} = Q - W$$



$$\Delta E_{\text{int}} = 0$$

## 20-11 Adiabatic Expansion

### Free expansions



**We do not know the volume and pressure at points between the initial and final states**

## 20-11 Adiabatic Expansion

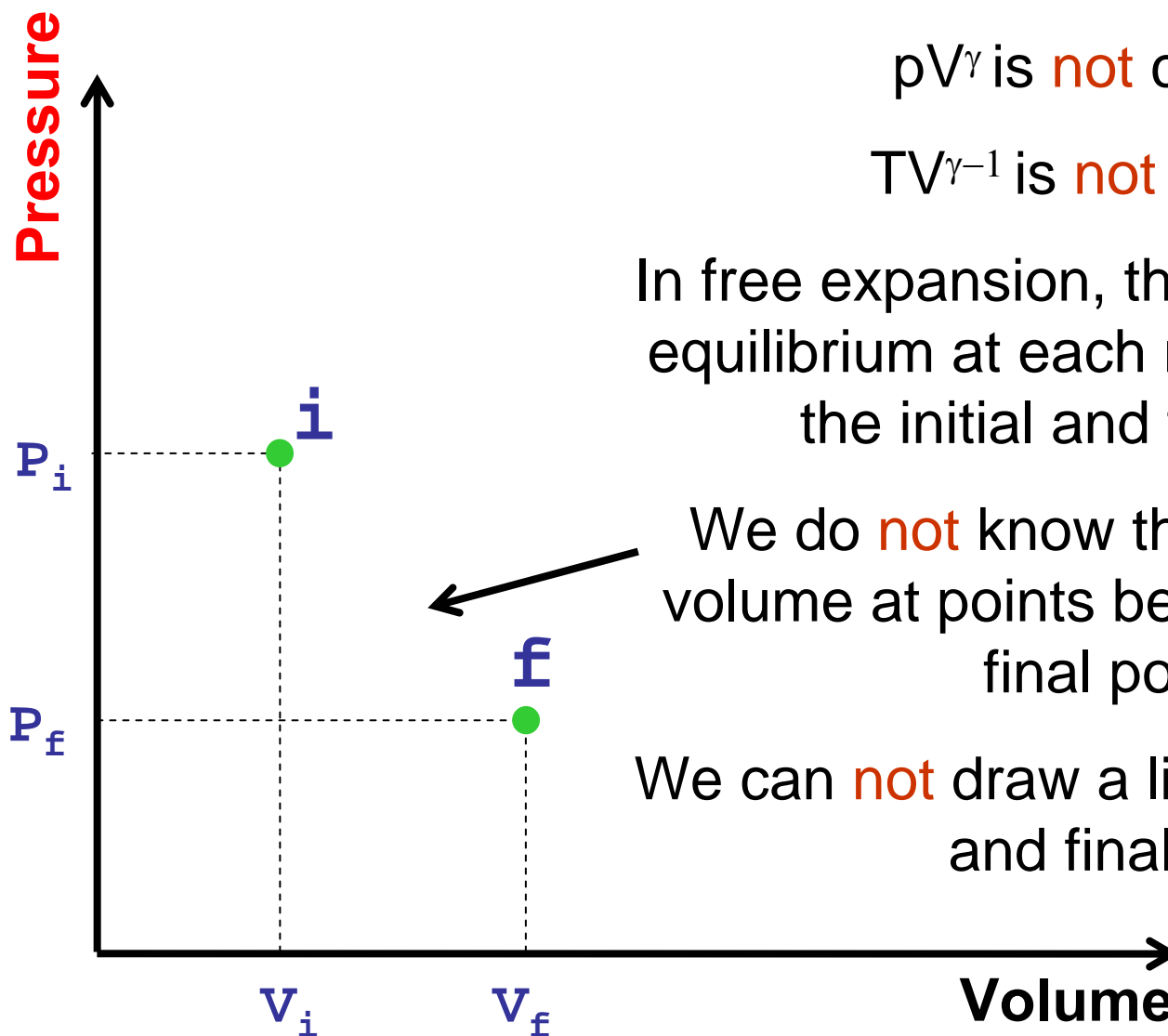
### Free expansions

Although, a free expansion is an adiabatic process,

$pV^\gamma$  is **not** constant

$TV^{\gamma-1}$  is **not** constant

In free expansion, the system is **not** in equilibrium at each moment between the initial and final states.



We do **not** know the pressure and volume at points between initial and final points.

We can **not** draw a line between initial and final point

## 20-11 Adiabatic Expansion

## Free expansions

$$\Delta E_{\text{int}} = n C_V \Delta T = 0 \rightarrow \Delta T = 0$$

$$T = \text{Constant}$$

$$T_i = T_f$$

$$pV = nRT \rightarrow pV = \text{Constant}$$

$$p_i V_i = p_f V_f$$

## 20-11 Adiabatic Expansion

### Sample Problem 20-10

1 mole of oxygen  
expands **adiabatically**  
Assume that it is ideal  
Oxygen is diatomic gas ( $\gamma = 7/5 = 1.4$ )

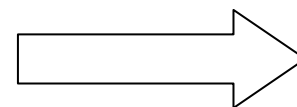
$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

$$T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma-1}$$

$$T_f = (310 \text{ K}) \left( \frac{12 \text{ L}}{19 \text{ L}} \right)^{1.4-1} = 256 \text{ K}$$

Initial State

$$\begin{aligned} V_i &= 12 \text{ L} \\ T_i &= 310 \text{ K} \\ P_i &= 2 \text{ Pa} \end{aligned}$$



Final State

$$\begin{aligned} V_f &= 19 \text{ L} \\ T_f &=? \\ P_f &= \end{aligned}$$

## 20-11 Adiabatic Expansion

Sample Problem 20-10

Continue

1 mole of oxygen  
expands **freely**  
Assume that it is ideal  
Oxygen is diatomic gas ( $\gamma = 7/5=1.4$ )

$$T_f = T_i = 310\text{K}$$

$$P_f V_f = P_i V_i$$

$$P_f = P_i \frac{V_i}{V_f}$$

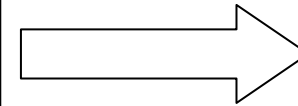
$$P_f = (2\text{Pa}) \left( \frac{12\text{ L}}{19\text{ L}} \right) = 1.3\text{Pa}$$

Initial State

$$V_i = 12\text{ L}$$

$$T_i = 310\text{K}$$

$$P_i = 2\text{ Pa}$$



Final State

$$V_f = 19\text{ L}$$

$$T_f = ?$$

$$P_f = ?$$

## 20-11 Adiabatic Expansion

Checkpoint 5