Solutions: H.W. Chapter # 4, PHYS 101 (011)

$\vec{r}(t) = (3.00t\,\hat{i} - 4.00t^2\,\hat{j} + 2.00\,\hat{k})$ m

a) $\vec{v}(t) = \dfrac{d\vec{r}}{dt} = (3.00\,\hat{i} - 8.00t\,\hat{j})$ m/s

b) $\vec{v}(t=2s) = (3.00\,\hat{i} - 8.00 * 2\,\hat{j}) = (3.00\,\hat{i} - 16.0\,\hat{j})$ m/s

c) $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(3.00)^2 + (16.0)^2} = 16.3$ m/s

d) The angle θ_x (between the velocity vector $\vec{v}(2s)$ and the positive x-axis is given by

$\theta_x = \cos^{-1}\dfrac{v_x}{|v|} = \cos^{-1}\left(\dfrac{3}{16.3}\right) = \pm 79^\circ \Rightarrow -79^\circ$ (clockwise)

or $\theta_x = 360 - \theta$; where $\theta = \tan^{-1}\left|\dfrac{3}{16}\right| = 79^\circ$
($\vec{v}$ is in the 4th Quadrant)

$\therefore$ The angle with the positive x-axis is 281° going Counterclockwise OR 79° going clockwise!!

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10. Given: $\vec{v}_0 = (4.0\,\hat{i} - 2.0\,\hat{j} + 3.0\,\hat{k})$ m/s

$t = 4.0$ s

$\vec{v} = (-2.0\,\hat{i} - 2.0\,\hat{j} + 5.0\,\hat{k})$ m/s

a) The change of velocity $\Delta\vec{v}$ in $\Delta t = 4.0$ s is given by

$\Delta\vec{v} = \vec{v} - \vec{v}_0 = (-6.0\,\hat{i} + 0\,\hat{j} + 2.0\,\hat{k})$ m/s

$\therefore \vec{a} = \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{(-6.0\,\hat{i} + 2.0\,\hat{k})\text{ m/s}}{4.0\text{ s}} = (-1.5\,\hat{i} + 0.50\,\hat{j})\ \frac{m}{s}$

b) $|\vec{a}| = \sqrt{(1.5)^2 + (0.5)^2} = 1.58\text{ m/s}^2 = 1.6\text{ m/s}^2$

$\theta_x = \cos^{-1}\left(\dfrac{-1.5}{1.58}\right) = 162$

15: ~~G~~Given : $\vec{v}_0 = (8.0\hat{j})\, m/s$; $\vec{r}_0 = 0$ (starts from origin)

$\vec{a} = (4.0\hat{i} + 2.0\hat{j})\, m/s^2$

$$\Delta\vec{r} = \vec{r} - \cancel{\vec{r}_0}^{\,0} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$\vec{r} = (8.0\hat{j})t + \frac{1}{2}(4.0\hat{i} + 2.0\hat{j})t^2$$

$$x\hat{i} + y\hat{j} = (2.0t^2)\hat{i} + (8.0t + 1.0t^2)\hat{j}$$

Now at the instant when $x = 29$, $t = ??$

$$\therefore x = 29 = 2.0t^2 \Longrightarrow t = 3.8\,s$$

a) $y(3.8s) = (8.0 * 3.8 + 1.0 * 3.8^2) = (30.4 + 14.50)$

$= \underline{45\,m}$

b) The velocity of the particle is

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t = 8.0\hat{j} + (4.0\hat{i} + 2.0\hat{j})t$$

$$\vec{v}(3.8s) = 8.0\hat{j} + (4.0\hat{i} + 2.0\hat{j}) * 3.8 = 15.2\hat{i} + 15.6\hat{j}$$

Speed of the particle is then given by

$$|\vec{v}| = \sqrt{(15.2)^2 + (15.6)^2} = \sqrt{231.04 + 243.36} = \underline{22\,\frac{m}{s}}$$

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28. Try to do the problem using $x = v_{0x}t$

$y = v_{0y}t - \frac{1}{2}gt^2$

a) $y = v_0 \sin 40^\circ t - \frac{1}{2}gt^2$, where $\boxed{t = \frac{x}{v_0 \cos 40^\circ} = \frac{22}{25\cos 40^\circ} = \underline{1.15\,s}}$

$y = 25 \sin 40^\circ * 1.15 - \frac{1}{2}(9.8)(1.15)^2$

$= 18.48 - 6.48 = \underline{12.0\,m}$

$\boxed{\text{c) Since } v_y = +ve\text{, it has not reached the highest point.}}$

b) $v_x = v_{0x} = v_0 \cos 40^\circ = 19.2\, m/s$

$v_y = v_{0y} - gt = 25 \sin 40^\circ - 9.8 * 1.15 = 16.07 - 11.27 = \underline{4.8\,\frac{m}{s}}$

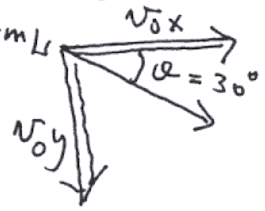
③

33. Given: $\Delta x = 700\,m$, $v_0 = 290.0\ km/hr$ or $v_0 = 80.56\ m/s$

30^0 below the horizontal

$v_{0x} = v_0 \cos 30^0 = (80.56)(0.866) = 69.76\ m/s$

$v_{0y} = -v_0 \sin 30^0 = -(80.56)(0.500) = -40.28\ m/s$



a) We know that $\Delta X = v_{0x}\, t$

$$t = \frac{\Delta X}{v_{0x}} = \frac{700\ m}{69.77\ m/s} = \underline{10.03\ Second}$$

b) $$\Delta y = v_{0y} t - \frac{1}{2} g t^2 = (-40.28)(10.03) - \frac{1}{2}(9.8)(10.03)^2$$

$$= -404.00 - 492.94 = \underline{-897\ m}$$ (This is displacement and is in the downward direction)

It was released from a height of 897 m above the ground.

44: a) $Distance = 2\pi R = 2\pi(0.15m) = \underline{0.94\ m}$

b) $$v = \frac{Total\ distance}{Time} = \left(1200 \frac{\cancel{rev}}{\cancel{min}}\right)\left(\frac{0.94m}{\cancel{rev}}\right)\left(\frac{1\ \cancel{min}}{60s}\right) = \underline{19\ m/s}$$

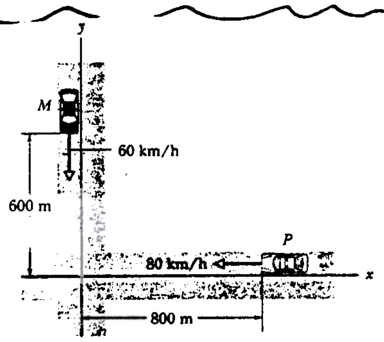
c) $$a_r = \frac{v^2}{R} = \frac{(19\ m/s)^2}{0.15m} = 2.4 \times 10^3\ m/s^2$$

d) $$Time\ period = T = \frac{2\pi R}{v} = \frac{0.94m}{19\ m/s} = \underline{0.049\ s}$$

58: a) $$\vec{v}_{MP} = \vec{v}_{MG} + \vec{v}_{GP} = \vec{v}_{MG} - \vec{v}_{PG}$$

$$= (-60\hat{j})\frac{km}{hr} - (-80\hat{i})\frac{km}{hr}$$

$$= (-60\hat{j} + 80\hat{i})\ km/hr$$



b) $\vec{v}_{MP}$ happens to be along the line of sight. This can be seen from this figure as $\frac{60\ km/hr}{80\ km/hr} = \frac{600\ m}{800\ m}$

c) NO, the answers to (a) & (b) remain the same.