RECITATION 10

Ch. 11

•5 ILW A 1000 kg car has four 10 kg wheels. When the car is moving, what fraction of its total kinetic energy is due to rotation of the wheels about their axles? Assume that the wheels have the same rotational inertia as uniform disks of the same mass and size. Why do you not need to know the radius of the wheels?

For the wheels,

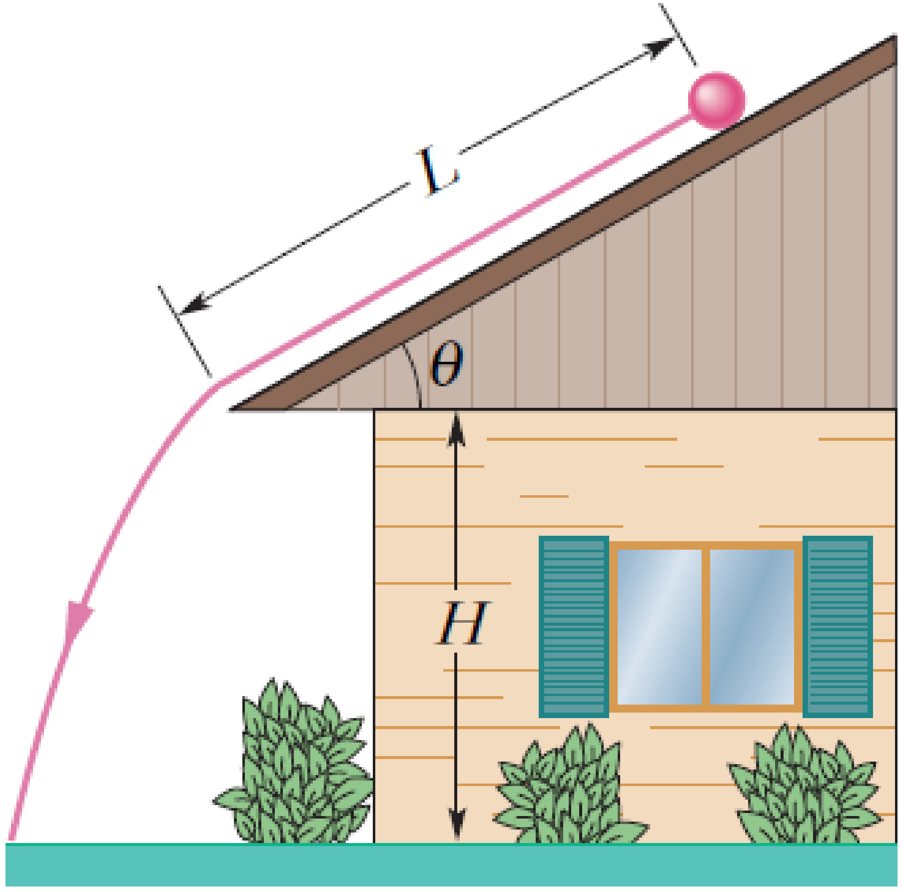
$$K_{\text{rot}} = 4\left(\frac{1}{2}I\omega^2\right) = 2\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = mv^2.$$

For the car,

$$K = \frac{1}{2}Mv^2$$

$$\frac{K_{\text{rot}}}{K_{\text{rot}} + K} = \frac{mv^2}{mv^2 + Mv^2/2} = \frac{m}{m + M/2} = \frac{10\text{ kg}}{10\text{ kg} + 1000\text{ kg}/2} = 0.020.$$

••7 ILW In Fig. 11-31, a solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance $L = 6.0$ m down a roof that is inclined at the angle $\theta = 30°$. (a) What is the angular speed of the cylinder about its center as it leaves the roof? (b) The roof's edge is at height $H = 5.0$ m. How far horizontally from the roof's edge does the cylinder hit the level ground?



a)

$$a_{\text{com},x} = -\frac{g \sin\theta}{1 + \frac{I_{\text{com}}}{MR^2}} = -\frac{\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)\sin 30°}{1 + \frac{\frac{1}{2}MR^2}{MR^2}} = -6.92\ \frac{\text{m}}{\text{s}^2}.$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta = \omega_0^2 + 2\left(\frac{a_{com,x}}{R}\right)\left(\frac{\Delta x}{R}\right) = \omega_0^2 + \frac{2a_{com,x}\Delta x}{R^2}$$

Substituting gives,

$$\omega^2 = 0 + \frac{2(-6.92\ \text{m s}^2)(-6.0\ \text{m})}{(0.10\ \text{m})^2} = 3920\ \text{rad}^2/\text{s}^2.$$

$$\omega = 63\ \text{rad/s}.$$

b) We first find the time when the cylinder hits the ground.

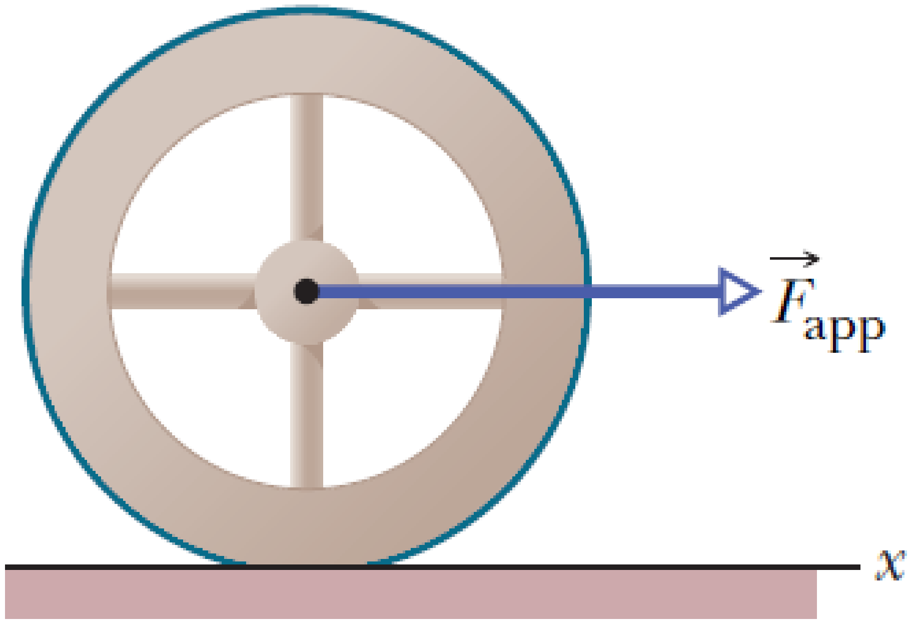
$$0 - H = v_{0y}t - \frac{1}{2}gt^2.$$

With $v_{0y} = v\sin 210° = \omega r \sin 210° = -3.13\ \mathrm{m/s}$. Solving for t gives that $t = 0.740\ \mathrm{s}$. To find the horizontal distance travelled, we write

$$x - x_0 = v_{0x}t = (\omega r \cos 210°)t = -4.0\ \mathrm{m}.$$

The horizontal distance travelled is thus 4.0 m.

••11 In Fig. 11-34, a constant horizontal force $\vec{F}_{app}$ of magnitude 10 N is applied to a wheel of mass 10 kg and radius 0.30 m. The wheel rolls smoothly on the horizontal surface, and the acceleration of its center of mass has magnitude 0.60 m/s^2. (a) In unit-vector notation, what is the frictional force on the wheel? (b) What is the rotational inertia of the wheel about the rotation axis through its center of mass?



a)

$$F_{\text{net},x} = F_{\text{app}} - f_s = M a_{\text{com},x}$$

$$f_s = F_{\text{app}} - M a_{\text{com},x} = 10\text{ N} - (10\text{ kg})\left(0.60\,\frac{\text{m}}{\text{s}^2}\right) = 4.0\text{ N}.$$

b)

$$\tau_{\text{net}} = r f_{\text{s}} = I_{\text{com}}\alpha$$

$$I_{\text{com}} = \frac{r f_{\text{s}}}{\alpha} = \frac{r^2 f_{\text{s}}}{a_{\text{com},x}} = \frac{(0.30\text{ m})^2(4.0\text{ N})}{0.60\,\frac{\text{m}}{\text{s}^2}} = 0.60\text{ kg}\cdot\text{m}^2.$$

•51 SSM ILW A wheel is rotating freely at angular speed 800 rev/min on a shaft whose rotational inertia is negligible. A second wheel, initially at rest and with twice the rotational inertia of the first, is suddenly coupled to the same shaft. (a) What is the angular speed of the resultant combination of the shaft and two wheels? (b) What fraction of the original rotational kinetic energy is lost?

a)

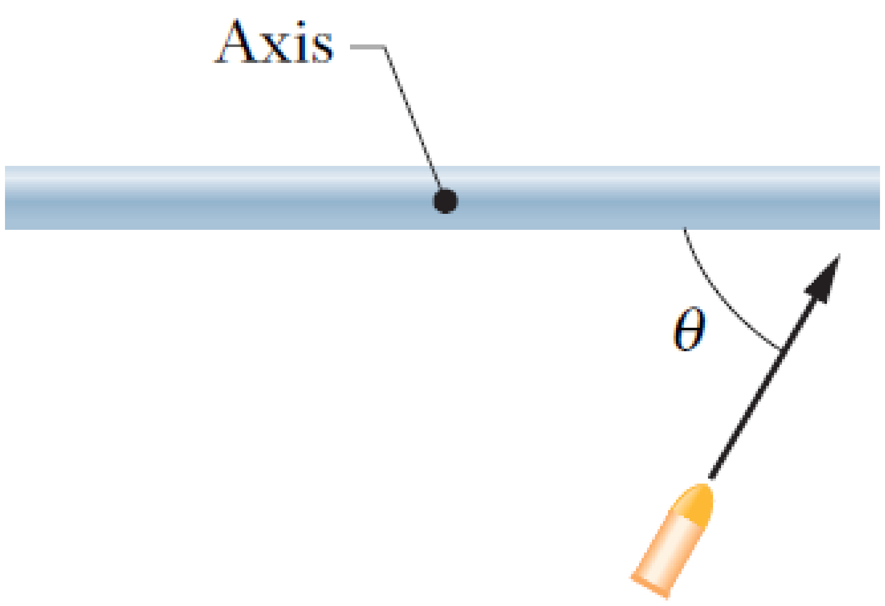
$$I_1\omega_1 = I_2\omega_2$$

$$\omega_2 = \frac{I_1\omega_1}{I_2} = \frac{I_1\omega_1}{I_1 + 2I_1} = \frac{1}{3}\omega_1 = \frac{800\ \text{rev/min}}{3} = 267\ \text{rev/min.}$$

b)

$$1 - \frac{K_2}{K_1} = 1 - \frac{\frac{1}{2}I_2\omega_2^2}{\frac{1}{2}I_1\omega_1^2} = 1 - \frac{\frac{1}{2}(3I_1)\left(\frac{\omega}{3}\right)^2}{\frac{1}{2}I_1\omega_1^2} = 1 - \frac{1}{3} = 0.667.$$

••53 GO A uniform thin rod of length 0.500 m and mass 4.00 kg can rotate in a horizontal plane about a vertical axis through its center. The rod is at rest when a 3.00 g bullet traveling in the rotation plane is fired into one end of the rod. As viewed from above, the bullet's path makes angle $\theta = 60.0°$ with the rod (Fig. 11-50). If the bullet lodges in the rod and the angular velocity of the rod is 10 rad/s immediately after the collision, what is the bullet's speed just before impact?



$$\vec{L}_{\mathrm{i}} = \vec{L}_{\mathrm{f}}.$$

$$-rmv\sin\theta + 0 = \omega_f I_{\mathrm{b+r}}$$

$$I_{\mathrm{b+r}} = I_{\mathrm{rod}} + I_{\mathrm{bllt}} = \frac{1}{12}ML^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12}(4.00\ \mathrm{kg})(0.500\ \mathrm{m})^2 + (3.00\times 10^{-3}\ \mathrm{kg})\left(\frac{0.500\ \mathrm{m}}{2}\right)^2$$

$$= 0.0852\mathrm{kg}\cdot\mathrm{m}^2.$$

$$v = \frac{\omega_f I_{\mathrm{b+r}}}{-rm\sin\theta} = \frac{(-10\ \mathrm{rad/s})(0.0852\mathrm{kg}\cdot\mathrm{m}^2)}{-\left(\frac{0.500\ \mathrm{m}}{2}\right)(3.00\times 10^{-3}\ \mathrm{kg})\sin 60.0^\circ}$$

$$= 1310\,\frac{\mathrm{m}}{\mathrm{s}}.$$