Chapter 4

Motion in Two and Three Dimensions

4.1 Position and Displacement

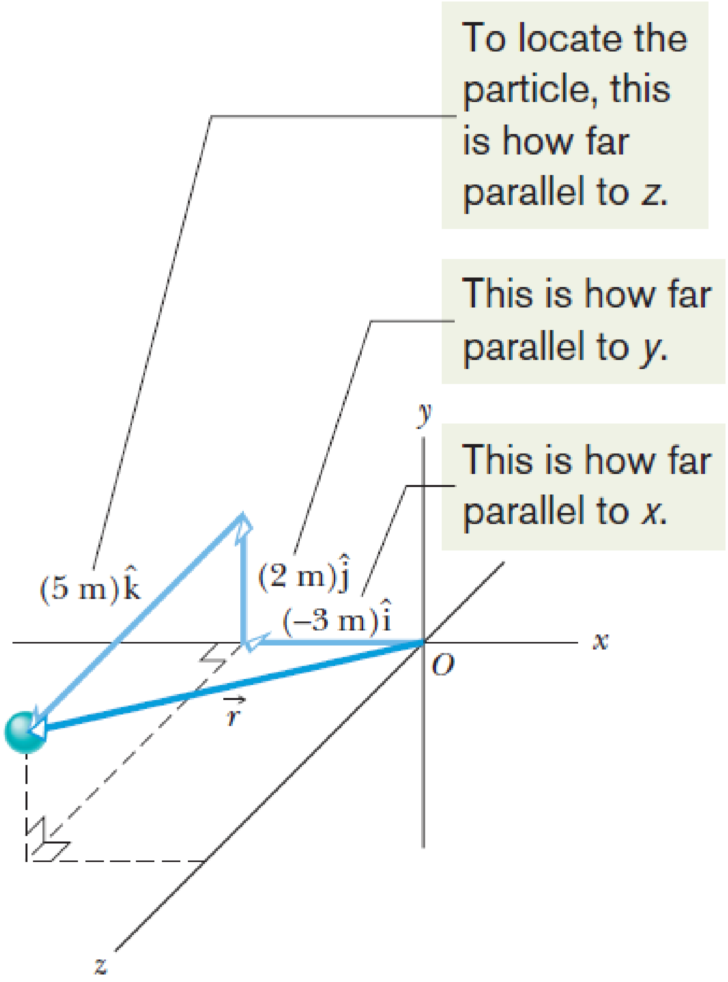
- One way to locate a particle is with a position vector $\vec{r}$.
- A position vector $\vec{r}$ extend from a reference point (usually the origin) to the particle. In unit-vector notation, $\vec{r}$ has the form

$$\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}.$$

The particle has the rectangular coordinates (x, y, z). For example, a particle with position vector

$$\vec{r} = (-3\text{ m})\hat{\imath} + (2\text{ m})\hat{\jmath} + (5\text{ m})\hat{k},$$

is located at the point $(-3\text{ m}, 2\text{ m}, 5\text{ m})$.



4.1 Position and Displacement

- As the particle moves $\vec{r}$ changes. If the position vector changes from $\vec{r}_1$ to $\vec{r}_2$ then the particle's displacement $\Delta\vec{r}$ is

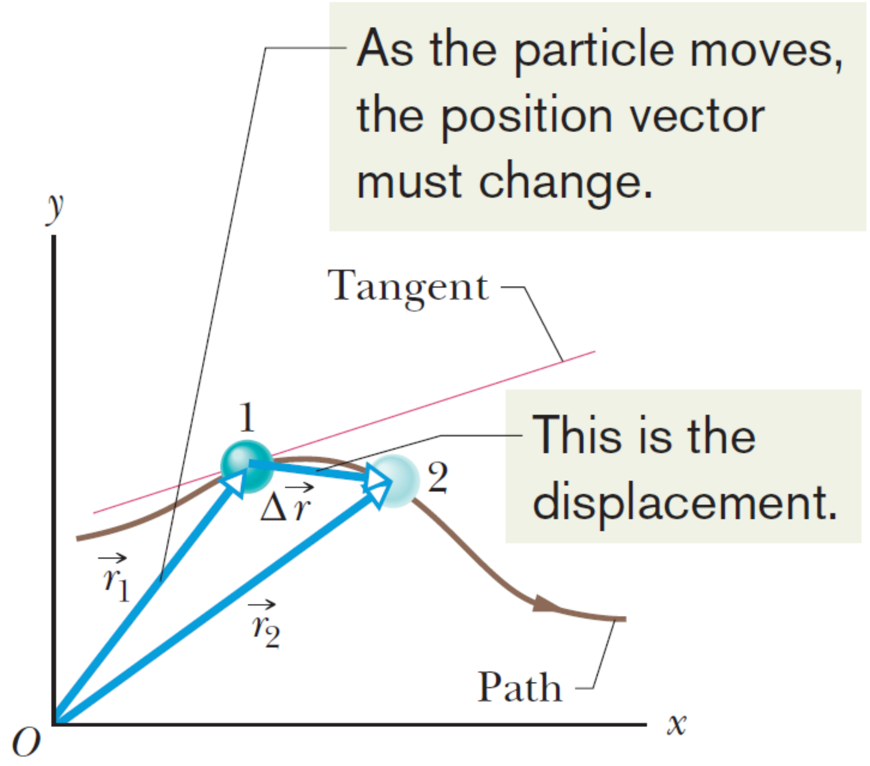
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

In unit-vector notation

$$\Delta\vec{r} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$
$$= \Delta x\,\hat{\imath} + \Delta y\,\hat{\jmath} + \Delta z\,\hat{k}.$$

where $\vec{r}_1 = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and

$\vec{r}_2 = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}.$



4.2 Position and Displacement

Example 1: The coordinates (meters) of a rabbit's position as functions of time t (seconds) are given by

$$x = -0.31\,t^2 + 7.2\,t + 28,$$
$$y = 0.22\,t^2 - 9.1\,t + 30.$$

At $t = 15$ s, what is the rabbit's position vector in unit vector notation and in magnitude-angle notation?

The position vector is

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}.$$

At $t = 15$ s, $x = 66$ m and $y = -57$ m, and therefore,

$$\vec{r} = (66\text{ m})\hat{\imath} - (57\text{ m})\hat{\jmath}.$$

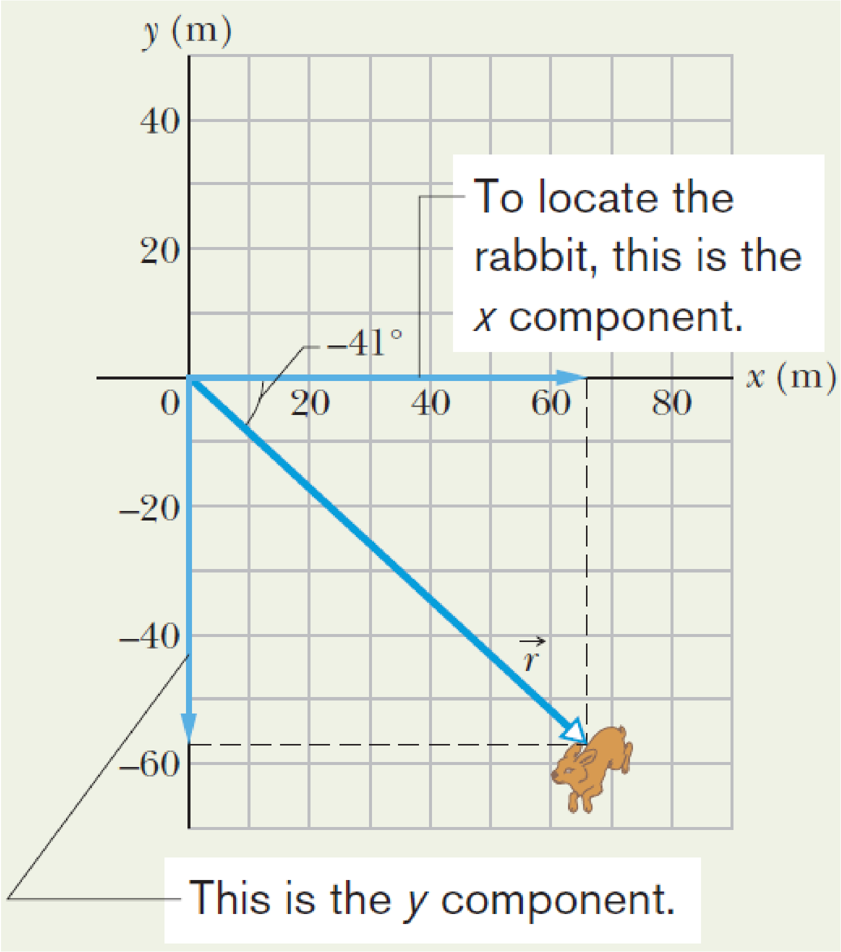
4.2 Position and Displacement

The magnitude of $\vec{r}$ is

$$r = \sqrt{x^2 + y^2} = \sqrt{(66\text{ m})^2 + (-57\text{ m})^2}$$
$$= 87\text{ m}.$$

The angle of $\vec{r}$ is

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{66\text{ m}}{-57\text{ m}} = -41°.$$



4.2 Average Velocity & Instantaneous Velocity

- As in one dimension, we can define a particle's average velocity and instantaneous velocity (velocity).
- **Average Velocity:** If a particle moves through a displacement $\Delta\vec{r}$ in a time interval Δt, then its average velocity is

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}.$$

In unit vector notation

$$\vec{v}_{\text{avg}} = \frac{\Delta x}{\Delta t}\hat{\text{i}} + \frac{\Delta y}{\Delta t}\hat{\text{j}} + \frac{\Delta z}{\Delta t}\hat{\text{k}}.$$

$\vec{v}_{\text{avg}}$ is in the direction of $\Delta\vec{r}$.

4.2 Average Velocity & Instantaneous Velocity

- **Instantaneous Velocity:** It is defined as

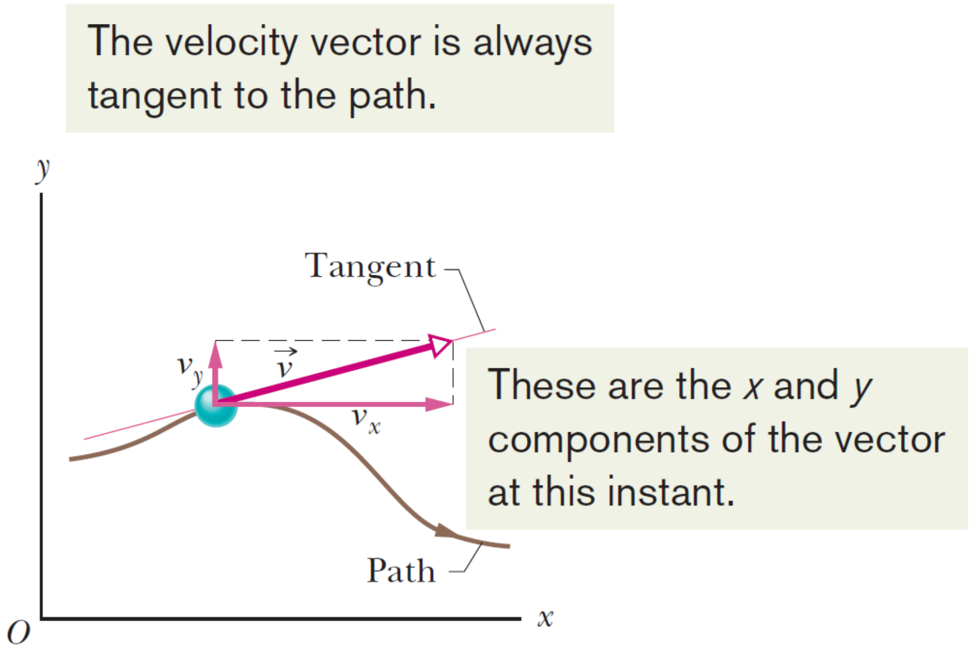
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}.$$

$\vec{v}$ is tangent to the particle's path at the particle's position.

In unit vector notation

$$\vec{v} = \frac{d}{dt}(x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}) = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$
$$= v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}.$$

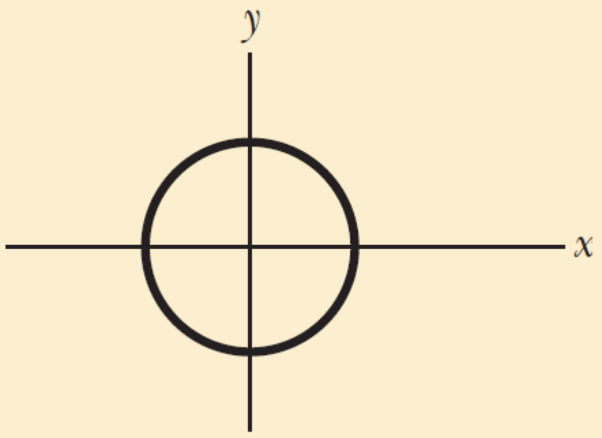
v_x, v_y and v_z are the components of $\vec{v}$.



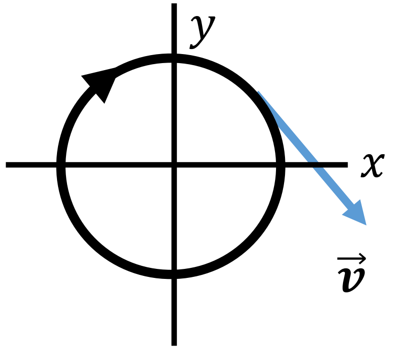
4.2 Average Velocity & Instantaneous Velocity

Checkpoint 1

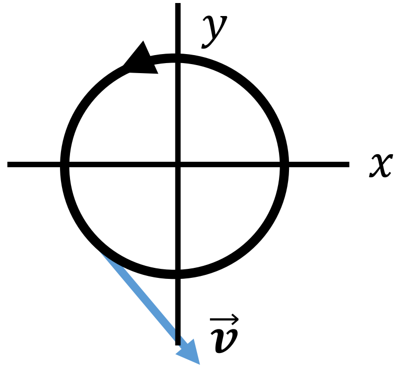
The figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v} = (2\text{ m/s})\hat{\text{i}} - (2\text{ m/s})\hat{\text{j}}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases, draw $\vec{v}$ on the figure.



(a) The first quadrant.



(b) The third quadrant.



4.2 Average Velocity & Instantaneous Velocity

Example 2: The coordinates (meters) of a rabbit's position as functions of time t (seconds) are given by

$$x = -0.31\,t^2 + 7.2\,t + 28,$$
$$y = 0.22\,t^2 - 9.1\,t + 30.$$

Find the rabbit's velocity $\vec{v}$ at $t = 15$ s.

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31\,t^2 + 7.2\,t + 28) = -0.62\,t + 7.2,$$
$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22\,t^2 - 9.1\,t + 30) = 0.44\,t - 9.1.$$

At $t = 15$ s, $v_x = -2.1$ m/s and $v_y = -2.5$ m/s.

4.2 Average Velocity & Instantaneous Velocity

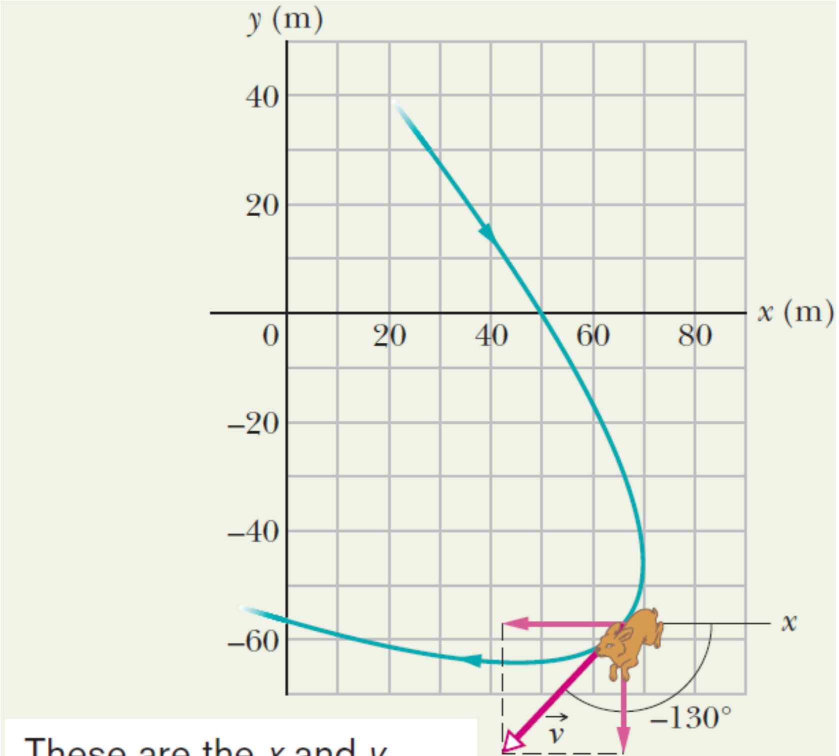
Therefore,

$$\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} = (-2.1\ \mathrm{m/s})\hat{\mathrm{i}} + (-2.5\ \mathrm{m/s})\hat{\mathrm{j}}.$$

The magnitude and angle of $\vec{v}$ are, respectively,

$$v = \sqrt{(-2.1\ \mathrm{m/s})^2 + (-2.5\ \mathrm{m/s})^2} = 3.3\frac{\mathrm{m}}{\mathrm{s}},$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-2.5\ \mathrm{m/s}}{-2.1\ \mathrm{m/s}} = -130°.$$



These are the *x* and *y* components of the vector at this instant.

4.3 Average Acceleration & Instantaneous Acceleration

- When a particle's velocity changes from $\vec{v}_1$ to $\vec{v}_2$ in a time interval Δt, its average acceleration is

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}.$$

- The instantaneous acceleration (acceleration) is

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}.$$

- A particle undergoes acceleration if:
  1. The magnitude of its velocity changes.
  2. The direction of its velocity changes.

4.3 Average Acceleration & Instantaneous Acceleration

- In unit-vector notation

$$\vec{a} = \frac{d}{dt}\left(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}\right)$$
$$= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}$$
$$= a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}.$$

  a_x, a_y and a_z are the components of $\vec{a}$.

- The direction of $\vec{a}$ is tangent to the particle's velocity curve $\vec{v}(t)$ at the particle's position.

4.3 Average Acceleration & Instantaneous Acceleration

CHECKPOINT 2

Here are four descriptions of the position (in meters) of a puck as it moves in an xy plane:

(1) $x = -3t^2 + 4t - 2$ and $y = 6t^2 - 4t$ (3) $\vec{r} = 2t^2\hat{i} - (4t + 3)\hat{j}$

(2) $x = -3t^3 - 4t$ and $y = -5t^2 + 6$ (4) $\vec{r} = (4t^3 - 2t)\hat{i} + 3\hat{j}$

Are the x and y acceleration components constant? Is acceleration $\vec{a}$ constant?

(1) $a_x = -6\ \mathrm{m/s^2}$, $a_y = 12\ \mathrm{m/s^2}$. (a_x, a_y and $\vec{a}$ are constant.)

(2) $a_x = -18t\ \mathrm{m/s^2}$, $a_y = -10\ \mathrm{m/s^2}$. ($a_y$ is constant.)

(3) $\vec{a} = (4\ \mathrm{m/s^2})\hat{i}$. ($a_x$, a_y and $\vec{a}$ are constant.)

(4) $\vec{a} = (24t)\hat{i}$. (a_y is constant.)

4.3 Average Acceleration & Instantaneous Acceleration

Example 3: The coordinates (meters) of a rabbit's position as functions of time t (seconds) are given by

$$x = -0.31\,t^2 + 7.2\,t + 28,$$
$$y = 0.22\,t^2 - 9.1\,t + 30.$$

Find the rabbit's acceleration $\vec{a}$ at $t = 15\text{ s}$.

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62\,t + 7.2) = -0.62\text{ m/s}^2,$$
$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44\,t - 9.1) = 0.44\text{ m/s}^2.$$

Combining the components gives that $\vec{a} = (-0.62\text{ m/s}^2)\hat{\imath} + (0.44\text{ m/s}^2)\hat{\jmath}$.

4.3 Average Acceleration & Instantaneous Acceleration

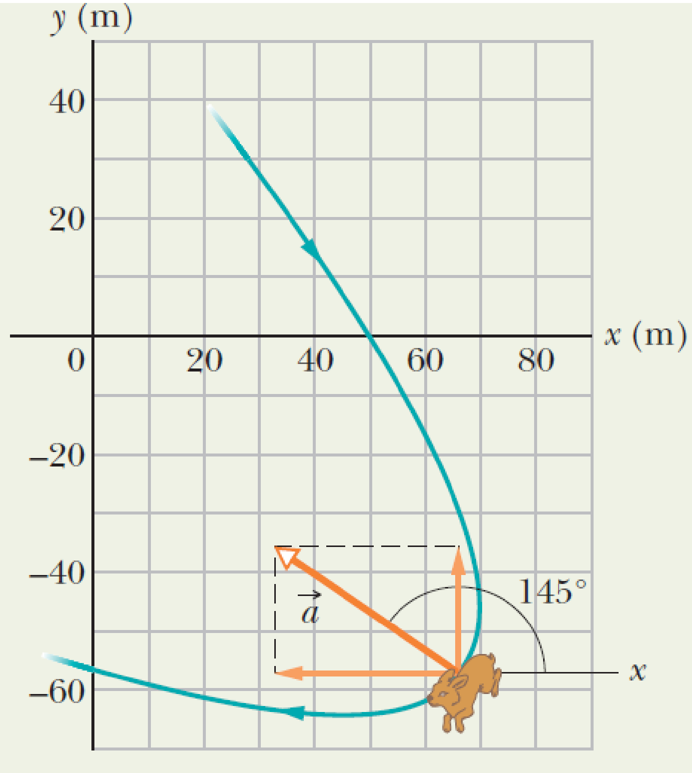
The magnitude and angle of $\vec{a}$ are, respectively,

$$a = \sqrt{{a_x}^2 + {a_y}^2}$$

$$= \sqrt{(-0.62\ \text{m/s}^2)^2 + (0.44\ \text{m/s}^2)^2}$$

$$= 0.76\,\frac{\text{m}}{\text{s}^2},$$

$$\theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{0.44\ \text{m/s}^2}{-0.62\ \text{m/s}^2} = 145°.$$



These are the *x* and *y* components of the vector at this instant.

4.4 Projectile Motion

- We analyze here the motion of a particle in a vertical plane thrown with some initial velocity $\vec{v}_0$ and always in free fall.
- Such a particle is called a **projectile** and its motion is called **projectile motion**. The air effect is neglected.

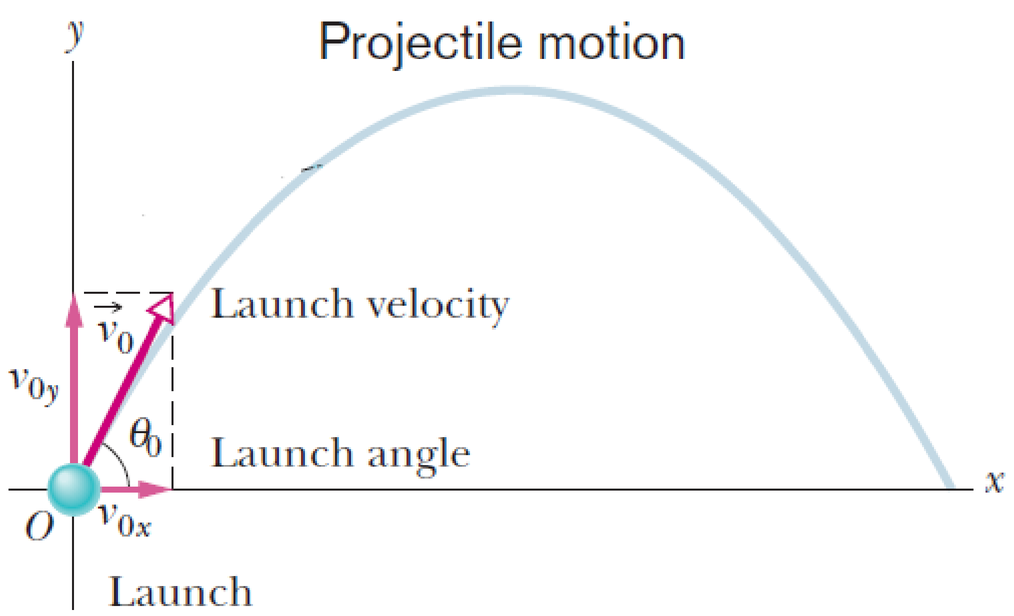
4.4 Projectile Motion

- The initial velocity $\vec{v}_0$ of the projectile has the form

$$\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}.$$

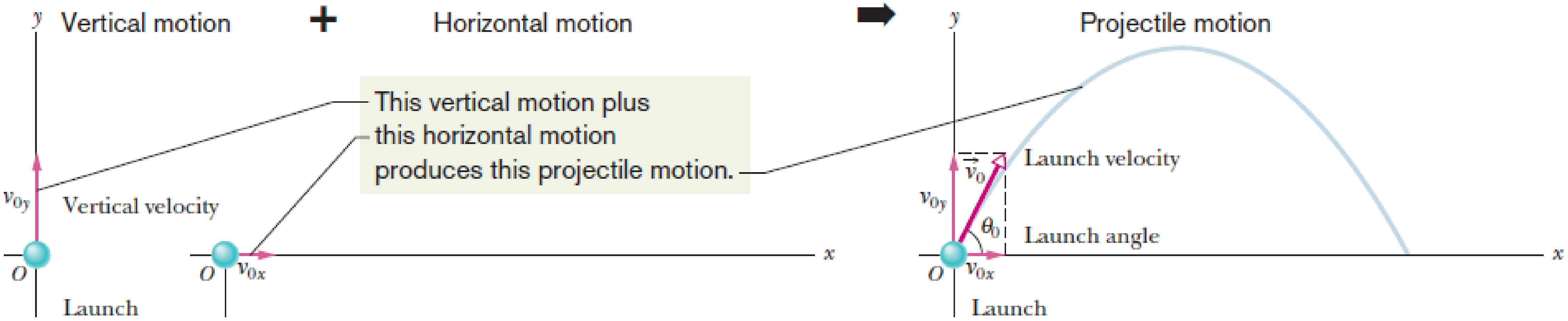
$$v_{0x} = v_0 \cos\theta_0 \text{ and } v_{0y} = v_0 \sin\theta_0 \,.$$

- Both $\vec{r}$ and $\vec{v}$ of the projectile change during the motion.
- $\vec{a} = -g\,\hat{\jmath} = -9.8\,\hat{\jmath}$ all the time during the flight.



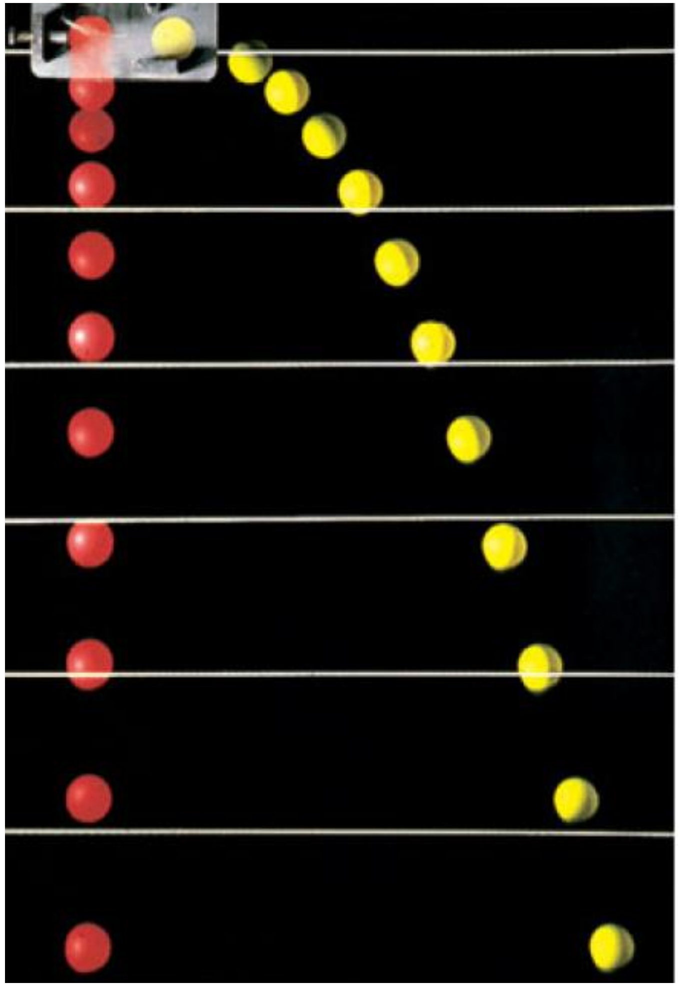
4.4 Projectile Motion

- The horizontal and vertical motions are independent.



https://www.youtube.com/watch?v=hlW6hZkgmkA

4.4 Projectile Motion



4.5 Projectile Motion

CHECKPOINT 3

At a certain instant, a fly ball has velocity $\vec{v} = 25\hat{i} - 4.9\hat{j}$ (the x axis is horizontal, the y axis is upward, and $\vec{v}$ is in meters per second). Has the ball passed its highest point?

Yes, since $v_y = -4.9 < 0$.

Reasoning: After the ball is launched, it travels upward ($v_y > 0$) then slows down vertically to rest ($v_y = 0$) and then starts to move downward ($v_y < 0$).

4.6 Projectile Motion Analyzed

1. The Horizontal Motion: $a_x = 0$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

or

$$x - x_0 = v_0 \cos\theta_0 \, t.$$

Velocities:

$$v_x = v_{0x} + a_x t$$

or

$$v_x = v_{0x} = v_0 \cos\theta_0.$$

2. The Vertical Motion: $a_y = -g$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

or

$$y - y_0 = v_0 \sin\theta_0 \, t - \frac{1}{2}gt^2$$

$$v_y = v_{0y} + a_y t$$

or

$$v_y = v_0 \sin\theta_0 - gt,$$

also,

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$

4.6 Projectile Motion Analyzed

3. The Equation of the path:

We can find the equation of the projectile trajectory by eliminating t between the vertical and horizontal equations of motion. Solving the horizontal equation of motion

$$x - x_0 = v_0 \cos\theta_0 \, t,$$

for t gives that

$$t = \frac{x - x_0}{v_0 \cos\theta_0}.$$

We then use this t expression in the vertical equation of motion:

$$y - y_0 = v_0 \sin\theta_0 \left(\frac{x - x_0}{v_0 \cos\theta_0}\right) - \frac{1}{2} g \left(\frac{x - x_0}{v_0 \cos\theta_0}\right)^2.$$

4.6 Projectile Motion Analyzed

3. The Equation of the path:

$$y - y_0 = v_0 \sin\theta_0 \left(\frac{x - x_0}{v_0 \cos\theta_0}\right) - \frac{1}{2} g \left(\frac{x - x_0}{v_0 \cos\theta_0}\right)^2 .$$

Choosing $x_0 = y_0 = 0$ and rearranging yield

$$y = \tan\theta_0 \, x - \frac{g x^2}{2(v_0 \cos\theta_0)^2} .$$

This is an equation of a parabola.

4.6 Projectile Motion Analyzed

4. The Horizontal Range:

The horizontal range (R) of the projectile is the distance it has traveled when it returns to its initial height. We write

$$R = v_0 \cos\theta_0 \, t,$$

$x_0 = 0, x = R$

$$0 = v_0 \sin\theta_0 \, t - \frac{1}{2} g t^2.$$

$y = y_0$

These equations yield

$$R = \frac{2v_0^2}{g} \sin\theta_0 \cos\theta_0 = \frac{v_0^2}{g} \sin 2\theta_0$$

R is maximum when $\sin 2\theta_0 = 1$ or $\theta_0 = 45°$.

This expression is valid only when the launch height y_0 and the final height y are the same!

4.6 Projectile Motion Analyzed

CHECKPOINT 4

A fly ball is hit to the outfield. During its flight (ignore the effects of the air), what happens to its (a) horizontal and (b) vertical components of velocity? What are the (c) horizontal and (d) vertical components of its acceleration during ascent, during descent, and at the topmost point of its flight?

(a) v_x is constant.

(b) v_y is initially positive, then decreases to zero, and then becomes increasingly negative.

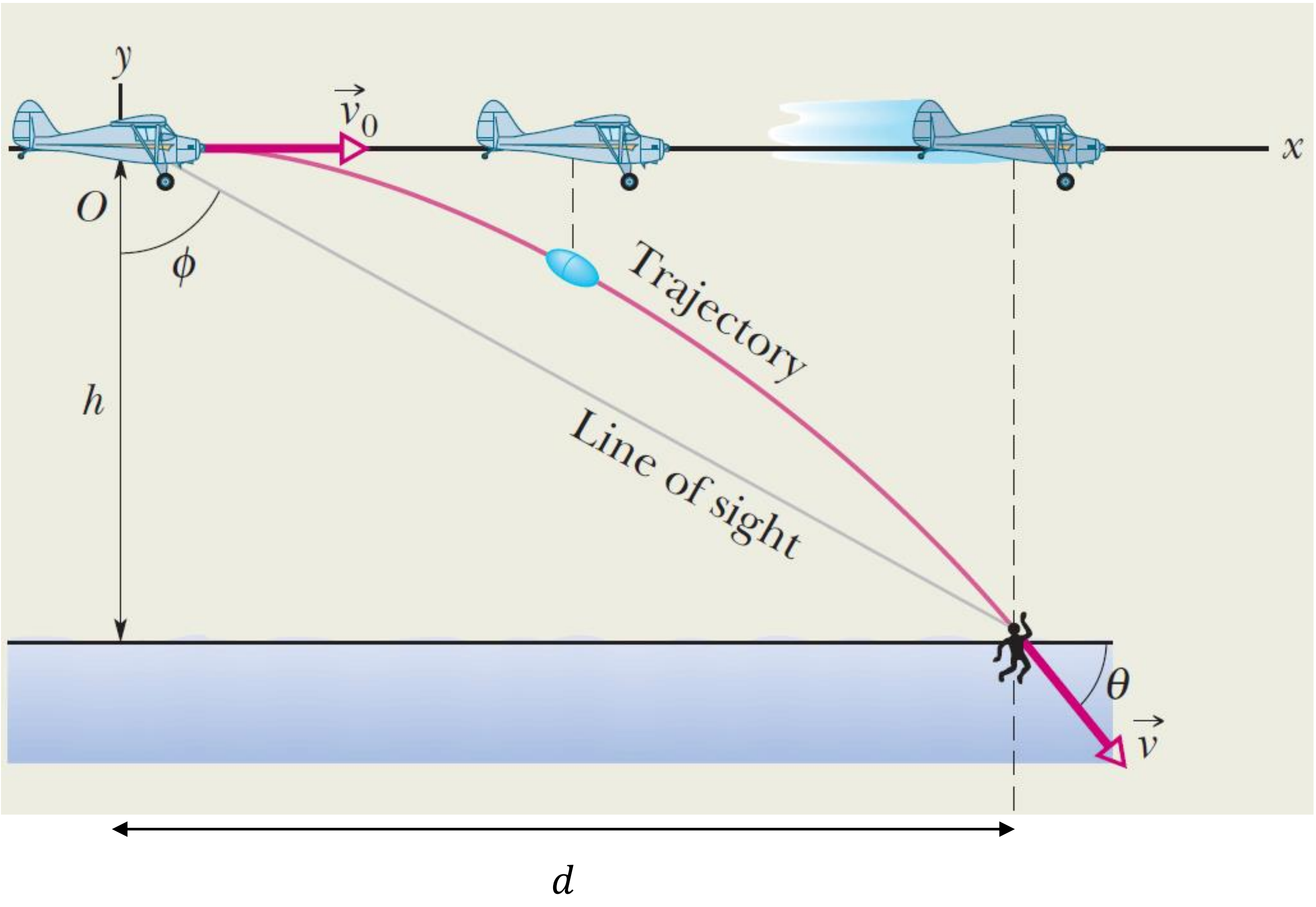
(c) a_x is always zero.

(d) a_y is always equal to $-\mathrm{g} = -9.8\ \mathrm{m/s^2}$.

4.6 Projectile Motion Analyzed

Example 4: a rescue plane flies at $198\,\mathrm{km/h}$ $(= 55.0\,\mathrm{m/s})$ and constant height $h = 500\,\mathrm{m}$ toward a point directly over a victim, where a rescue capsule is to land.

(a) What should be the horizontal distance d when the capsule release is made?



4.6 Projectile Motion Analyzed

We first find the time it takes the capsule to reach the water surface. Using the vertical motion equation

$$y - y_0 = v_0 \sin\theta_0\, t - \frac{1}{2} g t^2,$$

or

$$0 - 500\ \mathrm{m} = (55.0\ \mathrm{m/s}) \sin 0°\, t - \frac{1}{2}(9.8\ \mathrm{m}/s^2) t^2,$$

we get that $t = 10.1\ \mathrm{s}$. To find d we use the horizontal motion equation

$$x - x_0 = v_0 \cos\theta_0\, t,$$

or

$$d - 0 = (55.0\ \mathrm{m/s}) \cos 0°\, t,$$

which gives that $d = 556\ \mathrm{m}$.

4.6 Projectile Motion Analyzed

(b) As the capsule reaches the water, what is its velocity in unit-vector notation and in magnitude-angle notation?

$$v_x = v_0 \cos\theta_0 = (55.0\ \mathrm{m/s})\cos 0° = 55.0\ \mathrm{m/s}.$$

$$\begin{aligned} v_y &= v_0 \sin\theta_0 - gt \\ &= (55.0\ \mathrm{m/s})\sin 0° - (9.8\ \mathrm{m}/s^2)(10.1\ s) \\ &= -99.9\ \mathrm{m/s}. \end{aligned}$$

The capsule velocity at the surface is therefore $\vec{v} = (55.0\ \mathrm{m/s})\hat{\imath} - (99.9\ \mathrm{m/s})\hat{\jmath}$. The magnitude and angle of $\vec{v}$ are, respectively

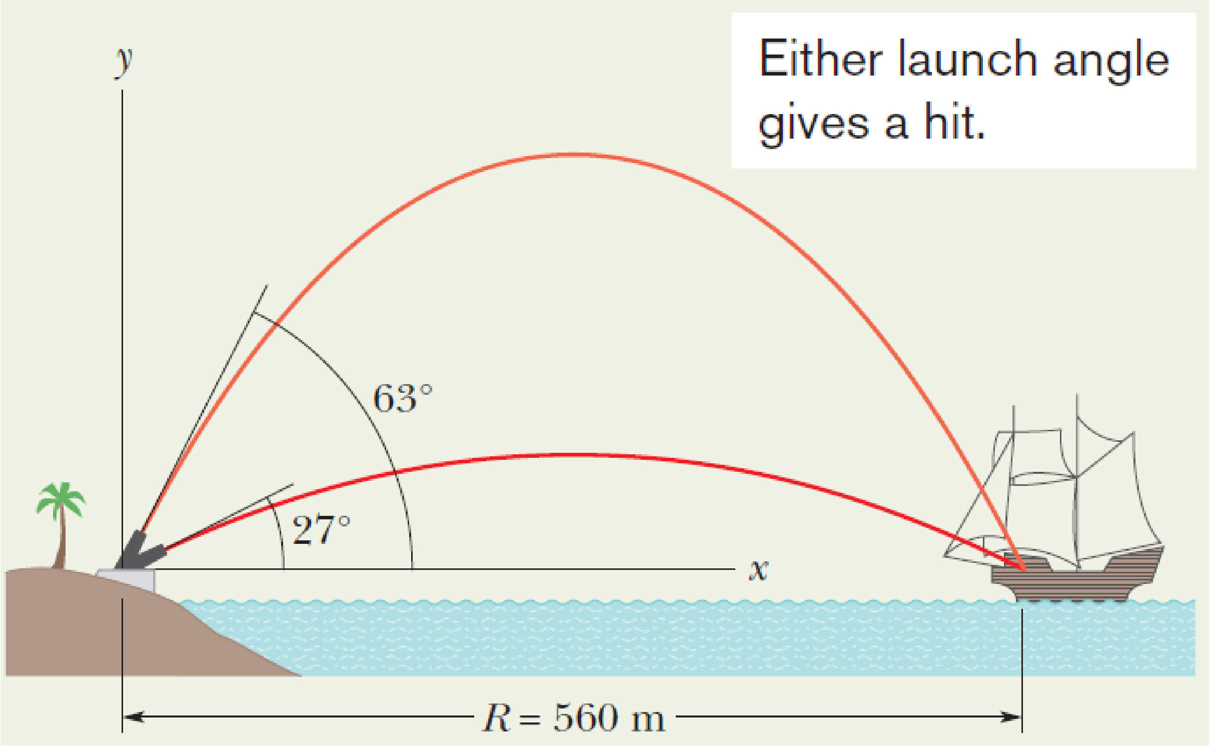
$$v = \sqrt{(55.0\ \mathrm{m/s})^2 + (-99.9\mathrm{m/s})^2} = 113\mathrm{m/s},$$

$$\theta = \tan^{-1}\frac{-99.9\mathrm{m/s}}{55.0\ \mathrm{m/s}} = -60.9°.$$

4.6 Projectile Motion Analyzed

Example 5: a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_0 = 82$ m/s.

(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?



4.6 Projectile Motion Analyzed

The cannon and the pirate ship are at the same height. The horizontal displacement is therefore the range. The cannon angle should be adjusted to make the range $R = 560\ \mathrm{m}$. The two are related by

$$R = \frac{v_0^2}{g}\sin 2\theta_0 \,.$$

Solving for θ_0 we get

$$\theta_0 = \frac{1}{2}\sin^{-1}\frac{gR}{v_0^2} = \frac{1}{2}\sin^{-1}\frac{(9.8\ \mathrm{m/s^2})(560\ \mathrm{m})}{(82\ \mathrm{m/s})^2} = \frac{1}{2}\sin^{-1} 0.816 \,.$$

Your calculator gives that $\sin^{-1} 0.816 = 54.7°$, which corresponds to $\theta_0 = 27°$.

Additionally, $\sin^{-1} 0.816 = 180° - 54.7° = 125.3°$, which corresponds to $\theta_0 = 63°$. This second solution is also acceptable since it is between $0°$ and $90°$.

4.6 Projectile Motion Analyzed

(b) What is the maximum range of the cannonballs?

The maximum range corresponds to the launch angle $\theta_0 = 45°$. Therefore,

$$R = \frac{(82 \text{ m/s})^2}{9.8 \text{ m/s}^2} \sin 2(45°) = 686 \text{ m} = 690 \text{ m}.$$

Reading!

Sample Problem 4.05 **Launched into the air from a water slide**

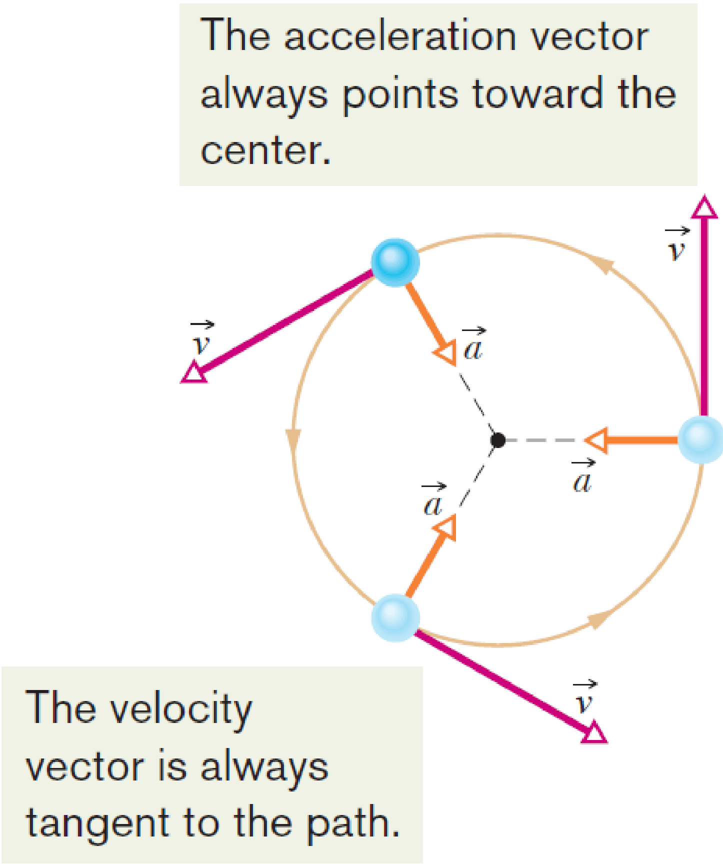
4.7 Uniform Circular Motion

- A particle is in uniform circular motion if it travels around a circle or a circular arc at a constant speed.
  - The particle is accelerating because the velocity changes direction.
  - The acceleration and velocity have constant magnitudes but they change direction.
  - $\vec{v}$ is tangent to the circle in the direction of motion.
  - $\vec{a}$ is always radially inward and therefore called **centripetal acceleration**.
  - Its magnitude is

$$a = \frac{v^2}{r},$$

Read the proof on p. 67

r is the radius of the circle or arc.



4.7 Uniform Circular Motion

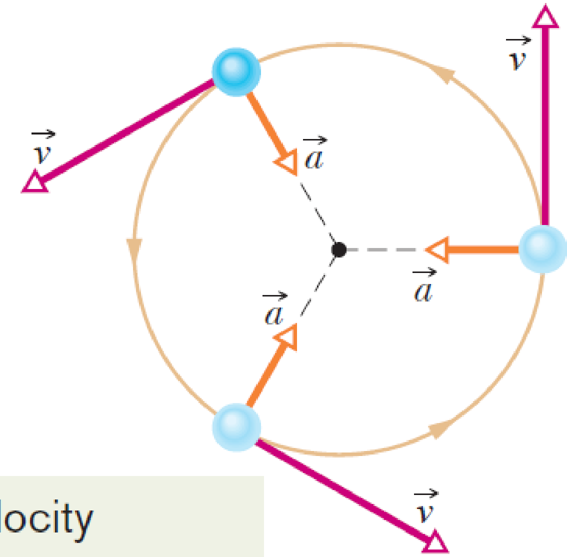
- The particle travels the circumference of the circl $(2\pi r)$ in time

$$T = \frac{2\pi r}{v}.$$

T is called the **period of revolution** or simply the **perio** of the motion.

- In general, the period is the time for a particle to go around a closed path exactly once.

The acceleration vector always points toward the center.



The velocity vector is always tangent to the path.

4.7 Uniform Circular Motion

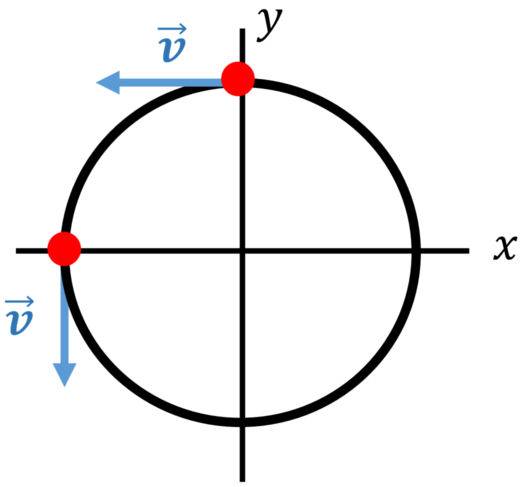
CHECKPOINT 5

An object moves at constant speed along a circular path in a horizontal xy plane, with the center at the origin. When the object is at $x = -2$ m, its velocity is $-(4 \text{ m/s})\hat{\text{j}}$. Give the object's (a) velocity and (b) acceleration at $y = 2$ m.

(a) $\vec{v} = -\left(4\,\frac{\text{m}}{\text{s}}\right)\hat{\text{i}}.$

(b) $a = \frac{v^2}{r} = \frac{\left(4\,\frac{\text{m}}{\text{s}}\right)^2}{2\text{ m}} = 8\,\frac{\text{m}}{\text{s}^2}.$

$\vec{a} = -\left(8\,\frac{\text{m}}{\text{s}^2}\right)\hat{\text{j}}.$



4.7 Uniform Circular Motion

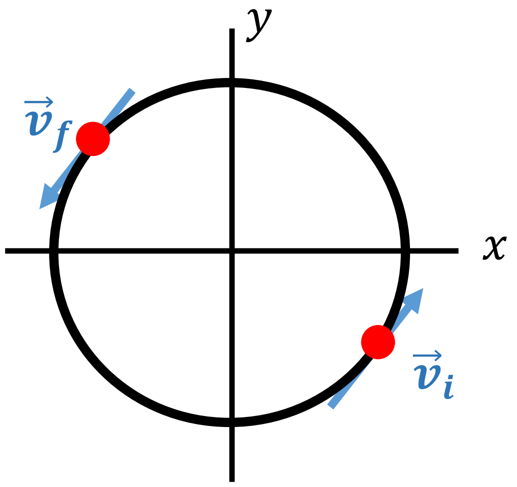
Example 5: What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\,\hat{\imath} + 500\,\hat{\jmath})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\,\hat{\imath} - 500\,\hat{\jmath})$ m/s?

Assuming the motion is uniform circular, the acceleration is centripetal and has the magnitude

$$a = \frac{v^2}{r}.$$

To find r we need to know the period T:

$$T = \frac{2\pi r}{v}.$$



4.7 Uniform Circular Motion

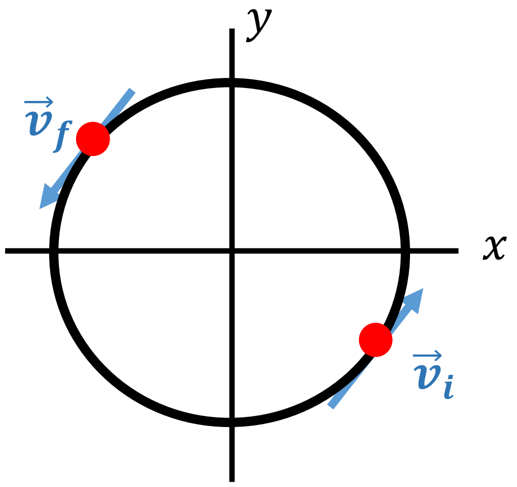
It took the aircraft $24.0\ \mathrm{s}$ to complete half the circle. The period T is therefore $2(24.0)\ \mathrm{s} = 48.0\ \mathrm{s}$.

Combining the above to expression we get

$$a = \frac{2\pi T}{v}.$$

$$v = \sqrt{(400\ \mathrm{m/s})^2 + (500\ \mathrm{m/s})^2} = 640.3\ \mathrm{m/s},$$

$$a = \frac{2\pi T}{v} = \frac{2\pi(48.0\ \mathrm{s})}{640.3\ \mathrm{m/s}} = 83.8\,\frac{\mathrm{m}}{\mathrm{s}^2} \approx 8.6\ g.$$



4.8 Relative Motion in One Dimension

- The velocity of a particle depends on the **reference frame** of the observer.
- From the figure

$$x_{PA} = x_{PB} + x_{BA}.$$

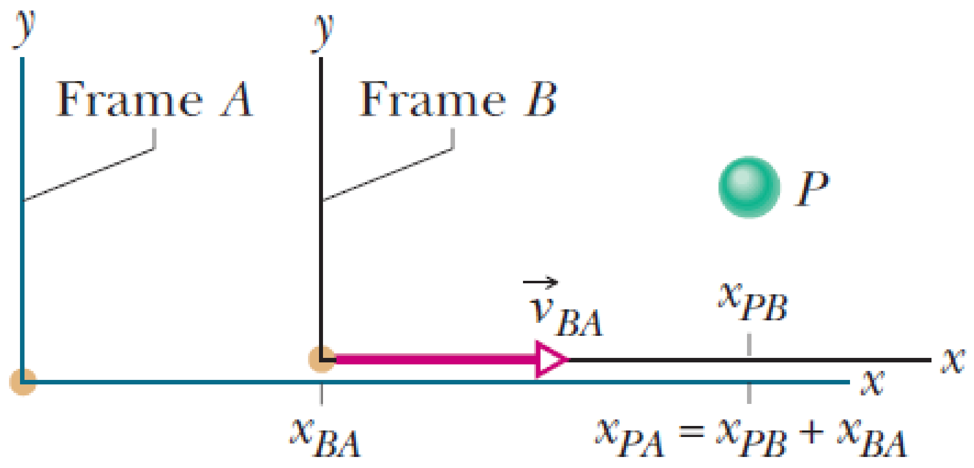
Differentiating with respect to time

$$\frac{d}{dt}x_{PA} = \frac{d}{dt}x_{PB} + \frac{d}{dt}x_{BA},$$

or

$$v_{PA} = v_{PB} + v_{BA}.$$

Frame B moves past frame A while both observe P.



$\vec{v}_{BA}$ is constant

4.8 Relative Motion in One Dimension

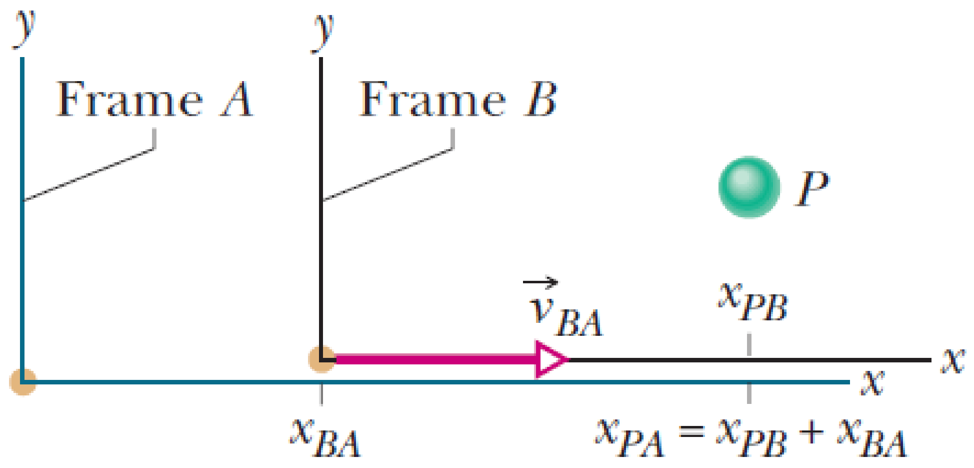
Differentiating again with respect to time gives

$$\frac{d}{dt}v_{PA} = \frac{d}{dt}v_{PB} + \frac{d}{dt}v_{BA}$$

$$a_{PA} = a_{PB}.$$

The acceleration of a particle is the same when measured in two frames in relative motion with constant velocity.

Frame B moves past frame A while both observe P.



$\vec{v}_{BA}$ is constant

4.8 Relative Motion in One Dimension

CHECKPOINT 6

A train is travelling at $60\ \mathrm{km/h}$ due north. At some instant, a student in the train measured the velocity and acceleration of a particle in the train to be 0 and $3\ \mathrm{m/s^2}$, respectively. What are (a) the velocity and (b) acceleration of the particle relative to another student on ground?

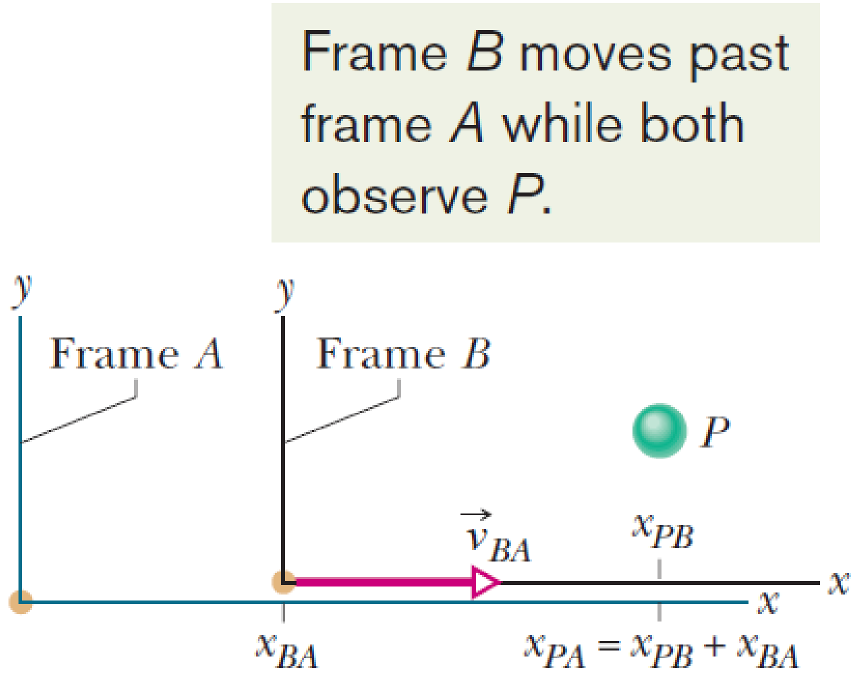
(a) $60\ \mathrm{km/h}$ due north.

(b) $3\ \mathrm{m/s^2}$.

4.8 Relative Motion in One Dimension

Example 6: Suppose that the velocity of a truck (frame B) relative to a standing person (frame A) is a constant $v_{BA} = 52\ \mathrm{km/h}$ and a car (particle P) is moving in the negative direction of the x axis.

(a) If the person measures a constant velocity $v_{PA} = -78\ \mathrm{km/h}$ for the car, what velocity v_{PB} will the truck measure?



4.8 Relative Motion in One Dimension

$$v_{PA} = v_{PB} + v_{BA},$$

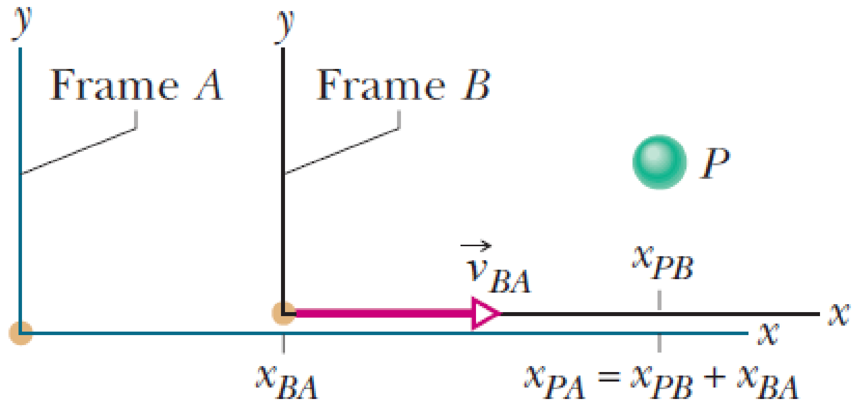
or

$$v_{PB} = v_{PA} - v_{BA} = -78\,\frac{\text{km}}{\text{h}} - 52\,\frac{\text{km}}{\text{h}} = -130\,\frac{\text{km}}{\text{h}}.$$

(b) If the car brakes to a stop relative to the standing person (and thus relative to the ground) in time $t = 10\text{ s}$ at constant acceleration, what is its acceleration a_{PA} relative to him?

$$a_{PA} = \frac{v - v_0}{t} = \frac{0 - (-78\text{ km/h})}{10\text{ s}} = 2.2\,\frac{\text{m}}{\text{s}^2}.$$

Frame *B* moves past frame *A* while both observe *P*.



4.8 Relative Motion in One Dimension

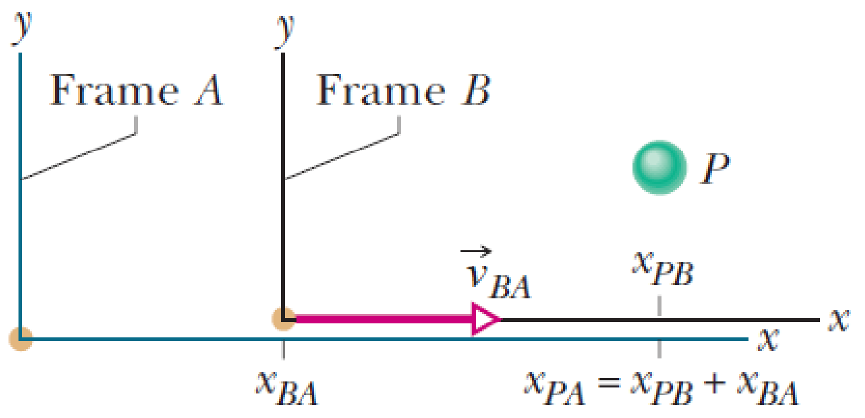
(c) What is the acceleration a_{PB} of the car relative to the truck during the braking?

The initial velocity of the car relative to the truck is -130 km/h and the final velocity is -52 km/h. Therefore,

$$a_{PA} = \frac{v - v_0}{t} = \frac{-52\ \mathrm{km/h} - (-130\ \mathrm{km/h})}{10\ \mathrm{s}}$$

$$= 2.2\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

Frame *B* moves past frame *A* while both observe *P*.



4.8 Relative Motion in Two Dimension

- From the figure

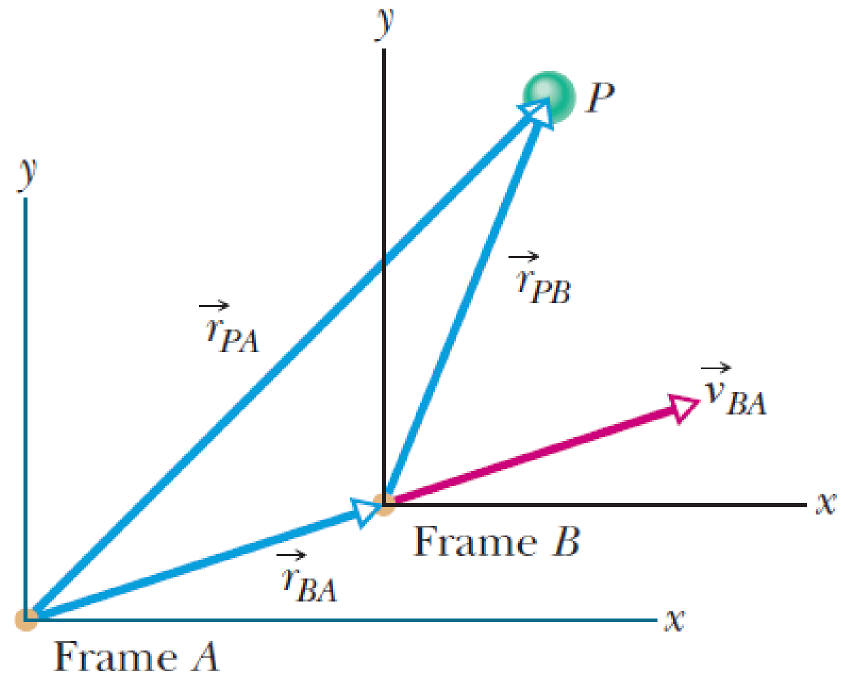
$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}.$$

Differentiating with respect to time

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}.$$

Differentiating again with respect to time gives

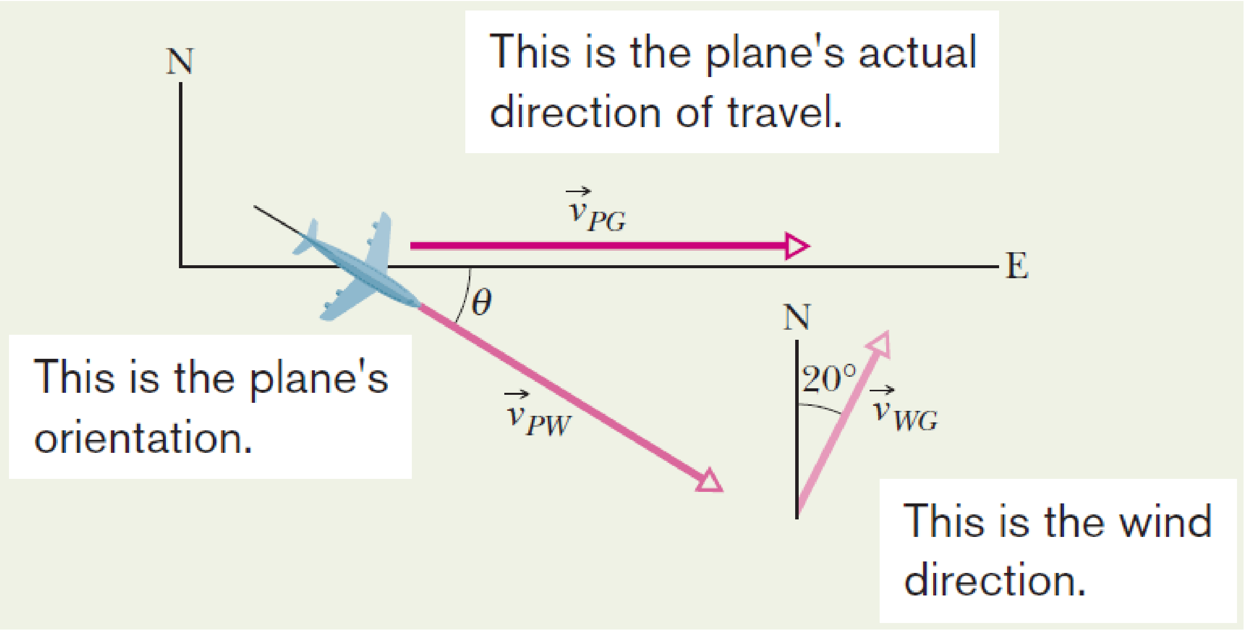
$$\vec{a}_{PA} = \vec{a}_{PB}.$$



$\vec{v}_{BA}$ is constant

4.8 Relative Motion in Two Dimension

Example 7: a plane moves due east while the pilot points the plane somewhat south of east, toward a steady wind that blows to the northeast. The plane has velocity $\vec{v}_{PW}$ relative to the wind of 215 km/h, directed at angle θ south of east. The wind has velocity $\vec{v}_{WG}$ relative to the ground with speed 65.0 km/h, directed 20.0° east of north. What is the magnitude of the velocity $\vec{v}_{PG}$ of the plane relative to the ground, and what is θ?



4.8 Relative Motion in Two Dimension

The three velocities are related by

$$\vec{v}_{PG} = \vec{v}_{PW} + \vec{v}_{WG}.$$

For the y-components, we have

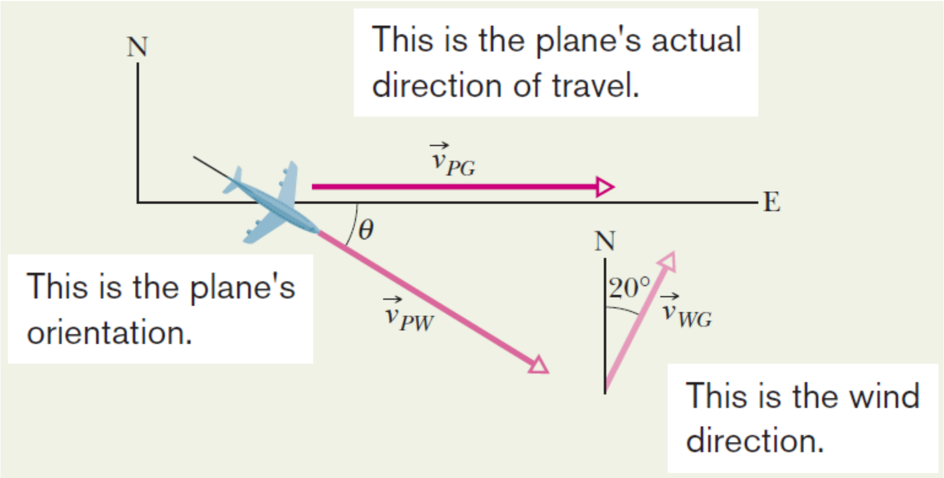
$$v_{PG,y} = v_{PW,y} + v_{WG,y},$$

or

$$0 = (215\ \mathrm{km/h})\sin(-\theta) + (65.0\ \mathrm{km/h})\sin 70°.$$

Solving for θ we find that

$$\theta = \sin^{-1}\frac{+(65.0\ \mathrm{km/h})\sin 70°}{(215\ \mathrm{km/h})} = 16.5°.$$



4.8 Relative Motion in Two Dimension

For the x-components, we have

$$v_{PG,x} = v_{PW,x} + v_{WG,x},$$

or

$$v_{PG} \cos 0 = (215 \text{ km/h}) \cos(-16.5°) + (65.0 \text{ km/h}) \cos 70°,$$

which gives $v_{PG} = 228$ km/h.