3. By Eq. 11-44, the work required to stop the hoop is the negative of the initial kinetic energy of the hoop. The initial kinetic energy is $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$ (Eq. 12-5), where $I = mR^2$ is its rotational inertia about the center of mass, $m = 140$ kg, and $v = 0.150$ m/s is the speed of its center of mass. Eq. 12-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K = \frac{1}{2}mR^2\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2 = mv^2 = (140)(0.150)^2$$

which implies that the work required is -3.15 J.

4. The rotational kinetic energy is $K = \frac{1}{2}I\omega^2$, where $I = mR^2$ is its rotational inertia about the center of mass (Table 11-2(a)), $m = 140$ kg, and $\omega = v_{\text{com}}/R$ (Eq. 12-2). The asked-for ratio is

$$\frac{K_{\text{transl}}}{K_{\text{rot}}} = \frac{\frac{1}{2}mv_{\text{com}}^2}{\frac{1}{2}\left(mR^2\right)\left(v_{\text{com}}/R\right)^2} = 1 \ .$$

18. If we write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\mathrm{i}} + (zF_x - xF_z)\,\hat{\mathrm{j}} + (xF_y - yF_x)\,\hat{\mathrm{k}}\ .$$

(a) In the above expression, we set (with SI units understood) $x = -2$, $y = 0$, $z = 4$, $F_x = 6$, $F_y = 0$ and $F_z = 0$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = 24\,\hat{\mathrm{j}}$ N·m.

(b) The values are just as in part (a) with the exception that now $F_x = -6$. We find $\vec{\tau} = \vec{r} \times \vec{F} = -24\,\hat{\mathrm{j}}$ N·m.

(c) In the above expression, we set $x = -2$, $y = 0$, $z = 4$, $F_x = 0$, $F_y = 0$ and $F_z = 6$. We get $\vec{\tau} = \vec{r} \times \vec{F} = 12\,\hat{\mathrm{j}}$ N·m.

(d) The values are just as in part (c) with the exception that now $F_z = -6$. We find $\vec{\tau} = \vec{r} \times \vec{F} = -12\,\hat{\mathrm{j}}$ N·m.

25. (a) We use $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the object, $\vec{v}$ is its velocity vector, and m is its mass. Only the x and z components of the position and velocity vectors are nonzero, so Eq. 3-30 leads to $\vec{r} \times \vec{v} = (-xv_z + zv_x)\,\hat{\jmath}$. Therefore,

$$\begin{aligned} \vec{\ell} &= m\,(-xv_z + zv_x)\,\hat{\jmath} \\ &= (0.25\,\text{kg})\,(-(2.0\,\text{m})(5.0\,\text{m/s}) + (-2.0\,\text{m})(-5.0\,\text{m/s}))\,\hat{\jmath} \\ &= 0\ . \end{aligned}$$

(b) If we write $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k}\ .$$

With $x = 2.0$, $z = -2.0$, $F_y = 4.0$ and all other components zero (and SI units understood) the expression above yields $\vec{\tau} = \vec{r} \times \vec{F} = \left(8.0\,\hat{\imath} + 8.0\,\hat{k}\right)$ N·m.

26. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{v}$ is equal to

$$(y'v_z - z'v_y)\,\hat{\mathrm{i}} + (z'v_x - x'v_z)\,\hat{\mathrm{j}} + (x'v_y - y'v_x)\,\hat{\mathrm{k}}\ .$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3\,\hat{\mathrm{i}} - 4\,\hat{\mathrm{j}}$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3$, $y = -4$, $z = 0$, $v_x = 30$, $v_y = 60$ and $v_z = 0$. Then (with $m = 2.0$ kg) we obtain $\vec{\ell} = m\,(\vec{r} \times \vec{v}) = 600\,\hat{\mathrm{k}}$ kg·m^2/s.

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = -2\,\hat{\mathrm{i}} - 2\,\hat{\mathrm{j}}$. Therefore, in the above expression, we set $x' = 5$, $y' = -2$, $z' = 0$, $v_x = 30$, $v_y = 60$ and $v_z = 0$. We get $\vec{\ell} = m\,(\vec{r}\,' \times \vec{v}) = 720\,\hat{\mathrm{k}}$ kg·m^2/s.

34. (a) Eq. 11-27 gives $\alpha = \tau/I$ and Eq. 11-12 leads to $\omega = \alpha t = \tau t/I$. Therefore, the angular momentum at $t = 0.033$ s is

$$I\omega = \tau t = (16\,\mathrm{N}\cdot\mathrm{m})(0.033\,\mathrm{s}) = 0.53\ \mathrm{kg{\cdot}m^2/s}$$

where this is essentially a derivation of the angular version of the impulse-momentum theorem.

(b) We find

$$\omega = \frac{\tau t}{I} = \frac{(16)(0.033)}{1.2 \times 10^{-3}} = 440\ \mathrm{rad}$$

which we convert as follows: $\omega = (440)(60/2\pi) \approx 4200$ rev/min.

41. (a) No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved. Let I_1 be the rotational inertia of the wheel that is originally spinning (at ω_i) and I_2 be the rotational inertia of the wheel that is initially at rest. Then $I_1\omega_i = (I_1 + I_2)\omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2}\,\omega_i$$

where ω_f is the common final angular velocity of the wheels. Substituting $I_2 = 2I_1$ and $\omega_i = 800\,\text{rev/min}$, we obtain $\omega_f = 267\,\text{rev/min}$.

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_1\omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2}(I_1 + 2I_1)\left(\frac{I_1\omega_i}{I_1 + 2I_1}\right)^2 = \frac{1}{6}I\omega_i^2\ .$$

Therefore, the fraction lost, $(K_i - K_f)/K_i$, is

$$1 - \frac{K_f}{K_i} = 1 - \frac{\frac{1}{6}I\omega_i^2}{\frac{1}{2}I\omega_i^2} = \frac{2}{3}\ .$$