Chapter 2

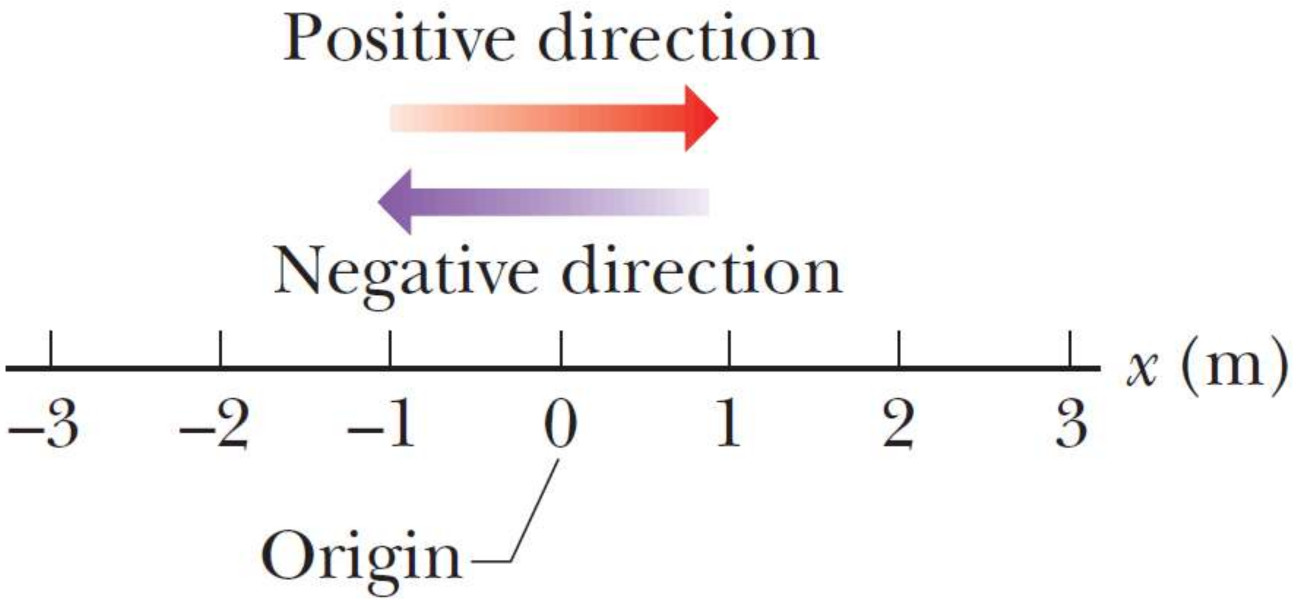
Motion along a Straight Line

2.1 Motion

- Everything in the universe, from atoms to galaxies, is in motion.
- A first step to study motion is to consider simplified cases. In this chapter we study motion with three restrictions:
    1. The motion is along a straight line. The line can be vertical, horizontal or slanted.
    2. Only motion and changes in it are discussed. Forces causing the motion are not discussed.
    3. The moving object is either a point-like particle (e.g. an electron) or an object that moves like a particle (e.g. a block or a car.)

2.2 Position and Displacement

- An object is located by finding its position relative to a reference point, often the **origin** of an axis. The **positive direction** of the axis is in the direction of increasing numbers. The opposite is the **negative direction**.



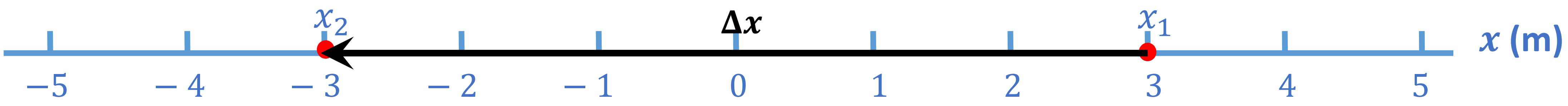
2.2 Position and Displacement

- A change from position x_1 to position x_2 is called a **displacement** Δx, where

$$\Delta x = x_2 - x_1.$$

Δx is positive when the displacement is in the positive direction and negative in the negative direction.

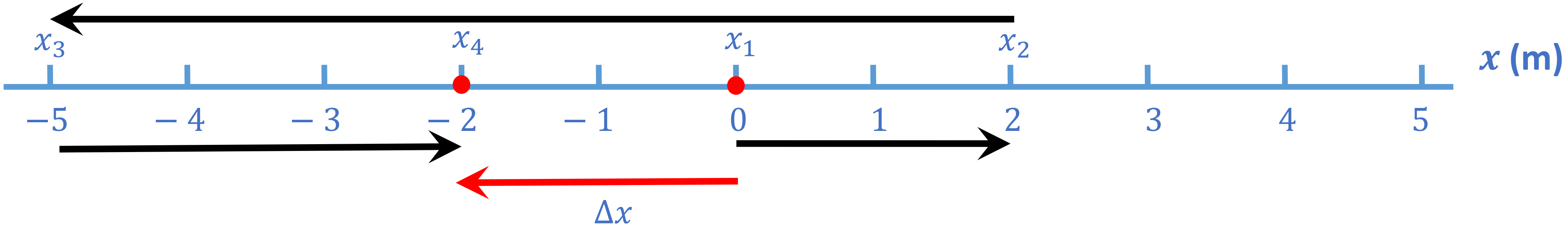
$$\Delta x = -1 - (-4) = 3 \text{ m}$$



$$\Delta x = -3 - (3) = -6 \text{ m}$$

2.2 Position and Displacement

- The displacement Δx is a **vector quantity**; it has
  1. A *magnitude* $|\Delta x|$ which represents the distance between the initial and final positions
  2. A *direction* which is usually given by the sign of Δx.
- The displacement is in general different from the <u>total distance</u> coved!



$$\Delta x = x_4 - x_1 = -2 - 0 = -2 \text{ m}$$

$$\text{Total distance} = 2 + 7 + 3 = 12 \text{ m}$$

2.3 Average Velocity and Average Speed

- The position of an object can be described with a graph of position x plotted as a function of time t; $x(t)$.

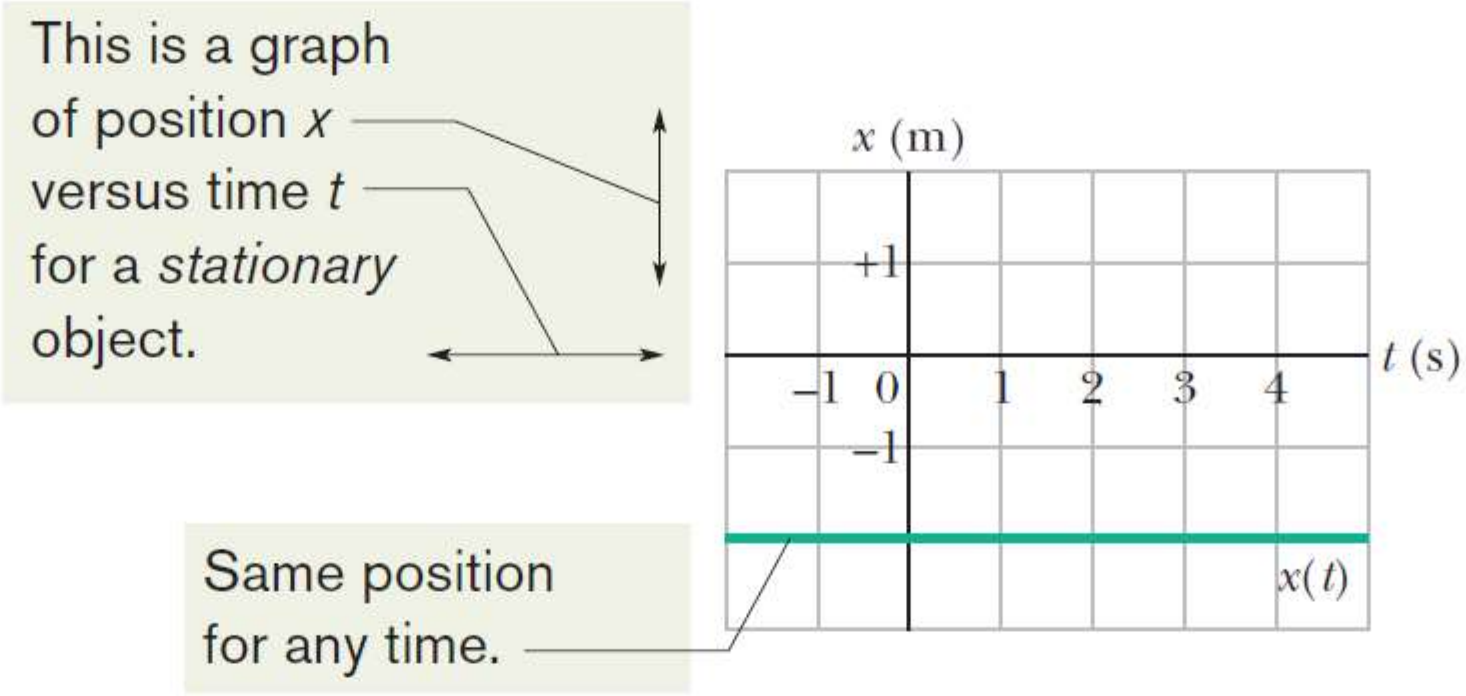


Fig. 2-2 The graph of $x(t)$ for an armadillo that is stationary at $x = -2$ m. The value of x is -2 m for all times t.

2.3 Average Velocity and Average Speed

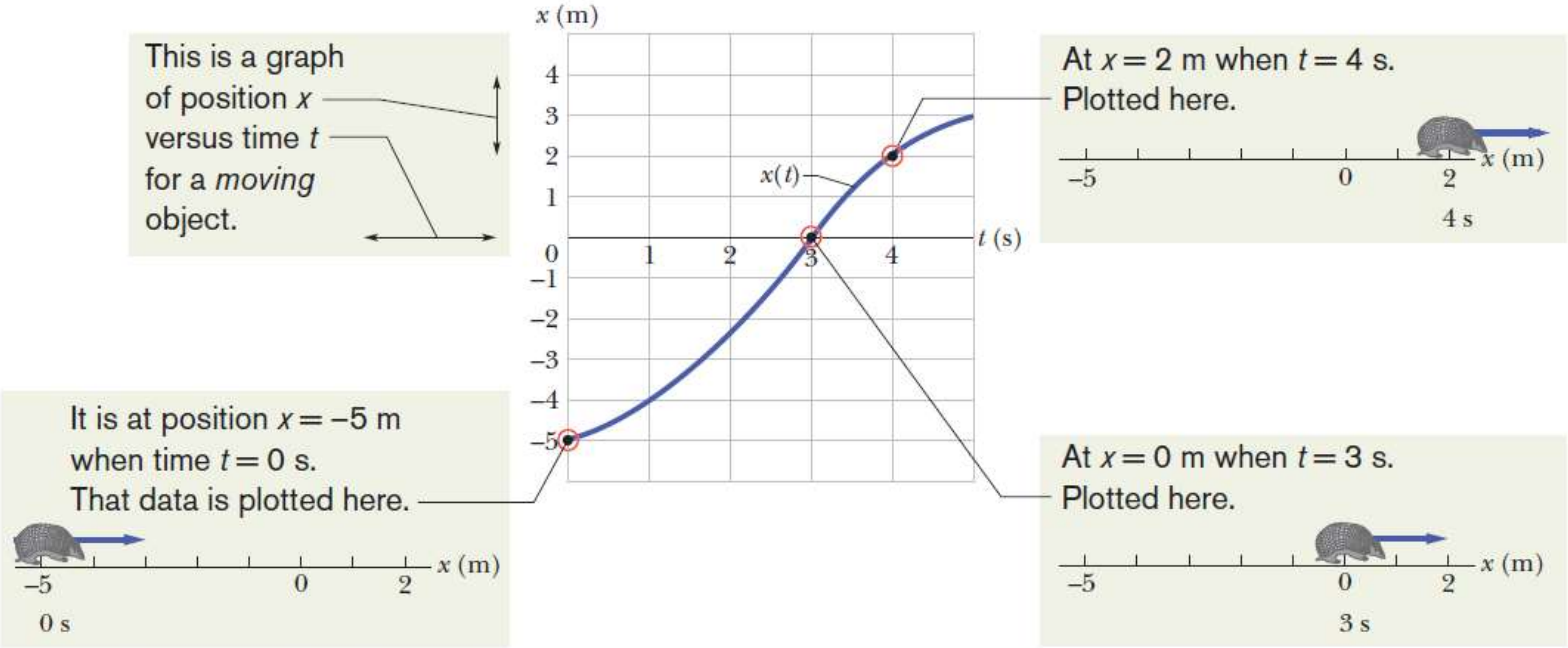


Fig. 2-3 The graph of $x(t)$ for a moving armadillo. The path associated with the graph is also shown, at three times.

2.3 Average Velocity and Average Speed

- There are several quantities that describe *how fast* an object is. One of them is the **average velocity** v_{avg}, which is the ratio of the displacement Δx during a time interval Δt to that interval:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

- A common unit for v_{avg} is (m/s). Generally, it has a unit of the form length/time.
- On the x vs t graph, v_{avg} is the **slope** of the straight line connecting the points (x_2, t_2) and (x_1, t_1).
- v_{avg} is a vector quantity and has the same sign as Δx.

2.3 Average Velocity and Average Speed

- Positive v_{avg}(and slope) means that the line goes *up* to the right
- Negative v_{avg} (and slope) means that the line goes *down* to the right

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{6\text{m}}{3\text{s}} = 2\text{ m/s}$$

This is a graph of position x versus time t.

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.

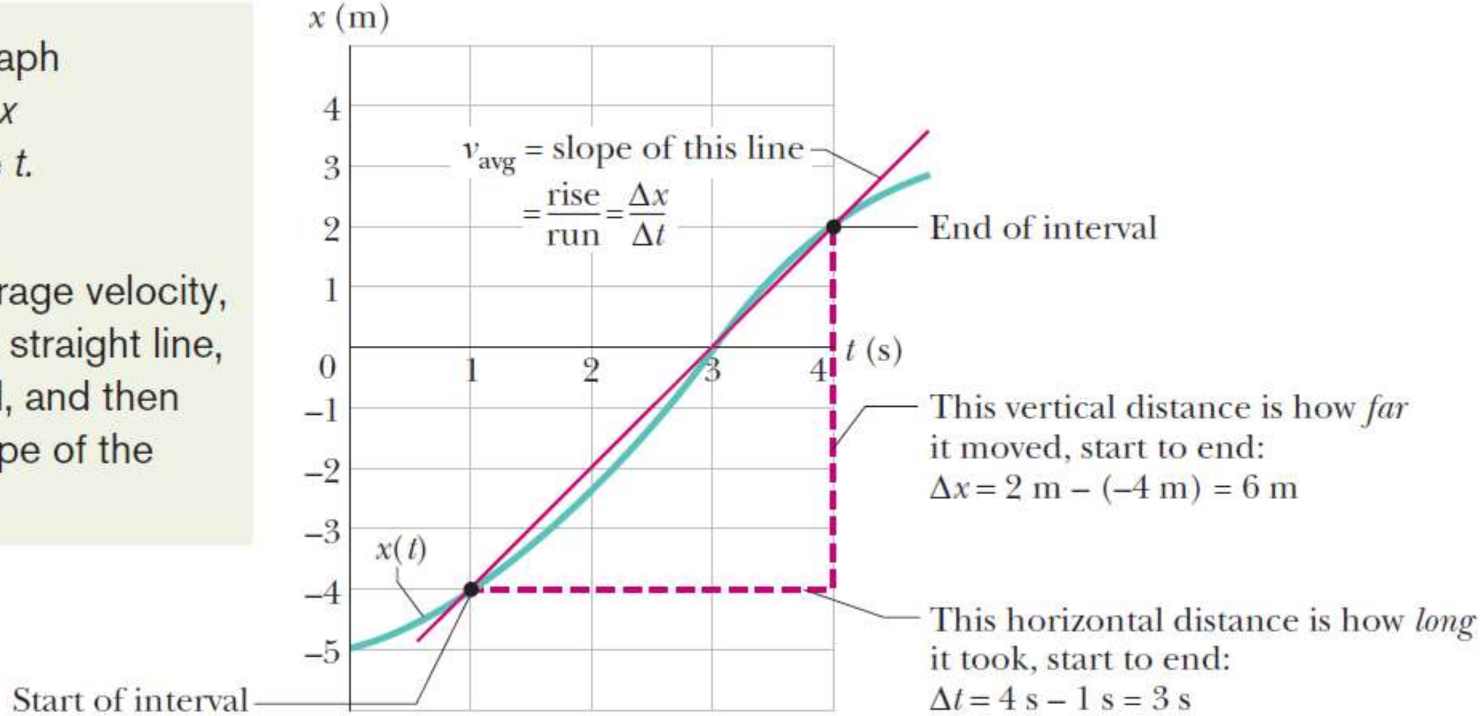


Fig. 2-4 Calculation of the average velocity between $t = 1$ s and $t = 4$ s as the slope of the line that connects the points on the $x(t)$ curve representing those times.

2.3 Average Velocity and Average Speed

- A second quantity that describes how fast an object is, is the **average speed** s_{avg}. It is given by

$$s_{avg} = \frac{\text{total distance}}{\Delta t}.$$

- s_{avg} lacks the algebraic sign. Like the total distance, s_{avg} is a *scalar* quantity.
- It is not always true that s_{avg} is the same as $\left|v_{avg}\right|$. That is because the total distance is not always the same as $|\Delta x|$.

2.3 Average Velocity and Average Speed

Example 1: You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

(a) We have that $x_1 = 0$ and $x_2 = 8.4\text{ km} + 2.0\text{ km} = 10.4\text{ km}$. Therefore,

$$\Delta x = x_2 - x_1 = 10.4\text{ km} - 0 = 10.4\text{ km}.$$

2.3 Average Velocity and Average Speed

(b) What is the time interval Δt from the beginning of your drive to your arrival at the station?

(b) The walking time interval is $\Delta t_{\mathrm{wlk}} = 0.50\ \mathrm{h}$. The driving interval Δt_{dr} is

$$\Delta t_{\mathrm{dr}} = \frac{\Delta x_{\mathrm{dr}}}{v_{\mathrm{avg,dr}}} = \frac{8.4\ \mathrm{km}}{70\mathrm{km/h}} = 0.12\ \mathrm{h}.$$

The total time interval is then

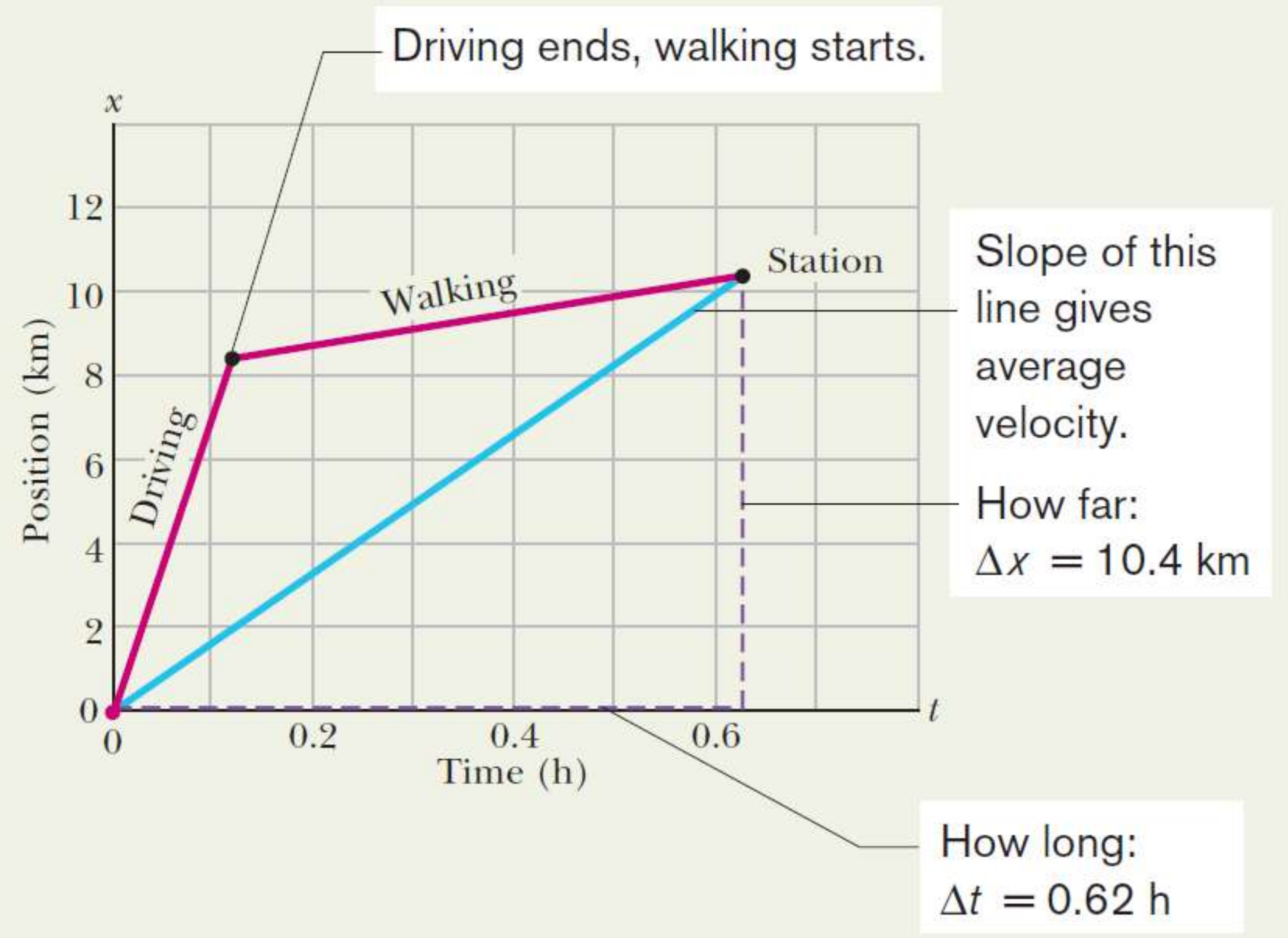
$$\Delta t = \Delta t_{\mathrm{dr}} + \Delta t_{\mathrm{wlk}} = 0.62\ \mathrm{h}.$$

2.3 Average Velocity and Average Speed

(c) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

(c)

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10.4\ \text{km}}{0.62\ \text{h}} = 17\,\frac{\text{km}}{\text{h}}.$$



2.3 Average Velocity and Average Speed

(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

(d) The total distance is $8.4\ \text{km} + 2.0\ \text{km} + 2.0\ \text{km} = 12.4\ \text{km}$. The total time is $0.12\ \text{h} + 0.50\ \text{h} + 0.75\ \text{h} = 1.37\ \text{h}$. Thus,

$$s_{avg} = \frac{\text{total distance}}{\text{time interval}} = \frac{12.4\ \text{km}}{1.37\ \text{h}} = 9.1\,\frac{\text{km}}{\text{h}}.$$

2.4 Instantaneous Velocity and Speed

- The instantaneous velocity (or simply velocity) v describes how fast an object is at a given instant of time, instead of a time interval Δt.
- The velocity is obtained from the average velocity as Δt approaches zero:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$

v is the rate at which x is changing with time at a given instant; or v is the derivative of x with respect to t. v is also the slope of the position-time curve at a point representing that instant.

2.4 Instantaneous Velocity and Speed

- Speed is the magnitude of velocity v:
$$\text{speed} = |v|.$$

2.4 Instantaneous Velocity and Speed

Checkpoint 2

The following equations give the position $x(t)$ of a particle in four situations (in each equation, x is in meters, t is in seconds, and $t > 0$): (1) $x = 3t - 2$; (2) $x = -4t^2 - 2$; (3) $x = 2/t^2$; and (4) $x = -2$. (a) In which situation is the velocity v of the particle constant? (b) In which is v in the negative x direction?

(a) 1 and 4.

(b) 2 and 3.

2.4 Instantaneous Velocity and Speed

Example 2: The position of a particle on the x axis is given by $x = t^3 - 27t + 4$, where x is in meters and t is in seconds. (a) Find the particle's velocity function $v(t)$. (b) At what point does the particle momentarily stop?

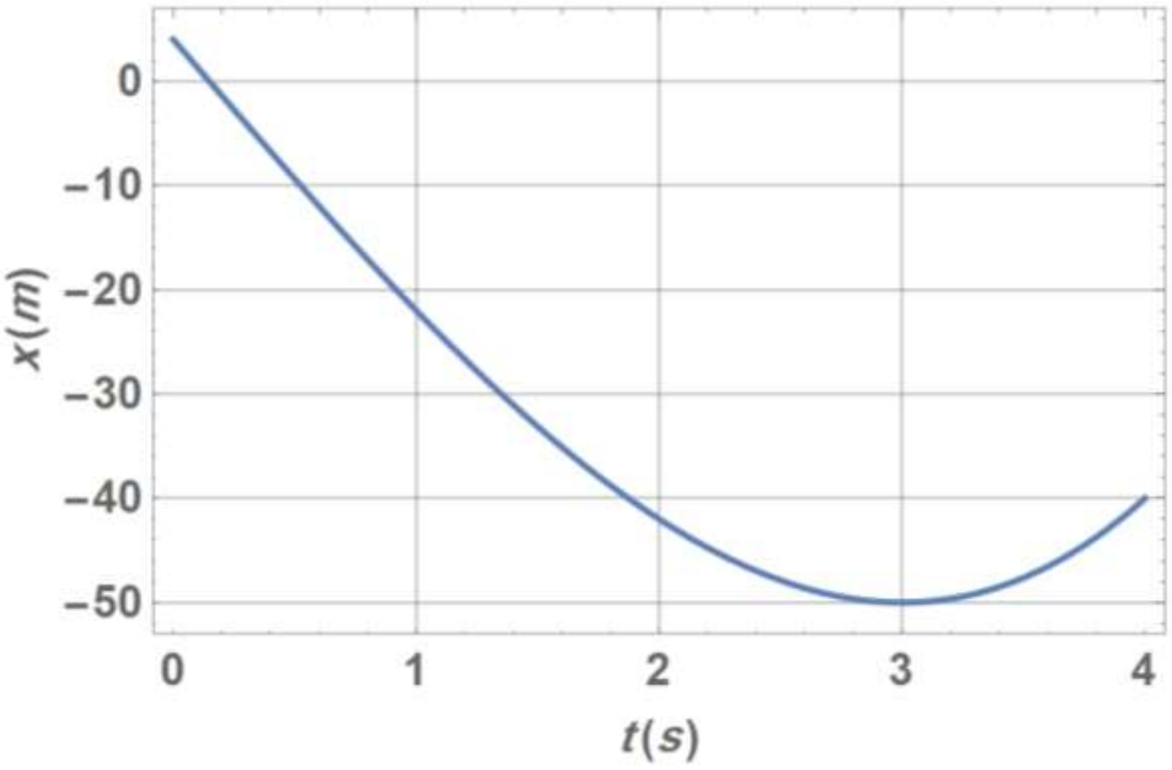
(a)

$$v = \frac{dx}{dt} = 3t^2 - 27.$$

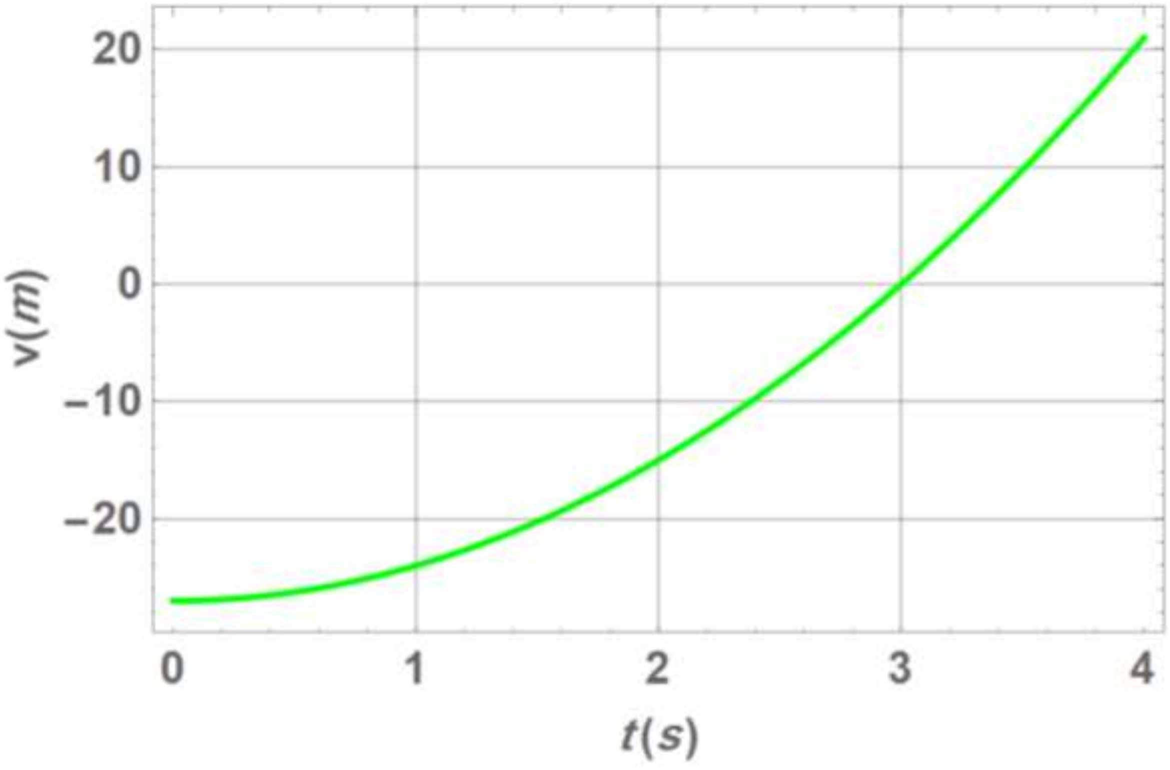
(b) The particle stops momentarily when $v = 0$ or $3t^2 - 27 = 0$. Solving for t and discarding the negative time root we get $t = 3$ s. Thus, the particle stops momentarily at

$$x(3) = 3^3 - 27(3) + 4 = -50 \text{ m}.$$

2.4 Instantaneous Velocity and Speed



$x = t^3 - 27t + 4$



$v = 3t^2 - 27$

2.5 Acceleration

- When a particle accelerates its velocity changes.
- The average acceleration a_{avg} over a time interval Δt is

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1},$$

where v_1 and v_2 are the particle's velocities at times t_1 and t_2, respectively.

- On the v vs t graph, a_{avg} is the **slope** of the straight line connecting the points (v_2, t_2) and (v_1, t_1).

2.5 Acceleration

- The instantaneous acceleration (or simply acceleration) is

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$

- The acceleration of a particle is the rate at which velocity changes at some instant of time.
- Graphically, a at any time t is the slope of the $v(t)$ curve at the point corresponding to that time.
- The acceleration is the second time derivative of the position function $x(t)$:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

2.5 Acceleration

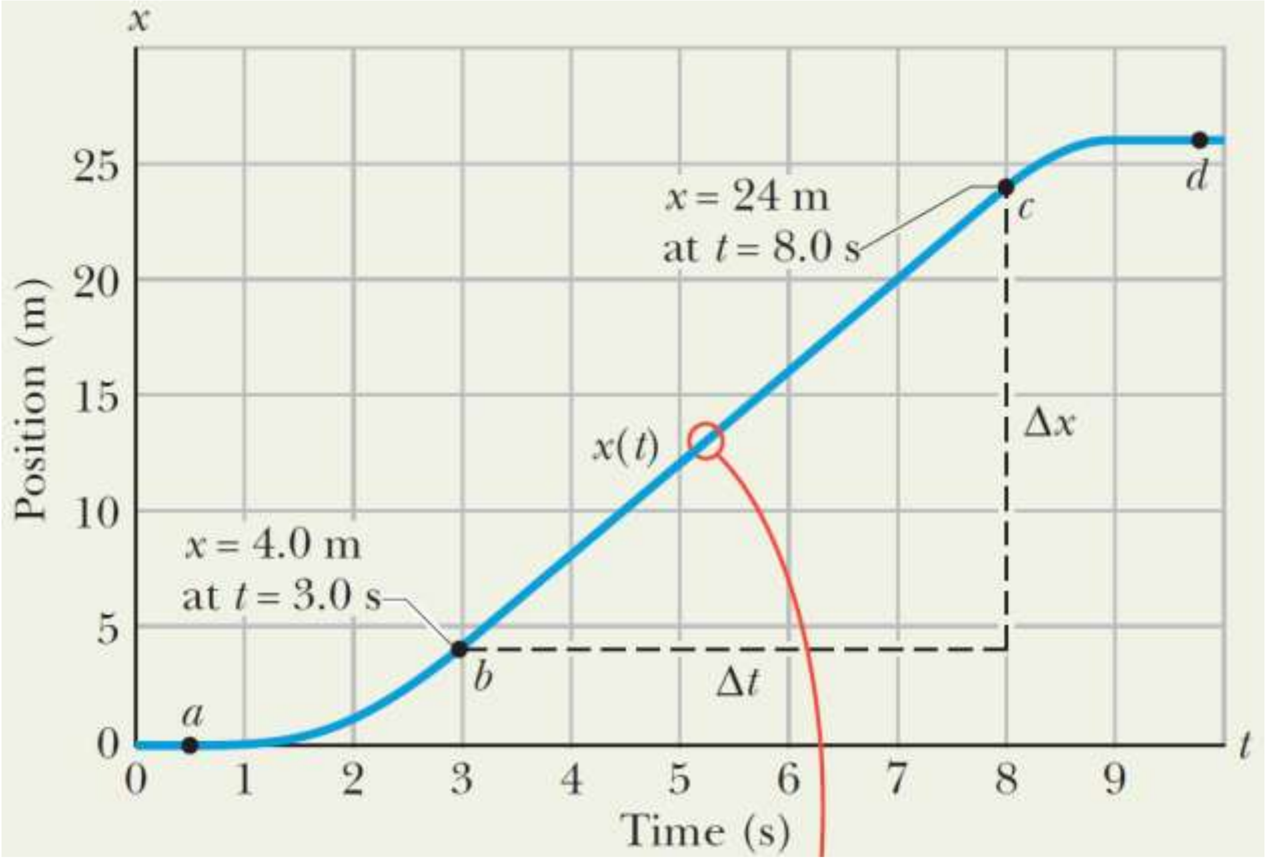
- Acceleration has both magnitude and direction; its is a vector quantity. It is positive to the right and negative to the left.
- The unit of acceleration has the form length/(time·time) and usually (m/s^2).

2.5 Instantaneous Velocity and Speed

Example 3: The figure is an $x(t)$ plot for an elevator cab that is initially stationary, then moves upward (which we take to be the positive direction of x), and then stops.

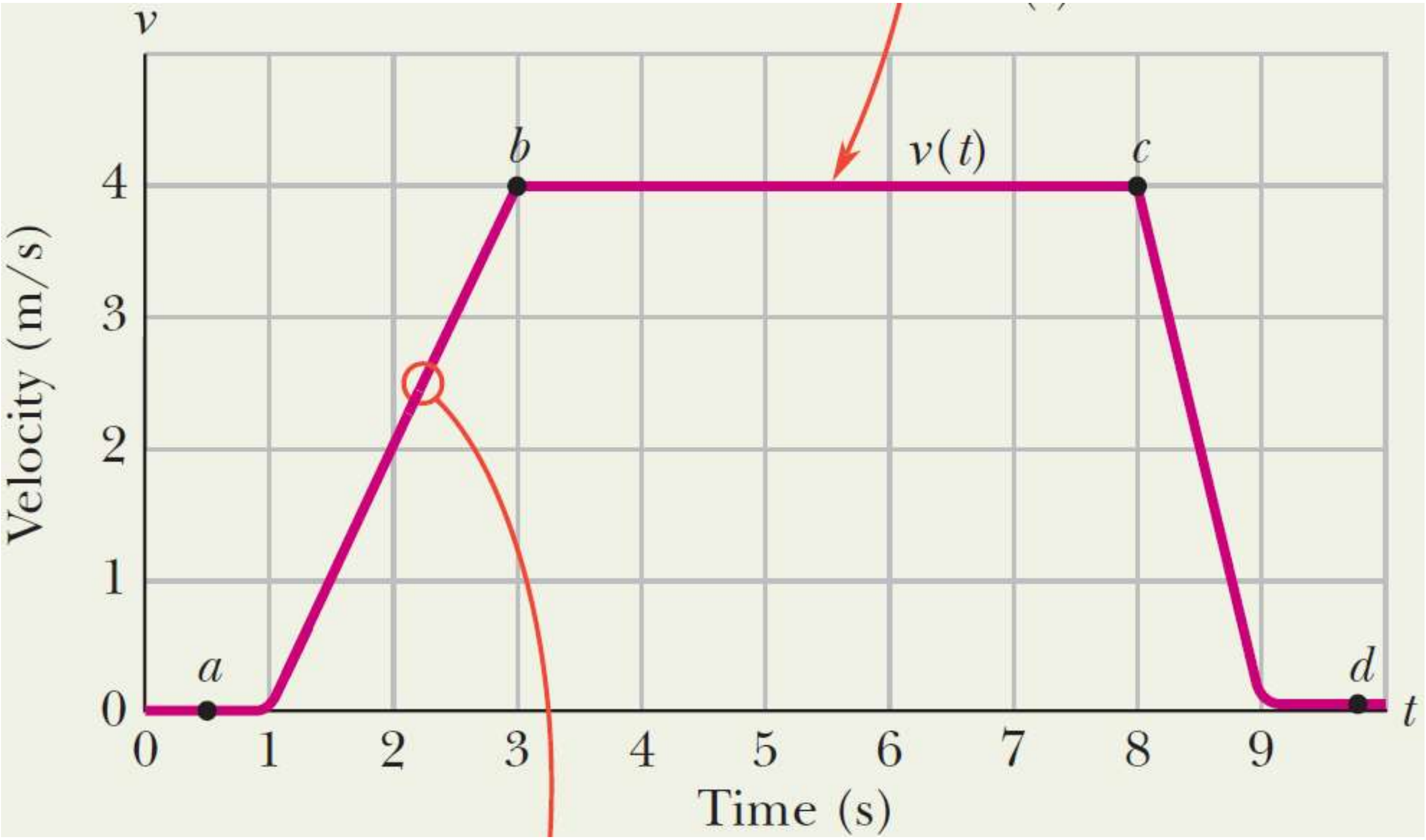
(a) Plot $v(t)$.

(b) Plot $a(t)$.



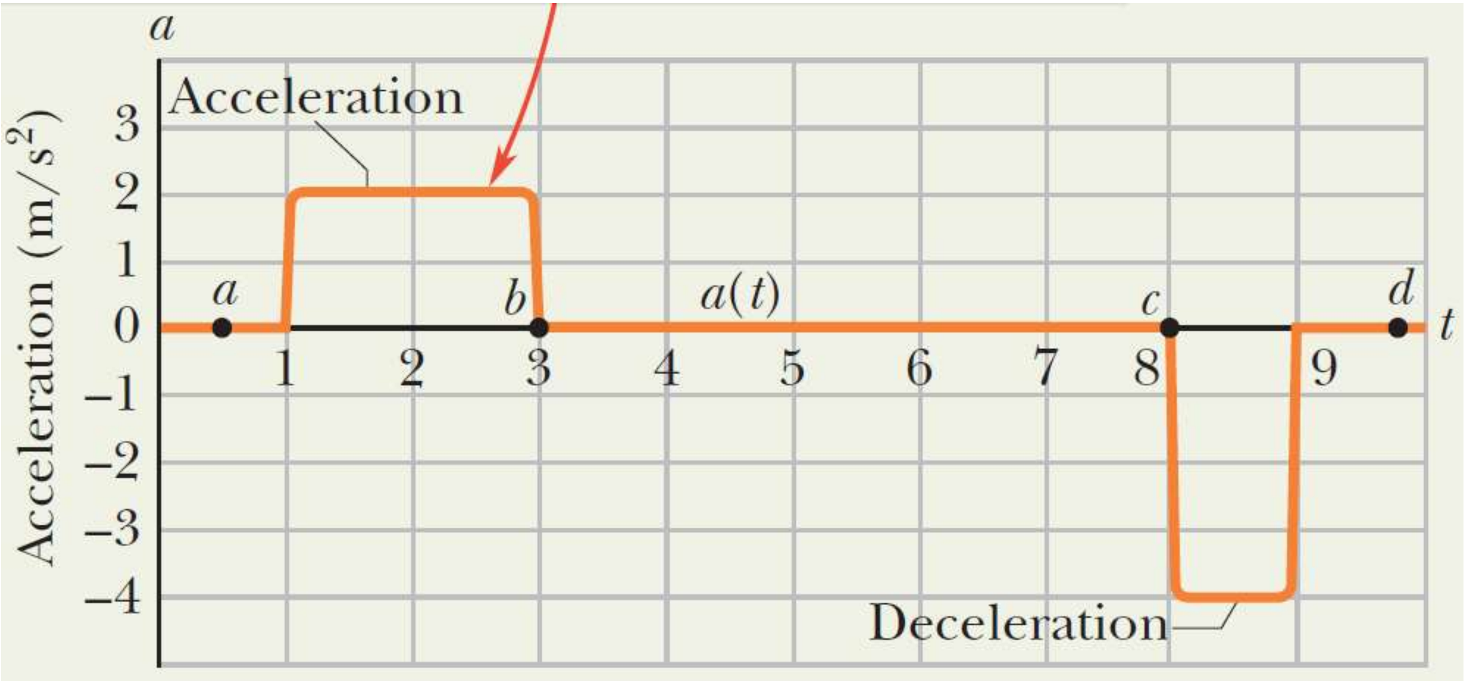
2.6 Acceleration

(a) We can find the velocity at any time from the slope of the $x(t)$ curve at that time.



2.6 Acceleration

(b) We can find the acceleration at any time from the slope of the $v(t)$ curve at that time.



2.6 Acceleration

Example 4: A particle's position on the x axis is given by $x = t^3 - 27t + 4$, where x is in meters and t is in seconds. Find the particle's acceleration function $a(t)$.

Acceleration $a(t)$ is the time derivative of velocity $v(t)$ and second time derivative of position $x(t)$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}.$$

Therefore,

$$a = \frac{d^2}{dt^2}(4 - 27t + t^3) = 6t.$$

2.6 Acceleration

- Large accelerations are sometimes expressed in terms of g units, with
$$1g = 9.8\frac{\text{m}}{\text{s}^2}.$$
For instance an acceleration of $3g$ is $3(9.8\ \text{m/s}^2) = 29\ \text{m/}s^2$.
- Unlike in common language, acceleration is change in velocity, not speed! For example, a car with $v = -25\ \text{m/}s$ that is brought to stop in 5.0 s undergoes an average acceleration $a_{avg} = +5\ \text{m/}s^2$. The speed of the car decreased but its velocity increased.

  If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

2.6 Acceleration

Checkpoint 3

A wombat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

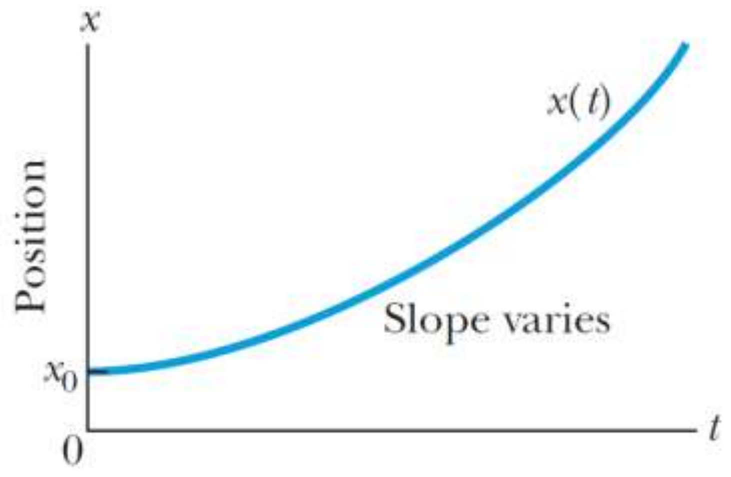
(a) plus

(b) minus

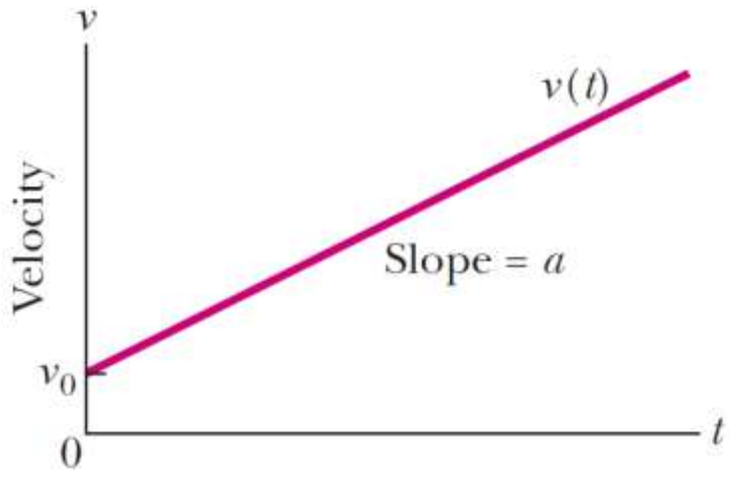
(c) minus

(d) plus

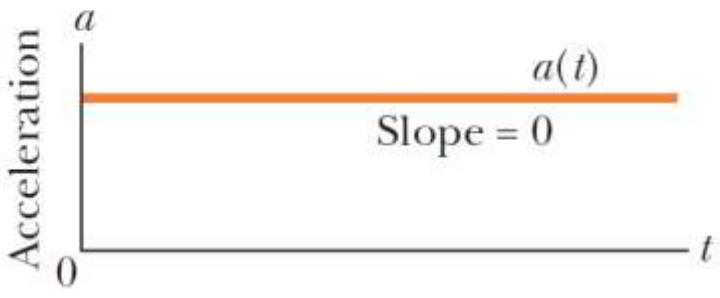
2.7 Constant Acceleration: A Special Case



Position-time graph for a particle starting at position x_0 that is moving with constant acceleration $a > 0$.



Velocity-time graph for a particle with initial velocity v_0 that is moving with constant acceleration $a > 0$.



Acceleration-time graph for a particle with constant acceleration $a > 0$.

2.7 Constant Acceleration: A Special Case

- In many situations, the acceleration is constant or nearly so. There is therefore a special set of equations for motion with constant acceleration.
- Because a is constant, we can write

$$a = a_{avg} = \frac{v - v_0}{t - 0},$$

$$v_2 = v \quad v_1 = v_0 \quad t_2 = t \quad t_1 = 0$$

or

$$v = v_0 + at.$$

Also, using

$$v_{avg} = \frac{x - x_0}{t - 0},$$

$$x_2 = x \quad x_1 = x_0$$

2.7 Constant Acceleration: A Special Case

we get

$$x = x_0 + v_{avg} t.$$

We know that

$$\begin{aligned} v_{avg} &= (v_0 + v)/2 \\ &= (v_0 + v_0 + at)/2 \\ &= v_0 + \frac{1}{2} at. \end{aligned}$$

Combining results yields

$$x - x_0 = v_0 t + \frac{1}{2} at^2.$$

2.7 Constant Acceleration: A Special Case

- These are the basic equations for motion with constant acceleration

$$x - x_0 = v_0 t + \frac{1}{2} a t^2,$$

$$v = v_0 + at.$$

- We can use these equations to write three useful equations:

$$v^2 = v_0^2 + 2a(x - x_0),$$ t eleminated

$$x - x_0 = \frac{1}{2}(v_0 + v)t,$$ a eleminated

$$x - x_0 = vt - \frac{1}{2} a t^2.$$ v_0 eleminated

2.8 Constant Acceleration: Another Look

- **Another Derivation**: Using a $= dv/dt$ we write

$$dv = a\,dt.$$

Integrating,

$$\int dv = \int a\,dt = a\int dt$$

which yields

$$v = at + c.$$

At time $t = 0$, $v = v_0$ which gives $c = v_0$.

2.8 Constant Acceleration: Another Look

Using $v = dx/dt$ we write

$$dx = v\,dt = a\,t\,dt + v_0 dt.$$

Integrating,

$$\int dx = a \int t\,dt + v_0 \int dt\,,$$

or

$$x = \frac{1}{2}at^2 + v_0 t + c'.$$

At time $t = 0$, $x = c'$ which gives $c' = x_0$

2.8 Constant Acceleration: Another Look

Checkpoint 4

The following equations give the position $x(t)$ of a particle in four situations: (1) $x = 3t - 4$; (2) $x = -5t^3 + 4t^2 + 6$; (3) $x = 2/t^2 - 4/t$; (4) $x = 5t^2 - 3$. To which of these situations do the equations of Table 2-1 apply?

1 and 4

2.9 Free-Fall Acceleration

- Near Earth surface, all objects fall in vacuum downward at a certain rate: the free-fall acceleration (g). It has the magnitude $g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$.
- During free fall the constant acceleration equations apply with

$$a = -g = -9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

- The motion is along the vertical axis with the positive direction of y upward.

2.9 Free-Fall Acceleration

- These are the basic equations for motion with constant acceleration

$$y - y_0 = v_0 t - \frac{1}{2} g t^2,$$

$$v = v_0 - g t.$$

- We can use these equations to write three useful equations:

$$v^2 = v_0^2 - 2g(y - y_0),$$ t eleminated

$$y - y_0 = \frac{1}{2}(v_0 + v)t,$$ a eleminated

$$y - y_0 = vt + \frac{1}{2} g t^2.$$ v_0 eleminated

2.9 Free-Fall Acceleration

- **Example 4**: A ball is thrown upward with an initial speed of $v_0 = 12\ \mathrm{m/s}$.

  (a) How long does the ball take to reach its maximum height?

  The ball is in free fall just after the release point. At the maximum height v is 0. Using $v = v_0 - gt$ we get

$$t = \frac{v_0 - v}{g} = \frac{v_0 - 0}{g} = \frac{12\ \mathrm{m/s}}{9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 1.2\ \mathrm{s}.$$

2.9 Free-Fall Acceleration

(b) What is the ball's maximum height above its release point?

Taking the ball's initial height to be $y_0 = 0$ we write

$$y = v_0 t - \frac{1}{2} g t^2 = \left(12 \frac{\text{m}}{\text{s}}\right)(1.2\ \text{s}) - \frac{1}{2}\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(1.2\ \text{s})^2 = 7.3\ \text{m}.$$

We could have used any of the other three equations to find y. Try them!

2.9 Free-Fall Acceleration

(c) How long does the ball take to reach a point 5.0 m above its release point?

Substituting in $y - y_0 = v_0 t - \frac{1}{2} g t^2$, we get

$$5.0\ \mathrm{m} = \left(12\,\frac{\mathrm{m}}{\mathrm{s}}\right) t - \frac{1}{2}\left(9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}\right) t^2.$$

Solving for t we find

$$t = 0.53\ \mathrm{s} \quad \text{and} \quad t = 1.9\ \mathrm{s}.$$

The ball passes through $\mathrm{y} = 5.0\ \mathrm{m}$ twice, once on the way up at $t = 0.53\ \mathrm{s}$ and once on the way down at $t = 1.9\ \mathrm{s}$.

2.10 Graphical Integration in Motion Analysis

Checkpoint 5

(a) If you toss a ball straight up, what is the sign of the ball's displacement for the ascent, from the release point to the highest point? (b) What is it for the descent, from the highest point back to the release point? (c) What is the ball's acceleration at its highest point?

a) Positive
b) Negative
c) $a = -g = -9.8\ \mathrm{m/s^2}$

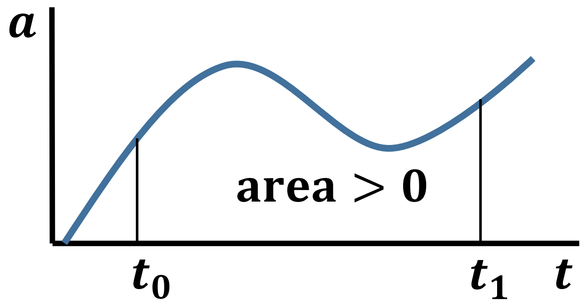
2.10 Graphical Integration in Motion Analysis

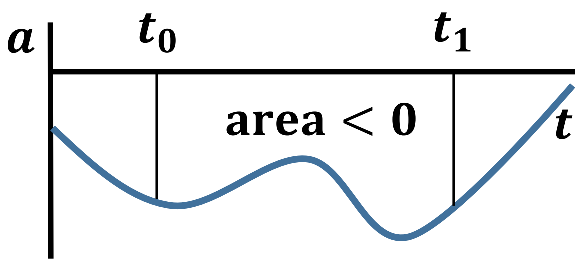
- **Acceleration**: from $a = \frac{dv}{dt}$, we have

$$v_1 - v_0 = \int_{t_0}^{t_1} a\, dt$$

a is not necessarily constant!

$$= \begin{bmatrix} \text{area between } a(t) \text{ curve} \\ \text{\& time axis, from } t_0 \text{ to } t_1 \end{bmatrix}.$$



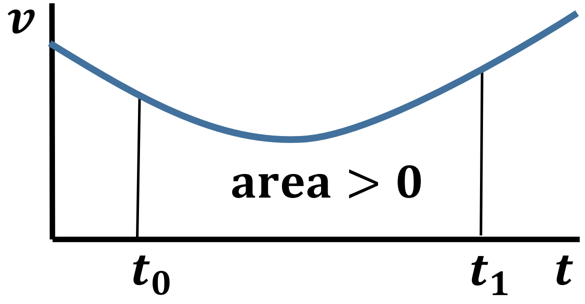


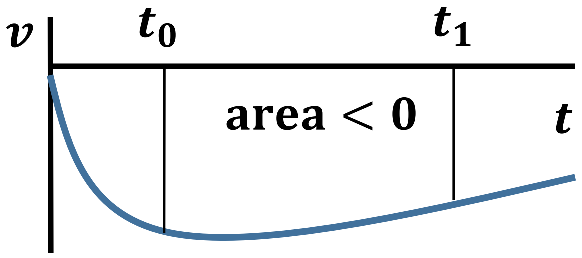
2.10 Graphical Integration in Motion Analysis

- **Velocity**: from $v = \frac{dx}{dt}$, we have

$$x_1 - x_0 = \int_{t_0}^{t_1} v\,dt$$

$$= \begin{bmatrix} \text{area between } v(t) \text{ curve} \\ \text{\& time axis, from } t_0 \text{ to } t_1 \end{bmatrix}.$$





2.10 Graphical Integration in Motion Analysis

••69 ILW How far does the runner whose velocity–time graph is shown in Fig. 2-37 travel in 16 s? The figure's vertical scaling is set by $v_s = 8.0$ m/s.

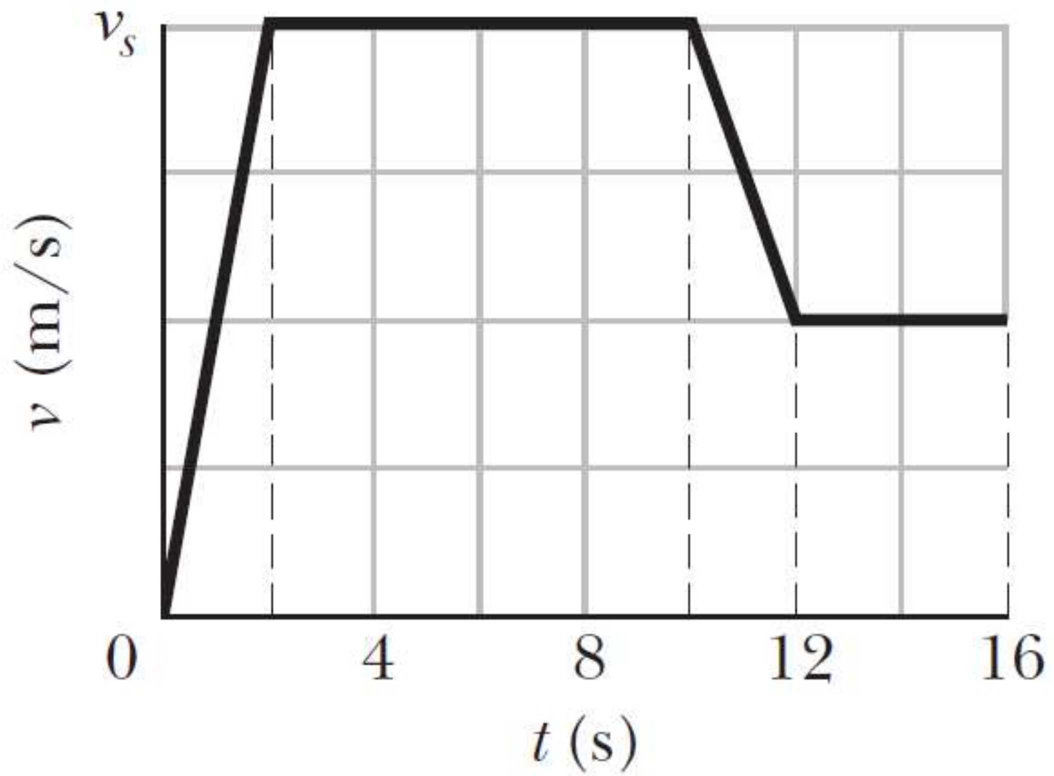


Fig. 2-37 Problem 69.

2.10 Graphical Integration in Motion Analysis

$$\Delta x = \int_0^{16} v dt = \text{the area under the } v(t) \text{ curve}$$

$$= 25 \text{ squares.}$$

The area of a square is $2.0 \text{ m/s} \times 2.0 \text{ s} = 4.0 \text{ m}$. Thus

$$\Delta x = 100 \text{ m.}$$