CH 16 Oscillations الاهتزاز

- Simple harmonic motion
  - displacement
  - velocity
  - acceleration
  - Force law
  - energy
- Angular simple harmonic motion.
- Pendulums
  - simple pendulum.
  - physical pendulum.
- Simple harmonic motion and uniform circular motion.

Simple harmonic motion :- الحركة التوافقية البسيطة

Displacement

The motion of an object is a simple harmonic motion (SHM), if the object's displacement is given by

$$\boxed{x(t) = x_m \cos(\overbrace{\omega t + \phi}^{\text{phase}})}$$

$x(t)$: displacement at time t

x_m: Amplitude (maximum displacement)

ω: Angular frequency

t: time

ϕ: phase constant (phase angle)

x_m, ω, ϕ } constants

ω: omega

ϕ: phi

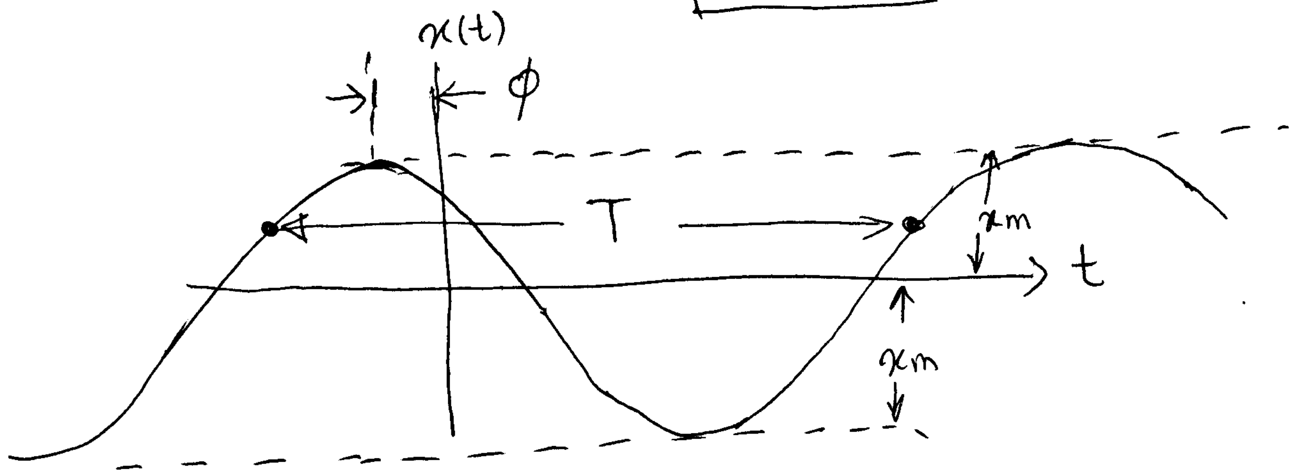
$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$

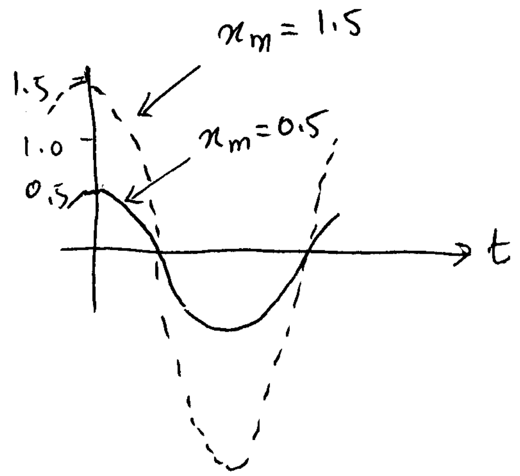
ω: Angular frequency (rad/s)

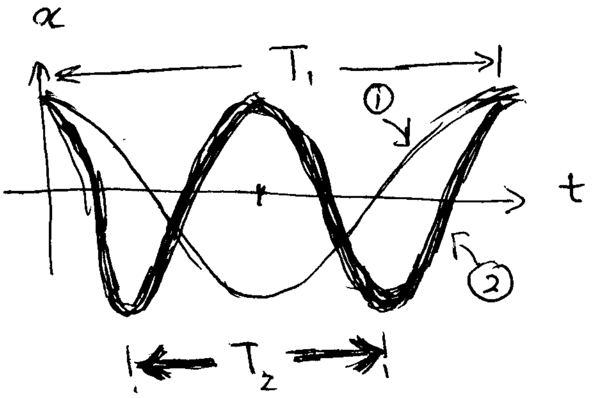
T: period

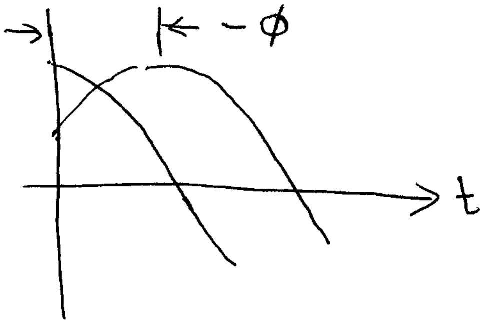
f: frequency (Hertz = Hz = Oscillation per second)

$$\boxed{T = \frac{1}{f}}$$









Decreasing ϕ shifts the curve right ward.

<u>Velocity</u>

$$v(t) = \frac{dx(t)}{dt} = -\omega x_m \sin(\omega t + \phi)$$

$$\boxed{v(t) = -v_m \sin(\omega t + \phi)}$$

$v_m = \omega x_m$

velocity amplitude.

<u>acceleration</u>

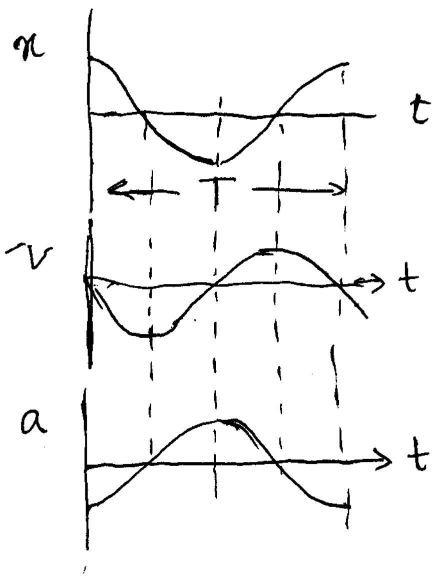
$$a(t) = \frac{d}{dt} v(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

$$\boxed{a(t) = -a_m \cos(\omega t + \phi)}$$

$a_m = \omega^2 x_m$ $\qquad$ $a(t) = -\omega^2 x(t)$

In simple harmonic motion,

$$a(t) = -\omega^2 x(t)$$



Displacement

velocity — shifted to the left by $T/4$ with respect to displacement

Acceleration — shifted to the left by $T/2$ with respect to displacement

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Force law :-

For simple harmonic motion $a(t) = -\omega^2 x(t)$

From Newton's second law $F = ma$

$F = m(-\omega^2 x)$

$$\boxed{F = -m\omega^2 x}$$ Hooke's law

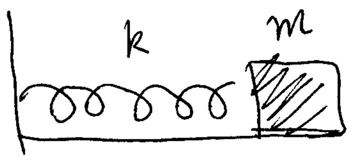
k

$$\boxed{k = m\omega^2}$$

spring constant

mass

angular ~~fre~~quency

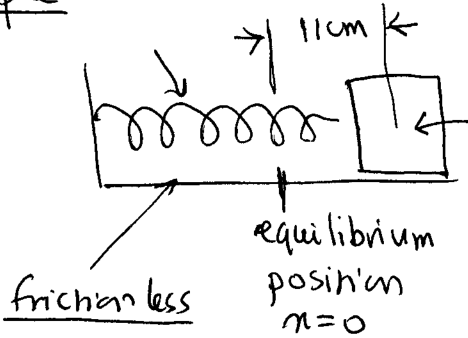


$\omega = \sqrt{\dfrac{k}{m}}$ $\qquad T = 2\pi\sqrt{\dfrac{m}{k}}$

Example

$k = 65\ N/m$

at $t=0$



Q what is the angular frequency, the frequency and the period of the motion?

A $$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65}{0.680}} = 9.8\ \text{rad/sec}$$

convert to kg.

$$f = \frac{\omega}{2\pi} = 1.6\ \text{Hz}$$

$$T = \frac{1}{f} = 0.64\ \text{s}$$

Q what is the amplitude of the motion?

A $x_m = 11\ \text{cm}.$

Q what is the maximum speed of the block and where is the block when it occurs?

convert to m

A $$v_m = \omega x_m = (9.8\ \text{rad/s})(0.11\ \text{m}) = 1.1\ \text{m/s}$$

this happens when $x = 0$

Q what is the maximum acceleration of the block?

A $$a_m = \omega^2 x_m = (9.8\ \text{rad/s})^2 (0.11\ \text{m}) = 11\ \text{m/s}^2$$

Q what is the phase constant of the motion?

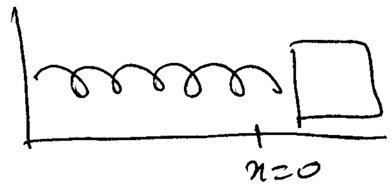
A at $t=0$, $x = 11$ cm

$$x(t) = x_m \cos(\cancel{\omega t} + \phi) \quad (\omega t = 0)$$

$$0.11 = 0.11 \cos(\phi)$$

$$1 = \cos(\phi) \Rightarrow \phi = 0$$

Example



At $t=0$

$x = -8.50$ cm

$v = -9.20$ m/s

$a = 47.0$ m/s²

Q what is the angular frequency of the system?

A
$$x(0) = x_m \cos\phi$$
$$v(0) = -\omega x_m \sin\phi$$
$$a(0) = -\omega^2 x_m \cos\phi$$

$$\frac{a(0)}{x(0)} = \frac{-\omega^2 x_m \cos\phi}{x_m \cos\phi} = -\omega^2$$

$$\omega = \sqrt{-\frac{a(0)}{x(0)}} = \sqrt{-\frac{47.0}{-8.50}} = 23.5 \text{ rad/s}$$

Q what is the phase constant and amplitude?

$$\frac{v(0)}{x(0)} = \frac{-\omega x_m \sin\phi}{x_m \cos\phi} = -\omega \tan\phi$$

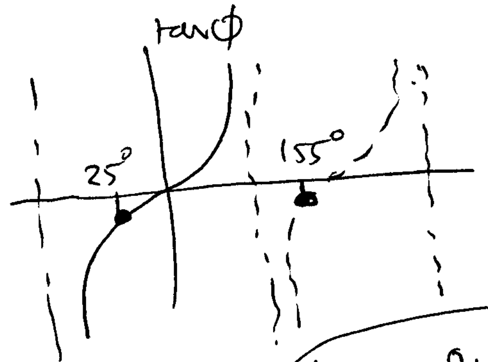
$$\tan\phi = -\frac{v(0)}{\omega x(0)} = \frac{-0.920}{(23.5)(-0.085)} = -0.461$$

$\phi = \tan^{-1}(-0.461)$

$\phi = -25°$ OR $\phi = 180° + (-25) = 155°$

not from Calculator

which one is correct?



we know that

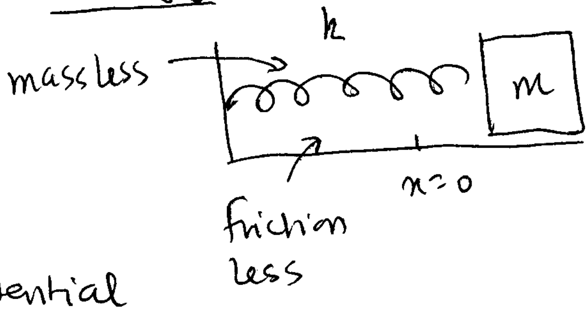
$x(0) = x_m \cos\phi$

$x_m = \dfrac{x(0)}{\cos\phi}$ $\begin{cases} \text{for } \phi = -25 & x_m = -0.094\text{ m} \text{ not acceptable} \\ \text{for } \phi = 155° & x_m = 0.094\text{ m} \end{cases}$

↑ always positive

so $\phi = 155°$ and $x_m = 0.094\text{ m}$

Energy

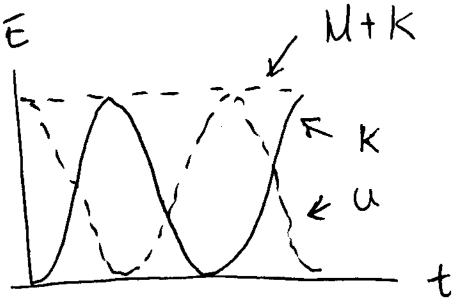


elastic potential energy $\longrightarrow U = \frac{1}{2} k x^2 = \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi)$

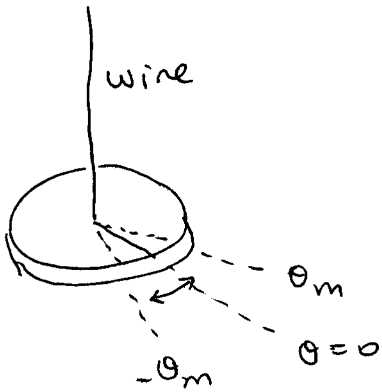
Kinetic energy $\longrightarrow K = \frac{1}{2} m v^2 = \frac{1}{2} m (-\omega x_m \sin(\omega t + \phi))^2$

$= \frac{1}{2} m\omega^2 x_m^2 \sin^2(\omega t + \phi)$

Mechanical energy $\longrightarrow E = U + K = \frac{1}{2} k x_m^2$



Angular simple harmonic motion:



$F \longrightarrow \tau$

$x \longrightarrow \theta$

$m \longrightarrow I$

$k \longrightarrow \kappa$

k: spring constant

κ: kappa, torsion constant

$$\boxed{k = m\omega^2}$$

$$\boxed{\kappa = I\omega^2}$$

$$F = -kx \implies \tau = -\kappa\theta$$

$$T = 2\pi\sqrt{\frac{m}{k}} \implies T = 2\pi\sqrt{\frac{I}{\kappa}}$$

Example

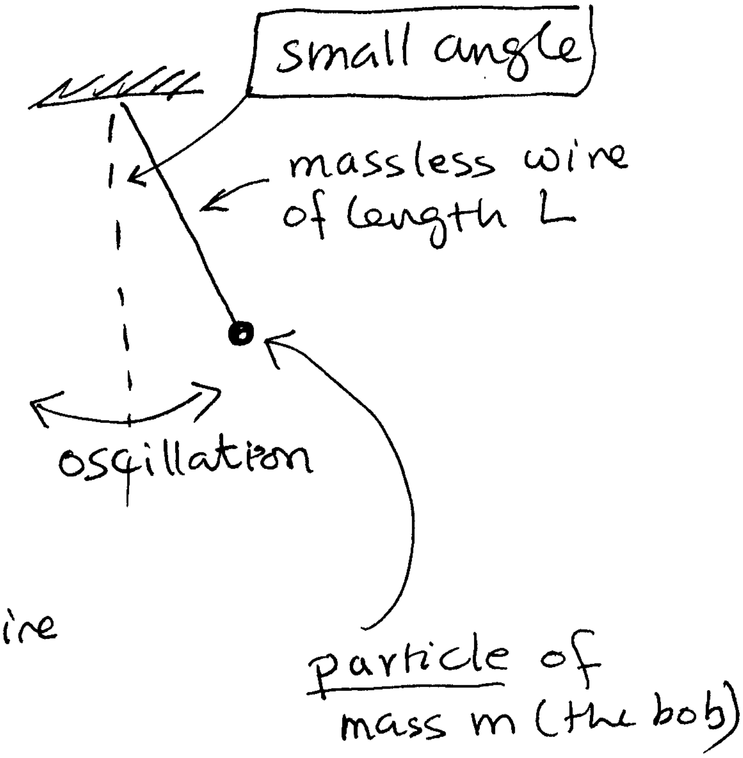
Pendulums

Simple pendulum

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

period of oscillation

Length of the wire



Physical pendulum

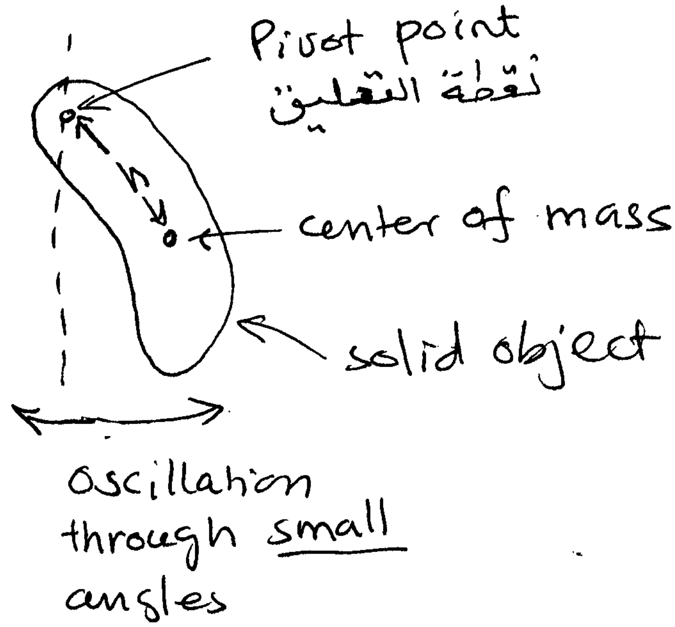
$$\boxed{T = 2\pi\sqrt{\frac{I}{mgh}}}$$

period of oscillation

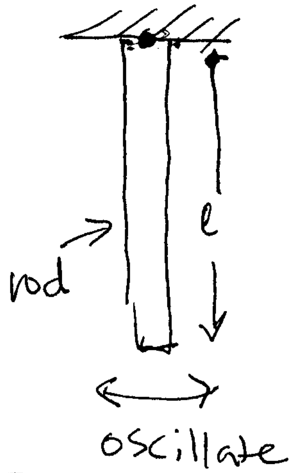
mass of the solid object

moment of inertia of the solid object about the pivot point

distance between pivot point and the center of mass



Example



Q what is the period of oscillation

A $T = 2\pi\sqrt{\dfrac{I}{mgh}}$, with $h = \dfrac{L}{2}$

$$I = \underbrace{\frac{1}{12}mL^2}_{\text{moment of Inertia about CoM}} + m\left(\frac{L}{2}\right)^2 = \frac{1}{3}mL^2$$

Parallel-axis theorem

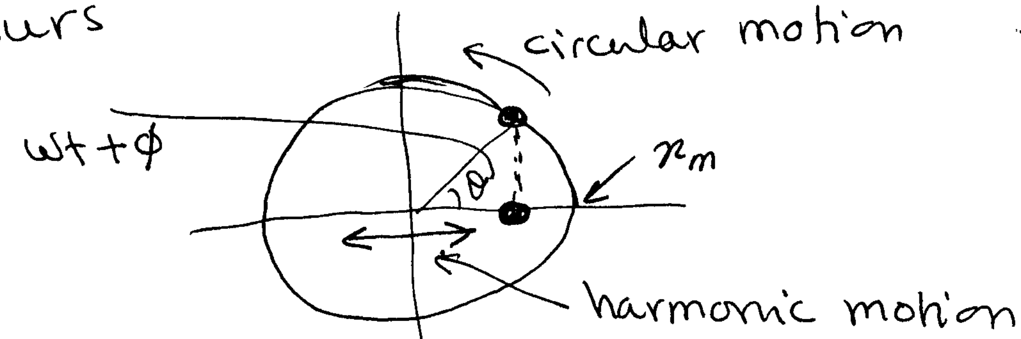
moment of Inertia about CoM. (CoM = center of mass)

$$= 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg\frac{L}{2}}}$$

$$= 2\pi\sqrt{\frac{2L}{3g}}.$$

Simple harmonic motion and uniform circular motion

Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the later motion occurs



$$x = x_m\cos(\omega t + \phi)$$