

Oct 29, 01

Rec-ch10-01

Q1

$$\text{Impulse} = \int_{t_1}^{t_2} F dt$$

= area under the force curve

(a) (Area) impulse =  $12 F_0 t_0$

(b) impulse =  $12 F_0 t_0$

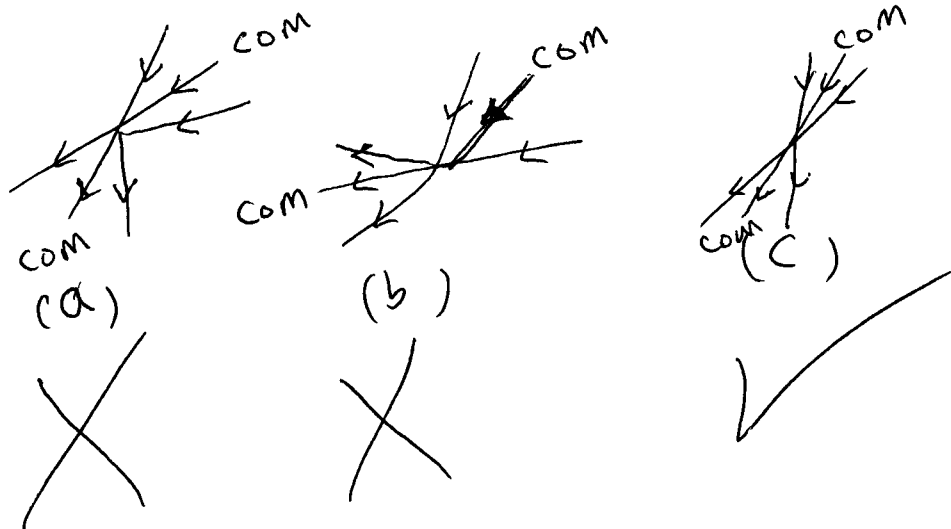
(c) impulse =  $\frac{1}{2} (24) F_0 t_0$   
=  $12 F_0 t_0$

All same.

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Q2

Since our system is isolated and closed, the linear momentum of the system is conserved.  $V_{\text{com, before}} = V_{\text{com, after}}$   
So (c) is represent the two bodies and the both of their center of mass.



5. We take the initial direction of motion to be positive and use  $F_{\text{avg}}$  to denote the magnitude of the average force,  $\Delta t$  as the duration of the force,  $m$  as the mass of the ball,  $v_i$  as the initial velocity of the ball, and  $v_f$  as the final velocity of the ball. The force is in the negative direction and the impulse-momentum theorem (Eq. 10-4 with Eq. 10-8) yields  $-F_{\text{avg}}\Delta t = mv_f - mv_i$ . Thus,

$$v_f = \frac{mv_i - F_{\text{avg}}\Delta t}{m} = \frac{(0.40 \text{ kg})(14 \text{ m/s}) - (1200 \text{ N})(27 \times 10^{-3} \text{ s})}{0.40 \text{ kg}} = -67 \text{ m/s} .$$

The final speed of the ball is 67 m/s. The negative sign indicates that the velocity is opposite to the initial direction of travel.

7. We choose  $+y$  upward, which means  $\vec{v}_i = -25$  m/s and  $\vec{v}_f = +10$  m/s. During the collision, we make the reasonable approximation that the net force on the ball is equal to  $F_{\text{avg}}$  – the average force exerted by the floor up on the ball.

(a) Using the impulse momentum theorem (Eq. 10-4) we find

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (1.2)(10) - (1.2)(-25) = 42 \text{ kg}\cdot\text{m/s} .$$

(b) From Eq. 10-8, we obtain

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{42}{0.020} = 2.1 \times 10^3 \text{ N} .$$

20. (a) We choose  $+x$  along the initial direction of motion and apply momentum conservation:

$$\begin{aligned} m_{\text{bullet}}\vec{v}_i &= m_{\text{bullet}}\vec{v}_1 + m_{\text{block}}\vec{v}_2 \\ (5.2\text{ g})(672\text{ m/s}) &= (5.2\text{ g})(428\text{ m/s}) + (700\text{ g})\vec{v}_2 \end{aligned}$$

which yields  $v_2 = 1.81\text{ m/s}$ .

- (b) It is a consequence of momentum conservation that the velocity of the center of mass is unchanged by the collision. We choose to evaluate it before the collision:

$$\vec{v}_{\text{com}} = \frac{m_{\text{bullet}}\vec{v}_i}{m_{\text{bullet}} + m_{\text{block}}} = \frac{(5.2\text{ g})(672\text{ m/s})}{5.2\text{ g} + 700\text{ g}}$$

which gives the result  $\vec{v}_{\text{com}} = 4.96\text{ m/s}$ .

29. Let  $m_F$  be the mass of the freight car and  $v_F$  be its initial velocity. Let  $m_C$  be the mass of the caboose and  $v$  be the common final velocity of the two when they are coupled. Conservation of the total momentum of the two-car system leads to  $m_F v_F = (m_F + m_C)v$ , so  $v = v_F m_F / (m_F + m_C)$ . The initial kinetic energy of the system is

$$K_i = \frac{1}{2} m_F v_F^2$$

and the final kinetic energy is

$$K_f = \frac{1}{2} (m_F + m_C) v^2 = \frac{1}{2} (m_F + m_C) \frac{m_F^2 v_F^2}{(m_F + m_C)^2} = \frac{1}{2} \frac{m_F^2 v_F^2}{(m_F + m_C)}.$$

Since 27% of the original kinetic energy is lost, we have  $K_f = 0.73 K_i$ . Thus,

$$\frac{1}{2} \frac{m_F^2 v_F^2}{(m_F + m_C)} = (0.73) \left( \frac{1}{2} m_F v_F^2 \right).$$

Simplifying, we obtain  $m_F / (m_F + m_C) = 0.73$ , which we use in solving for the mass of the caboose:

$$m_C = \frac{0.27}{0.73} m_F = 0.37 m_F = (0.37) (3.18 \times 10^4 \text{ kg}) = 1.18 \times 10^4 \text{ kg}.$$

36. We use  $m_1$  for the mass of the electron and  $m_2 = 1840m_1$  for the mass of the hydrogen atom. Using Eq. 10-31,

$$v_{2f} = \frac{2m_1}{m_1 + 1840m_1} v_{1i} = \frac{2}{1841} v_{1i}$$

we compute the final kinetic energy of the hydrogen atom:

$$K_{2f} = \frac{1}{2}(1840m_1) \left( \frac{2 v_{1i}}{1841} \right)^2 = \frac{(1840)(4)}{1841^2} \left( \frac{1}{2}(1840m_1)v_{1i}^2 \right)$$

so we find the fraction to be  $(1840)(4)/1841^2 \approx 2.2 \times 10^{-3}$ , or 0.22%.

41. (a) Let  $m_1$  be the mass of the body that is originally moving,  $v_{1i}$  be its velocity before the collision, and  $v_{1f}$  be its velocity after the collision. Let  $m_2$  be the mass of the body that is originally at rest and  $v_{2f}$  be its velocity after the collision. Then, according to Eq. 10-30,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} .$$

We solve for  $m_2$  to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1f} + v_{1i}} m_1 .$$

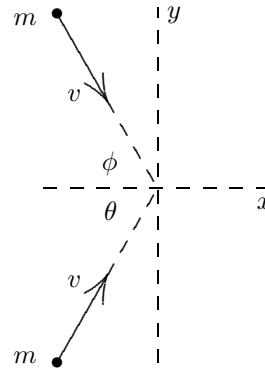
We combine this with  $v_{1f} = v_{1i}/4$  to obtain  $m_2 = 3m_1/5 = 3(2.0)/5 = 1.2 \text{ kg}$ .

- (b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(2.0)(4.0)}{2.0 + 1.2} = 2.5 \text{ m/s} .$$

51. Suppose the objects enter the collision along lines that make the angles  $\theta > 0$  and  $\phi > 0$  with the  $x$  axis, as shown in the diagram below. Both have the same mass  $m$  and the same initial speed  $v$ .

We suppose that after the collision the combined object moves in the positive  $x$  direction with speed  $V$ . Since the  $y$  component of the total momentum of the two-object system is conserved,  $mv \sin \theta - mv \sin \phi = 0$ . This means  $\phi = \theta$ . Since the  $x$  component is conserved,  $2mv \cos \theta = 2mV$ . We now use  $V = v/2$  to find that  $\cos \theta = 1/2$ . This means  $\theta = 60^\circ$ . The angle between the initial velocities is  $120^\circ$ .





52. We orient our  $+x$  axis along the initial direction of motion, and specify angles in the “standard” way – so  $\theta = +60^\circ$  for one ball (1) which is assumed to go into the first quadrant with speed  $v'_1 = 1.1$  m/s, and  $\phi < 0$  for the other ball (2) which presumably goes into the fourth quadrant. The mass of each ball is  $m$ , and the initial speed of one of the balls is  $v_0 = 2.2$  m/s. We apply the conservation of linear momentum to the  $x$  and  $y$  axes respectively.

$$\begin{aligned} mv_0 &= mv'_1 \cos \theta + mv'_2 \cos \phi \\ 0 &= mv'_1 \sin \theta + mv'_2 \sin \phi \end{aligned}$$

The mass  $m$  cancels out of these equations, and we are left with two unknowns and two equations, which is sufficient to solve.

- (a) With SI units understood, the  $y$ -momentum equation can be rewritten as

$$v'_2 \sin \phi = -v'_1 \sin 60^\circ = -0.95$$

and the  $x$ -momentum equation yields

$$v'_2 \cos \phi = v_0 - v'_1 \cos 60^\circ = 1.65$$

Dividing these two equations, we find  $\tan \phi = -0.577$  which yields  $\phi = -30^\circ$ . If we choose to measure this as a positive-valued angle (as the textbook does in §10-6), then this becomes  $30^\circ$ . We plug  $\phi = -30^\circ$  into either equation and find  $v'_2 \approx 1.9$  m/s.

- (b) One can check to see if this an elastic collision by computing

$$\frac{2K_i}{m} = v_0^2 \quad \text{and} \quad \frac{2K_f}{m} = v_1'^2 + v_2'^2$$

and seeing if they are equal (they are), but one must be careful not to use rounded-off values. Thus, it is useful to note that the answer in part (a) can be expressed “exactly” as  $v'_2 = \frac{1}{2}v_0\sqrt{3}$  (and of course  $v'_1 = \frac{1}{2}v_0$  “exactly” – which makes it clear that these two kinetic energy expressions are indeed equal).