Chapter 6

FORCE AND MOTION II

Friction

- Here we deal with the frictional forces that exist between dry surfaces, either stationary relative to each other or moving across each other at slow speeds.
- Consider the following thought experiments:
  1. A block sliding across a long horizontal counter slows down and then stops. The block encountered acceleration in the direction opposite to the block's velocity. By Newton's 2nd law, a force acted on the block parallel to the counter surface, opposite to the velocity direction. The force is friction.
  2. Push horizontally on the book to make it move at a constant velocity. The net force on the book is zero since it is not accelerating, by Newton's 2nd law. There must be another force opposing and balancing your force. That is the force of friction.

Friction

3. Push horizontally on a crate. The crate does not move which means that a second force, the force of friction, is opposing your push. Push harder. The crate still does not move. The frictional force can change itself to balance the new push. Push as hard as you can. The crate starts to move. There is maximum magnitude of frictional force. The crate accelerates when you exceed it.

- The frictional force that prevents the block from moving is called the **static frictional force** $\vec{f}_s$.
- When the external force exceeds the maximum of $\vec{f}_s$, the block breaks away from its contact with the surface and accelerate. The new weaker frictional force that opposes the motion is called the **kinetic frictional force**.

Read the text for the nature of friction.

Properties of Friction

- When a dry unlubricated body presses against a surface in the same condition and a force $\vec{F}$ attempts to slide the body along the surface, the resulting frictional force has properties:
  1. If the body does not move, the static friction $\vec{f_s}$ and the component of $\vec{F}$ along the surface balance each other.
  2. The magnitude of $\vec{f_s}$ has a maximum value $f_{s,max}$, given by

     $$f_{s,max} = \mu_s F_N,$$

     where μ_s is the coefficient of static friction and F_N is magnitude of the normal force.

     If the component of $\vec{F}$ along the surface is greater than $\vec{f}_{s,max}$ the body starts to move.

Properties of Friction

3. After the body starts to slide, the frictional force decreases to a constant value f_k,

$$f_k = \mu_k F_N,$$

where μ_k is the coefficient of kinetic friction.

- Both $f_{s,max}$ and f_k are directly proportional to F_N or how hard the body presses against the surface.
- The above equations are not vector equations: $\vec{f}_k$ and $\vec{f}_s$ are perpendicular to $\vec{F}_N$.
- Both μ_k and μ_s are determined experimentally between two particular surfaces (e.g. rubber and wood).

Properties of Friction

CHECKPOINT 1

A block lies on a floor. (a) What is the magnitude of the frictional force on it from the floor? (b) If a horizontal force of 5 N is now applied to the block, but the block does not move, what is the magnitude of the frictional force on it? (c) If the maximum value $f_{s,\max}$ of the static frictional force on the block is 10 N, will the block move if the magnitude of the horizontally applied force is 8 N? (d) If it is 12 N? (e) What is the magnitude of the frictional force in part (c)?

(a) Zero.

(b) 5 N, since $\vec{F}_{net} = 0$.

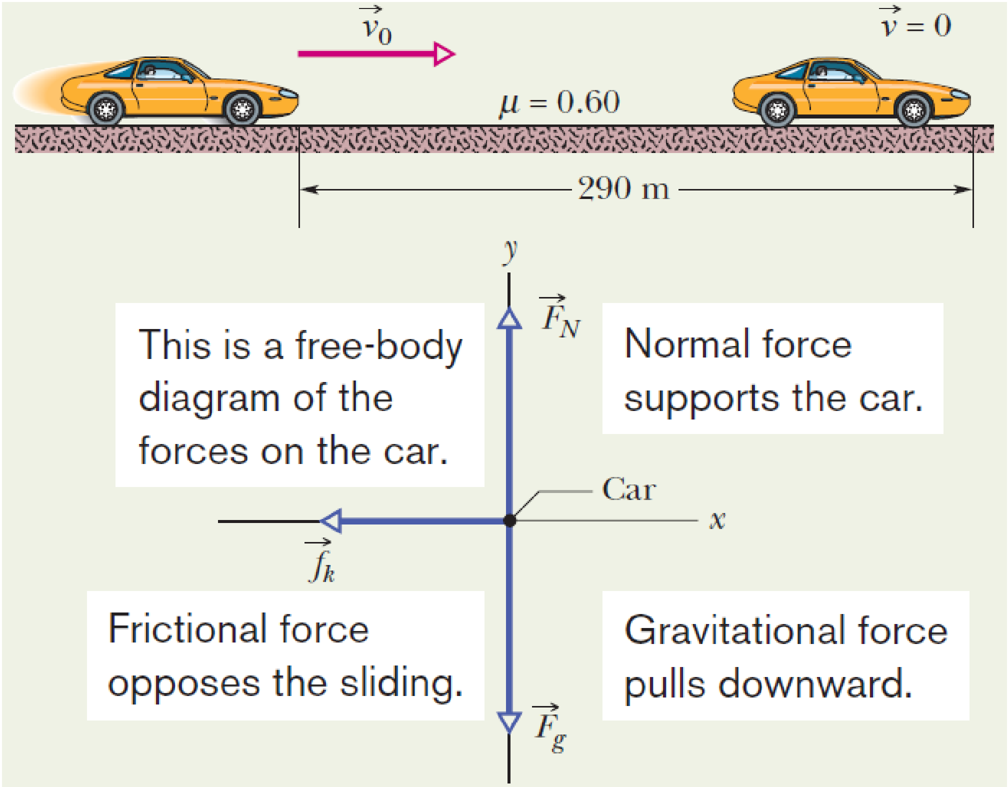
(c) No.

(d) Yes.

(e) 8 N.

Properties of Friction

Example 1: A car slid along a road with its wheels locked (not rolling) for a distance of 290 m. Assuming that $\mu_k = 0.60$ and the car's acceleration was constant during the braking, how fast was the car going when the wheels became locked?



We use Newton's 2nd law to relate the force of friction to the acceleration:

$$F_{net,x} = ma_x,$$

or

$$-f_k = ma_x.$$

Properties of Friction

Using that $f_k = \mu_k F_N = \mu_k mg$ we get

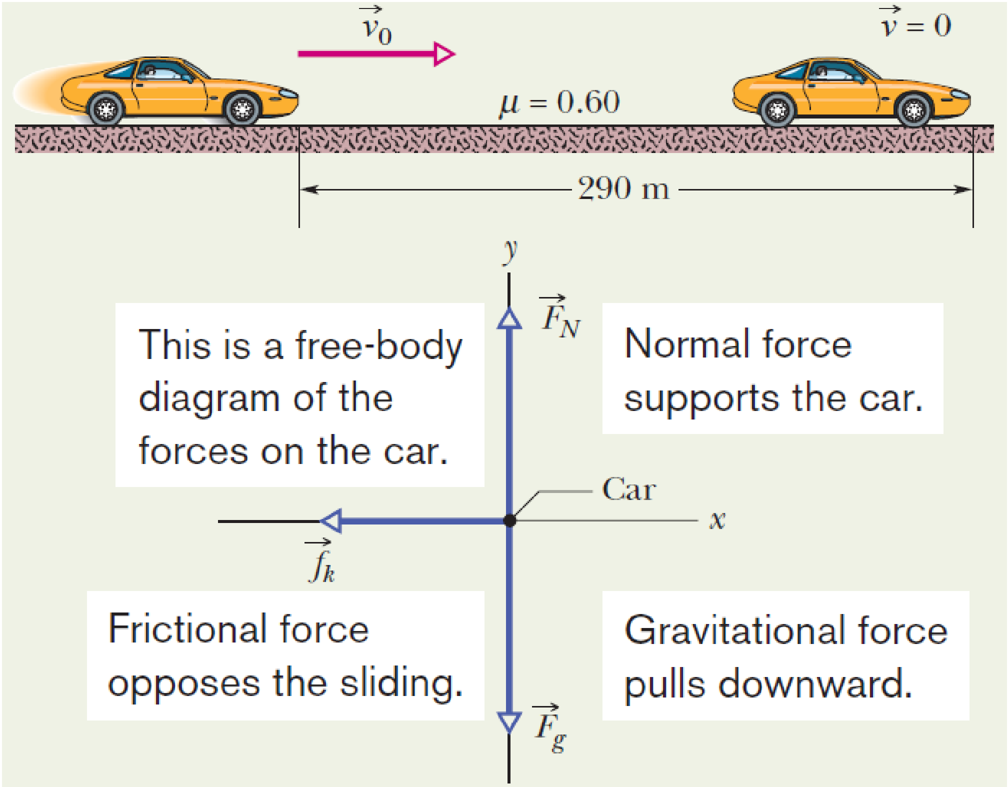
$$-\mu_k mg = ma_x,$$

which gives

$$a_x = -\mu_k g.$$

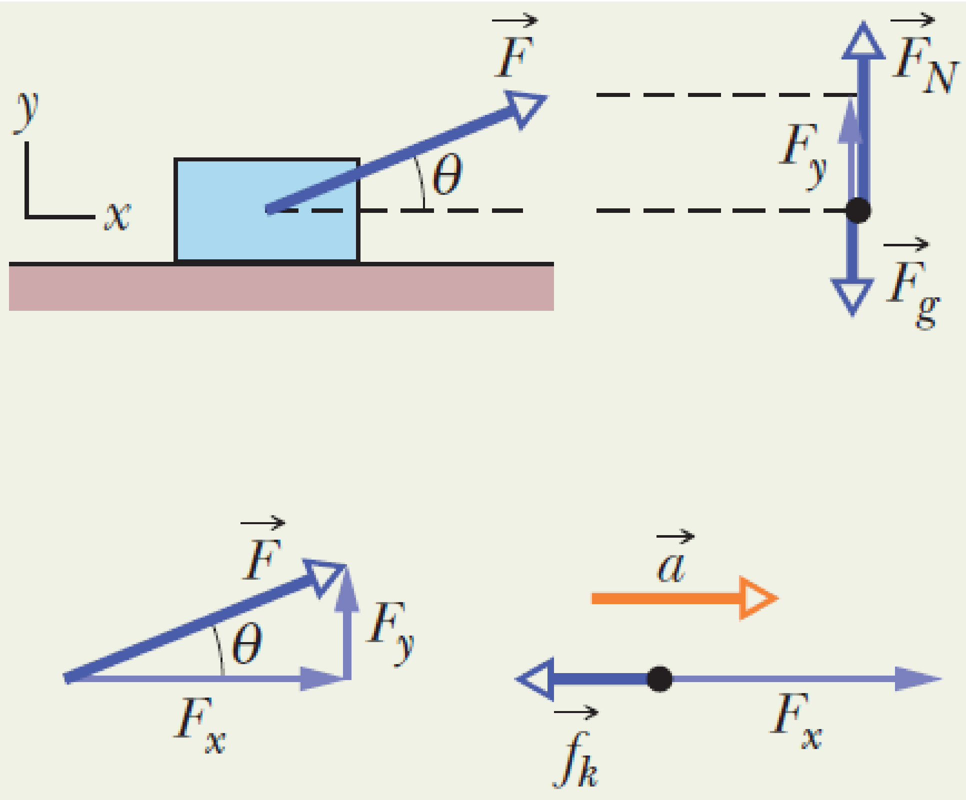
Then using $v^2 = v_0^2 + 2a_x(x - x_0)$ with $v = 0$ we get

$$v_0 = \sqrt{2\mu_k g(x - x_0)}$$
$$= \sqrt{2(0.60)(9.8\ \mathrm{m/s^2})(290\ \mathrm{m})}$$
$$= 58\ \mathrm{m/s} = 210\ \mathrm{km/h}.$$



Properties of Friction

Example 2: a block of mass $m = 3.0\,\mathrm{kg}$ slides along a floor while a force of magnitude $12.0\,\mathrm{N}$ is applied to it at an upward angle $\theta = 21.8°$. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.40$. What is the block's acceleration magnitude a?



To find f_k we need to find F_N first. Using $F_{net,y} = ma_y$ we have

$$F_N + F\sin\theta - mg = m(0).$$

Therefore, $F_N = mg - F\sin\theta$.

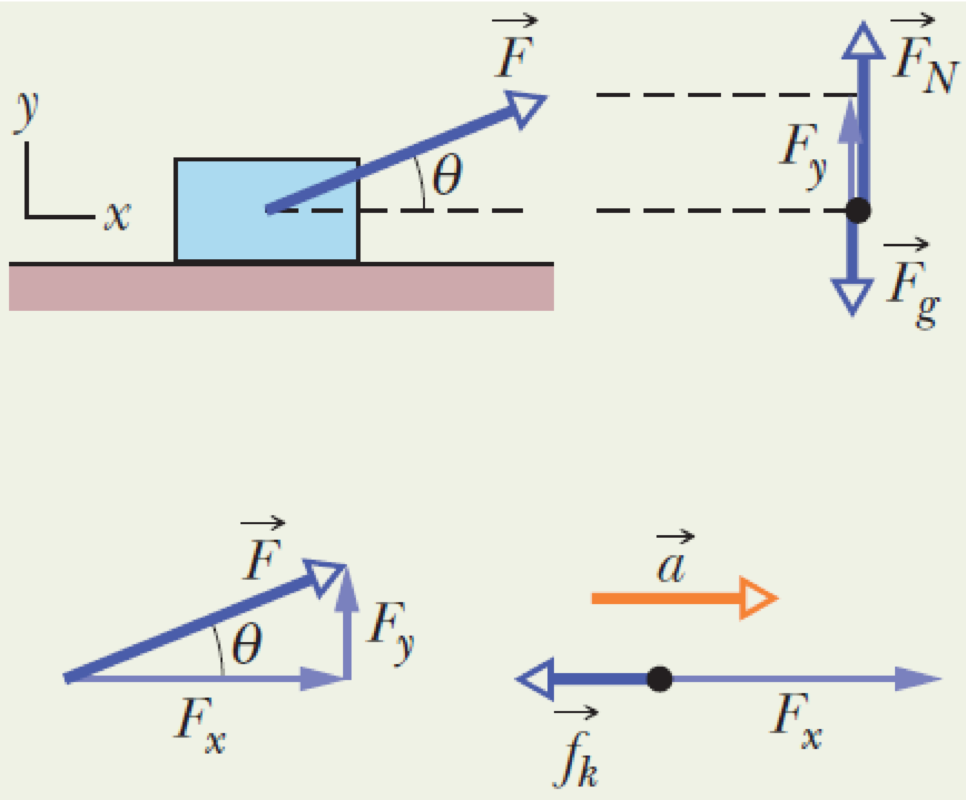
Properties of Friction

Now applying $F_{net,x} = ma_x$ yields

$$F\cos\theta - \mu_k F_N = ma.$$

Solving for a and substituting we have

$$\begin{aligned} a &= \frac{1}{m}[F\cos\theta - \mu_k(mg - F\sin\theta)] \\ &= -\mu_k g + \frac{F}{m}(\cos\theta + \mu_k\sin\theta) \\ &= -(0.40)(9.8\ \mathrm{m/s^2}) \\ &\quad + \frac{12.0\ \mathrm{N}}{3.0\ \mathrm{kg}}(\cos 21.8° + 0.40\sin 21.8°) \\ &= 0.39\ \mathrm{m/s^2}. \end{aligned}$$



Uniform Circular Motion

- Recall from Ch. 4 that a body moving in a circle (or a circular arc) with constant speed v, is in uniform circular motion. The body has a centripetal acceleration of constant magnitude given by

$$a = \frac{v^2}{R},$$

where R is the radius of the circle or a circular arc.

Let us examine 3 examples of uniform circular motion:

1. <u>Rounding a curve in a car</u>: Both the car and the backseat passenger are in uniform circular motion. The centripetal force on the car is the force of static friction on the tires from the road. The centripetal force on the backseat passenger is the normal force from the right wall of the car.

Uniform Circular Motion

2. <u>Orbiting Earth</u>: An astronaut in a space shuttle orbiting Earth is floating. The centripetal force is the gravitational force (on the shuttle and astronaut).

   The floating astronaut is not feeling the gravitational force because it pulls equally on every atom in his body; there is no stretching or compression on any part of the astronaut body.

3. <u>A hockey puck in uniform circular motion</u>: The centripetal force is the tension from the string.

Uniform Circular Motion

- The centripetal force is not a new kind of force. The name merely indicates the direction. The centripetal force can be frictional, gravitational, tension or any other force.
- A centripetal force accelerates a body by changing the direction of the body's velocity, without changing the body's speed.

  Using Newton's 2nd law, the magnitude of the centripetal force is given by

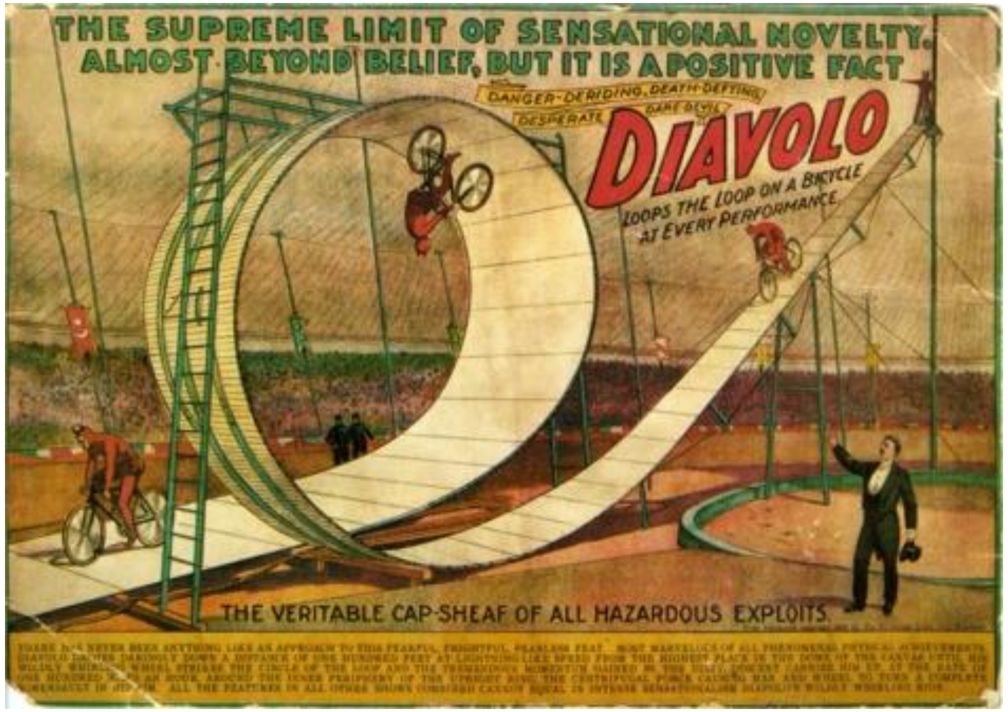
$$F = m\frac{v^2}{R}.$$

- The speed and the magnitudes of acceleration and force are constant.
- The direction of the acceleration and force are changing. They point from the body to the center of the circle.

Properties of Friction

Example 3: Consider a circular loop with radius $R = 2.7\ \mathrm{m}$, what is the least speed v that a biker could have at the top of the loop to remain in contact with it there?

We can assume that the biker and bicycle travel through the top of the loop as a single particle in uniform circular motion. Thus, at the top, the acceleration of this particle must have the magnitude $a = v^2/R$.



Properties of Friction

Newton's 2nd law gives

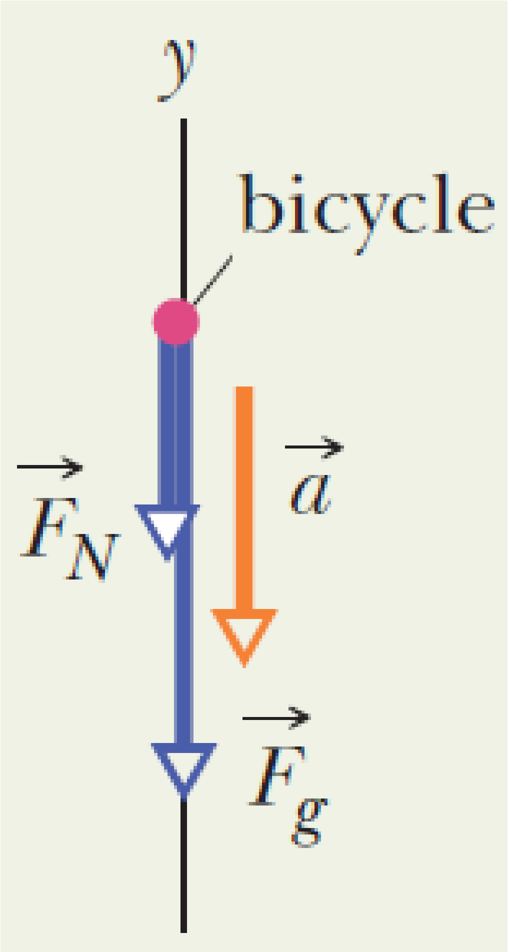
$$-F_N - F_g = m(-a),$$

or

$$-F_N - mg = m\left(-\frac{v^2}{R}\right).$$

Just before loosing contact with the loop $F_N = 0$. Solving for v we get

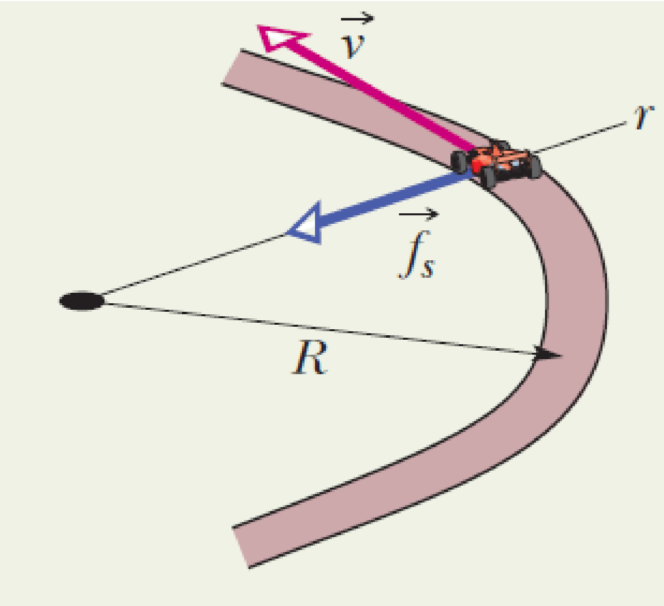
$$v = \sqrt{gR} = \sqrt{(9.8\,\mathrm{m/s^2})(2.7\,\mathrm{m})}$$
$$= 5.1\,\mathrm{m/s}.$$



Properties of Friction

Example 4: A modern race car is designed so that the passing air pushes down on it, allowing the car to travel much faster through a flat turn without friction failing. This downward push is called *negative lift.*

A race car of mass $m = 600\ \mathrm{kg}$ travels on a flat track in a circular arc of radius $R = 100\ \mathrm{m}$. The coefficient of static friction between the tires and the track is 0.75. (Assume that the forces on the four tires are identical.) If the car is on the verge of sliding out of the turn when its speed is $28.6\ \mathrm{m/s}$, what is the magnitude of the negative lift $\vec{F}_L$ acting downward on the car?



The toward-the-center force is the frictional force.

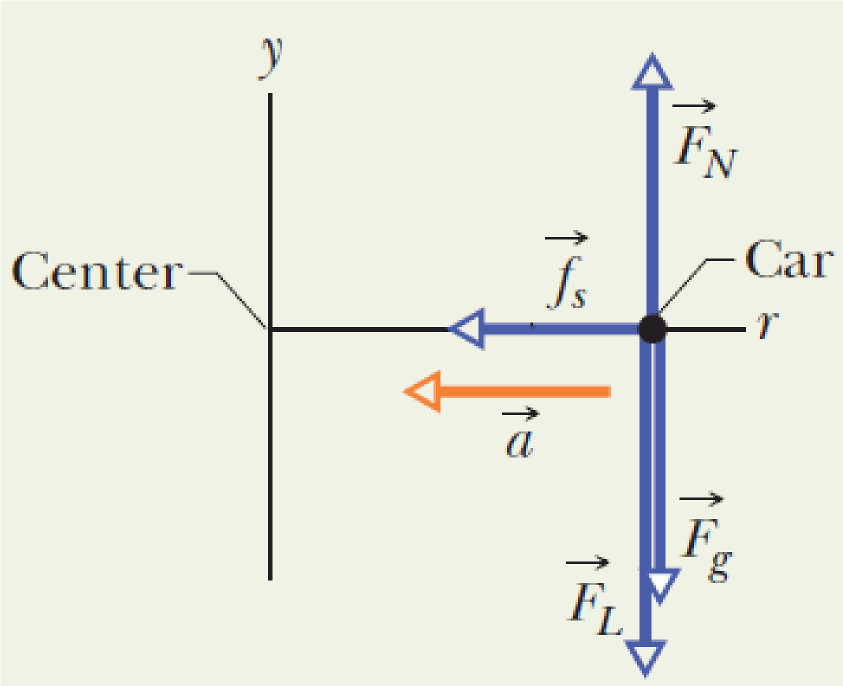
Properties of Friction

The centripetal force is static frictional f_s . Newton's 2nd law along the r axis ($F_{net,r} = ma_r$) gives

$$-f_s = m\left(-\frac{v^2}{R}\right),$$

Because the car is on the verge of sliding $f_s = f_{s,max} = \mu_s F_N$. Hence,

$$-\mu_s F_N = m\left(-\frac{v^2}{R}\right).$$



Properties of Friction

Newton's 2nd law along the y axis ($F_{net,y} = ma_y$) gives

$$F_N - mg - F_L = 0,$$

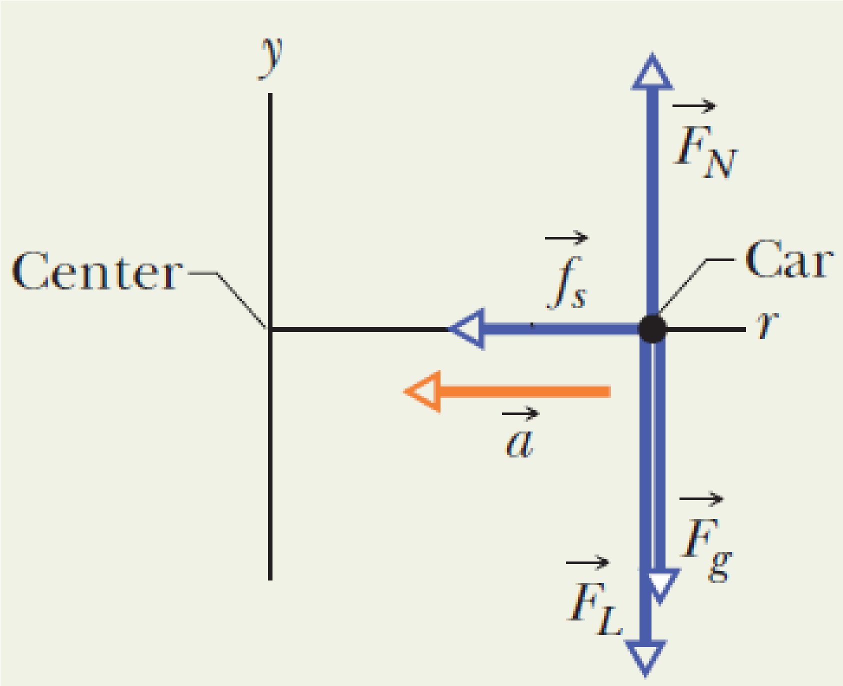
or

$$F_L = F_N - mg.$$

We then substitute for F_N to get

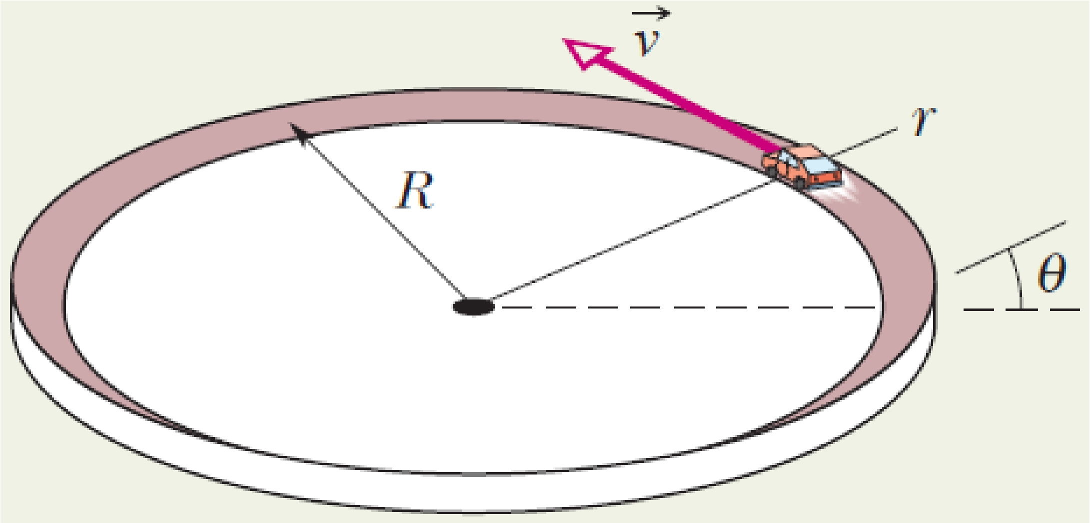
$$F_L = (600\ \text{kg})\left(\frac{(28.6\ \text{m/s})^2}{(0.75)(100\ \text{m})} - 9.8\ \text{m/s}^2\right)$$

$$\approx 660\ \text{N}.$$



Properties of Friction

Example 5: a car of mass m moves at a constant speed v of $20\,\mathrm{m/s}$ around a banked circular track of radius $R = 190\,\mathrm{m}$. If the frictional force from the track is negligible, what bank angle θ prevents sliding?



Here the track is banked so as to tilt the normal force $\vec{F}_N$ on the car toward the center of the circle. Thus, $\vec{F}_N$ now has a centripetal component of magnitude F_{Nr}, directed inward along a radial axis r.

Properties of Friction

Newton's 2nd law along the r axis ($F_{net,r} = ma_r$) gives

$$-F_N \sin\theta = m\left(-\frac{v^2}{R}\right).$$

Newton's 2nd law along the y axis ($F_{net,y} = ma_y$) gives

$$F_N \cos\theta - mg = m(0),$$

or

$$F_N \cos\theta = mg.$$

Eliminating F_N we get $\tan\theta = \frac{v^2}{gR}$ which gives

$$\theta = \tan^{-1}\frac{v^2}{gR} = \tan^{-1}\frac{(20\ \mathrm{m/s})^2}{(9.8\ \mathrm{m/s^2})(190\ \mathrm{m})} = 12°.$$