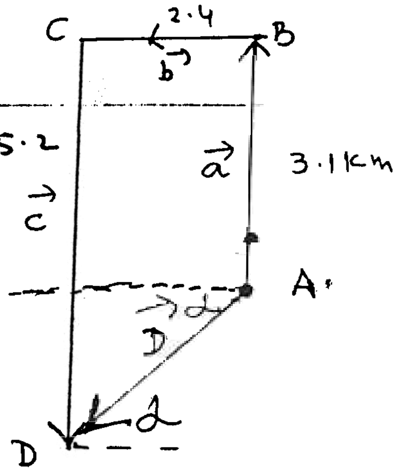
Solutions: CH #3 Fall 2001 (011)

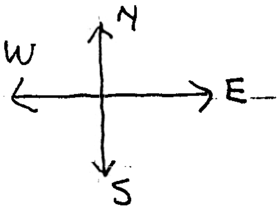
12: $\vec{D} = \vec{a} + \vec{b} + \vec{c}$

$\vec{a} = 3.1\,\hat{j}$ km

$\vec{b} = -2.1\,\hat{i}$ km

$\vec{c} = -5.2\,\hat{j}$ km





$$\vec{D} = 3.1\hat{j} - 2.4\hat{i} - 5.2\hat{j} = -2.4\hat{i} - 2.1\hat{j}$$

$$|\vec{D}| = \text{Distance} = \left[(-2.4)^2 + (-2.1)^2\right]^{\frac{1}{2}} = 3.2\text{ km}$$

$$\theta_x = 180^0 + \alpha \quad \text{and} \quad \alpha = \tan^{-1}\frac{2.1}{2.4} = 4^0$$

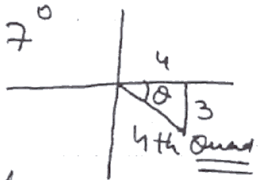
It makes an angle of 221^0 with the East direction.

Or 41^0 South of West

18: $\vec{a} = (4.0\text{m})\hat{i} - (3.0\text{m})\hat{j}$ & $\vec{b} = (6.0\text{m})\hat{i} + (8.0\text{m})\hat{j}$

a & b): $|\vec{a}| = \sqrt{4^2 + 3^2} = \underline{5\text{ m}}$; $\theta = \tan^{-1}\frac{3}{4} = 37^0$

$\theta_x = 360^0 - \theta = \underline{323^0}$



c & d): $|\vec{b}| = \sqrt{36+64} = \underline{10\text{ m}}$; $\theta = \theta_x = \underline{53^0}$ (First quadrant)

e & f): $\vec{a} + \vec{b} = (10\text{ m})\hat{i} + (5.0\text{ m})\hat{j} \Rightarrow |\vec{a} + \vec{b}| =$

$\theta_x = \theta = 27^0$ (First quadrant)

g & h $\vec{b} - \vec{a} = (2.0\text{m})\hat{i} + (11.0\text{m})\hat{j} \Rightarrow |\vec{b} - \vec{a}| = \underline{11.2\text{m}}$

$\theta_x = \theta = 80^0$

$\vec{a} - \vec{b} = (-2.0\text{m})\hat{i} - (11.0\text{m})\hat{j} \Rightarrow |\vec{a} - \vec{b}| = 11.2\text{m}$

$\theta_x = \theta + 180^0 = 260^0$

18: (Continued)

The angle between $\vec{b}-\vec{a}$ and $\vec{a}-\vec{b}$ is $260^\circ - 80^\circ = \underline{180^\circ}$

* You can tell without doing any calculations as

$$\underline{\vec{a}-\vec{b}} = -(\underline{\vec{b}-\vec{a}})$$

* You can also find it by using

$$\theta = \cos^{-1} \frac{(\vec{a}-\vec{b})\cdot(\vec{b}-\vec{a})}{|\vec{a}-\vec{b}|\,|\vec{b}-\vec{a}|} = \cos^{-1}\frac{-4-121}{11 * 11.2} = \cos^{-1}(-1)$$

$$\theta = 180^\circ$$

31:

$$\vec{a} = 3.0\hat{i} + 3.0\hat{j} + 3.0\hat{k}$$
$$\vec{b} = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k}$$

$|\vec{a}| = \sqrt{27}$, $|\vec{b}| = \sqrt{14}$

$\vec{a}\cdot\vec{b} = 6 + 3 + 9 = 18$

$$\theta = \cos^{-1}\frac{(\vec{a}\cdot\vec{b})}{|\vec{a}|\,|\vec{b}|} = \cos^{-1}\frac{18}{\sqrt{27}\sqrt{14}} = \cos^{-1}(0.926)$$

$$\underline{\theta = 22^\circ}$$

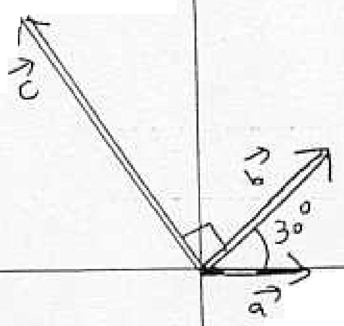
36:

$$\vec{A} = 2.00\hat{i} + 3.00\hat{j} - 4.00\hat{k}$$
$$\vec{B} = -3.00\hat{i} + 4.00\hat{j} + 2.00\hat{k}$$
$$\vec{C} = 7.00\hat{i} - 8.00\hat{j}$$

$$\therefore\ 3(7.00\hat{i} - 8.00\hat{j})\cdot\left[2(2.00\hat{i} + 3.00\hat{j} - 4.00\hat{k}) \times (-3.00\hat{i} + 4.00\hat{j} + 2.00\hat{k})\right]$$

$$2\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & -8 \\ -3 & 4 & 2 \end{vmatrix} = (12+32)\hat{i} - \hat{j}(8-24) + \hat{k}(16+18)$$
$$= 44\hat{i} + 16\hat{j} + 34\hat{k}$$

$$3\vec{C}\cdot(2\vec{A}\times\vec{B}) = 3 * [7 * 44 - 8 * (+16)] = \cancel{1692}\ \underline{\underline{540}}$$



37: $a_x = 3.00\,m$; $a_y = 0$

$b_x = 4\cos 30^\circ$; $b_y = 4\sin 30^\circ$

$b_x = 3.46\,m$; $b_y = 2.00\,m$

$C_x = 10.0\cos(120^\circ)$; $C_y = 10.0\sin(120^\circ)$

$C_x = -5.00\,m$; $C_y = 8.66\,m$

$$\vec{C} = p\vec{a} + q\vec{b}$$

$$-5.00\,\hat{i} + 8.66\,\hat{j} = p\left(3.00\,\hat{i} + 0.00\,\hat{j}\right) + q\left(3.46\,\hat{i} + 2.00\,\hat{j}\right)$$

$\therefore \quad 3.00\,p + 3.46\,q = -5.00$ &

$2.00\,q = 8.66 \Longrightarrow q = \underline{\underline{4.33}}$

Hence $\quad 3p + 3.46 \times 4.33 = -5.00$

$$p = \underline{\underline{-6.66}}$$