CH 12 Rolling, torque, and angular momentum

- Rolling
  - Kinetic Energy of rolling
  - friction and rolling
- Torque
- Angular Momentum
  - Newton's second law
  - Conservation

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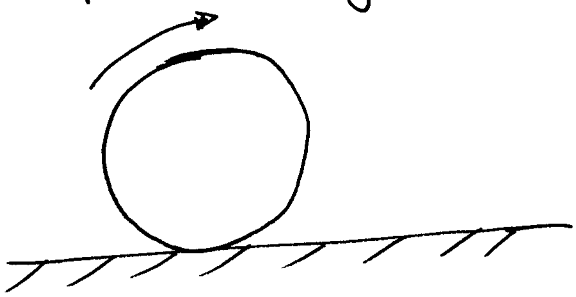
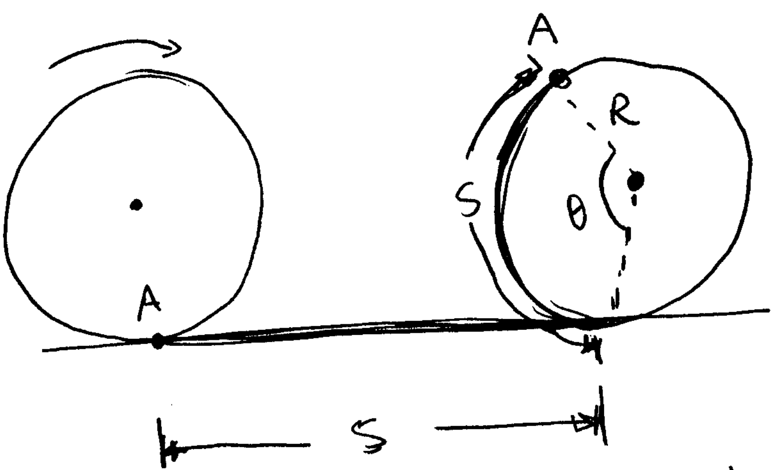
Rolling

A wheel moving along a straight track is an example of a rolling object.



We say an object rolls <u>smoothly</u>, when it does <u>not</u> slide along its path.



$$S = R\theta$$

$$\frac{ds}{dt} = R\frac{d\theta}{dt}$$

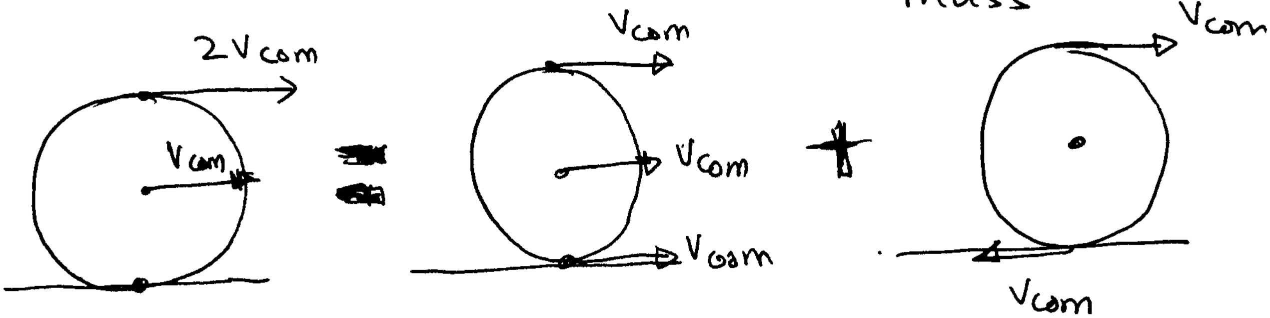
$$\boxed{v_{com} = R\omega}$$

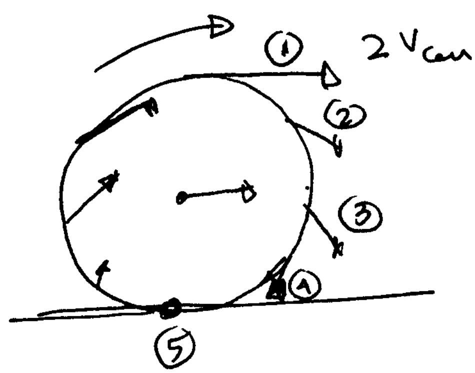
v_{com}: speed of center of mass

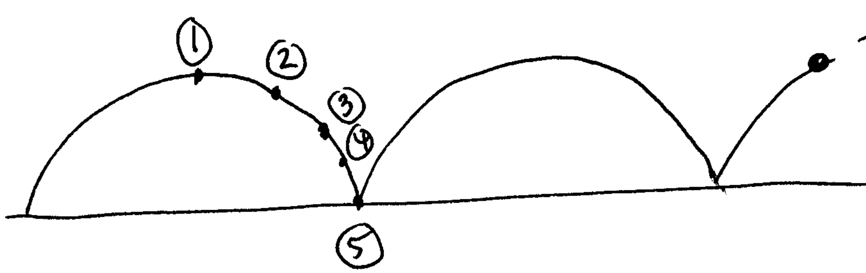
ω: angular velocity about the com.

com ≡ center of mass

Rolling = Pure translation + Pure Rotation about the center of mass







trace of a point on the rim of the rolling object

The kinetic energy of rolling :-

moment of inertia about com

$$\boxed{K = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M v_{com}^2}$$

rotational kinetic energy due to rotation about center of mass

translational kinetic energy due to translation of the center of mass

Example :-

A disk of mass $M = 1.4\ kg$ and radius $R = 8.5\ cm$ rolls smoothly across a horizontal table at a speed of $15\ cm/s$. What is K?

$$K = \frac{1}{2} I_{com} \omega^2 + \frac{1}{2} M v_{com}^2$$

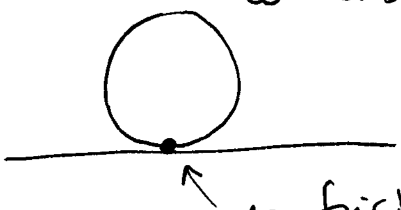
$$= \frac{1}{2}\left(\frac{1}{2} MR^2\right)\left(\frac{v_{com}}{R}\right)^2 + \frac{1}{2} M v_{com}^2$$

$$= \frac{3}{4} M v_{com}^2 = \frac{3}{4}(1.4\ kg)(0.15\ m/s)^2$$

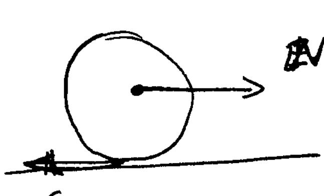
$$= 24\ mJ.$$

Friction and rolling:-

- If a body rolls at <u>constant</u> speed ($\alpha = 0$), frictional force at the point of contact $= 0$. no external Force makes the body tend to slide.

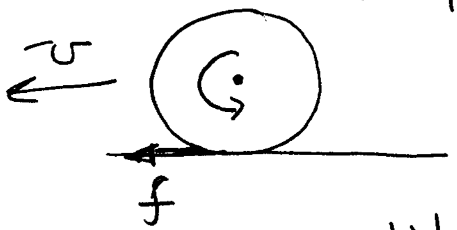


- If an external Force is applied on the center of mass to make the body tend to rotate faster or slower, the frictional force will be opposite to the applied net force parallel to the track.

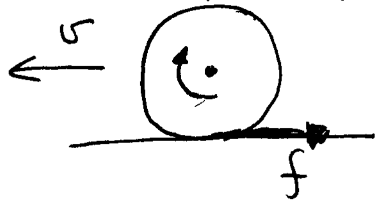


- If a torque is applied about the center of mass to make the body rotate faster or slower, the frictional force will be in a direction to oppose the change in angular speed.

torque on com to increase speed



torque on c.o.m. to decrease speed

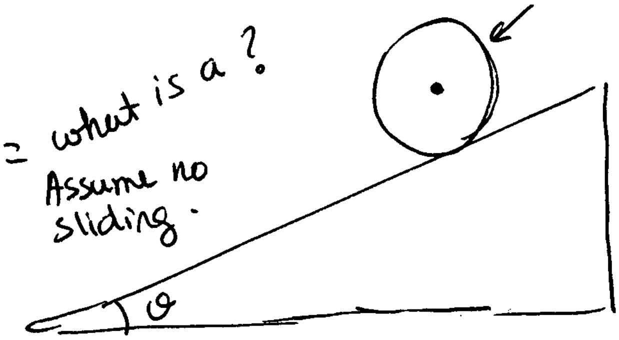


- If body does not slide the friction = static. → $\alpha = \frac{a}{R}$
  If body slide, friction = Kenitic → $\alpha \neq \frac{a}{R}$

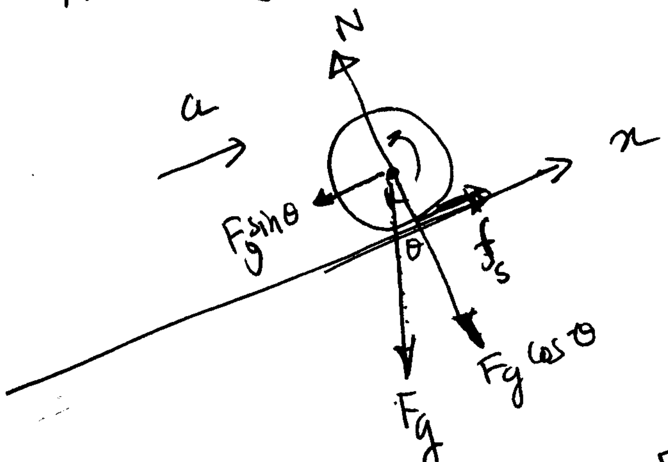
Example

radius R
mass M

Q = what is a ?
Assume no sliding.



A: Free-body diagram



Newton's second law along x : $-F_g \sin\theta + f_s = ma$

Newton's second law about the c.o.m.

$$\tau_{com} = I_{com}\alpha$$

$$f_s R = I_{com}\alpha$$

$\alpha = -\frac{a}{R}$
no sliding

negative because we choose a in opposite direction to the direction of rotation.

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$$-F_g \sin\theta + f_s = ma$$

$$f_s R = I_{com}\left(-\frac{a}{R}\right)$$

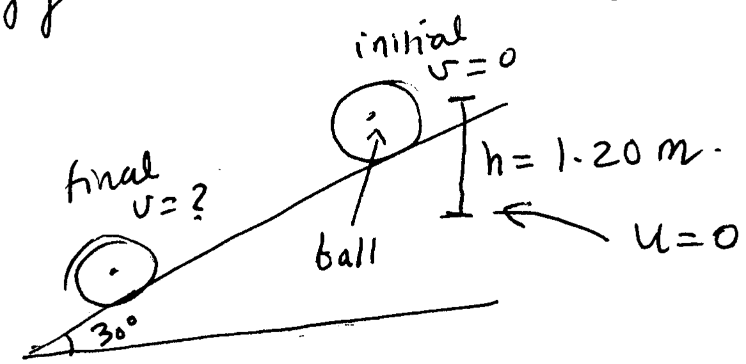
$$-mg\sin\theta - \frac{I_{com}\,a}{R^2} = ma$$

$$a = \frac{-mg\sin\theta}{m + \frac{I_{com}}{R^2}}.$$

For a rolling body that does not slide, the static frictional force does not tranfer any energy to thermal energy.

Example:

body does not slide



Q. $v = ?$

A. Since the ball does not slide, the static frictional force does not transfer energy to themal energy. In our ball-Earth system, no external frce does work on the system ⇒ Mechanical energy is conserved

$$K_i + U_i = K_f + U_f$$

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$$K_i + U_i = K_f + U_f$$

$$0 + \cancel{M}gh = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2 + 0$$

$$= \frac{1}{2}\,\frac{2}{5}\cancel{M}R^2\left(\frac{v}{R}\right)^2 + \frac{1}{2}\cancel{M}v_{com}^2$$

$$gh = \left(\frac{1}{5} + \frac{1}{2}\right) v_{com}^2$$

$$gh = \frac{7}{10} v_{com}^2$$

$$v_{com} = 4.1 \text{ m/s}.$$

Tor:

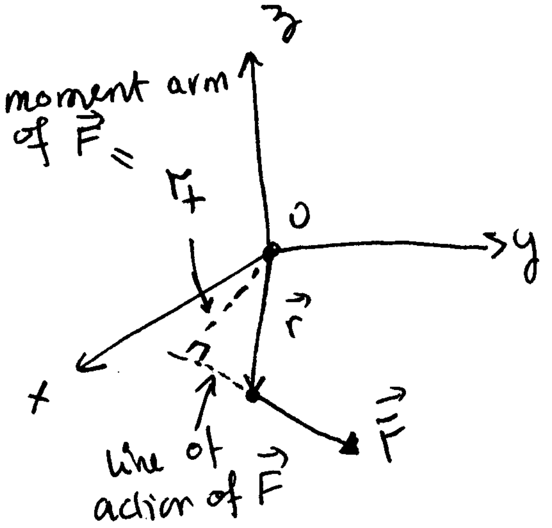
$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

Torque about a point O

position vector from point O to $\vec{F}$

force

magnitude:



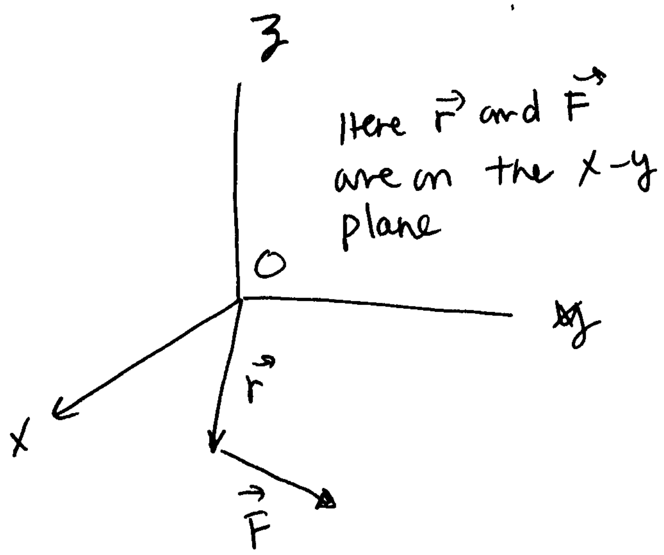
$$\tau = rF\sin\theta$$

$$\tau = rF_\perp$$

$$\tau = r_\perp F$$

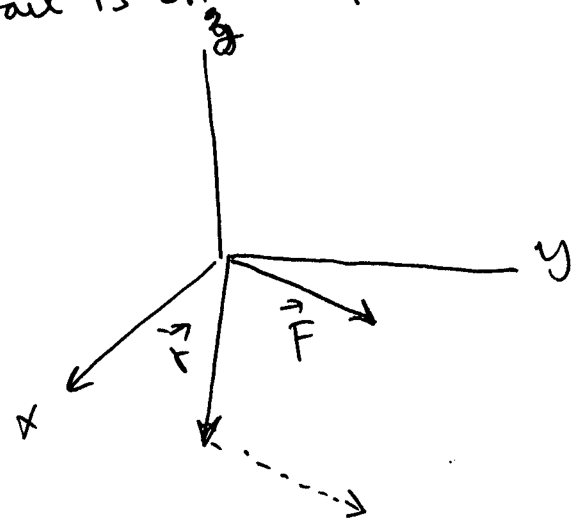
moment arm of F.

<u>direction</u>: the torque about point O is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$. To determine whether it is up or down the plane, you can use the right-hand rule.

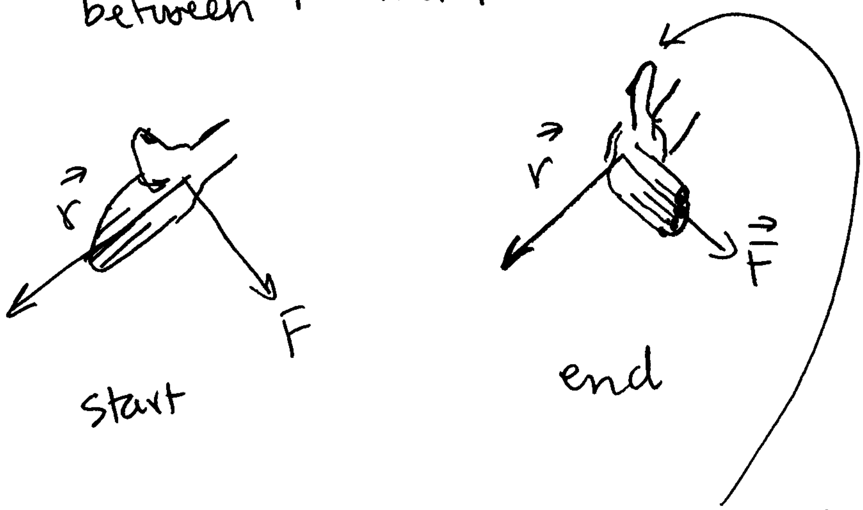


<u>To use right-hand rule</u>:-

1- slide vector $\vec{F}$ (do not change its direction) until its tail is on the point O

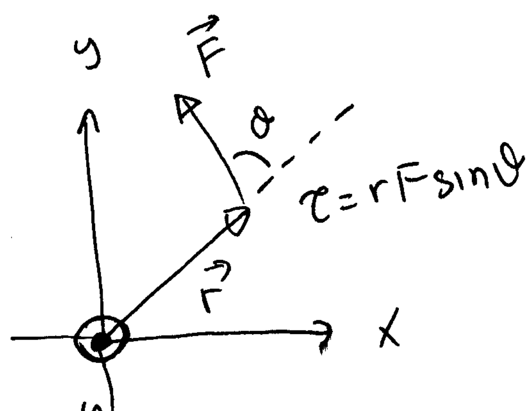


2 - sweep your fingers of your <u>right</u> hand from $\vec{r}$ into $\vec{F}$ through the smallest angle between $\vec{r}$ and $\vec{F}$.

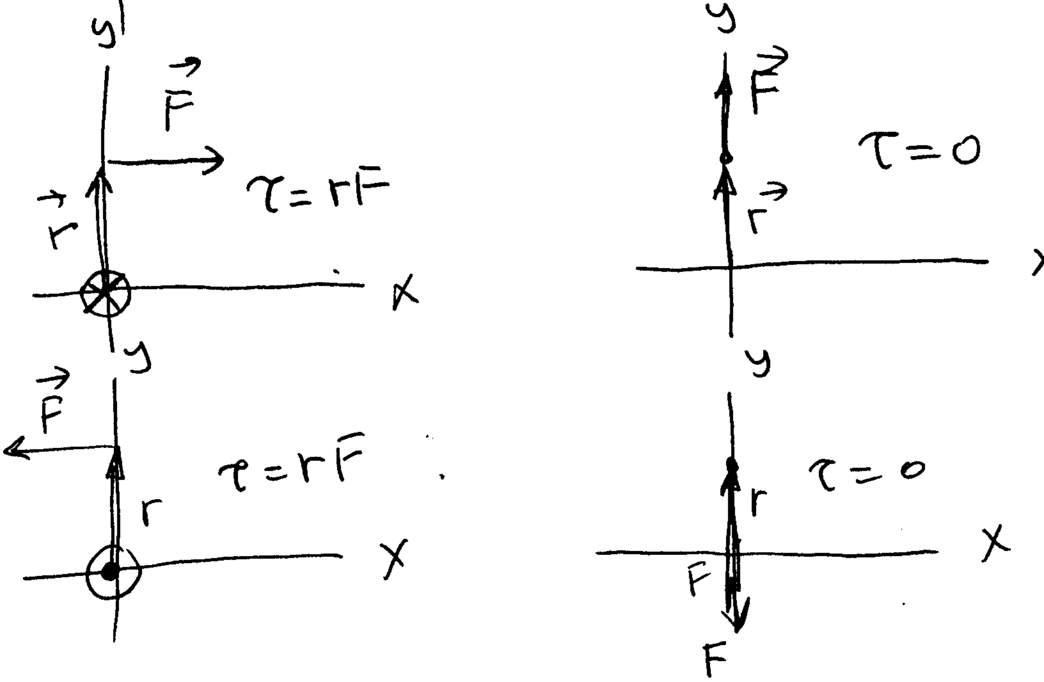


your thumb points to the direction of $\vec{\tau}$.

Example



⊙	out of the page
⊗	into the page



Angular momentum:-

for a particle about point O : $\boxed{\vec{l} = \vec{r} \times \vec{p}}$

$\vec{r}$: position vector between point O and the particle

$\vec{p}$: linear momentum of the particle

For collection of particle about point O :

$$\vec{L} = \vec{l_1} + \vec{l_2} + \dots$$

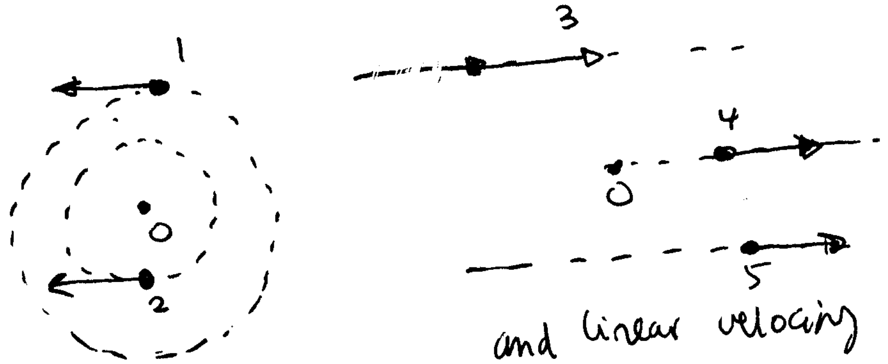
For a rigid body about a fixed axis:

$$\boxed{L = I\omega}$$

I: moment of inertia about the fixed axis

ω: angular velocity about the fixed axis.

Example



All particles have the same mass and linear velocity. Rank according to magnitude of angular momentum about O.

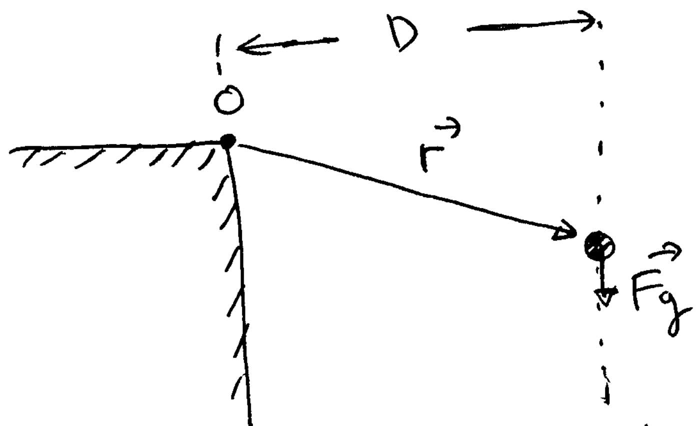
$l_1 = l_3 > l_2 = l_5 > l_4 = 0$

Newton's Second law in angular form :-

$$\boxed{\vec{\tau}_{net} = \frac{d}{dt}\vec{\ell}}$$ single particle

$$\boxed{\vec{\tau}_{net} = \frac{d}{dt}\vec{L}}$$ system of particles

Example

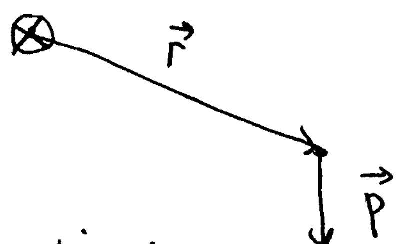


Q what is the angular momentum $\vec{\ell}$ of the falling object about O? at $t=0$, $v_0 = 0$.

A $\vec{\ell} = \vec{r} \times \vec{p}$

$\ell = r_\perp p$

$= D m v = D m g t$

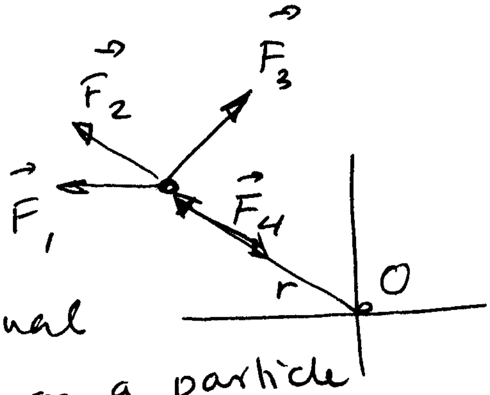


Q what is the torque on the object due to gravity about O?

A method 1 $\vec{\tau} = \frac{d}{dt}\vec{\ell} = Dmg$.

method 2 $\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \tau_1 = r_\perp F = Dmg$

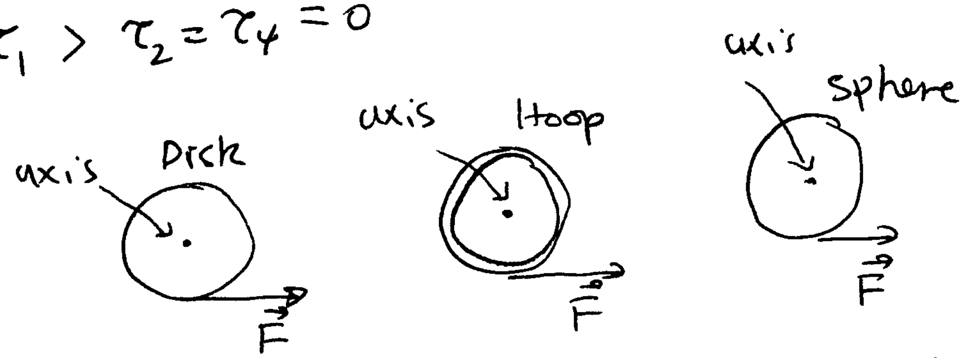
Example



Q four forces of equal magnitude act on a particle at $\vec{r}$. Rand their torque about O.

A $\tau_3 > \tau_1 > \tau_2 = \tau_4 = 0$

Example



All object have the same mass and radius.
They spin about fixed central axis and all started from rest.

Q Rank according to their angular momentum about their axis?

A $$\vec{\tau}_{net} = RF = \frac{dL}{dt}$$

all tie, same τ_{net} and same time.

Q Rand according to their angular velocity

$$L = I\omega$$

we know $I_s < I_D < I_h$ and $L_s = L_D = L_h$

$\Rightarrow \omega_s > \omega_D > \omega_h$

Conservation of angular momentum :-

If the torque on a system is zero, then the angular momentum is conserved.

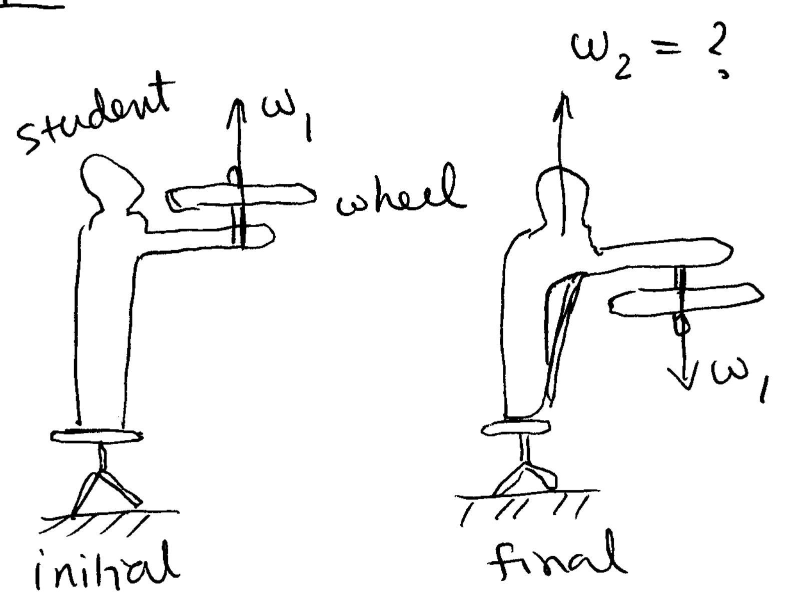
$$\text{If } \vec{\tau} = 0 \Rightarrow \vec{L} = \text{const}$$

$$\vec{L}_i = \vec{L}_f \,.$$

If the torque along an axis is zero, then the angular momentum along this axis is conserved.

for example if $\tau_z = 0 \Rightarrow L_z = \text{Constant}$

Example



moment of inertia for the wheel is I_1 and for student and chair is I_2

NO torque on our system (wheel + student + chair)

$$L_i = L_f$$

$$I_1 \omega_1 = I_1(-\omega_1) + I_2 \omega_2$$

$$\omega_2 = \frac{2 I_1 \omega_1}{I_2}$$