Let the absorption spectrum $\alpha(\omega)$ of a sample be centered at ω_0 and have a Lorentzian profile with a maximum value of one and a full-width at half maximum of γ. Also, let the length of the sample be one.

Suppose a laser beam with frequency ω_{L0} and power P_0 is sent through the sample and the laser frequency is modulated sinusoidally such that the laser frequency becomes $\omega_L = \omega_{L0} + a \sin \Omega t$, where a is the modulation depth and Ω is the angular frequency of modulation. Suppose you are using an ideal lock-in amplifier to detect the transmitted power. Your lock-in amplifier multiplies the transmitted power by $\sin(n\Omega t + \phi)$, and take the average over on period of the modulation frequency and divides the result by the average of P_0 over one modulation period. Here n is a positive integer and ϕ is a phase constant that needs to be adjusted for each order n to get the maximum value and the right sign of the derivative. When modulating with $\sin \Omega t$ and assuming no phase is accumulated during measurement process, multiply by $\sin(1\Omega t + 0)$ to extract the first derivative, multiply by $\sin(2\Omega t - \pi/2)$ to extract the second derivative, and multiply by $\sin(2\Omega t - \pi)$ to extract the third derivative.

Q1.

Use Mathematica to simulate your ideal lock-in amplifier action and plot the output of the amplifier as a function of a dimensionless quantity $x = \frac{\omega - \omega_0}{\gamma}$ from $x = -2$ to $x = +2$ for the following cases

$a = 0.05\,\gamma$ and $n = 1$.

$a = 0.05\,\gamma$ and $n = 2$.

$a = 0.05\,\gamma$ and $n = 3$.

Q2.

Find expressions for the first, second and third derivatives of $\alpha(\omega)$ with respect to ω.

Use the last equation in page 10 of the 5th edition of Demtröder book "laser spectroscopy 2 Experimental Techniques" to find an approximate expression for the output of your lock-in amplifier for $n = 1, 2$ and 3. Use only the first term of each square bracket.

Q3.

Plot the expressions you obtain in Q2 as function of x from $x = -2$ to $x = +2$ for the same cases in Q1. Compare the approximate result of Q3 and the exact result of Q1 by overlapping the corresponding plots.

For $a = 0.05$, we obtain same plots as in Q1.

Q4.

Plot the exact and approximate result of your lock-in amplifier for the case $n = 1$ and modulation depth of 0.2.

Q5.

Plot the exact result of your lock-in amplifier for the case $n = 1$ and for different modulation depths and find the modulation depth that results in the maximum signal of the lock-in amplifier.