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CH 10 Collisions

- Impulse

- Elastic Collisions

$$\vec{P}_i = \vec{P}_f$$
$$K_i = K_f$$

momentum and kinetic energy are conserved.

- Inelastic Collisions

$$\vec{P}_i = \vec{P}_f$$ momentum is conserved

$$K_i \neq K_f$$ Kinetic energy is <u>not</u> conserved.

Impulse $\vec{J}$

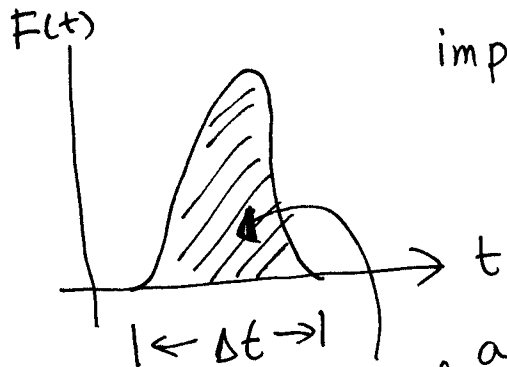
impulse = change in momentum

$$\vec{J} = \vec{P}_f - \vec{P}_i$$

$$\vec{J} = \int_{t_i}^{t_f} F(t)\,dt$$

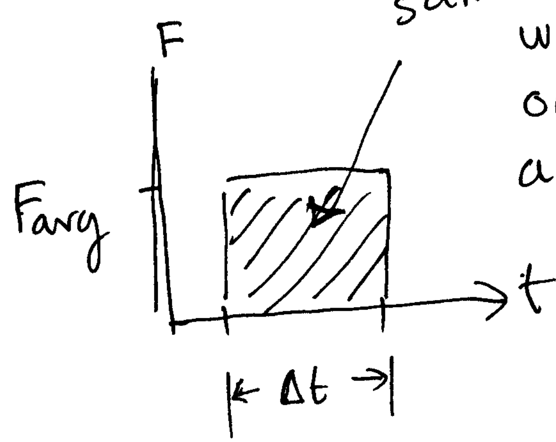
$$\vec{J} = \vec{F}_{avg}\,\Delta t$$

impulse = area under the force curve

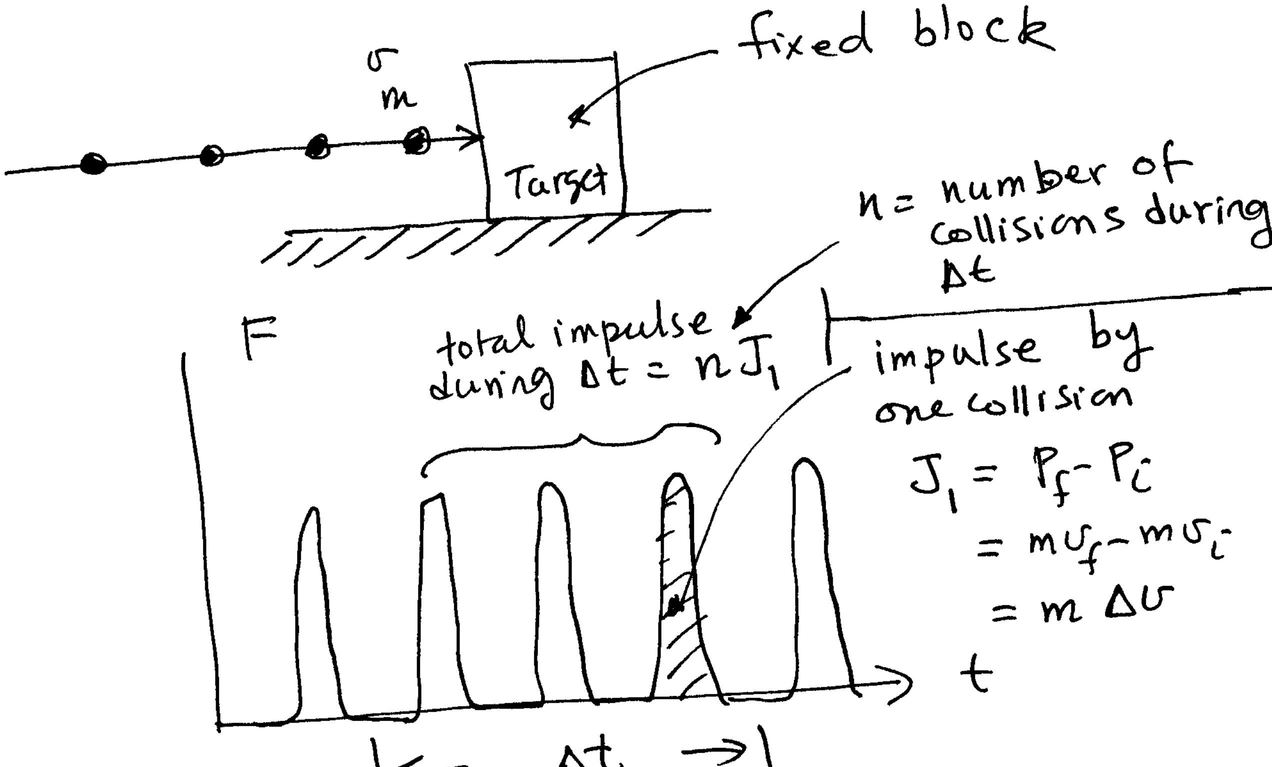


same area

we can approximate our force by an average force F_{avg}



Example :- average force due to series of collisions



$$F_{avg} = \frac{J}{\Delta t} = \frac{nJ_1}{\Delta t} = \frac{n\,m\,\Delta v}{\Delta t}$$

This is the force of the block on the particles.

The force of the particles on the block $= -F_{avg}$.

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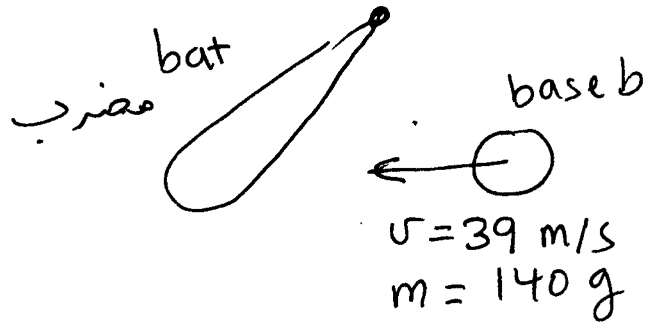
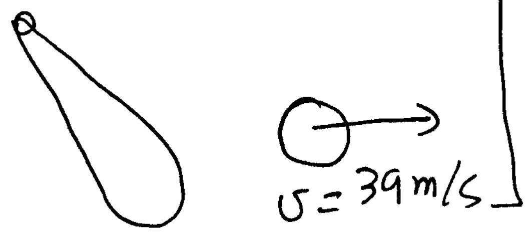
Example



Initial



Final

Q What impulse J acts on the ball

A $$\vec{J} = \vec{P}_f - \vec{P}_i$$

$$= m\vec{v}_f - m\vec{v}_f$$

$$J = (0.14)(39) - (0.14)(-39) = 10.9 \text{ kg m/s}$$

The impulse is on $+x$ direction: so the force acted on the ball is also on the $+x$ direction.

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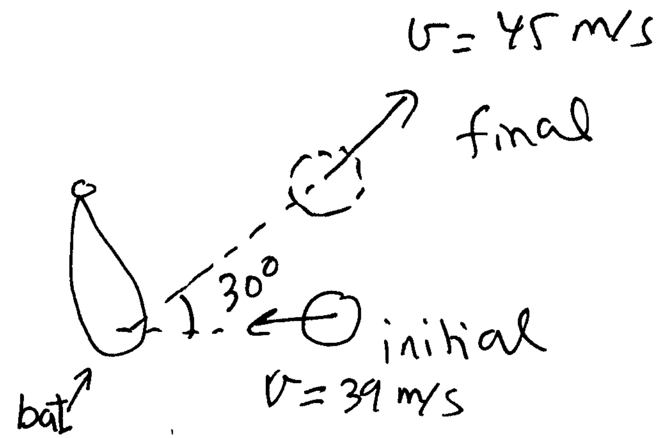
Q. suppose that the impact time $\Delta t = 1.20$ m/s, what is F_{avg}.

A $$F_{avg} = \frac{J}{\Delta t} = \frac{10.9}{1.20\times10^{-3}} = 9080 \text{ N}$$

Example



Q. what is the impulse on the ball?

$$\vec{J} = \vec{P_f} - \vec{P_i}$$

$$= m\vec{v_f} - m\vec{v_i}$$

$$= (0.14)(45)\cos 30\ \hat{i} + (0.14)(45)\sin 30\ \hat{j} - (0.14)(-39)\ \hat{i}$$

$$= (10.9\ \hat{i} + 3.15\ \hat{j})\ \text{kg m/s}$$

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Elastic Collisions

In elastic collision both momentum and kinetic energy are conserved.

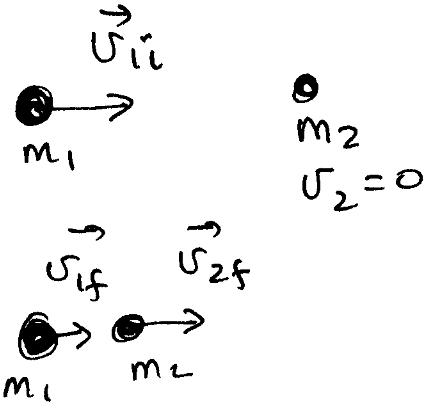
$$\vec{P}_i = \vec{P}_f$$

$$K_i = K_f$$

Stationary target

initial

final



Q what is $\vec{v}_{1f}$ and $\vec{v}_{2f}$

A $\boxed{\vec{P}_i = \vec{P}_f}$

$$m_1 \vec{v}_{1i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$m_1 (v_{1i} - v_{1f}) = m_2 v_{2f} \quad \text{— (1)}$$

$\boxed{K_i = K_f}$

$$\cancel{\tfrac{1}{2}} m_1 v_{1i}^2 = \cancel{\tfrac{1}{2}} m_1 v_{1f}^2 + \cancel{\tfrac{1}{2}} m_2 v_{2f}^2$$

$$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 v_{2f}^2$$

$$m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 v_{2f}^2$$

divide

$$v_{1i} + v_{1f} = v_{2f}$$

Substitute $v_{1i} + v_{1f} = v_{2f}$ in equation (1)

$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{1i} + v_{1f})$$

$$v_{1i} (m_1 - m_2) = v_{1f} (m_1 + m_2)$$

$$\boxed{v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}}$$

substitute in equation (1)

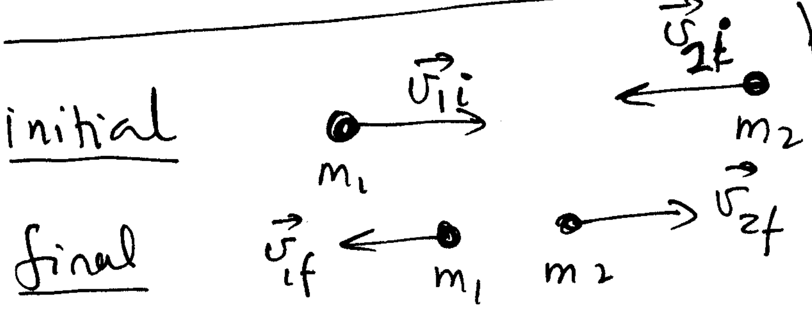
$$\boxed{v_{2f} = \frac{2 m_1}{m_1 + m_2} v_{1i}}$$

note
if $m_1 = m_2$
$\Rightarrow v_{1f} = 0$
$v_{2f} = v_{1i}$
particle 1 stops and particle 2 moves with speed of v_{1i}.

If $m_1 > m_2$
projectile (m_1) continue to move in same direction

If $m_1 < m_2$
projectile moves in opposite direction of its initial motion.

moving target



Note If particle moves to left then $v < 0$.

Q what is v_{1f} and v_{2f} ?

A $\vec{P}_i = \vec{P}_f$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$K_i = K_f$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

After some algebra

$$\boxed{\begin{aligned} v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2 m_2}{m_1 + m_2} v_{2i} \\ v_{2f} &= \frac{2 m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i} \end{aligned}}$$

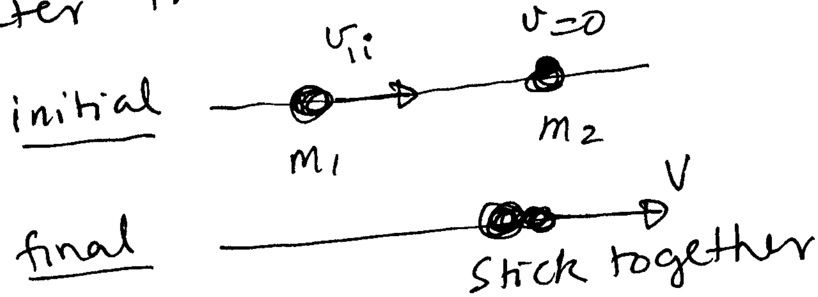
Inelastic Collisions:-

For inelastic collisions, the momentum is conserved but the kinetic energy is <u>not</u>.

$$\vec{P_i} = \vec{P_f}$$

$$K_i \neq K_f$$

We say that a collision is <u>completely inelastic</u> when both particles <u>stick together</u> after the collision

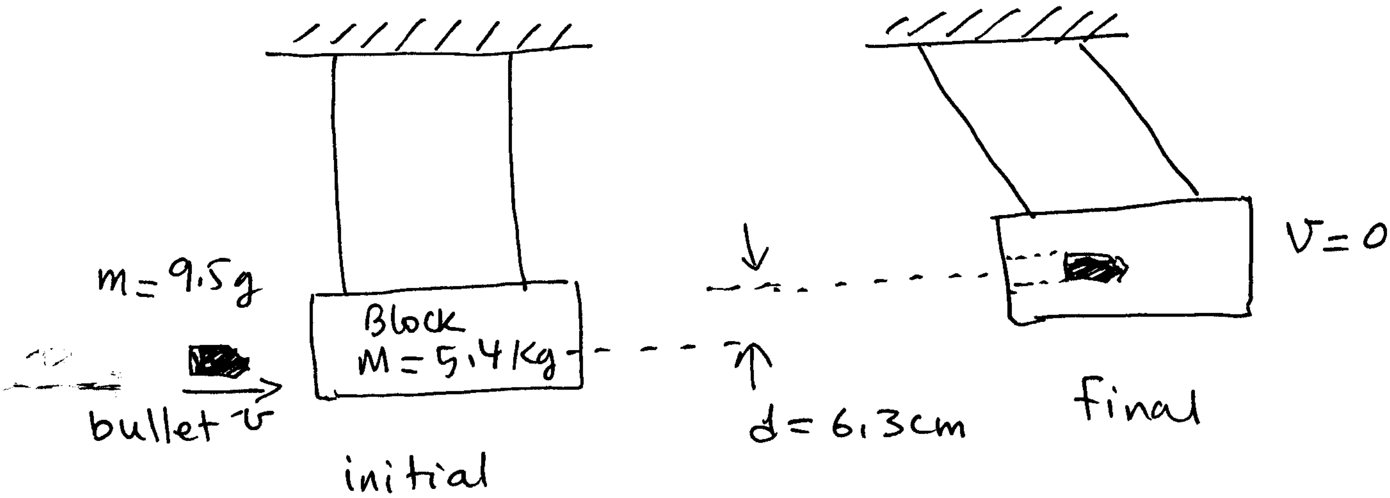


$$\vec{P_i} = \vec{P_f}$$

$$m_1 v_{1i} = (m_1 + m_2) V$$

$$V = \frac{m_1}{m_1 + m_2} v_{1i}$$

Example



Q what is the speed of the bullet before it hits the block?

A We will solve this problem in two steps. In step 1, we will find the speed of the block + bullet system just after the collision. In step 2, we have a swinging pendulum.

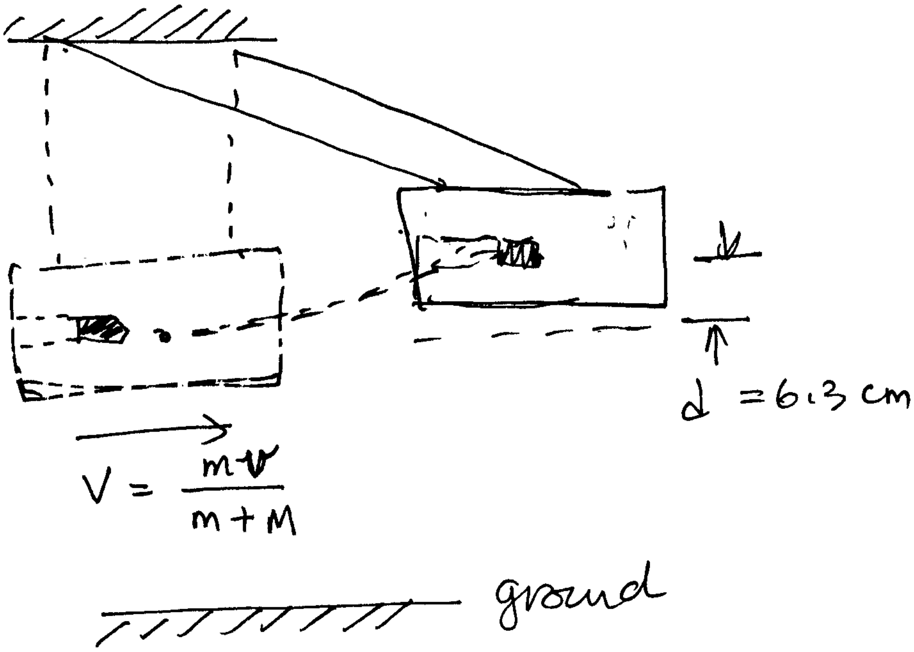
Step 1 :- Here we assume that the collision takes plase in a very short time such that the block does not move much from its position.

Since the is no external force acting on our system along x direction, the linear momentum is conserved along x direction

$$P_{i,x} = P_{f,x}$$

$$mv = (m+M)V$$

Step 2



our system is bullet + block + ground.
Tension is the only external force on our system. Since the path of motion of the bullet + block is always perpendicular to the tension, the work done by the tension on our system is zero.

$\Rightarrow$ Mechanical Energy is conserved

$$E_{mech} = K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m+M)V^2 + 0 = 0 + (m+M)gd$$

final velocity = 0

we assume our refrence point at this position

From Step 1

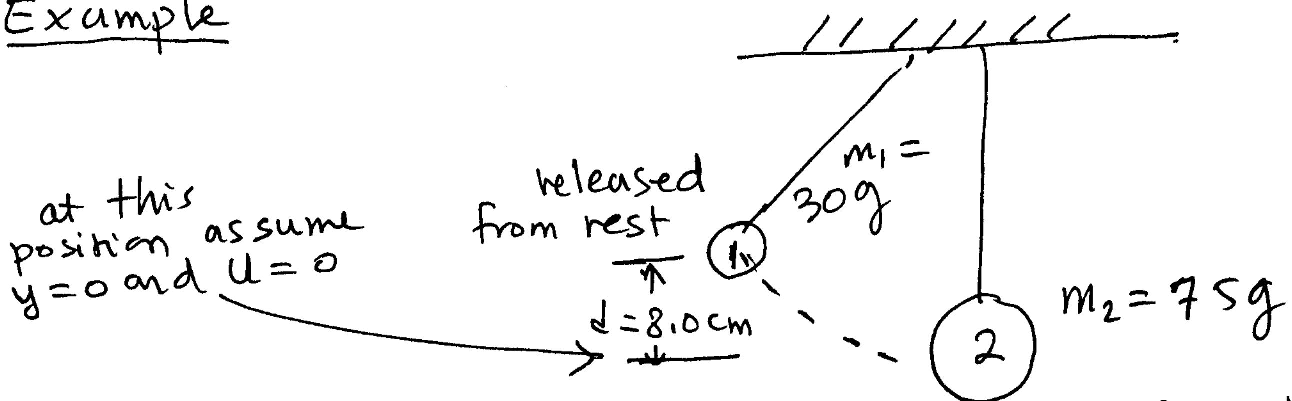
$$\frac{1}{2}V^2 = gd$$

$$\frac{1}{2}\left(\frac{mv}{m+V}\right)^2 = gd$$

$$v = \frac{m+M}{m}\sqrt{2gd}$$

$$= \frac{0.0095 + 5.4}{0.0095}\sqrt{2(9.8)(0.063)} = 630 \text{ m/s}$$

Example



Q what is the velocity of ball 1 just after the collision. (Assume elastic collision)

A just before the collision, the velocity of ball 1 can be found from conservation of mechanical energy. Our system consists of two balls and the earth. The only external force on our system; tension, does not do work on our system because it is always perpen- perpendicular to the motion of the balls. So the mechanical energy is conserved.

$$K_i + U_i = K_f + U_f$$

$$0 + mgd = \frac{1}{2}mv_1^2 + 0$$

ball released from rest $v=0$

we choose this position to have zero U.

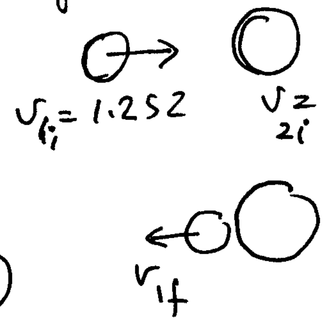
$$v_1 = \sqrt{2gd} = \sqrt{2(9.8)(0.08)} = 1.252\ m/s$$

now, we have two balls colliding with each other

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$= \frac{0.030 - 0.075}{0.030 + 0.075}(1.252)$$

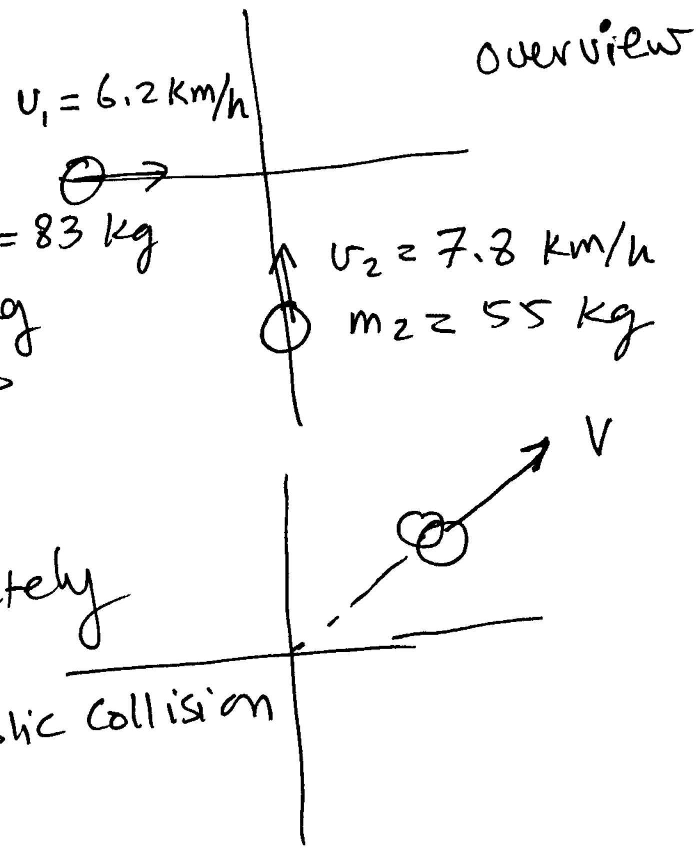
$$= -0.54\ m/s$$



Example

initial

two objects sliding on a frictionless surface



final

completely inelastic collision

Q what is V ?

A no external force on our system

$\Rightarrow$ Linear momentum is conserved.

$$\vec{P_i} = \vec{P_f}$$

$$\vec{P_i} = \vec{P_f}$$

$$m_1\vec{v_1} + m_2\vec{v_2} = (m_1+m_2)\vec{V}$$

$$m_1(6.2\,\hat{i}) + m_2(7.8\,\hat{j}) = (m_1+m_2)\vec{V}$$

$$\vec{V} = \left(\frac{83(6.2)}{83+55}\,\hat{i} + \frac{55(7.8)}{83+55}\,\hat{j}\right) \text{ km/h}$$

Q what is the velocity of the center of mass of the two objects before collision.

A Since there is no external force on our system, $a_{com} = 0$ OK V_{com} = constant.

We know after collision $V_{com} = V$

$\Rightarrow V_{com,\ \text{before collision}} = V$