Chapter 8

Potential Energy and Conservation of Energy

Introduction

Potential energy U is energy that can be associated with the *configuration* (arrangement) of a *system* of objects that exert conservative forces on each other.

Consider a block attached to a spring that is attached to the ceiling. The block starts to fall after it is released from rest where the spring is in its relaxed state. The system then consists of Earth and the block. The configuration of the system changes (the distance between Earth and the block).

We can account for the block motion by defining a **gravitational potential energy** U.

This is the energy associated with the state of separation between two objects that attract each other by the gravitational force, here the block and Earth.

Introduction

The block then stretches the spring, the system of objects consists of the block and the spring. The force between the objects is elastic. The configuration of the system changes (the spring stretches).

We can account for the block's decrease in kinetic energy and the spring's increase in length by defining an **elastic potential energy** U.

This is the energy associated with the state of compression or extension of an elastic object, here the spring.

Work and Potential Energy

- In CH. 7 we discussed the relation between work and a change in kinetic energy. Here we discuss the relation between work and change in potential energy.
- Throw a object upward. As the object rises, the work W_g done on the object by the gravitational force is negative because the force transfers energy *from* the kinetic energy of the object. We can now say that this energy is transferred by the gravitational force *to* the gravitational potential energy of the object–Earth system.

  The object slows, stops, and then begins to fall back down because of the gravitational force. During the fall, the transfer is reversed: The work W_g done on the object by the gravitational force is now positive—that force transfers energy *from* the gravitational potential energy of the object–Earth system *to* the kinetic energy of the object.

Work and Potential Energy

- For either rise or fall, the change ΔU in gravitational potential energy is defined as being equal to the negative of the work done on the object by the gravitational force:

$$\Delta U = -W,$$

  which is true for any type of work done by conservative forces.

- Consider a block–spring system. If we give the block a shove to send it moving rightward, the spring force acts leftward and thus does negative work on the block, transferring energy from the kinetic energy of the block to the elastic potential energy of the spring–block system.

  The block slows and eventually stops, and then begins to move leftward because the spring force is still leftward. The transfer of energy is then reversed—it is from potential energy of the spring–block system to kinetic energy of the block.

Work and Potential Energy

- Conservative and Nonconservative Forces:

  The two previous systems discussed share the following key elements:

  1. The system consists of two or more objects.
  2. A *force* acts between an object in the system and the rest of the system.
  3. When the system configuration changes, the force does work W_1 on the object, transferring energy between the kinetic energy K of the object and some other type of energy of the system.
  4. When the configuration change is revered, the force reverses the energy transfer, doing work W_2 in the process.

  In situations when $W_1 = -W_2$ is always true, the other type of energy is **potential energy** and the force is said to be a **conservative force**.

Work and Potential Energy

- Conservative and Nonconservative Forces:

  The gravitational force and the spring force are both conservative. A force that is not conservative is called a **nonconservative force**.

  The kinetic friction force and the drag force (e.g. air resistance) are nonconservative.

  Consider a block sliding across a floor that is not frictionless. The kinetic frictional force from the floor slows the block transferring energy from its kinetic energy to **thermal energy**. This energy transfer cannot be reversed. Thermal energy cannot be transferred back to the block's kinetic energy by the kinetic frictional force. The kinetic frictional force is the nonconservative. Therefore, thermal energy is not a potential energy.

Path Independence of Conservative Force

Path independence is the primary test for determining whether a force is conservative or not.

Let a force act on a particle that moves along a closed path. If the total energy it transfers to and from the particle during the round trip is ZERO, the force is conservative. In other words:

The net work done by a conservative force on a particle moving around any closed path is ZERO.

The gravitational force is conservative. Throw an object upward with speed v_0 and kinetic energy $mv_0^2/2$. The object slows and stops then falls black down. When the object returns to the launch point it again has speed v_0 and kinetic energy $mv_0^2/2$. The net work done on the object by the gravitational force is zero.

Path Independence of Conservative Force

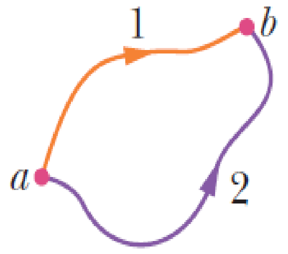
An important result of the closed–path test is that:

The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

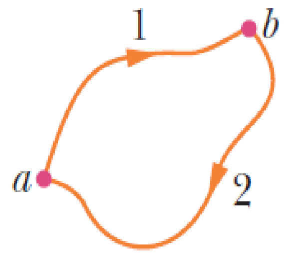
In the figure a particle moves from point a to point b along either path 1 or path 2. If only a conservative force acts on the particle, the work done on the particle is the same along the two paths:

$$W_{ab,1} = W_{ab,2}.$$

This result is very helpful in simplifying some problem that involve conservative forces only.



The force is conservative. Any choice of path between the points gives the same amount of work.

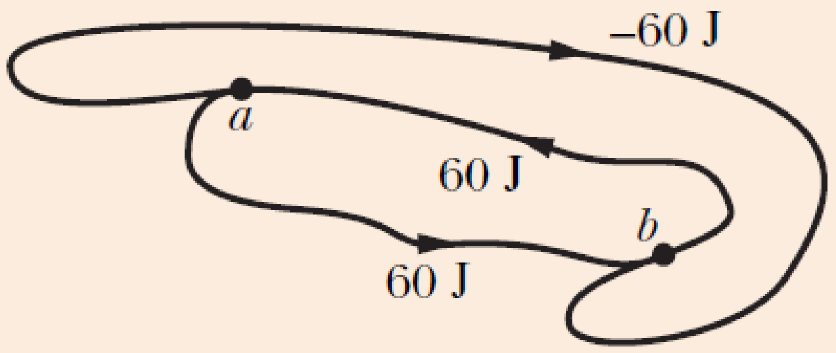


And a round trip gives a total work of zero.

Path Independence of Conservative Force

CHECKPOINT 1

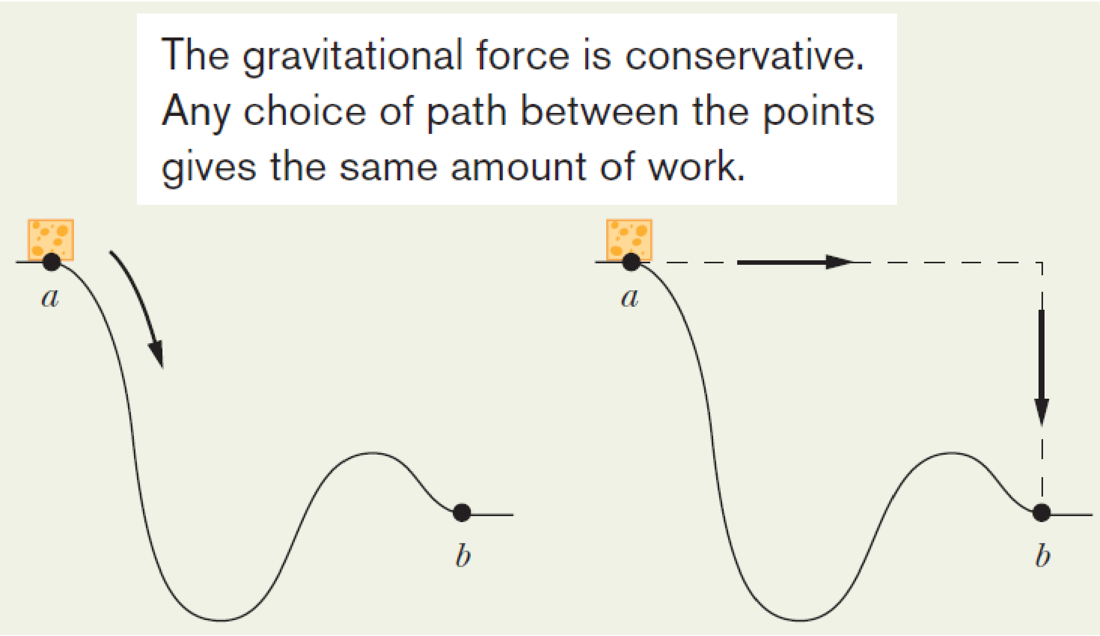
The figure shows three paths connecting points a and b. A single force $\vec{F}$ does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force $\vec{F}$ conservative?



The net work around the large loop is zero. However, the net work around the small loop is $60\text{ J} + 60\text{ J} = 120\text{ J}$. Therefore, the force is not nonconservative.

Path Independence of Conservative Force

Example 1: The figure shows a $2.0\ \mathrm{kg}$ block of slippery cheese that slides along a frictionless track from point a to point b. The cheese travels through a total distance of $2.0\ \mathrm{m}$ along the track, and a net vertical distance of $0.80\ \mathrm{m}$. How much work is done on the cheese by the gravitational force during the slide?



We cannot use $W_g = mgd\cos\phi$, because ϕ varies along the track in unknown way.

Luckily, we can find the work by choosing another simple path between a and b since the gravitational force is conservative.

Path Independence of Conservative Force

We choose the dashed path shown in the figure. Along the horizontal segment

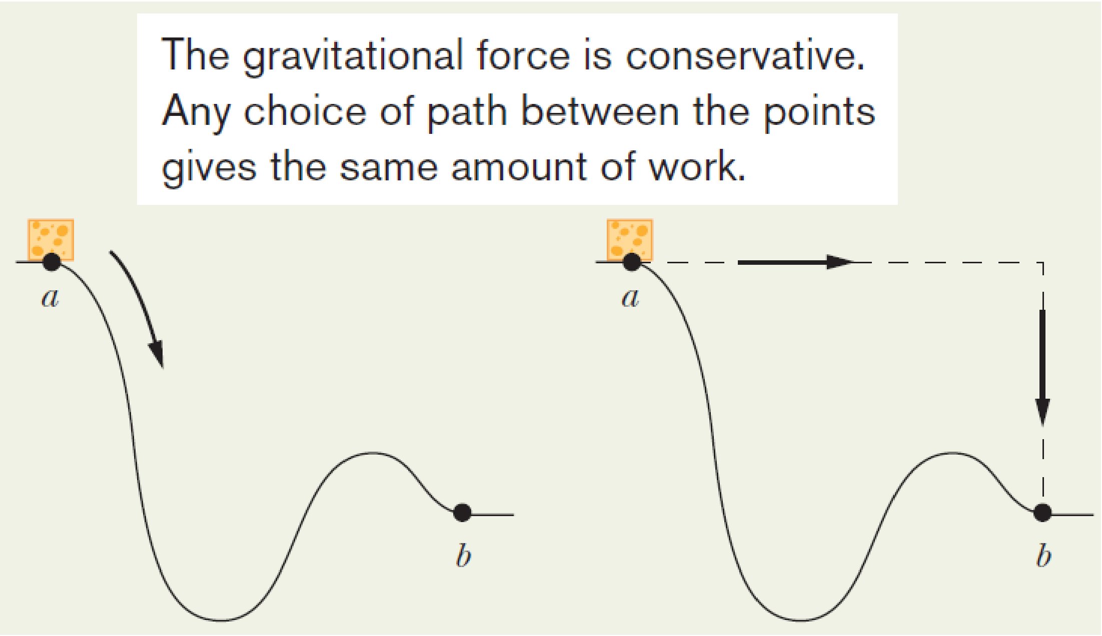
$$W_{g,h} = mgd \cos 90° = 0.$$

Along the vertical segment

$$W_{g,v} = mgd \cos 0° = (2.0\,kg)(9.8\,m/s^2)(0.80\text{ m}) = 15.7\text{ J}.$$

The total work done by the gravitational force is

$$W_g = W_{g,h} + W_{g,v} \approx 16\text{ J}.$$



Determining Potential Energy Values

Here we want to find equations for gravitational potential energy and elastic potential energy. Before that we find a general relation between a conservative force and the associated potential energy.

We found before that

$$\Delta U = -W.$$

The work done by a variable force is given by

$$W = \int_{x_i}^{x_f} F(x)\, dx\,.$$

Combining the results we can write

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx$$

Determining Potential Energy Values

- Gravitational Potential Energy:

  When a particle of mass m moving vertically along the y axis from point y_i to point y_f, the change in the gravitational potential energy of the particle–Earth system is given by

$$\Delta U = -\int_{y_i}^{y_f} (-mg)\, dy = mg \int_{y_i}^{y_f} dy = mg[y]_{y_i}^{y_f},$$

  or

$$\Delta U = mg\left(y_f - y_i\right) = mg\Delta y.$$

Determining Potential Energy Values

- Gravitational Potential Energy:

  Only changes ΔU in gravitational potential energy are physically meaningful. However, it is useful to associate certain gravitational potential value U with a certain particle–Earth system when the particle is at a certain height y. We then write

$$U - U_i = mg(y - y_i).$$

  We take U_i to be the gravitational potential energy of the system when it is in a **reference configuration** in which the particle is at a reference point y_i. We usually take U_i and y_i to be zero. Therefore, we write

$$U(y) = mgy.$$

Determining Potential Energy Values

- Gravitational Potential Energy:

  The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

Determining Potential Energy Values

- Elastic Potential Energy:

  Consider a block of mass m that is attached to the free end of a spring of spring constant k. When the block moves from point x_i to x_f, the corresponding change in the elastic potential energy of the block–mass system is

$$\Delta U = -\int_{x_i}^{x_f} (-kx)\, dx = k \int_{x_i}^{x_f} x\, dx = \frac{1}{2} k [x^2]_{x_i}^{x_f},$$

  or

$$\Delta U = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2.$$

Determining Potential Energy Values

- Elastic Potential Energy:

  To associate a potential energy value U with the block at position x, we choose the reference configuration to be where the spring is at its relax length and the block is at $x_i = 0$. Then $U_i = 0$ and the previous expression reduces to

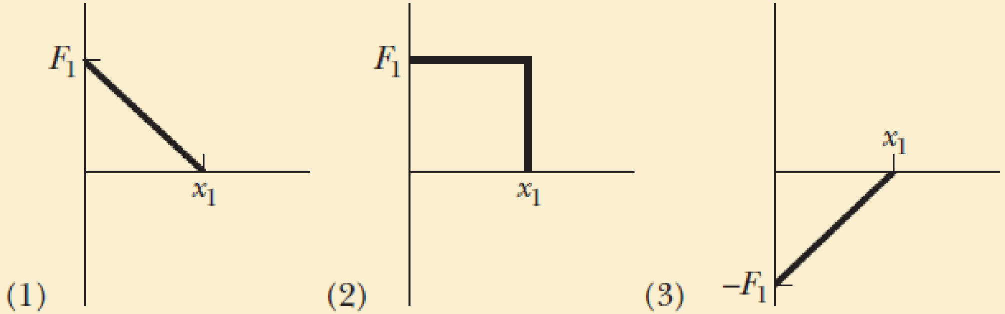
$$U - 0 = \frac{1}{2}kx^2 - 0,$$

or

$$U(x) = \frac{1}{2}kx^2.$$

Checkpoint 2

A particle is to move along an x axis from $x = 0$ to x_1 while a conservative force, directed along the x axis, acts on the particle. The figure shows three situations in which the x component of that force varies with x. The force has the same maximum magnitude F_1 in all three situations. Rank the situations according to the change in the associated potential energy during the particle's motion, most positive first.



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Determining Potential Energy Values

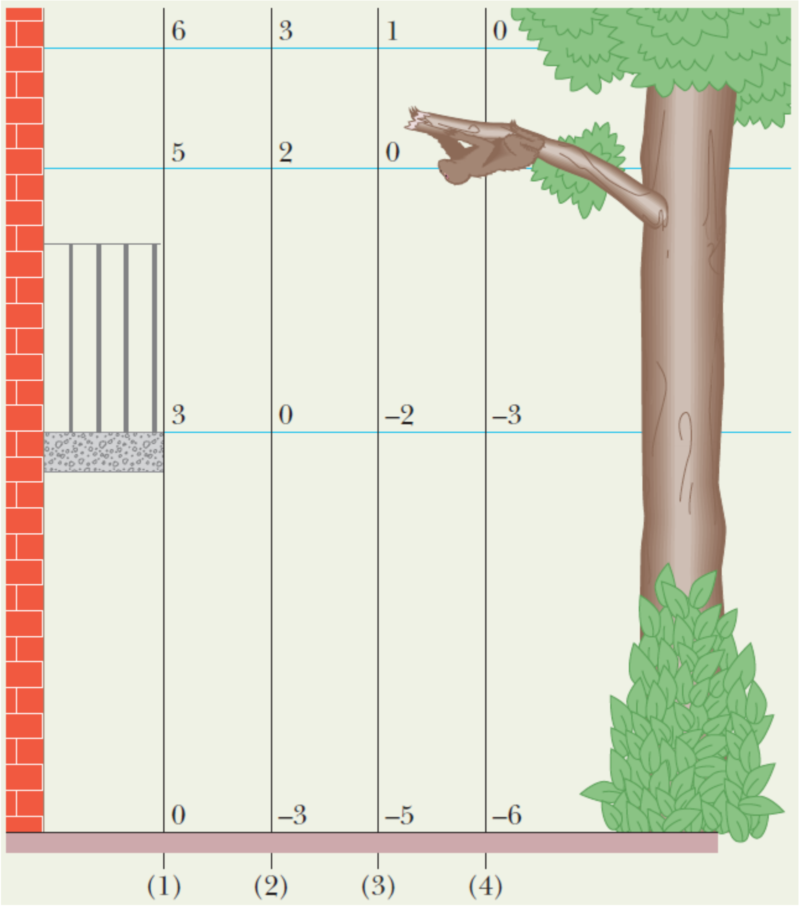
Example 2: A $2.0\ \mathrm{kg}$ sloth hangs $5.0\ \mathrm{m}$ above the ground. (a) What is the gravitational potential energy U of the sloth–Earth system if we take the reference point $y = 0$ to be (1) at the ground, (2) at a balcony floor that is $3.0\ \mathrm{m}$ above the ground, (3) at the limb, and (4) $1.0\ \mathrm{m}$ above the limb? Take the gravitational potential energy to be zero at $y = 0$.

(1) $U = mgy = (2.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(5.0\ \mathrm{m}) = 98\ \mathrm{J}.$

(2) $U = mgy = (2.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(2.0\ \mathrm{m}) = 39\ \mathrm{J}$

(3) $U = mgy = (2.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(0\ \mathrm{m}) = 0\ \mathrm{J}$

(4) $U = mgy = (2.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(-1.0\ \mathrm{m}) \approx -20\ \mathrm{J}$



Determining Potential Energy Values

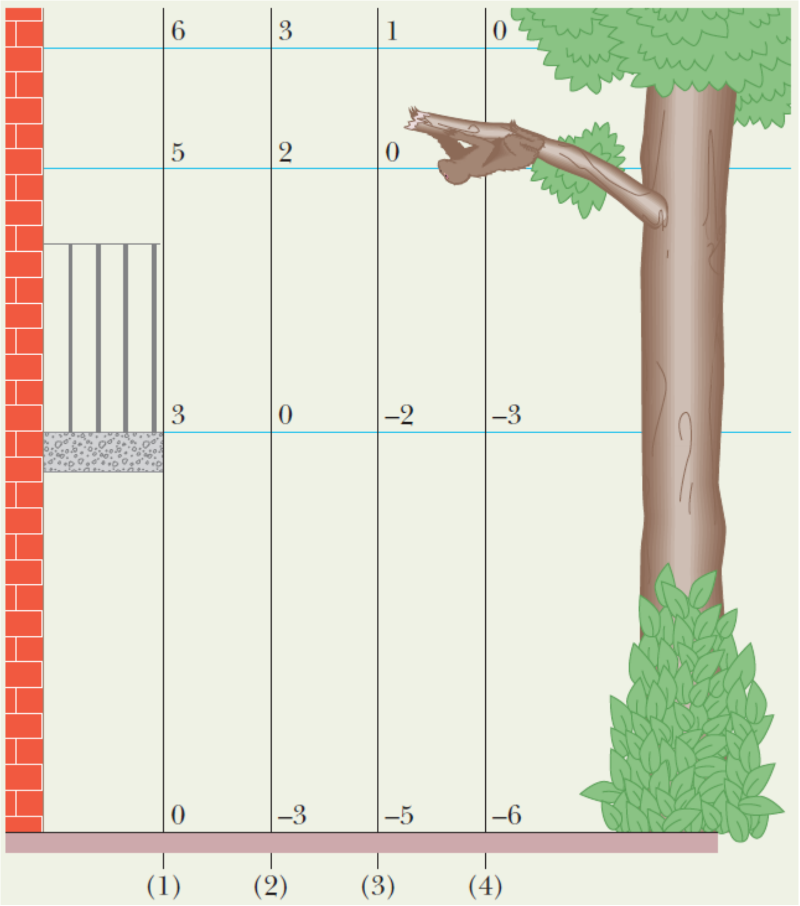
(b) The sloth drops to the ground. For each choice of reference point, what is the change ΔU in the potential energy of the sloth–Earth system due to the fall?

In all cases $\Delta y = -5.0 \text{ m}$. Therefore,

$$\Delta U = mg\Delta y$$
$$= (2.0 \text{ kg})\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(-5.0 \text{ m}) = -98 \text{ J}.$$

(c) What is the work done by the gravitational force during dropping?

$$W_g = -\Delta U = 98 \text{ J}.$$



Conservation of Mechanical Energy

- The mechanical energy E_{mec} of a system is the sum of its potential energy U and kinetic energy K of the objects within the system:

$$E_{\mathrm{mec}} = K + U.$$

- We study here what happens to this mechanical energy when only conservative forces cause energy transfers within the system (on frictional or drag forces). We also assume that the system is isolated; no external force from an object outside the system causes changes inside the system.
- When a constant force does work W on an object within the system, that force transfers energy between kinetic energy K of the object and potential energy U of the system. According to the work–energy theorem:

Conservation of Mechanical Energy

$$\Delta K = W,$$

and from a previous section, we found that the change in potential energy is

$$\Delta U = -W.$$

Combining these equations gives

$$\Delta K = -\Delta U.$$

In words, the increase in one of these energies is equal to the decrease in the other.

We can rewrite the above expression as

$$K_2 - K_1 = -(U_2 - U_1),$$

or

Conservation of Mechanical Energy

$$K_2 + U_2 = K_1 + U_1.$$

In words,

$$\begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system}\end{pmatrix} = \begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for any} \\ \text{other state of the system}\end{pmatrix}.$$

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

- This result is known as the principle of **conservation of mechanical energy**. With the aid of the relation $\Delta K = -\Delta U$, we can express this principle as

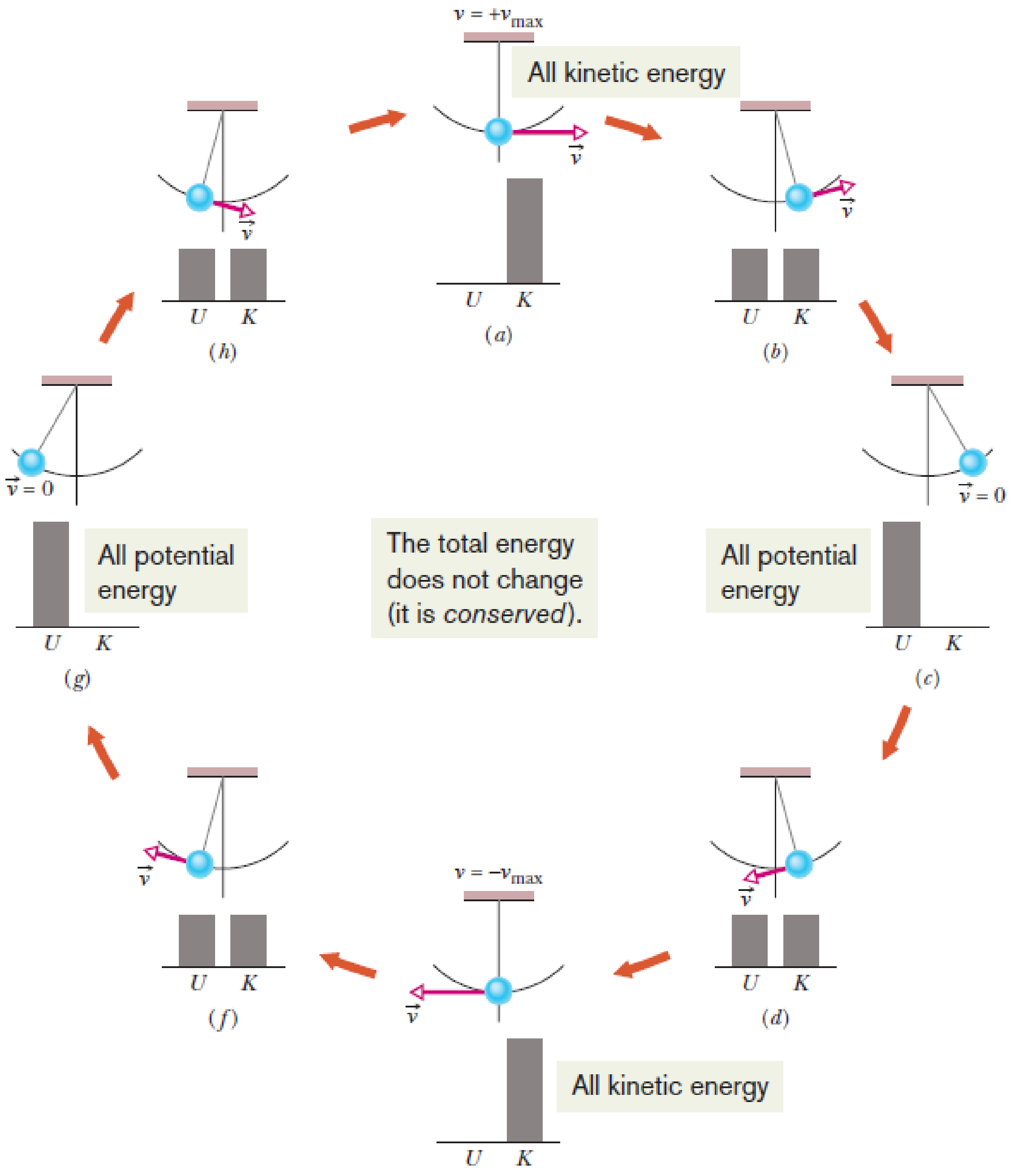
$$\Delta E_{\mathrm{mec}} = \Delta K + \Delta U = 0.$$

Conservation of Mechanical Energy

The principle of conservation of mechanical energy enables us to solve problems that would be difficult to solve using Newtonian mechanics.

When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion* and *without finding the work done by the forces involved.*

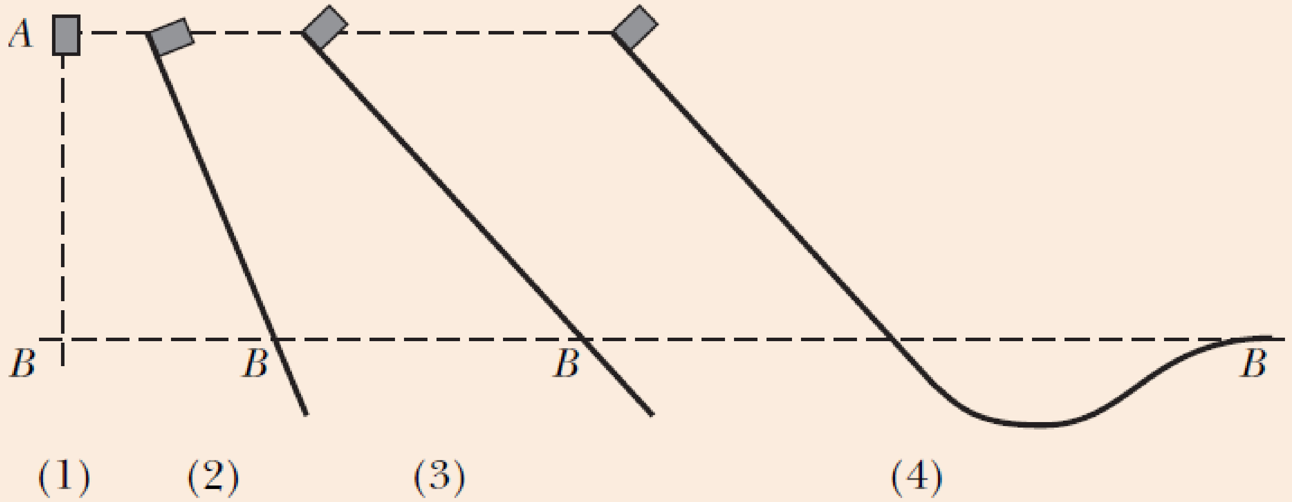
In all cases, $E_{\text{mec}} = K + U$.



Conservation of Mechanical Energy

CHECKPOINT 3

The figure shows four situations—one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. (a) Rank the situations according to the kinetic energy of the block at point B, greatest first. (b) Rank them according to the speed of the block at point B, greatest first.

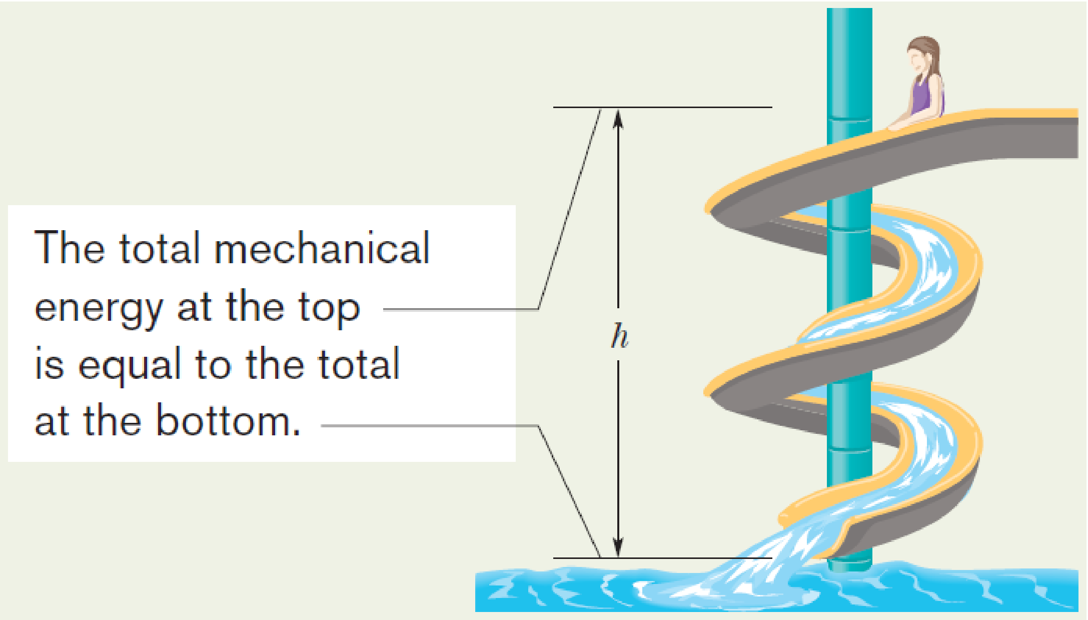


$$\Delta K = -\Delta U = -mg\Delta y.$$

In all cases ΔK (and v_f) is the same, since Δy is the same.

Conservation of Mechanical Energy

Example 3: a child of mass m is released from rest at the top of a water slide, at height $\mathrm{h} = 8.5\ \mathrm{m}$ above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



There are to forces that act on the child. A normal force which does no work, and a gravitational force which is a conservative force. The mechanical energy of the isolated, child-Earth system is therefore conserved.

Conservation of Mechanical Energy

The mechanical energy of the system at the top of the slide $E_{\text{mec,t}}$ is equal to mechanical energy of the system at the bottom of the slide $E_{\text{mec},b}$:

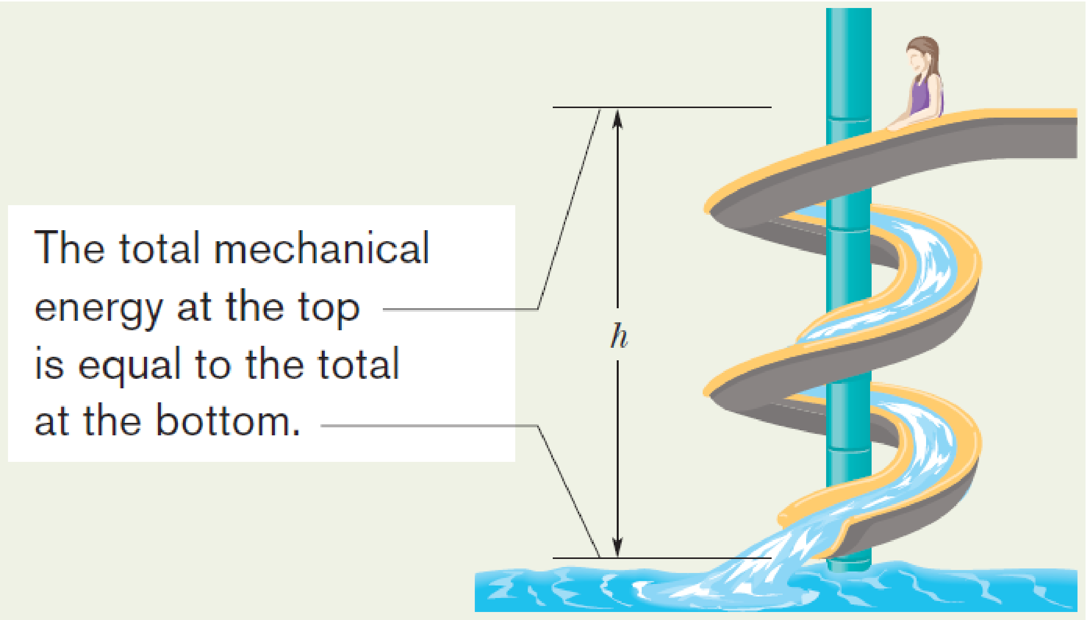
$$E_{\text{mec,t}} = E_{\text{mec},b},$$

or

$$K_{\text{t}} + U_{\text{t}} = K_{\text{b}} + U_{\text{b}}.$$

Using $K = 1/2\,mv^2$ and $U = mgy$, we get that

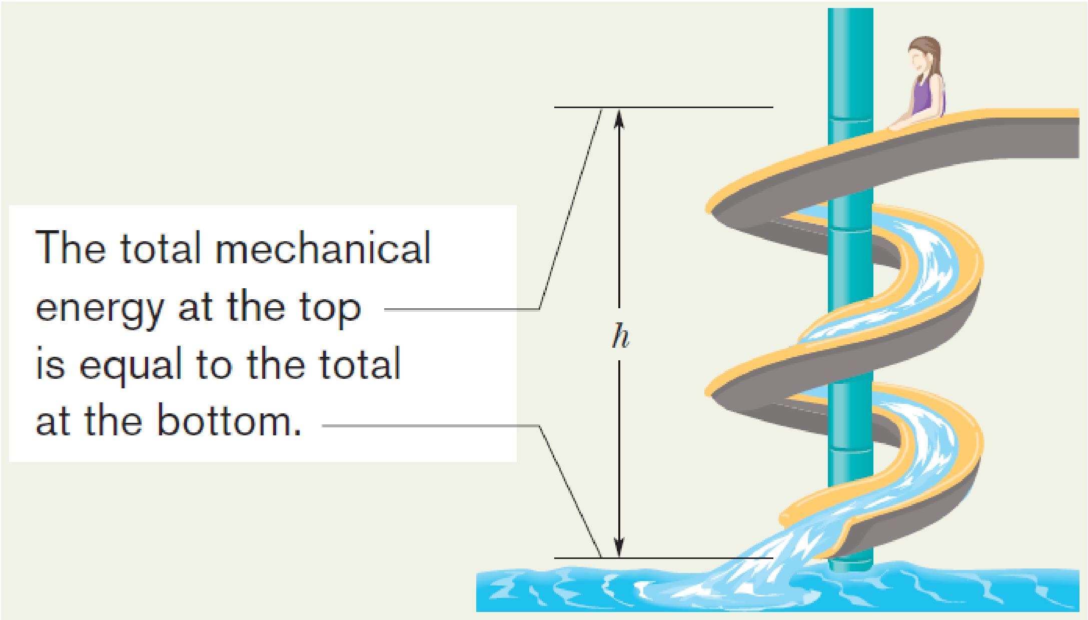
$$0 + mgh = \frac{1}{2}mv_b^2 + 0.$$



Conservation of Mechanical Energy

Solving for v_b and substituting we get that

$$v_b = \sqrt{2gh} = \sqrt{2(9.8\ \mathrm{m/s^2})(8.5\ \mathrm{m})}$$
$$= 13\ \mathrm{m/s}.$$



Potential Energy Curve and Force

- Suppose that we know the potential energy function $U(x)$ of a particular system and want to find the corresponding conservative force.
- For one dimensional motion, the work W done by a force that acts on a particle as it moves through a distance Δx is $F(x)\Delta x$. We can then write

$$\Delta U(x) = -W = -F(x)\Delta x.$$

  Solving for $F(x)$ and taking the limit as Δx approaches zero we find that

$$F(x) = -\frac{dU(x)}{dx}.$$

  Lets us check this expression for the spring force and gravitational force.

Potential Energy Curve and Force

- The elastic potential energy function for a spring force is $U(x) = \frac{1}{2}kx^2$, and the spring force is

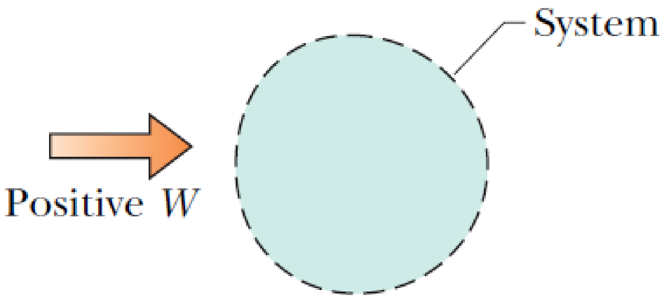
$$F(x) = -\frac{d(1/2\,kx^2)}{dx} = -\frac{1}{2}k\frac{d(x^2)}{dx} = -\frac{1}{2}k(2x) = -kx.$$

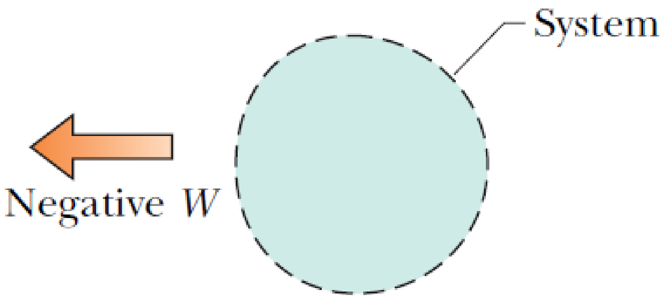
- The gravitational potential energy function for a particle–Earth system is $U(y) = mgy$, and the gravitation force is

$$F(y) = -\frac{d(mgy)}{dy} = -mg\frac{d(y)}{dy} = -mg.$$

Work Done on a System by an External Force

- In Ch. 7, we defined work as being energy, transferred to or from an object via a force acting on the object. We can now extend that definition to an external force acting on a system of objects.
- Work is energy transferred to or from a system by means of an external force acting on that system.
- If a system is a single particle or particle like object, the work done on the system can change only the kinetic energy of the system (work–energy theorem). The particle has only one energy channel; kinetic energy. If a system is more complicated, an external force can change other forms of energy such as potential energy; a more complicated system can have multiple energy channels.





Work Done on a System by an External Force

- <u>No Friction Involved</u>

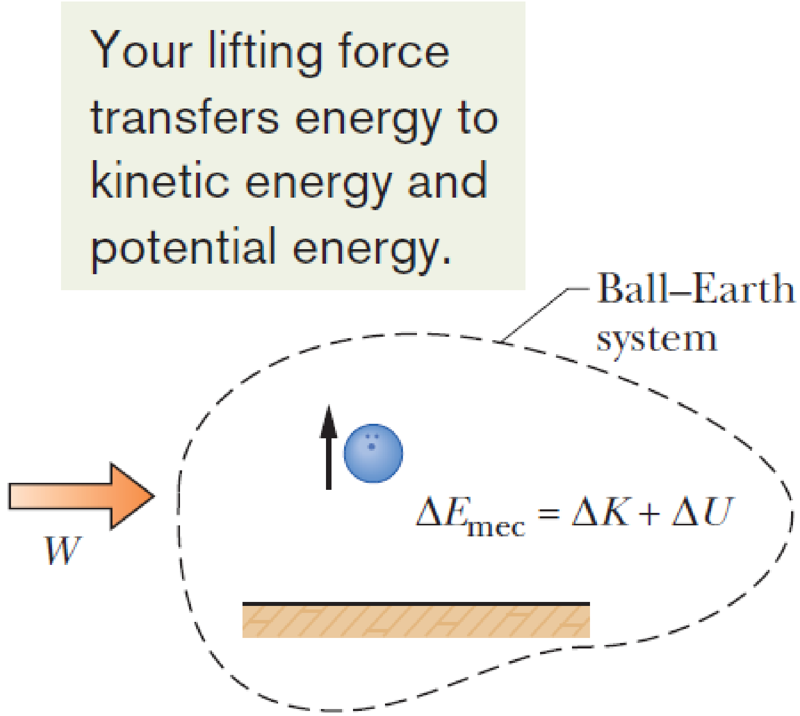
  Consider the process of throwing a bowling–ball. The work W done by the applied (external) force can result in a change ΔK in the ball's kinetic energy. If the ball and Earth become more separated, there is a change ΔU in the gravitational potential energy of the ball–Earth system, and the work is

$$W = \Delta K + \Delta U,$$

or

$$W = \Delta E_{\text{mec}},$$

where ΔE_{mec} is the change in mechanical energy.



Work Done on a System by an External Force

- <u>Friction Involved</u>

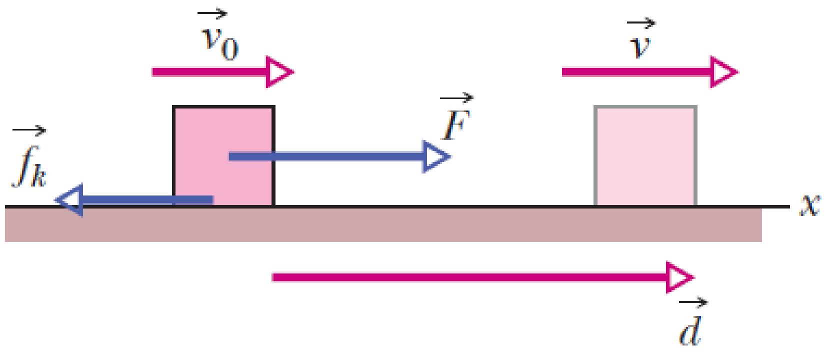
  Consider the system shown in the figure. A constant horizontal force pulls a block along an x axis and through a displacement of magnitude d, increasing the block's velocity from $\vec{v}_0$ to $\vec{v}$. Newton's 2nd law for the components along x axis reads

$$F - f_k = ma.$$

  The acceleration is constant since the forces are constant. Therefore, $\vec{v}_0$ to $\vec{v}$ are related by

$$v^2 = v_0^2 + 2ad.$$

The applied force supplies energy. The frictional force transfers some of it to thermal energy.



Work Done on a System by an External Force

- Friction Involved

  Solving this relation for a, substituting the result in the expression for v, and rearranging give

$$Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + f_k d,$$

  or

$$Fd = \Delta K + f_k d.$$

  In a more general situation, there can be a change in potential energy as well. The previous equation is generalized to

$$Fd = \Delta E_{\text{mec}} + f_k d.$$

  As the block slides, the frictional force increases the thermal energy E_{th} of the block and floor.

Work Done on a System by an External Force

- Friction Involved

  Through experiment, we find that the increase ΔE_{th} in thermal energy is equal to the product of the magnitudes f_{k} and d:

$$\Delta E_{\mathrm{th}} = f_{\mathrm{k}} d.$$

  Thus we can write that

$$Fd = \Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}}.$$

  Fd is the work W done by the external force $\vec{F}$ on the block–floor system, since the block's mechanical energy changes, and the thermal energy of the block and floor changes as well. That work is

$$W = \Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}}.$$

Work Done on a System by an External Force

CHECKPOINT 5

In three trials, a block is pushed by a horizontal applied force across a floor that is not frictionless, as in Fig. 8-13*a*. The magnitudes F of the applied force and the results of the pushing on the block's speed are given in the table. In all three trials, the block is pushed through the same distance d. Rank the three trials according to the change in the thermal energy of the block and floor that occurs in that distance d, greatest first.

Trial	F	Result on Block's Speed
a	5.0 N	decreases
b	7.0 N	remains constant
c	8.0 N	increases

All equal.

Conservation of Mechanical Energy

Example 4: A man pushes a wood crate of total mass $m = 14\ \mathrm{kg}$ across a concrete floor with a constant horizontal force $\vec{F}$ of magnitude $40\ \mathrm{N}$. In a straight-line displacement of magnitude $d = 0.50\ \mathrm{m}$, the speed of the crate decreases from $v_0 = 0.60\ \mathrm{m/s}$ to $v = 0.20\ \mathrm{m/s}$.

(a) How much work is done by force $\vec{F}$, and on what system does it do the work?

The force $\vec{F}$ is constant and therefore we can write

$$W = Fd\cos 0° = (40\ \mathrm{N})(0.50\ \mathrm{m}) = 20\ \mathrm{J}.$$

The decrease in the crate's speed indicates that there is a kinetic frictional force. The work is done on the crate–floor system, resulting in change in both ΔE_{th} and ΔK.

Conservation of Mechanical Energy

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

We know that the work W done by the force $\vec{F}$ is related to ΔK and ΔE_{th} by

$$W = \Delta K + \Delta E_{\text{th}}.$$

Because there are no potential energy changes,

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Substituting for ΔK in the first equation and solving for ΔE_{th} we get that

$$\begin{aligned}\Delta E_{\text{th}} &= W + \frac{1}{2}m(v_0^2 - v^2)\\ &= 20\ \text{J} + \frac{1}{2}(14\ \text{kg})[(0.60\ \text{m/s})^2 - (0.20\ \text{m/s})^2] \approx 22\ \text{J}.\end{aligned}$$

Conservation of Energy

- The total energy E of a system is conserved. It can change only by amounts of energy that are transferred to or from the system.
- The only type of energy transfer that we have considered is work W done on a system. The law of energy conservation states that:

$$W = \Delta E = \Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}} + \Delta E_{\mathrm{int}},$$

  where ΔE_{int} is any other type of internal energy.
- This law is not derived from basic physics principles. It is a law based on countless number of experiments.

Conservation of Energy

- Isolated System
  - If a system is isolated from its environment, there can be no energy transfers to or from the system.
  - The law of conservation of energy states that: The total energy E of an isolated system cannot change.
  - Many energy transfers maybe going within an isolated system between different types of energy. However, the total of all types of energy in the system cannot change.
  - For an isolated system, the law of conservation of energy can be written in two ways.

Conservation of Energy

- Isolated System
  - The first is:

$$\Delta E_{\mathrm{mec}} + \Delta E_{\mathrm{th}} + \Delta E_{\mathrm{int}} = 0.$$

We can let $\Delta E_{\mathrm{mec}} = E_{\mathrm{mec},2} - E_{\mathrm{mec},1}$ and write that

$$E_{\mathrm{mec},2} = E_{\mathrm{mec},1} - \Delta E_{\mathrm{th}} - \Delta E_{\mathrm{int}}.$$

This equation says:

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times.*

Conservation of Energy

- Isolated System
  - This fact can be very helpful in solving problems about isolated systems, when you need to relate energies before and after a process occurring in the system.
  - When we discussed conservation of mechanical energy, we discussed a special case of isolated systems in which nonconservative forces do not act within them. In that special case ΔE_{th} and ΔE_{int} are both zero. The mechanical energy of an isolated system is conserved when nonconservative forces do not act on it.

Conservation of Energy

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Conservation of Energy

- External forces and Internal Energy Transfers
  - An external force can change the kinetic energy or potential energy of an object without doing work on the object. The role of the force is to transfer energy from one type to another inside the object.
  - Consider the example of a stationary roller skater. The roller skater then moves away from a railing by pushing against it. His kinetic energy increases because of an external force $\vec{F}$ from the railing. However, that force does not transfer energy to him from the railing, and hence does no work on him. Rather, his kinetic energy increases as a result of internal transfers from the biochemical energy in his muscles.
  - In situations like this, we can sometimes relate the external force $\vec{F}$ on an object to the change in the object's mechanical energy if we can simplify the situation.

Conservation of Energy

- External forces and Internal Energy Transfers
  - We may relate the change in the roller skater kinetic energy to the external force $\vec{F}$ by

$$\Delta K = Fd\cos\phi\,.$$

If the situation involves potential energy change then this equation becomes

$$\Delta K + \Delta U = Fd\cos\phi\,.$$

Conservation of Energy

- <u>Power</u>

  In a more general sense, power P is the rate at which energy is transferred (instead of work) by a force from one type to another. If an amount of energy E is transferred in an amount of time t, the **average power** due to the force is

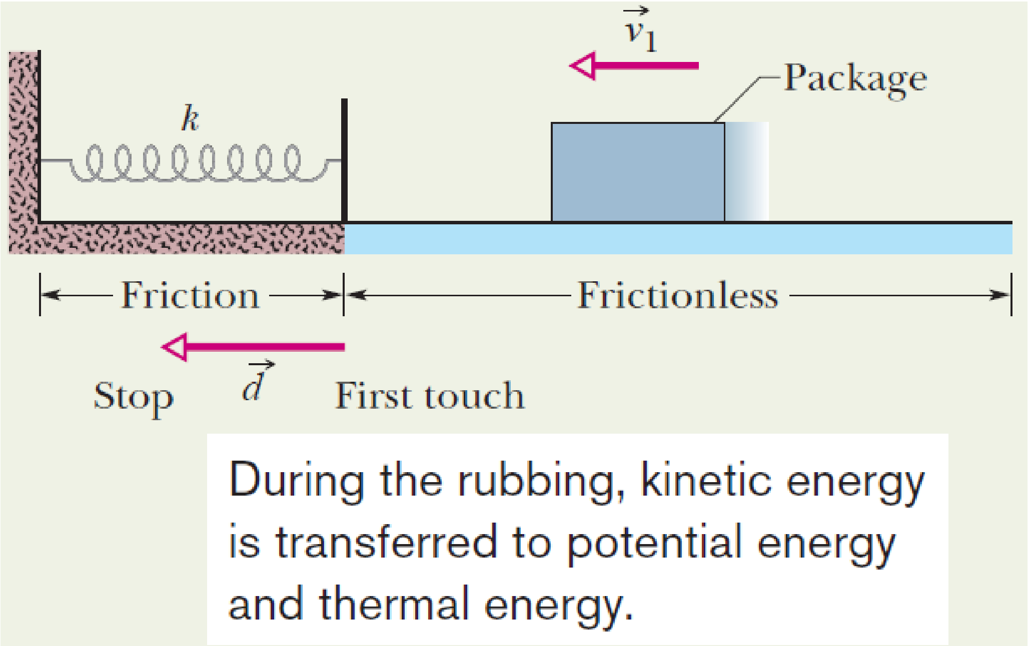
$$P_{avg} = \frac{\Delta E}{\Delta t}.$$

  Similarly, the **instantaneous power** due to the force is

$$P = \frac{dE}{dt}.$$

Conservation of Mechanical Energy

Example 5: $2.0\,\mathrm{kg}$ package slides along a floor with speed $v_1 = 4.0\,\mathrm{m/s}$. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude $15\,\mathrm{N}$, acts on the package. If $\mathrm{k} = 10000\,\mathrm{N/m}$, by what distance d is the spring compressed when the package stops?



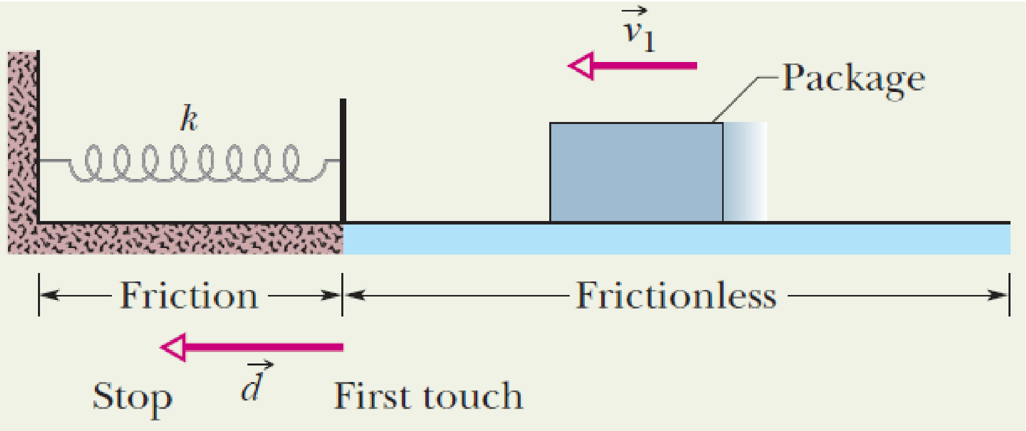
Conservation of Mechanical Energy

The forces that do work on the isolated system of the package–spring–floor–wall system are the kinetic frictional force and the spring force. can then apply the law of conservation of energy in the form

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}}.$$

For the initial state

$$E_{\text{mec},1} = K_1 + U_1 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(0)$$
$$= \frac{1}{2}mv_1^2$$



During the rubbing, kinetic energy is transferred to potential energy and thermal energy.

Conservation of Mechanical Energy

For the final state

$$E_{\text{mec},2} = K_2 + U_2 = \frac{1}{2}m(0) + \frac{1}{2}kd^2$$
$$= \frac{1}{2}kd^2.$$

ΔE_{th} is the work done by the frictional force $f_k d$. We can now write

$$\frac{1}{2}kd^2 = \frac{1}{2}mv_1^2 - f_k d.$$

After substituting we find that

$$5000d^2 + 15d - 16 = 0,$$

which give that $d = 0.055$ m.