5. Let $M$ be the mass of the car (presumably including the mass of the wheels) and $v$ be its speed. Let $I$ be the rotational inertia of one wheel and $\omega$ be the angular speed of each wheel. The kinetic energy of rotation is

$$K_{\text{rot}} = 4\left(\frac{1}{2}I\omega^2\right)$$

where the factor 4 appears because there are four wheels. The total kinetic energy is given by $K = \frac{1}{2}Mv^2 + 4\left(\frac{1}{2}I\omega^2\right)$. The fraction of the total energy that is due to rotation is

$$\text{fraction} = \frac{K_{\text{rot}}}{K} = \frac{4I\omega^2}{Mv^2 + 4I\omega^2} \ .$$

For a uniform disk (relative to its center of mass) $I = \frac{1}{2}mR^2$ (Table 11-2(c)). Since the wheels roll without sliding $\omega = v/R$ (Eq. 12-2). Thus the numerator of our fraction is

$$4I\omega^2 = 4\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = 2mv^2$$

and the fraction itself becomes

$$\text{fraction} = \frac{2mv^2}{Mv^2 + 2mv^2} = \frac{2m}{M + 2m} = \frac{2(10)}{1000} = \frac{1}{50} \ .$$

The wheel radius cancels from the equations and is not needed in the computation.

22. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$(y'F_z - z'F_y)\,\hat{\mathrm{i}} + (z'F_x - x'F_z)\,\hat{\mathrm{j}} + (x'F_y - y'F_x)\,\hat{\mathrm{k}}\ .$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3\,\hat{\mathrm{i}} - 2\,\hat{\mathrm{j}} + 4\,\hat{\mathrm{k}}$, and $\vec{F} = \vec{F}_1$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3$, $y = -2$, $z = 4$, $F_x = 3$, $F_y = -4$ and $F_z = 5$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F}_1 = \left(6.0\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{j}} - 6.0\,\hat{\mathrm{k}}\right)$ N·m.

(b) This is like part (a) but with $\vec{F} = \vec{F}_2$. We plug in $F_x = -3$, $F_y = -4$ and $F_z = -5$ and obtain $\vec{\tau} = \vec{r} \times \vec{F}_2 = \left(26\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} - 18\,\hat{\mathrm{k}}\right)$ N·m.

(c) We can proceed in either of two ways. We can add (vectorially) the answers from parts (a) and (b), or we can first add the two force vectors and then compute $\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$ (these total force components are computed in the next part). The result is $\left(32\,\hat{\mathrm{i}} - 24\,\hat{\mathrm{k}}\right)$ N·m.

(d) Now $\vec{r}\,' = \vec{r} - \vec{r}_{\mathrm{o}}$ where $\vec{r}_{\mathrm{o}} = 3\,\hat{\mathrm{i}} + 2\,\hat{\mathrm{j}} + 4\,\hat{\mathrm{k}}$. Therefore, in the above expression, we set $x' = 0$, $y' = -4$, $z' = 0$, $F_x = 3 - 3 = 0$, $F_y = -4 - 4 = -8$ and $F_z = 5 - 5 = 0$. We get $\vec{\tau} = \vec{r}\,' \times \left(\vec{F}_1 + \vec{F}_2\right) = 0$.

32. Both $\vec{r}$ and $\vec{v}$ lie in the $xy$ plane. The position vector $\vec{r}$ has an $x$ component that is a function of time (being the integral of the $x$ component of velocity, which is itself time-dependent) and a $y$ component that is constant ($y = -2.0$ m). In the cross product $\vec{r} \times \vec{v}$, all that matters is the $y$ component of $\vec{r}$ since $v_x \neq 0$ but $v_y = 0$:

$$\vec{r} \times \vec{v} = -y v_x \,\hat{\mathrm{k}} \ .$$

(a) The angular momentum is $\vec{\ell} = m\,(\vec{r} \times \vec{v})$ where the mass is $m = 2.0$ kg in this case. With SI units understood and using the above cross-product expression, we have

$$\vec{\ell} = (2.0)\left(-(-2.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}} = -24t^2\,\hat{\mathrm{k}}$$

in kg·m$^2$/s. This implies the particle is moving clockwise (as observed by someone on the $+z$ axis) for $t > 0$.

(b) The torque is caused by the (net) force $\vec{F} = m\vec{a}$ where

$$\vec{a} = \frac{d\vec{v}}{dt} = -12t\,\hat{\mathrm{i}}\ \mathrm{m/s^2}\ .$$

The remark above that only the $y$ component of $\vec{r}$ still applies, since $a_y = 0$. We use $\vec{\tau} = \vec{r} \times \vec{F} = m\,(\vec{r} \times \vec{a})$ and obtain

$$\vec{\tau} = (2.0)\left(-(-2.0)(-12t)\right)\hat{\mathrm{k}} = -48t\,\hat{\mathrm{k}}$$

in N·m. The torque on the particle (as observed by someone on the $+z$ axis) is clockwise, causing the particle motion (which was clockwise to begin with) to increase.

(c) We replace $\vec{r}$ with $\vec{r}\,'$ (measured relative to the new reference point) and note (again) that only its $y$ component matters in these calculations. Thus, with $y' = -2.0 - (-3.0) = 1.0$ m, we find

$$\vec{\ell}\,' = (2.0)\left(-(1.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}} = 12t^2\,\hat{\mathrm{k}}$$

in kg·m$^2$/s. The fact that this is positive implies that the particle is moving counterclockwise relative to the new reference point.

(d) Using $\vec{\tau}\,' = \vec{r}\,' \times \vec{F} = m\,(\vec{r}\,' \times \vec{a})$, we obtain

$$\vec{\tau} = (2.0)\left(-(1.0)\left(-12t\right)\right)\hat{\mathrm{k}} = 24t\,\hat{\mathrm{k}}$$

in N·m. The torque on the particle (as observed by someone on the $+z$ axis) is counterclockwise, relative to the new reference point.

50. The axis of rotation is in the middle of the rod, $r = 0.25$ m from either end. By Eq. 12-19, the initial angular momentum of the system (which is just that of the bullet, before impact) is $rmv \sin\phi$ where $m = 0.003$ kg and $\phi = 60°$. Relative to the axis, this is counterclockwise and thus (by the common convention) positive. After the collision, the moment of inertia of the system is $I = I_{\text{rod}} + mr^2$ where $I_{\text{rod}} = ML^2/12$ by Table 11-2(e), with $M = 4.0$ kg and $L = 0.5$ m. Angular momentum conservation leads to

$$rmv \sin\phi = \left(\frac{1}{12}ML^2 + mr^2\right)\omega \ .$$

Thus, with $\omega = 10$ rad/s, we obtain

$$v = \frac{\left(\frac{1}{12}(4.0)(0.5)^2 + (0.003)(0.25)^2\right)(10)}{(0.25)(0.003)\sin 60°} = 1.3 \times 10^3 \text{ m/s} \ .$$

54. The initial rotational inertia of the system is $I_i = I_{\text{disk}} + I_{\text{student}}$ where $I_{\text{disk}} = 300$ kg·m$^2$ (which, incidentally, does agree with Table 11-2(c)) and $I_{\text{student}} = mR^2$ where $m = 60$ kg and $R = 2.0$ m. The rotational inertia when the student reaches $r = 0.5$ m is $I_f = I_{\text{disk}} + mr^2$. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \quad \Longrightarrow \quad \omega_f = \omega_i \frac{I_{\text{disk}} + mR^2}{I_{\text{disk}} + mr^2}$$

which yields, for $\omega_i = 1.5$ rad/s, a final angular velocity of $\omega_f = 2.6$ rad/s.