Nov 3, 01

CH 11 Rotation

- Rotation and translation
- Moment of intertia
- Kinetic Energy of rotation
- Torque
- Newton's second law of rotation
- Work and rotational kinetik energy

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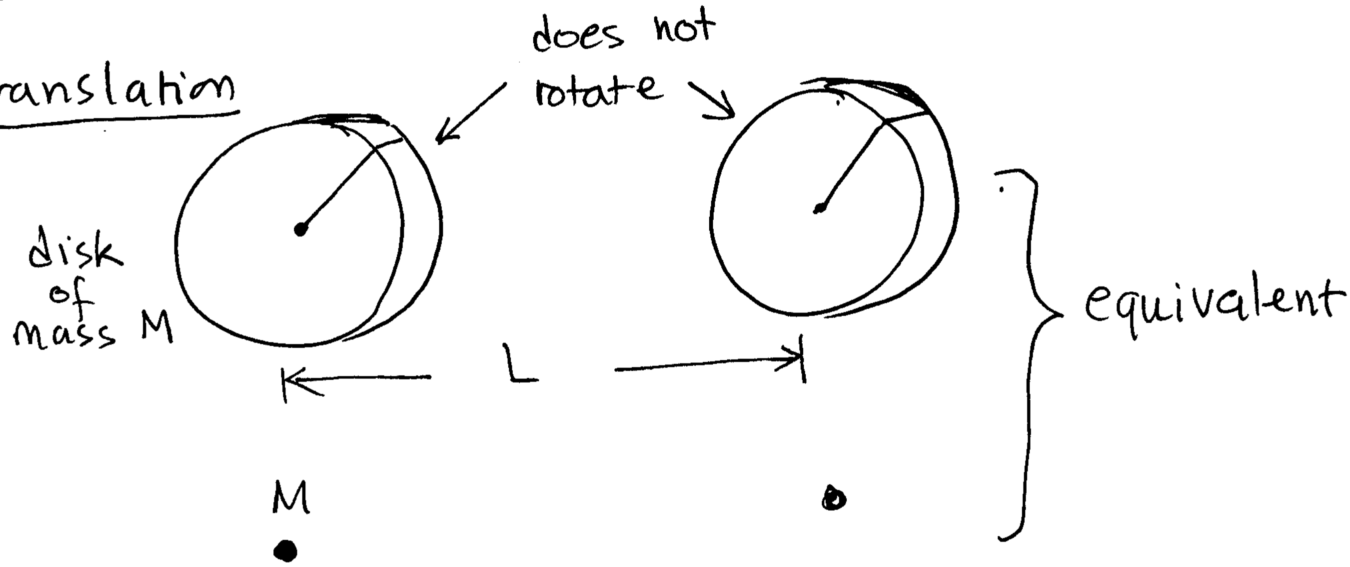
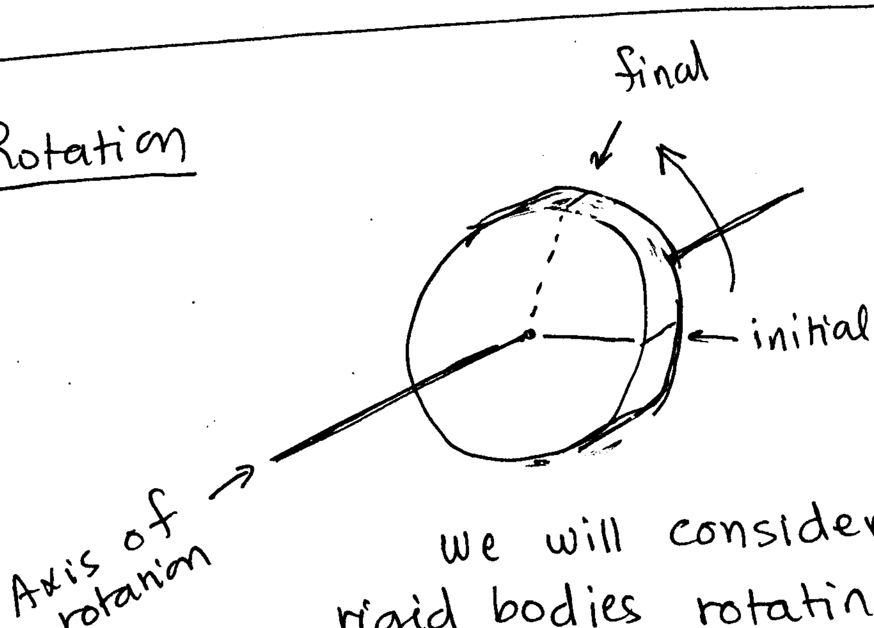
Rotation and translation :-

Translation



In case of translation, we can treat our solid object as a point-like particle Located at the center of mass.

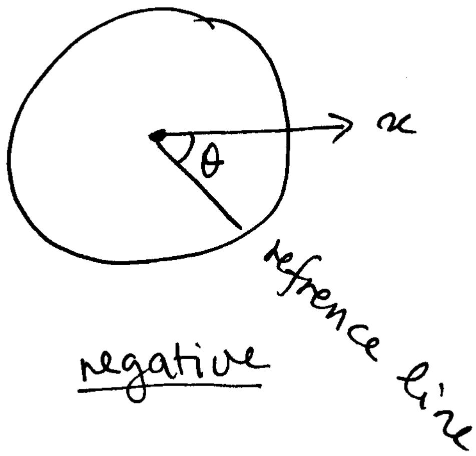
Rotation



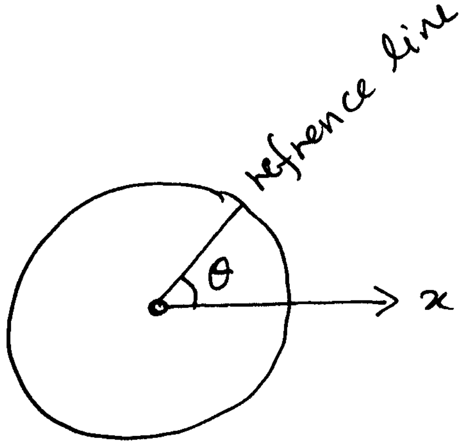
we will consider only rigid bodies rotating around a fixed axis (Axis of rotation)

When we study rotation, we can not treat our solid object as a point particle

Angular position θ ← theta



negative



positive

We measure angular position in radians

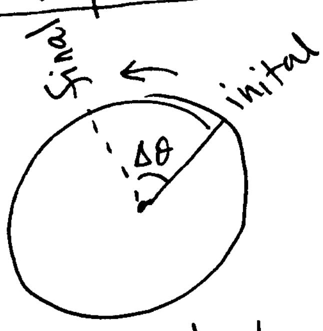
$$\boxed{\begin{aligned} \pi \text{ radian} &= 180^\circ \\ 2\pi \text{ radian} &= 1 \text{ revolution} \end{aligned}}$$

radian is dimensionless quantity.

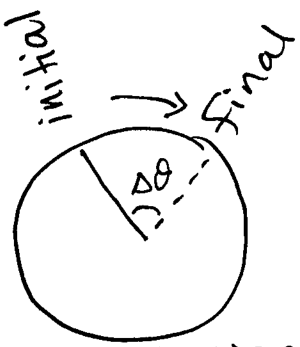
Example: a solid object rotates 2.5 rev. what is θ in radians?

$$2.5 \, \cancel{\text{rev}} \left(\frac{2\pi}{1 \, \cancel{\text{rev}}} \right) = 5\pi \text{ rad.}$$

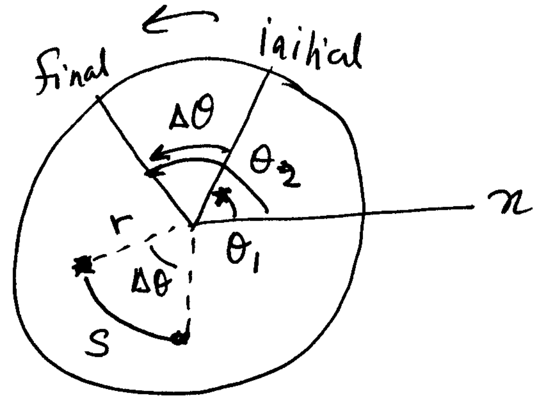
Angular displacement $\Delta\theta$



counterclockwise
positive angular displacement



clockwise
negative angular displacement.



$$\Delta\theta = \theta_2 - \theta_1$$

$$\Delta\theta = \frac{s}{r}$$

All parts of the solid object have the same angular displacement

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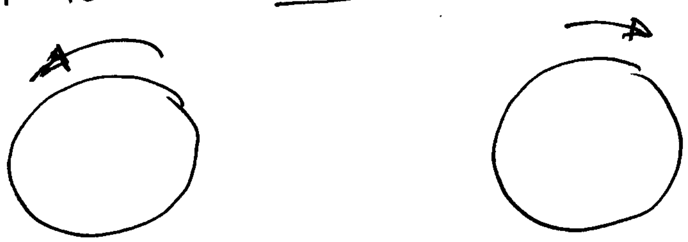
Angular velocity ω ← omega

average $\omega_{avg} = \dfrac{\theta_2 - \theta_1}{t_2 - t_1} = \dfrac{\Delta\theta}{\Delta t}$

instantaneous $\omega = \dfrac{d\theta}{dt}$

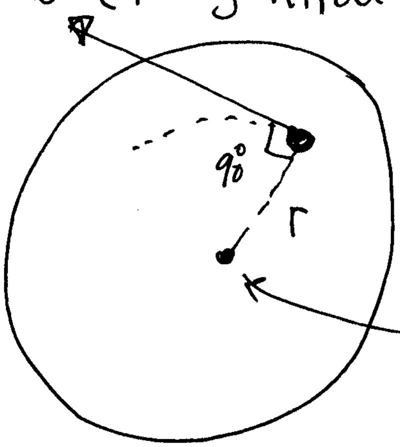
units rad / s

angular velocity is positive if our solid object rotates counterclockwise



$\omega > 0$ $\omega < 0$

All parts of our solid object have the same angular velocity



$$\boxed{\omega r = v}$$

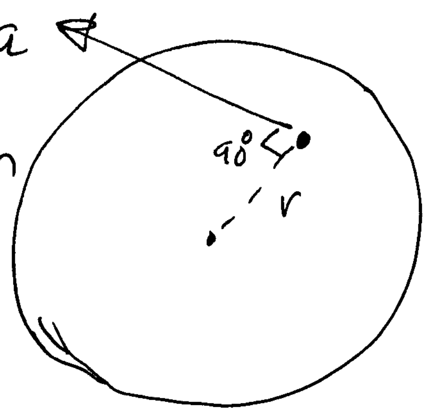
tangential velocity

Angular Acceleration α ← alpha

average $\alpha_{avg} = \dfrac{\omega_2 - \omega_1}{t_2 - t_1} = \dfrac{\Delta\omega}{\Delta t}$

instantaneous $\alpha = \dfrac{d}{dt}\omega$.

units rad / s^2

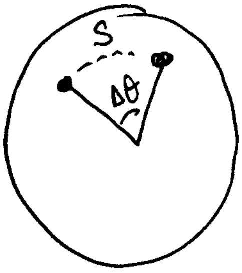


$$\boxed{\alpha r = a}$$

tangential acceleration

All parts of our solid object have the same angular acceleration.

linear (Tangential)	units	Angular	units	
S	m	$\Delta\theta$	rad	$S = \Delta\theta\, r$
v	m/s	ω	rad/s	$v = \omega r$
a	m/s^2	α	rad/s^2	$a = \alpha r$



Rotation with constant angular acceleration:

Linear	Angular
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2} a t^2$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2} at$	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Example :- a solid object rotates with constant acceleration $\alpha = 0.35\ rad/s^2$

at $t = 0$ $\quad \omega_0 = -4.6\ rad/s$

$\theta_0 = 0$

Q at what time $\theta = 5.0$ rev?

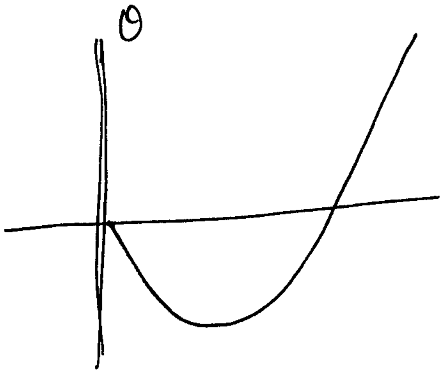
A $\quad \theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$

$$5\,rev\left(\frac{2\pi}{1\,rev}\right) = -4.6\,t + \frac{1}{2}(0.35)\,t^2$$

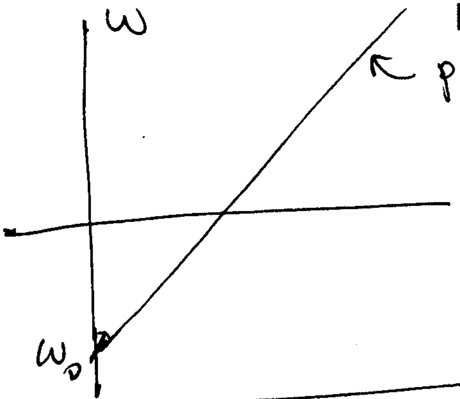
$$t = \frac{4.6 + \sqrt{(4.6)^2 - 4\left(\frac{0.35}{2}\right)(10\pi)}}{2\left(\frac{0.35}{2}\right)} = 32 \text{ sec}$$

Nov 3, 01

Q plot θ as a function of time
plot ω as a function of time



$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$$



positive angular acceleration
positive slope

$$\omega = \omega_0 + \alpha t$$

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moment of inertia (Rotational inertia) I

for collection of particles

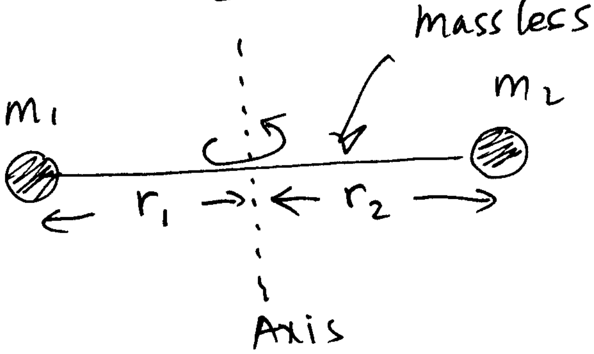
$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$

number of particles

for solid object

$$\boxed{I = \int r^2 dm}$$

Example



Q find moment of inertia about the axis ?

Nov 3, 01

<u>A</u>

$$I = m_1 r_1^2 + m_2 r_2^2$$

See your book for a table for moment of inertia for some solid objects.

Parallel-axis theorem

Suppose you know the moment of inertia of a solid object about an axis passing through the center of mass of this object. You can find the moment of inertia about another axis by Parallel-axis theorem

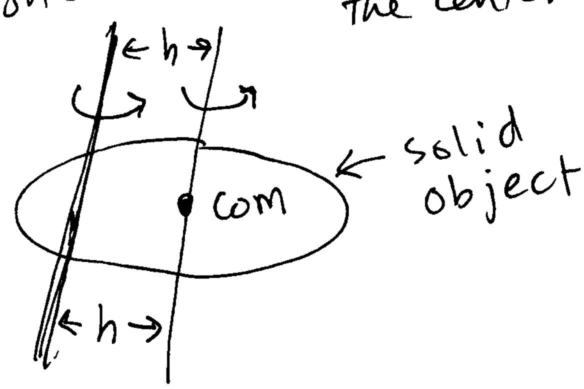
$$\boxed{I = I_{com} + M h^2}$$

moment of inertia about an axis which is parallel to an axis passing through com

moment of inertia about an axis passing through the center of mass

mass of the object

perpendicular distance between the two parallel axes



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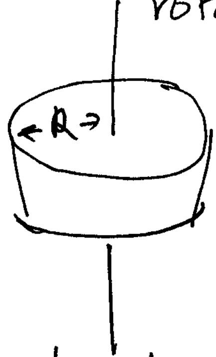
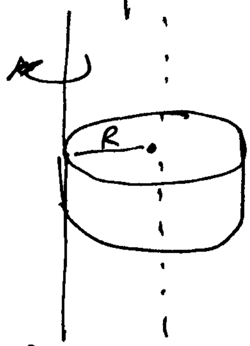
Example

Axis of rotation

We know



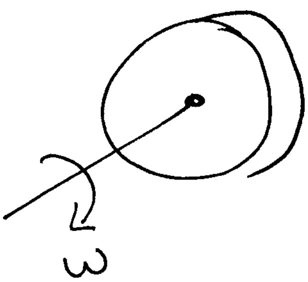
$$I_{com} = \frac{1}{2} MR^2$$



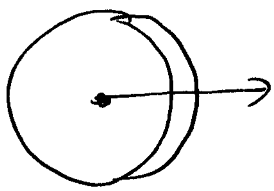
Axis of rotation

$$I = I_{com} + MR^2$$
$$= \frac{1}{2} MR^2 + MR^2$$
$$= \frac{3}{2} MR^2$$

Kinetic energy of rotation



$$K = \frac{1}{2} I \omega^2$$ rotation



$$K = \frac{1}{2} M v_{com}^2$$ translation

tau

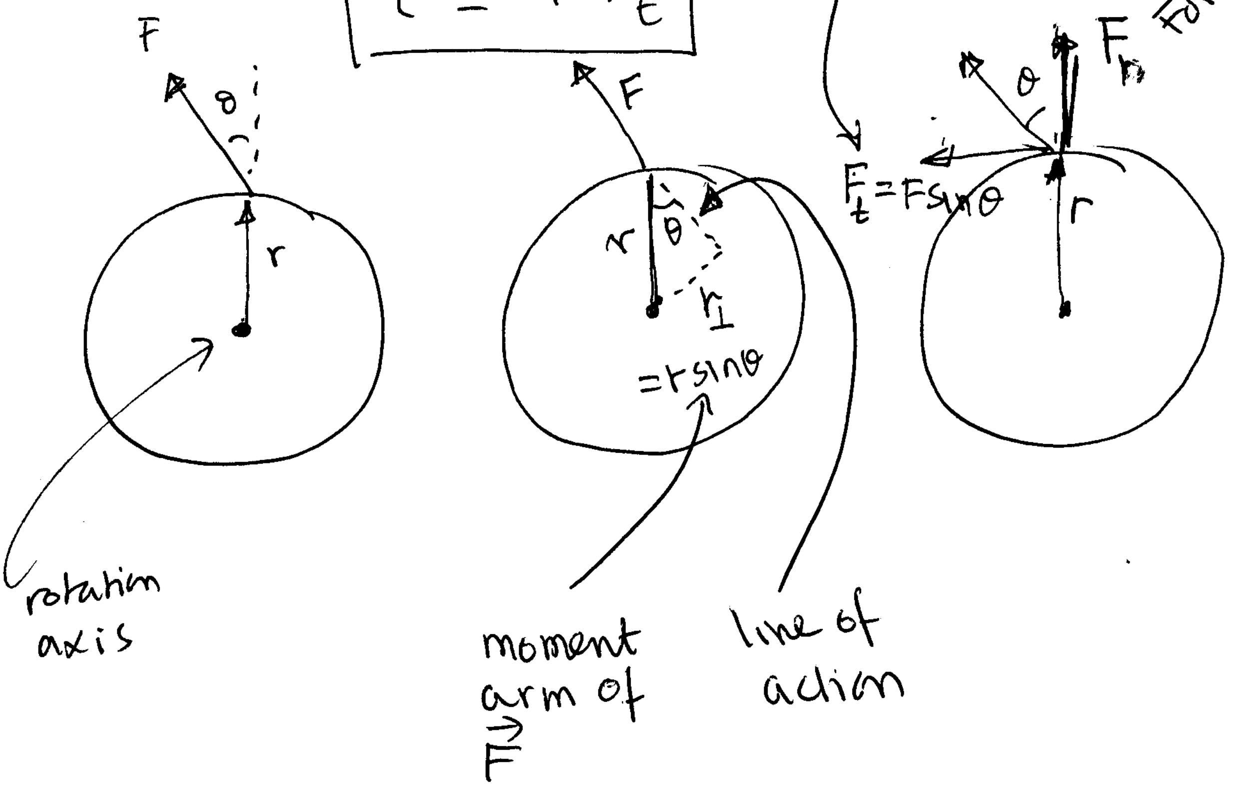
Torque τ

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\boxed{\tau = r F \sin\theta}$$

$$\boxed{\tau = r_\perp F}$$

$$\boxed{\tau = r F_t}$$



maximum torque when $\vec{F}$ perpendicular to $\vec{r}$

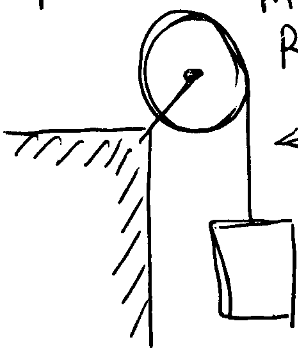
zero torque when $\vec{F}$ parallel to $\vec{r}$.

If the body would rotate <u>counter clock wise</u>, torque is <u>positive</u>.

Newton's Second Law of rotation:

$$\boxed{\tau_{net} = I\alpha}$$

Example



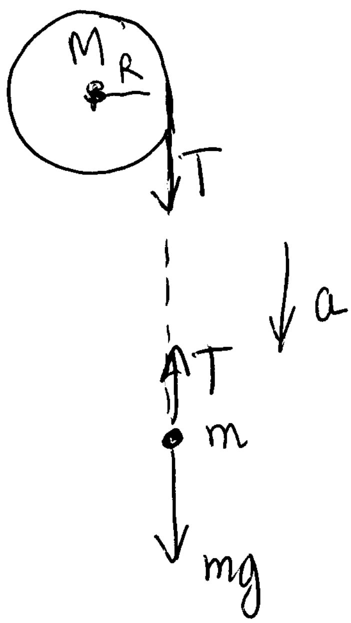
$\left.\begin{array}{l} M = 2.5\ \text{kg} \\ R = 20\ \text{cm} \end{array}\right\}$ uniform disk

massless cord wrapped around the rim of the disk

$m = 1.2$ kg block

Q What is the acceleration of the block

A Free-body diagram



Newton's second law

block $\boxed{T - mg = -ma}$

disk $\tau = I\alpha$

$$RT = \left(\tfrac{1}{2}MR^2\right)\frac{a}{R}$$

$$\boxed{T = \tfrac{1}{2}Ma}$$

$$\tfrac{1}{2}Ma - mg = -ma$$

$$\left(\tfrac{1}{2}M + m\right)a = mg$$

$$a = \frac{mg}{\tfrac{1}{2}M + m}$$

$$= \frac{1.2(9.8)}{\tfrac{1}{2}(2.5) + 1.2} = 4.8 \text{ m/s}^2$$

Q Find T.

A $T = \tfrac{1}{2}Ma = 6.0$ N

Work and Rotational kintec Energy

$$W = \int_{\theta_0}^{\theta} \tau \, d\theta$$

Newton's second law

block $\boxed{T - mg = -ma}$

disk $\tau = I\alpha$

$$RT = \left(\tfrac{1}{2}MR^2\right)\frac{a}{R}$$

$$\boxed{T = \tfrac{1}{2}Ma}$$

$$\tfrac{1}{2}Ma - mg = -ma$$

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Q Find T.

A $T = \tfrac{1}{2}Ma = 6.0\ N$

Work and Rotational kintec Energy

$$W = \int_{\theta_0}^{\theta} \tau\, d\theta$$

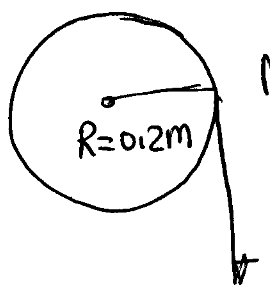
$$W = \Delta K$$

$$W = K_f - K_i$$

$$\boxed{W = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2}$$

Work - kinetic energy theorem

Example



at $t=0$ $\quad \omega_0 = 0$

$M = 2.5$ kg

$F = 6.0$ N

Q what is its rotational kinetic energy K at $t = 2.5$ s ?

A method 1 = $\quad K = \frac{1}{2} I \omega^2$

I: $\frac{1}{2} M R^2$

ω: we need to find

$$\omega = \omega_0 + \alpha t$$

ω_0: 0

α: we need to find

t: 2.5 s

$$\tau_{net} = I \alpha$$

$$R F \sin 90^\circ = R F = \frac{1}{2} M R^2 \alpha$$

Nov 3, 01

~~Q~~ #11-14

$$\alpha = \frac{RF}{I}$$

$$\omega = \frac{RF}{I} t$$

$$K = \frac{1}{2} I \left(\frac{RF}{I} t \right)^2$$

$$= \frac{1}{2} \frac{(RFt)^2}{I} = \frac{1}{2} \frac{(RFt)^2}{\frac{1}{2} M R^2}$$

$$= \frac{(Ft)^2}{M} = \frac{(6(2.5))^2}{2.5} = 90 J$$

Method II

$$W = K_f - K_i$$

K_i 0 because it starts from rest

$$\int_{\theta_0}^{\theta} \tau_{net} \, d\theta = RF(\theta - \theta_0)$$

$$= RF\left(\omega_0 t + \tfrac{1}{2} \alpha t^2 \right)$$

$$= RF\left(\tfrac{1}{2} \frac{RF}{I} t^2 \right)$$

$$= \frac{(Ft)^2}{M} = 90 J$$