Chapter 7

Kinetic Energy and Work

1. What is Energy

- Energy is a scalar quantity (number) associated with the state of one or more objects.
- The concept of energy can be used to predict the outcome of experiments, and design and build machines.
- The usefulness of the concept of energy is based on a wonderful property of the universe: Energy can be transformed from one type to another and transferred from one object to another, but the total amount of energy is the same. This property is called the *principle of energy conservation*.
- In this chapter we focus on one type of energy (kinetic energy) and on only one way in which energy can be transferred (work).

2. Kinetic Energy

- Kinetic energy K is energy associated with the state of motion of an object.
- For an object of mass m and speed v,

$$K = \frac{1}{2}mv^2.$$

  The faster the object moves, the greater is its kinetic energy.
- The SI unit of kinetic energy (and any other type of energy) is joule (J), named for James Joules (1818-1889).

  In terms of the SI base units

$$1 \text{ joule} = 1\text{ J} = 1\text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2} = 1\text{ N} \cdot \text{m}.$$

2. Kinetic Energy

Example 1: A $3.0\ \mathrm{kg}$ duck is flying at $2.0\ \mathrm{m/s}$.

(a) What is its kinetic energy?

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(3.0\ \mathrm{kg})(2.0\ \mathrm{m/s})^2 = 6.0\ \mathrm{J}.$$

(b) If the duck increases its speed to $4.0\ \mathrm{m/s}$, what is its new kinetic energy?

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(3.0\ \mathrm{kg})(4.0\ \mathrm{m/s})^2 = 24\ \mathrm{J}.$$

The kinetic energy is 4 times more when the speed is doubled.

2. Kinetic Energy

Example 2: Two locomotives $6.4\ \mathrm{km}$ apart were accelerated to each other and then allowed to crash head-on at full speed. Assuming each locomotive weighed $1.2 \times 10^6\ \mathrm{N}$ and its acceleration was a constant $0.26\ \mathrm{m/s^2}$, what was the total kinetic energy of the two locomotives just before the collision?

We need to find the locomotives speed just before the collision (after travelling $3.2\ \mathrm{km}$);

$$v^2 = v_0^2 + 2a(x - x_0)$$

or

$$v = \sqrt{0 + 2(0.26\ \mathrm{m/s^2})(3200\ \mathrm{m})} = 40.8\ \mathrm{m/s}.$$

2. Kinetic Energy

The mass of each locomotive is

$$m = \frac{1.2 \times 10^6 \text{ N}}{9.8 \text{ m/s}^2} = 1.22 \times 10^5 \text{ kg}.$$

The total kinetic energy of the two locomotives is then

$$K = 2\left(\frac{1}{2} m v^2\right)$$

$$= (1.22 \times 10^5 \text{ kg})(40.8 \text{ m/s})^2$$

$$= 2.0 \times 10^8 \text{ J}$$

https://www.youtube.com/watch?v=xNGGuxMEtQU

3. Work

- When you accelerate an object to a greater speed, you increase its kinetic energy $K\left(=\frac{1}{2}mv^2\right)$. If you decelerate the object to a lesser speed, you decrease its kinetic energy.
- In the first case your force transferred energy to the object from yourself, and from the object to yourself in the second.
- In such a transfer of energy via a force, work W is said to be *done on the object by the force*.
- Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

3. Work

- Work is transferred energy; it has the same unit as energy and is a scalar quantity.

4. Work and Kinetic Energy

- <u>Finding an Expression for Work:</u>

  Consider a bead of mass m that can slide along a frictionless wire that is stretched along the x axis. A **constant** force $\vec{F}$, direct at an angle ϕ to the wire, accelerates the bead. Newton's second law for components along the x axis reads

$$F_x = ma_x.$$

  As the bead moves through a displacement $\vec{d}$, the force changes the bead's velocity from $\vec{v}_0$ to $\vec{v}$, where

$$v^2 = v_0^2 + 2a_x d$$

4. Work and Kinetic Energy

- Finding an Expression for Work:

  Solving the for a_x, substituting in $F_x = ma_x$ and rearranging we obtain

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d.$$

  $\frac{1}{2}mv^2$ is the kinetic energy K_f of the bead at the end of the displacement d, and $\frac{1}{2}mv_0^2$ is the kinetic energy K_i of the bead at the start at the start of the displacement. The kinetic energy has been changed by the force and the change is equal to $F_x d$. Therefore, the work done W on the bead by the force is

$$W = F_x d.$$

4. Work and Kinetic Energy

- <u>Finding an Expression for Work:</u>

  To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

  Because $F_x = F\cos\phi$,

$$\begin{aligned} W &= F_x d \\ &= Fd\cos\phi \\ &= \vec{F}\cdot\vec{d}. \end{aligned}$$

  <u>Caution:</u> The expression for W here are valid when (1) the force is constant and (2) the object is particle-like (all parts of it move together).

4. Work and Kinetic Energy

Signs for work: The work done on an object can be either positive or negative. When ϕ is less than $90°$ the work is positive. When ϕ is between $90°$ and $180°,$ the work is negative.

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

Net work done by several forces: When there two or more forces acting on an object, the net work is the sum of the works done by the individual forces, or equivalently, the work done by the net force.

4. Work and Kinetic Energy

- <u>Work-Kinetic Energy Theorem:</u>

  Let ΔK be the change in the kinetic energy of an object, and let W be the net work done on it. Then

$$\Delta K = K_f - K_i = W,$$

  which says that

$$\begin{pmatrix}\text{change in the kinetic} \\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done} \\ \text{on the particle}\end{pmatrix}.$$

4. Work and Kinetic Energy

- Work-Kinetic Energy Theorem:

We can also write

$$K_f = K_i + W,$$

which says

$$\begin{pmatrix}\text{kinetic energy after} \\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy} \\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net} \\ \text{work done}\end{pmatrix}.$$

These statements are known traditionally as the *work-kinetic energy theorem* for particles. The kinetic energy of a particle increases if the net work done on it is positive. The kinetic energy of a particle decreases if the net work done on it is negative.

4. Work and Kinetic Energy

CHECKPOINT 1

A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from -3 m/s to -2 m/s and (b) from -2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

(a) v decreases; K decreases.

(b) v is the same; K is the same.

(c) In part a, W is negative since ΔK is negative.

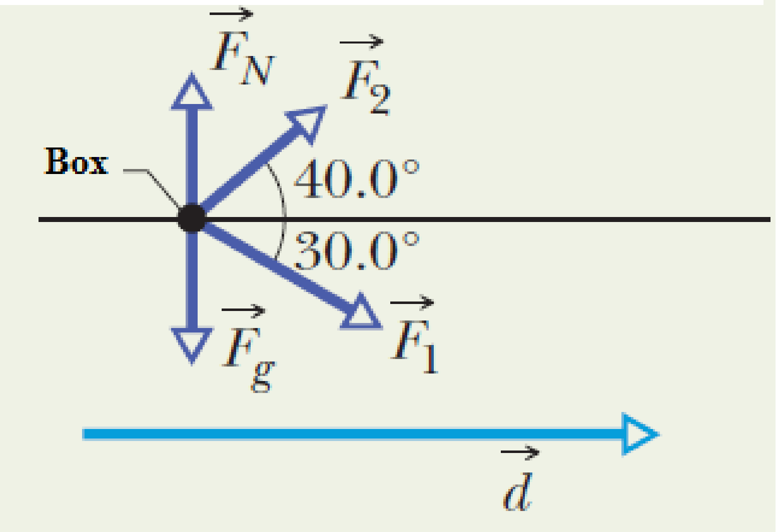
In part b, W is zero since ΔK is zero.

4. Work and Kinetic Energy

Example 3: Two forces slide an initially stationary 225 kg box a straight displacement of magnitude 8.50 m. F_1 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; F_2 is 10.0 N, directed at 40.0° above the horizontal. The floor and box make frictionless contact.

(a) What is the net work done on the box by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?

Only force components parallel to the displacement do work.



4. Work and Kinetic Energy

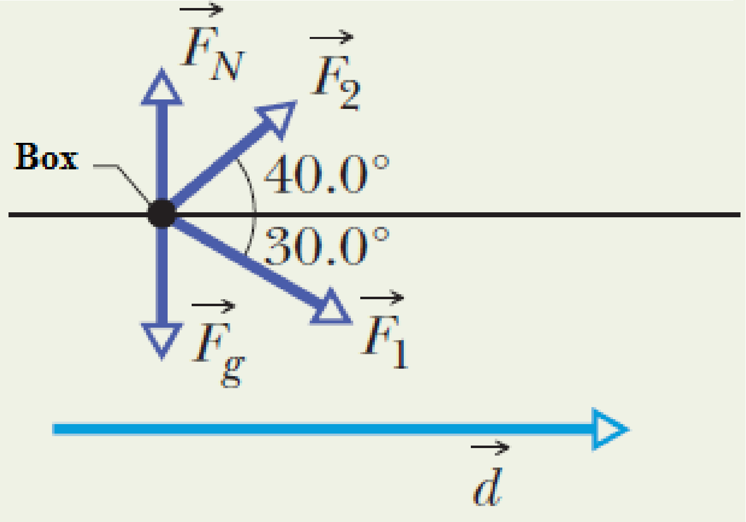
$$\begin{aligned} W_1 &= F_1 d \cos\phi_1 \\ &= (12.0\ \mathrm{N})(8.5\ \mathrm{m})(\cos 30.0°) \\ &= 88.33\ \mathrm{j}. \end{aligned}$$

$$\begin{aligned} W_2 &= F_2 d \cos\phi_2 \\ &= (10.0\ \mathrm{N})(8.5\ \mathrm{m})(\cos 40.0°) \\ &= 65.11\ \mathrm{j}. \end{aligned}$$

The net work W is

$$\begin{aligned} W_1 + W_2 &= 88.33\ \mathrm{j} + 65.11\ \mathrm{j} \\ &\approx 153\ \mathrm{j}. \end{aligned}$$

Only force components parallel to the displacement do work.

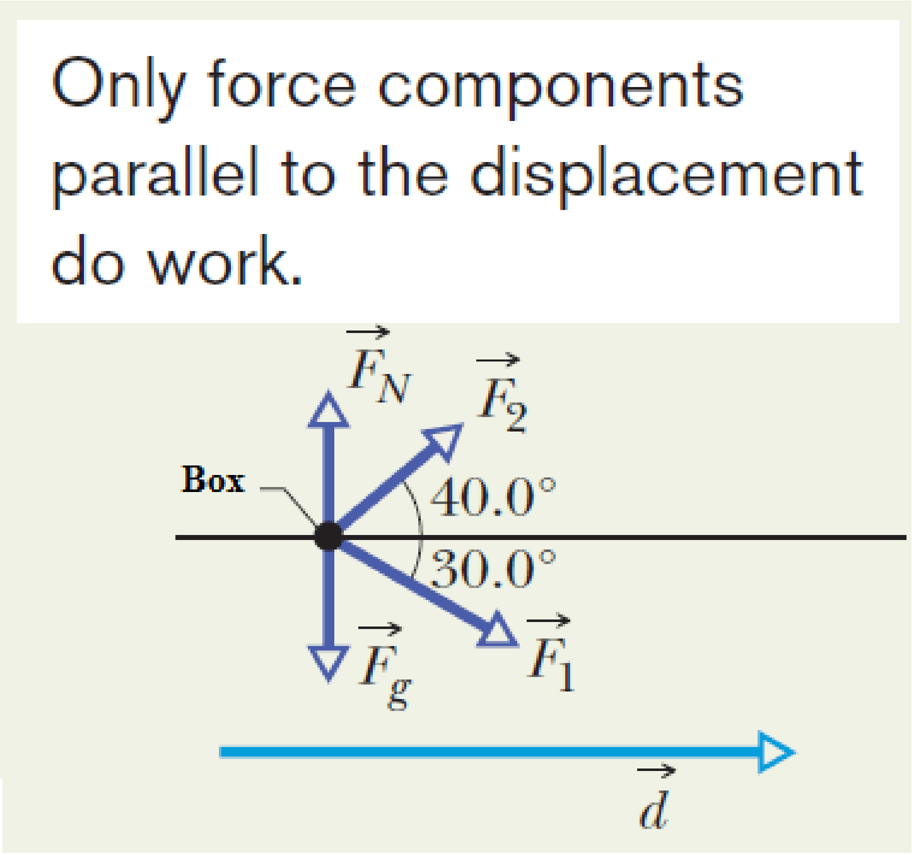


4. Work and Kinetic Energy

(b) During the displacement, what is the work W_g done on the box by the gravitational force F_g and what is the work W_N done on the box by the normal force F_N from the floor?

$$W_g = F_g d \cos 0 = 0$$

$$W_N = F_N d \cos 0 = 0$$



4. Work and Kinetic Energy

c) The box is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

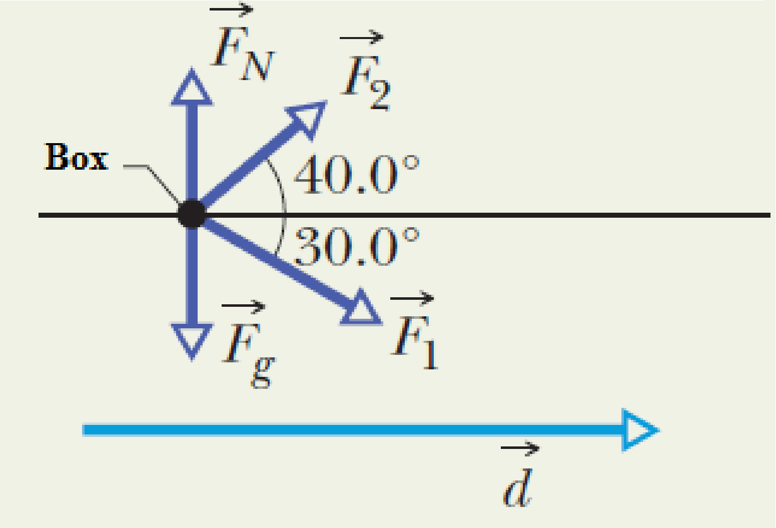
We relate the speed to the net work done.

$$W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2.$$

Solving for v_f we get

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\ \text{J})}{225\ \text{kg}}} = 1.17\ \text{m/s}$$

Only force components parallel to the displacement do work.



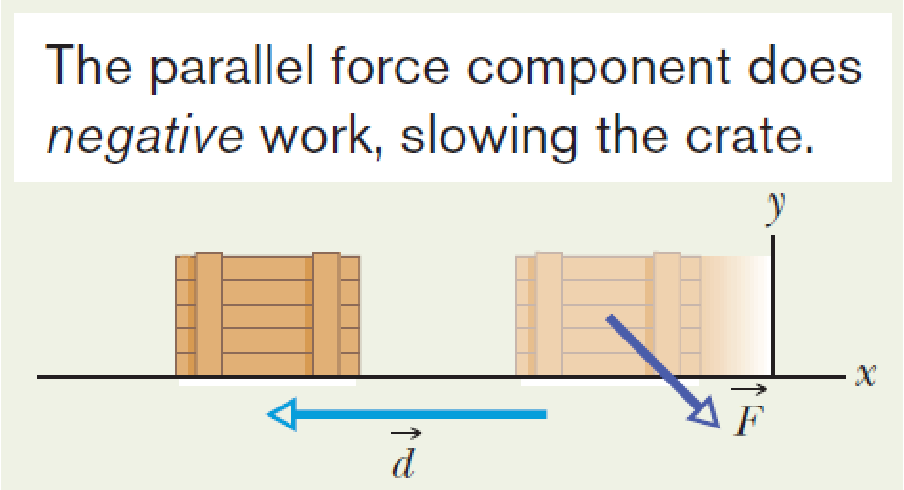
4. Work and Kinetic Energy

Example 4: A crate is sliding across a frictionless surface through a displacement $\vec{d} = (-3.0\,m)\hat{\imath}$ while a steady force pushes against the crate with a force $\vec{F} = (2.0\text{ N})\hat{\imath} + (-6.0\text{ N})\hat{\jmath}$?

(a) How much work does this force do on the crate during the displacement?

$$W = \vec{F} \cdot \vec{d} = [(-3.0\,m)\hat{\imath}] \cdot [(2.0\text{ N})\hat{\imath} + (-6.0\text{ N})\hat{\jmath}]$$
$$= -6.0\text{ J}.$$

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

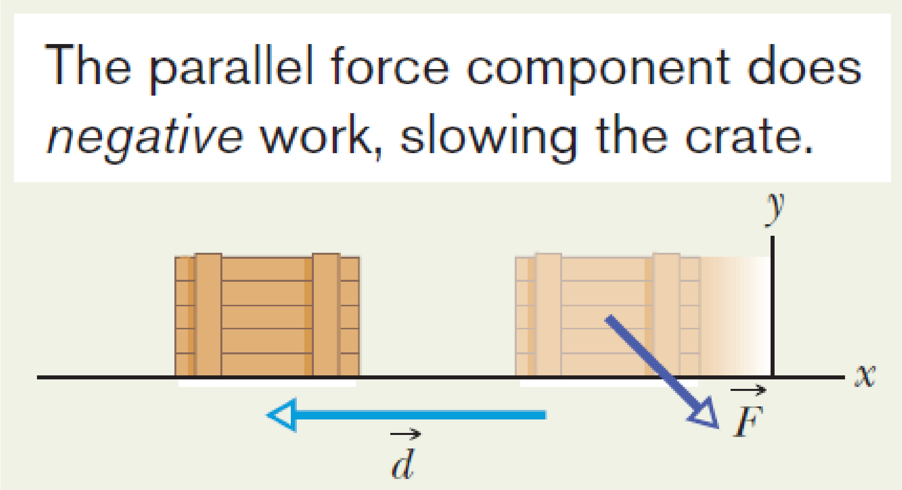


4. Work and Kinetic Energy

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

Using the work–kinetic energy theorem

$$K_f = K_i + W = 10 \text{ J} + (-6.0 \text{ J})$$
$$= 4.0 \text{ J}.$$



5. Work Done by the Gravitational Work

- Consider an object of mass m, thrown up with initial speed v_0. Its kinetic energy is $K_i = \frac{1}{2}mv_0^2$. As the object rises, it is slowed by the gravitational force $\vec{F}_g$; the object's kinetic energy decreases because $\vec{F}_g$ does work on the object as it rises.Using $W = Fd\cos\phi$ with $F = F_g = mg$ we express the express the work W_g done by a gravitational force $\vec{F}_g$ during a displacement $\vec{d}$ as

$$W_g = mgd\cos\phi\,.$$

- For a rising object, $\phi = 180°$ and

$$W_g = mgd(-1) = -mgd.$$

  $\vec{F}_g$ transfers energy in the amount mgd from the kinetic energy of the object. This is consistent with the slowing down of the object as it rises.

5. Work Done by the Gravitational Work

- When an object is falling down, $\phi = 0°$ and

$$W_g = mgd(+1) = +mgd.$$

  $\vec{F}_g$ transfers energy in the amount mgd to the kinetic energy of the object. This is consistent with the speeding up of the object as it falls.

- <u>Work Done in Lifting and Lowering an Object:</u>

  Suppose we lift the object by applying a vertical upward force $\vec{F}$ to it. During the upward displacement, $\vec{F}$ does positive work W_a on the object while the gravitational force does a negative work W_g on it. $\vec{F}$ transfers energy to the object while $\vec{F}_g$ transfers energy from it. The change ΔK in kinetic energy is

$$\Delta K = K_f - K_i = W_a + W_g.$$

5. Work Done by the Gravitational Work

$$\Delta K = K_f - K_i = W_a + W_g.$$

This equation is also valid if we lower the object, but then $\vec{F}_g$ transfers energy to the object while $\vec{F}$ transfers energy from it.

In some cases, the object is stationary before and after the lift. Then both K_f and K_i are zero, and the above relation reduces to

$$W_a + W_g = 0,$$

or

$$W_a = -W_g.$$

This result is valid when K_f and K_i are not the same but still equal.

In words, this relation says that the applied force transfers the same amount of energy to the object as the gravitational force transfers from the object.

5. Work Done by the Gravitational Work

We can rewrite the above relation as

$$W_a = -W_g = -mgd\cos\phi\,.$$

If $\vec{d}$ is vertically upward then $\phi = 180°$ and the work done by the applied force equals mgd. If $\vec{d}$ is vertically downward then $\phi = 0°$ and the work done by the applied force equals $-mgd$.

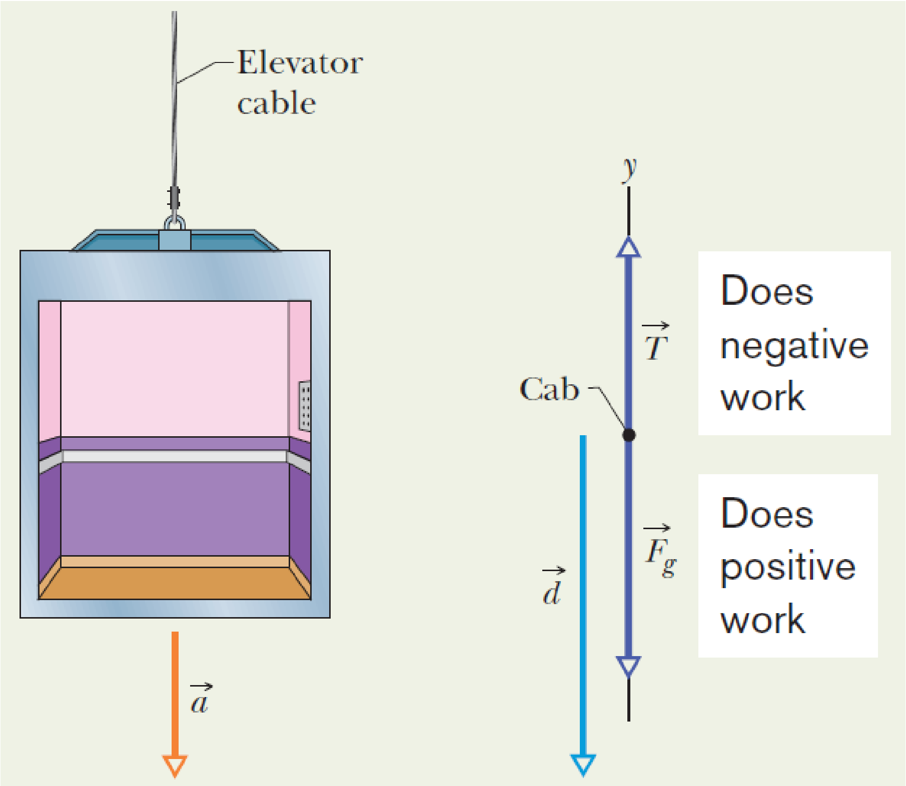
This applies to any situation in which an object is lifted or lowered of $K_f = K_i$. It is independent of the magnitude of the applied force. For example, if you lift a box from the ground to a height d, your force varies during the lift, but the work your force does is mgd.

5. Work Done by the Gravitational Work

Example 5: An elevator cab of mass $m = 500\,\text{kg}$ is descending with speed $v_i = 4.0\,\text{m/s}$ when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = -g/5$.

(a) During the fall through a distance $d = 12\,\text{m}$, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

$$\begin{aligned} W_g &= mgd\cos\phi \\ &= (500\,\text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(12\,\text{m})(\cos 0°) \\ &= 5.9 \times 10^4\,\text{J}. \end{aligned}$$



5. Work Done by the Gravitational Work

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull of the elevator cable?

Newton's 2nd law ($F_y = ma_y$) gives

$$T - F_g = -ma,$$

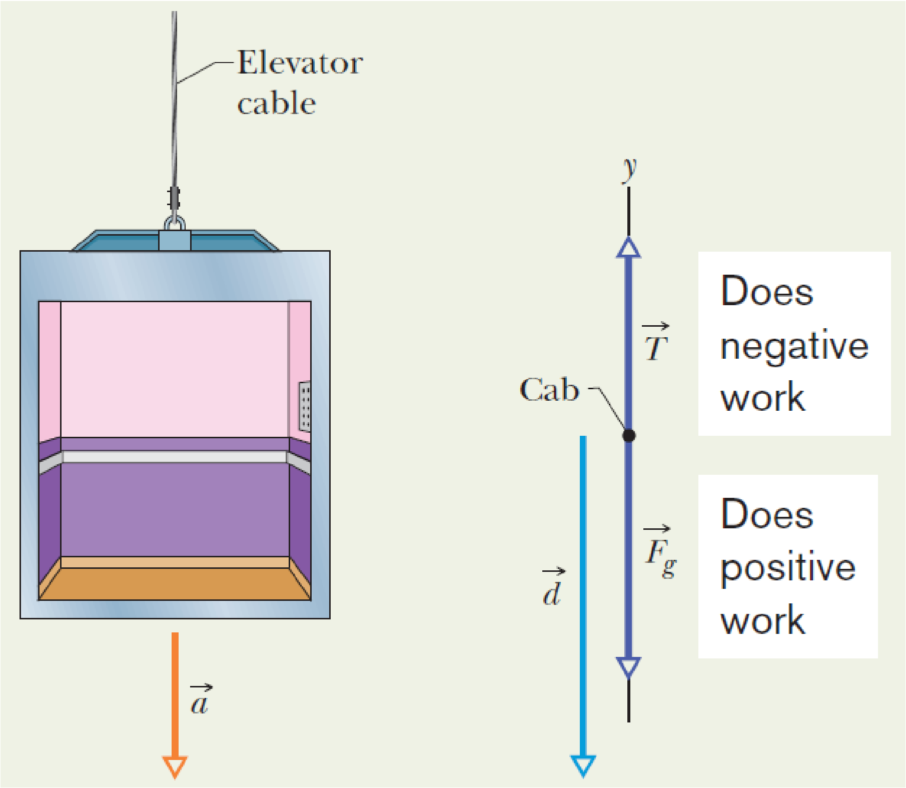
or

$$T = F_g - ma = mg - m\left(\frac{g}{5}\right) = \frac{4}{5}mg.$$

Using $W = Fd\cos\phi$ for T gives

$$W_T = Td\cos 180° = -\frac{4}{5}mgd$$

$$= -\frac{4}{5}(500\ \text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)(12\ \text{m}) = -4.7\times10^4\ \text{J}$$



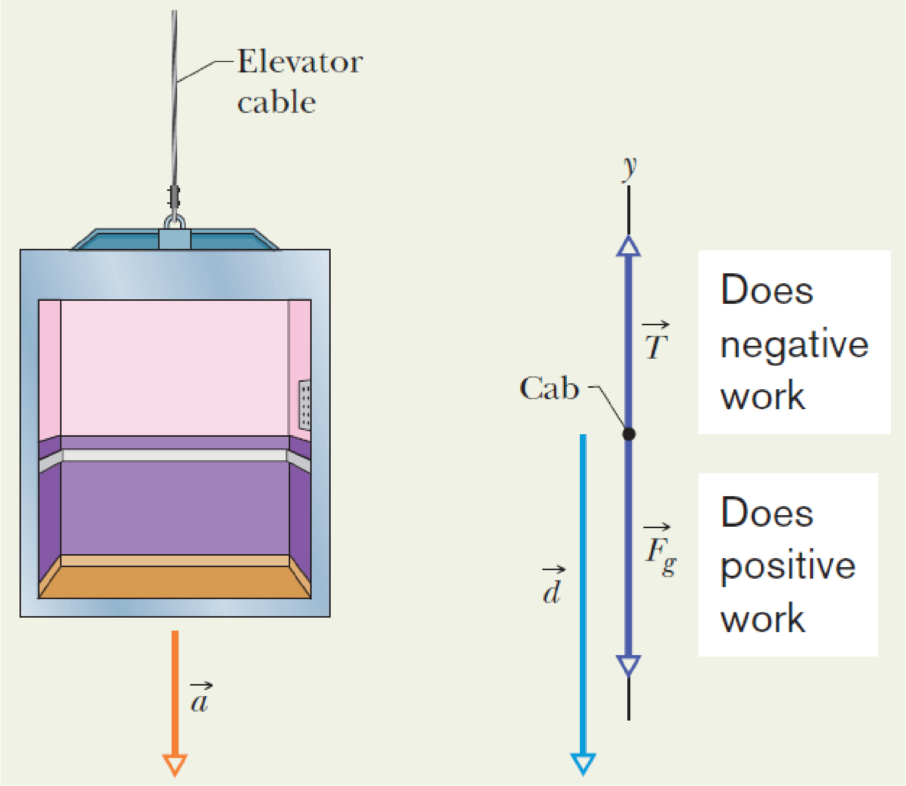
5. Work Done by the Gravitational Work

(c) What is the net work W done on the cab during the fall?

$$\begin{aligned} W &= W_g + W_T \\ &= 5.9 \times 10^4\ \mathrm{J} - 4.7 \times 10^4\ \mathrm{J} \\ &= 1.2 \times 10^4\ \mathrm{J}. \end{aligned}$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

$$\begin{aligned} K_f &= K_i + W = \frac{1}{2}mv_i^2 + W \\ &= \frac{1}{2}(500\ \mathrm{kg})(4.0\ \mathrm{m/s})^2 + 1.2 \times 10^4\ \mathrm{J} \\ &= 1.6 \times 10^4\ \mathrm{J}. \end{aligned}$$

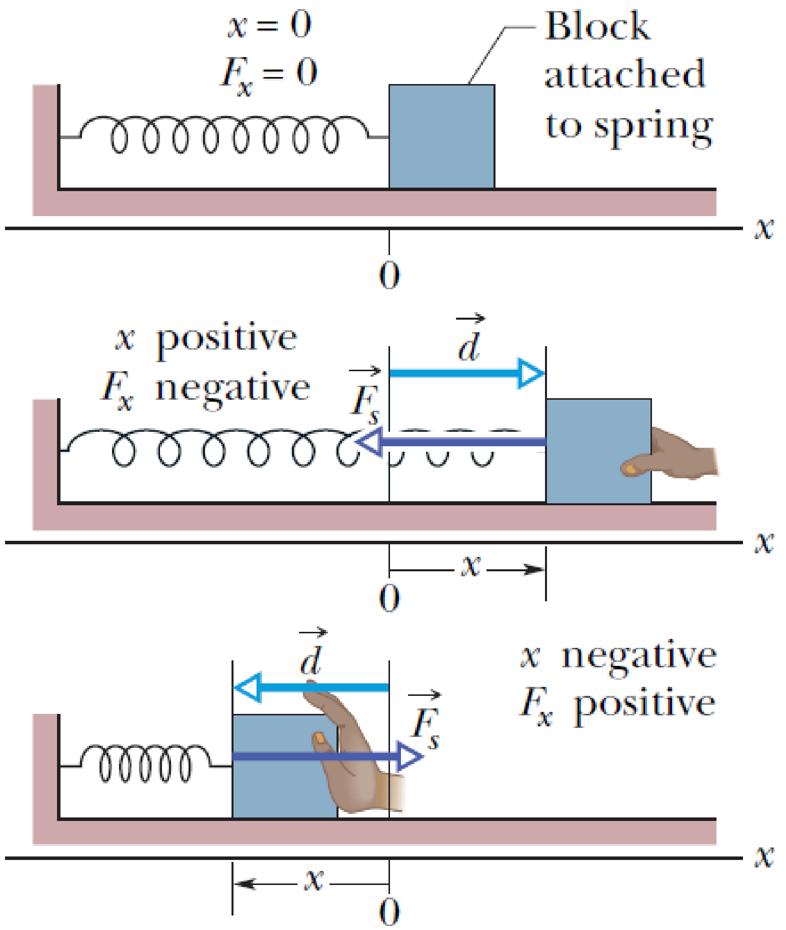


6. Work Done by a Spring

- Now we want to examine the work done by a particular *variable force*, the spring force. Many other forces in nature have the same mathematical form as the spring force.

- <u>The Spring Force</u>:

  The figure show a spring in its relaxed state; neither compressed not extended. One end is fixed, and a particle-like object is attached to the other free end. If we stretch the spring by pulling the block to the right, the spring pulls on the block back toward the left. If we compress the spring by pushing the block toward the left, the spring pushes on the block toward the right.



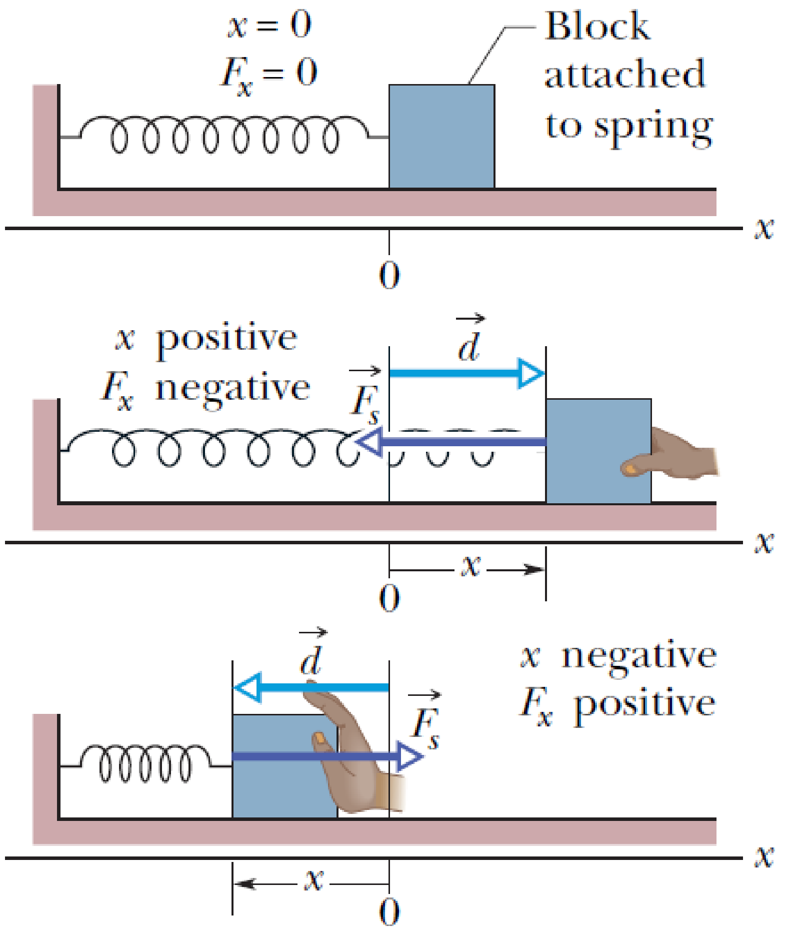
6. Work Done by a Spring

To a good approximation, the force $\vec{F}_s$ from a spring is proportional to the displacement $\vec{d}$ of the free end from the relaxed position. The spring force is given by

$$\vec{F}_s = -k\vec{d},$$

which is known as Hooke's law. The minus sign indicates that the direction of $\vec{F}_s$ is opposite to that of $\vec{d}$. The constant k is called **the spring constant** (or force constant); it is a measure of the stiffness of the spring.

What is the SI unit of k?

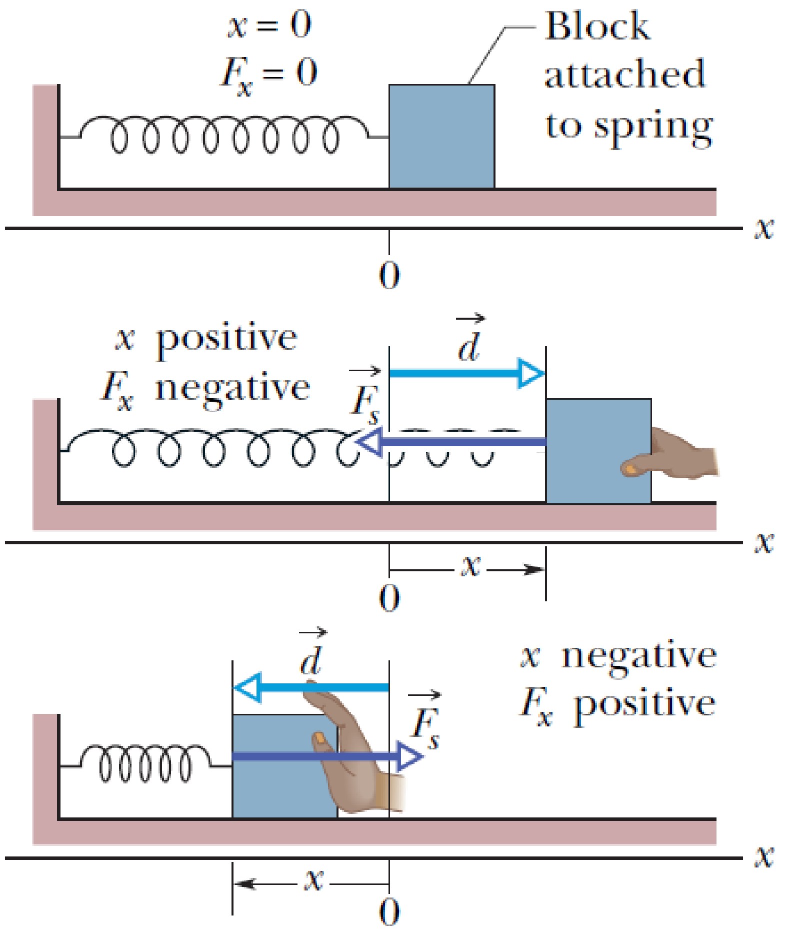


6. Work Done by a Spring

A common arrangement is when the length of the spring is parallel to the x axis, and the origin ($x = 0$) at the position of the free end when the spring is relaxed. Hooke's law in this case is written as

$$F_x = -kx.$$

Note that F_x is a *variable force* because it is a function of x; $F_x(x)$. Additionally, F_x is *linear* in x.



6. Work Done by a Spring

- <u>Work Done by a Spring Force</u>:

  We make two simplifying assumptions about the spring. (1) It is massless (its mass is negligible relative to the block's mass) and (2) it is an ideal spring (obeys Hooke's law).

  We give the block a rightward push. As it moves rightward, the spring force F_x does work on the block, decreasing the kinetic energy and slowing the block. However we cannot use $W = Fd\cos\phi$ since the spring force is a variable force.

  We use calculus to find the work done by a spring. (See page 160 for details.)

  The work W_s done by a spring force on a block moved from an initial position x_i to a final position x_f is

$$W_s = \int_{x_i}^{x_f} -|F_x|\, dx\,.$$

6. Work Done by a Spring

Substituting kx for $|F_x|$ leads to

$$W_s = \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx = \left(-\frac{1}{2}k\right)[x^2]_{x_i}^{x_f} = \left(-\frac{1}{2}k\right)\left(x_f^2 - x_i^2\right).$$

Rearranging yields

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2.$$

W_s can be positive or negative, depending on whether the net transfer of energy is to or from the block as it moves from x_i to x_f.

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

6. Work Done by a Spring

If $x_i = 0$ and we call the final position x we have

$$W_s = -\frac{1}{2}kx^2.$$

- The Work Done by an Applied Force:

Suppose that we displace the block while continuing to apply a force $\vec{F}_a$ to it. During the displacement, our applied force does work W_a on the block while the spring force does work W_s. The change ΔK in kinetic energy of the block due to these energy transfers is

$$\Delta K = K_f - K_i = W_a + W_s.$$

If the block is stationary before and after the displacement ($K_f = K_i = 0$) then

$$W_a = -W_s.$$

6. Work Done by a Spring

$$W_a = -W_s.$$

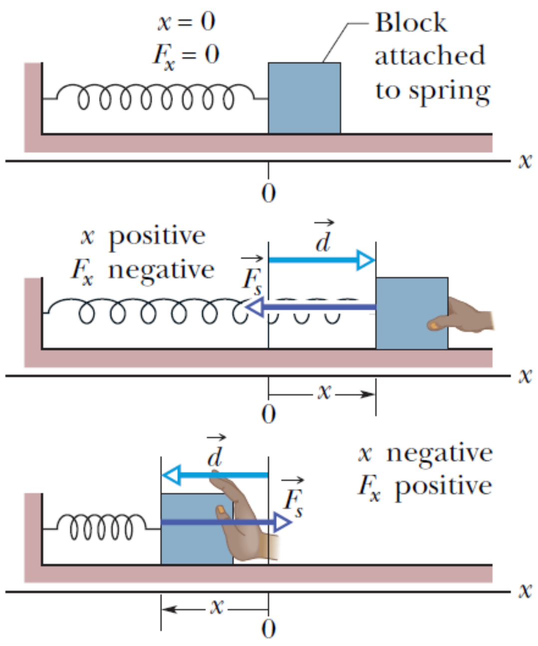
If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

6. Work Done by a Spring

CHECKPOINT 2

For three situations, the initial and final positions, respectively, along the x axis for the block in Fig. 7-9 are (a) -3 cm, 2 cm; (b) 2 cm, 3 cm; and (c) -2 cm, 2 cm. In each situation, is the work done by the spring force on the block positive, negative, or zero?

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$



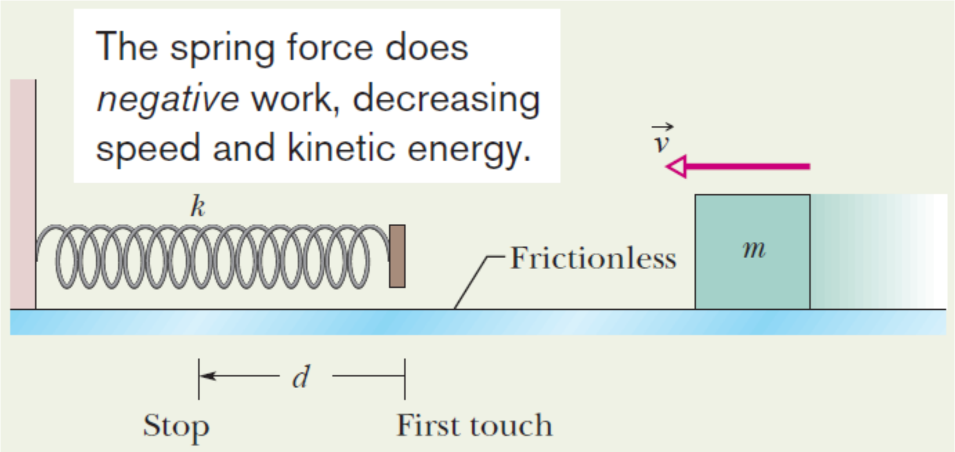
(a) $x_i^2 > x_f^2$; W_s is positive.

(b) $x_i^2 < x_f^2$; W_s is negative.

(c) $x_i^2 = x_f^2$; W_s is zero.

6. Work Done by a Spring

Example 6: a block of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.40$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the block is momentarily stopped by the spring, by what distance d is the spring compressed?



The work W_s done by the spring force on the block is related to the change ΔK in the block's kinetic energy by

$$\Delta K = K_f - K_i = W_s,$$

or

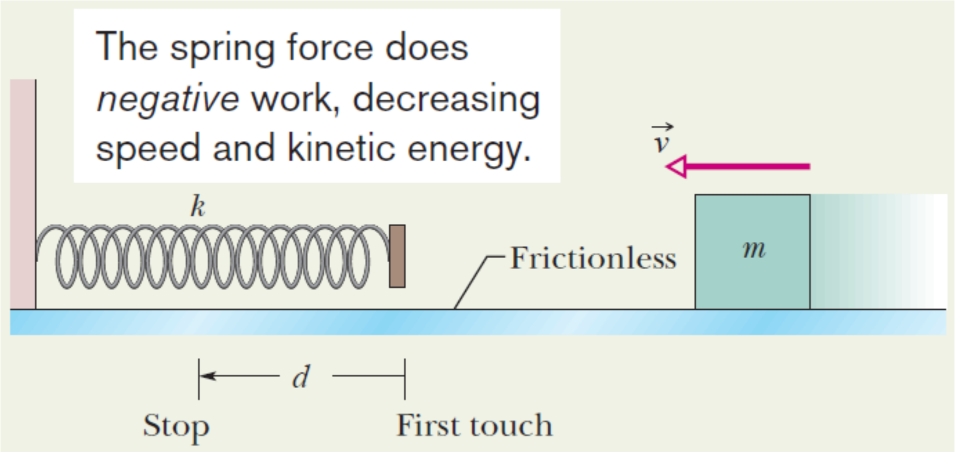
$$0 - \frac{1}{2}mv^2 = -\frac{1}{2}kd^2.$$

6. Work Done by a Spring

Solving for d and substituting

$$d = v\sqrt{\frac{m}{k}} = (0.40\ \mathrm{m/s})\sqrt{\frac{0.40\ \mathrm{kg}}{750\ \mathrm{N/m}}}$$

$$= 1.2 \times 10^{-2}\ \mathrm{m}.$$



7. Work Done by a General Variable Force

- <u>One-Dimensional Analysis</u>:

  Consider the situation of the bead in the wire, but with the force to be in the positive x direction and the force magnitude to vary with position x.

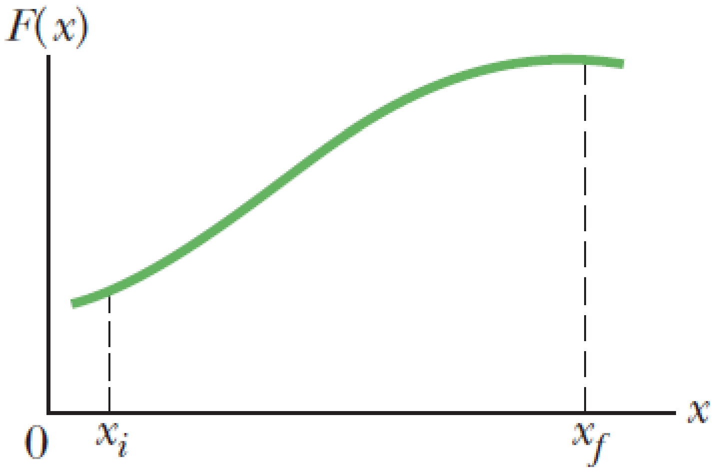
  We want to find the work done on the particle by this force as the particle moves from initial position x_i to final position x_f. We cannot use $W = Fd\cos\phi$ because $\vec{F}$ is a variable force.

  We need to use calculus. (See page 163 for details.)

  The work done in this case is given by

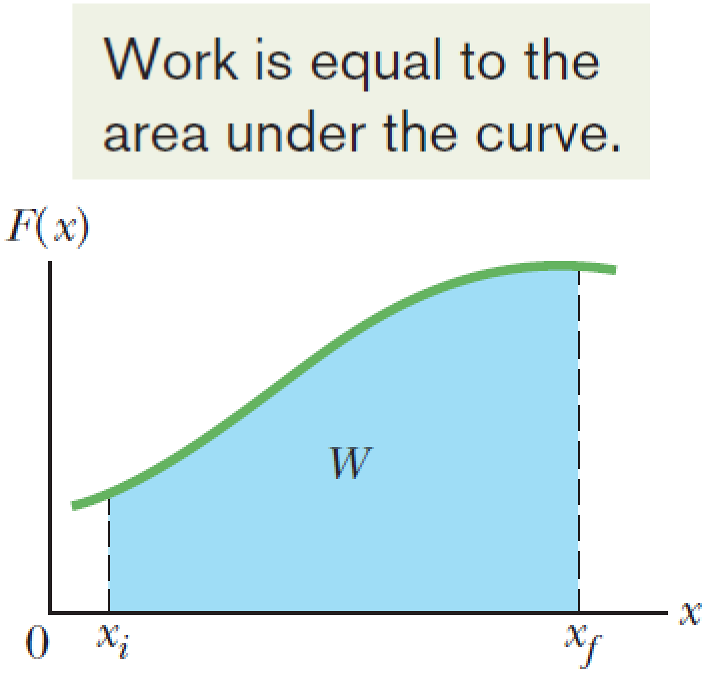
$$W = \int_{x_i}^{x_f} F(x)\, dx\,.$$

Work is equal to the area under the curve.



7. Work Done by a General Variable Force

Geometrically, the work is equal to the area between the $F(x)$ curve and the x axis, between x_i and x_f.



- <u>Work-Kinetic Energy Theorem with a Variable force</u>:

Let us make certain that the work is equal to the change in kinetic energy. Consider a particle of mass m, moving along an x axis acted on by a net force $F(x)$ that is directed along that axis. The work is

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} ma\,dx\,.$$

We can write

$$ma\,dx = m\frac{dv}{dt}dx.$$

7. Work Done by a General Variable Force

Using the chain rule of calculus, we have

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v,$$

which gives

$$ma\,dx = mv\,dv.$$

The integral becomes

$$W = m\int_{v_i}^{v_f} v\,dv = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$= K_f - K_i = \Delta K,$$

which is the work-kinetic energy theorem.

7. Work Done by a General Variable Force

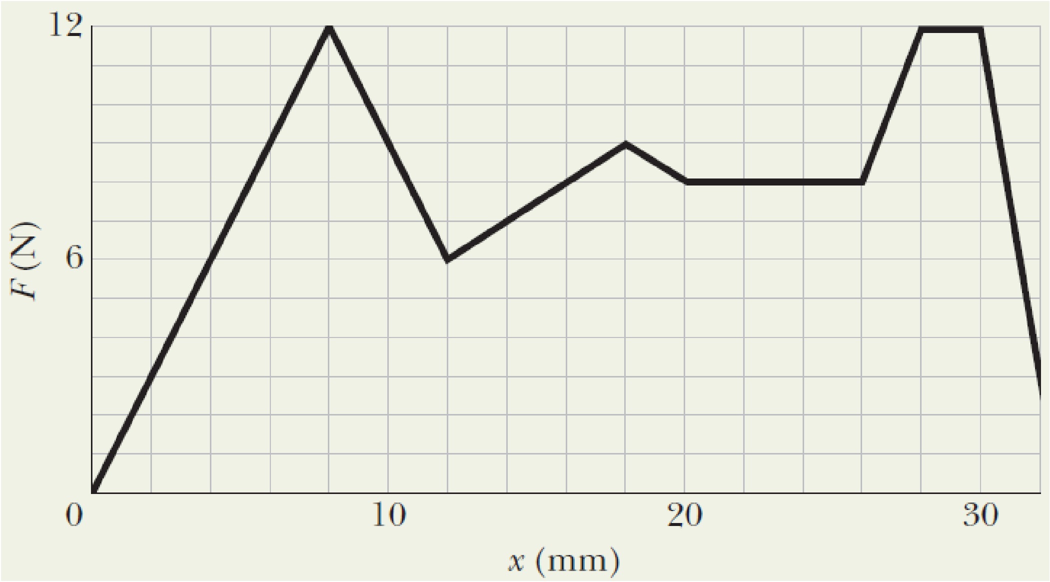
- Three-Dimensional Analysis:

  The work W done by a three-dimensional force $\vec{F} = F_x\,\hat{\mathrm{i}} + F_y\,\hat{\mathrm{j}} + F_z\,\hat{\mathrm{k}}$ while the particle moves from an initial position r_i having coordinates $(x_i,\, y_i,\, z_i)$ to a final position r_f having coordinates $(x_f\,,\, y_f\,,\, z_f)$ is

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z\, dz\,.$$

7. Work Done by a General Variable Force

Example 7: The graph shows the applied force magnitude F versus displacement x for a particle. How much work W is done by the force exerted on the particle between $x = 0$ and $x = 30\text{ mm}$?



The work done by a variable force is given by

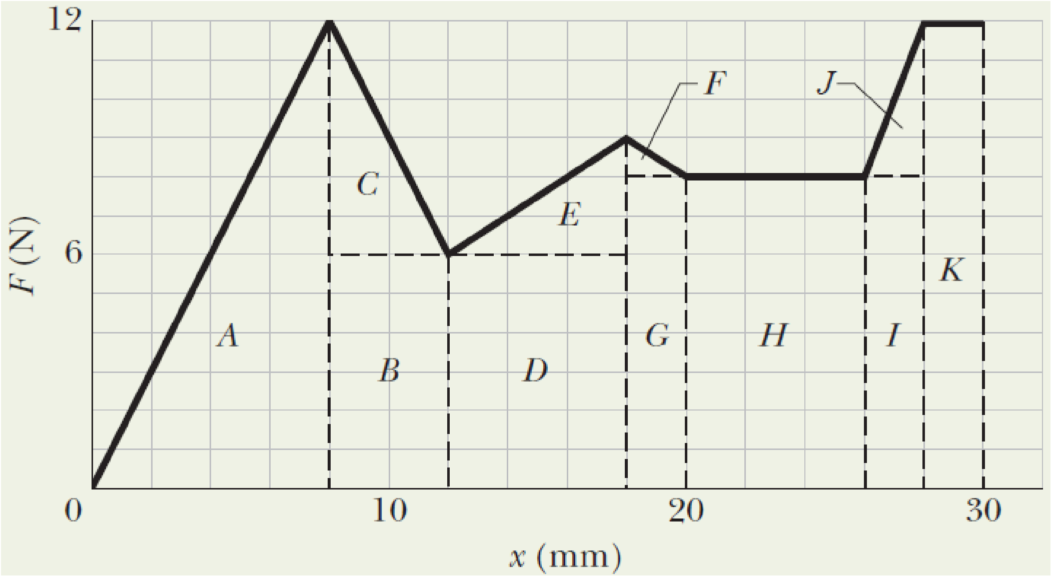
$$W = \int_{x_i}^{x_f} F(x)\, dx$$

$$= \begin{pmatrix} \text{area between force curve} \\ \text{and } x \text{ axis, from } x_i \text{ to } x_f \end{pmatrix}.$$

7. Work Done by a General Variable Force

To find the area under the curve, we divide it into rectangles and triangles and calculate the area of each. For example, the area of triangle A is

$$\text{area}_A = \frac{1}{2}(0.008\ \text{m})(12\ \text{N}) = 0.048\ \text{J}.$$



Working all the subareas we find that

$$\begin{aligned} W &= 0.048 + 0.024 + 0.012 + 0.036 \\ &\quad +0.009 + 0.001 + 0.016 + 0.048 \\ &\quad +0.016 + 0.004 + 0.024 \\ &= 0.238\ J. \end{aligned}$$

7. Work Done by a General Variable Force

Example 8: A force $F(x) = 3x^2\ \mathrm{N}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from $x = 2\ \mathrm{m}$ to $x = 3\ \mathrm{m}$? Does the speed of the particle increase, decrease, or remain the same?

The work done by this variable force is given by

$$W = \int_{x_i}^{x_f} F(x)\, dx = \int_{2}^{3} 3x^2\, dx = 3\left(\frac{1}{3}[x^3]_2^3\right) = 3^3 - 2^3 = 19\ \mathrm{J}.$$

The positive work means that energy is transferred to the particle increasing its speed.

8. Power

Power is the time rate at which work is done by a force. If a force does an amount of work W in an amount of time Δt, the average power due to the work during that time interval is

$$P_{avg} = \frac{W}{\Delta t}.$$

The **instantaneous power** P is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt}.$$

The SI unit of power is joule per second. It is given the special name **watt** (W), named after James Watt (1736-1819). A common unit of power is the horsepower:

$$1\ \text{horsepower} = 1\ \text{hp} = 746\ \text{W}.$$

8. Power

Work can be expressed as the power multiplied by time, as in the common unit of kilowatt-hour:

$$1 \text{ kilowatt-hour} = 1\ \mathrm{kW \cdot h} = (10^3\ \mathrm{W})(3600\ \mathrm{s})$$
$$= 3.60 \times 10^6\ \mathrm{J} = 3.60\ \mathrm{MJ}.$$

We can also express the power a constant force does on a particle in terms of that force and the particle's velocity:

$$P = \frac{dW}{dt} = \frac{F\cos\phi\, dx}{dt} = F\cos\phi\left(\frac{dx}{dt}\right) = Fv\cos\phi\,,$$

or

$$P = \vec{F}\cdot\vec{v}.$$

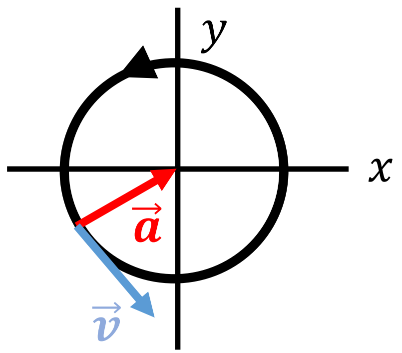
8. Power

CHECKPOINT 3

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

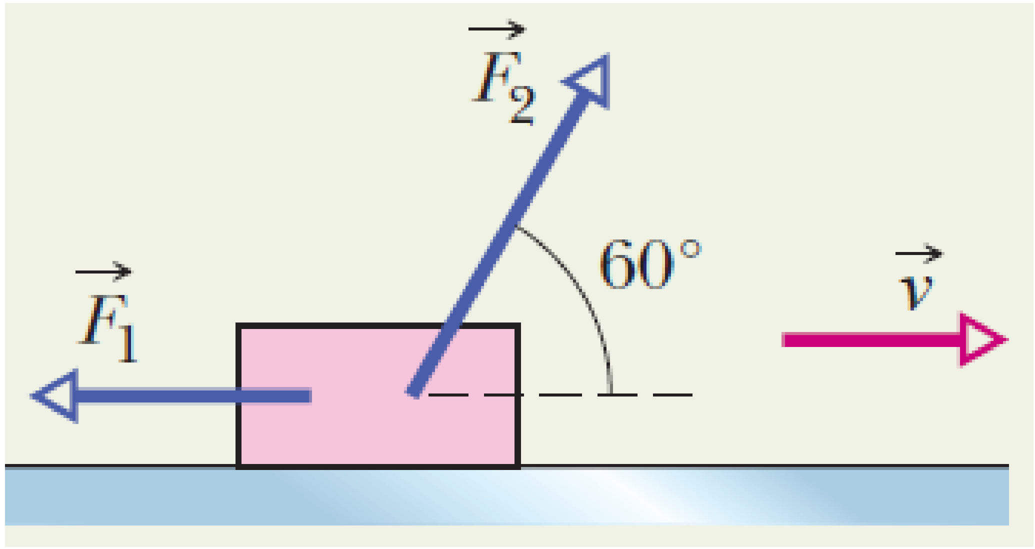
$$P = \vec{F} \cdot \vec{v}$$

Zero, since $\vec{F}$ and $\vec{v}$ are perpendicular.



8. Power

Example 9: Two constant forces $\vec{F}_1$ and $\vec{F}_2$ acting on a box as the box slides rightward across a frictionless floor. Force $\vec{F}_1$ is horizontal, with magnitude $2.0\ \mathrm{N}$; force $\vec{F}_2$ is angled upward by $60°$ to the floor and has magnitude $4.0\ \mathrm{N}$. The speed v of the box at a certain instant is $3.0\ \mathrm{m/s}$. What is the power due to each force acting on the box at that instant, and what is the net power?



8. Power

The power due to $\vec{F}_1$ is

$$P_1 = \vec{F}_1 \cdot \vec{v} = F_1 v \cos\phi_1$$

$$= (2.0\ \mathrm{N})\left(3.0\,\frac{\mathrm{m}}{\mathrm{s}}\right)\cos 180° = -6.0\ \mathrm{W}.$$

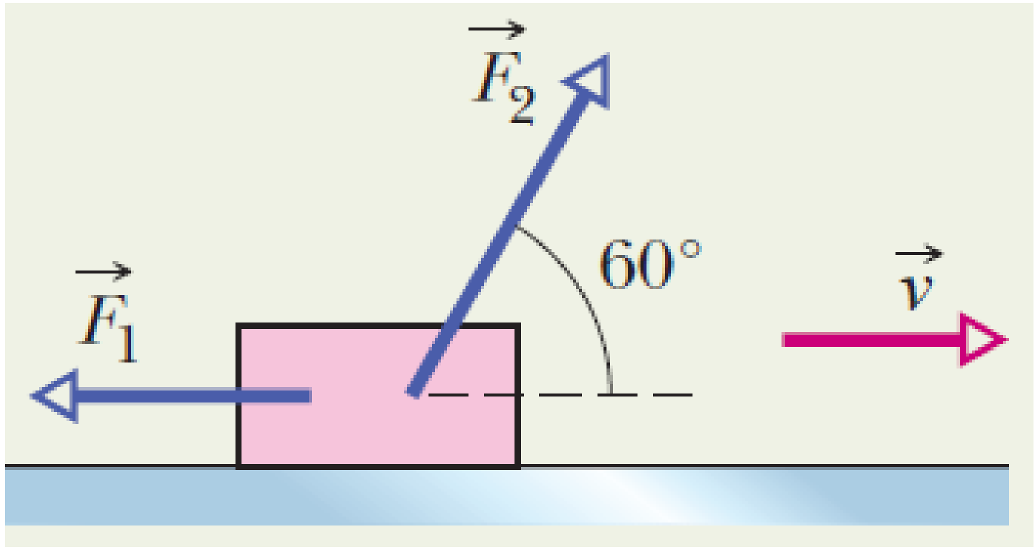
The power due to $\vec{F}_2$ is

$$P_2 = \vec{F}_2 \cdot \vec{v} = F_2 v \cos\phi_2$$

$$= (4.0\ \mathrm{N})\left(3.0\,\frac{\mathrm{m}}{\mathrm{s}}\right)\cos 60° = 6.0\ \mathrm{W}.$$

Therefore,

$$P_{\mathrm{net}} = P_1 + P_2 = -6.0\ \mathrm{W} + 6.0\ \mathrm{W} = 0$$



8. Power

Is the net power changing at that instant?

$P_{\text{net}} = 0$ tells us that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy of the box is not changing, and so the speed of the box will remain the same. With neither the forces not the velocity changing, we see that P_{net} is constant.