4. If we make the units explicit, the function is

$$\theta = (4.0\,\text{rad/s})t - \left(3.0\,\text{rad/s}^2\right)t^2 + \left(1.0\,\text{rad/s}^3\right)t^3$$

but generally we will proceed as shown in the problem – letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

   (a) Eq. 11-6 leads to

$$\omega = \frac{d}{dt}\left(4t - 3t^2 + t^3\right) = 4 - 6t + 3t^2 \ .$$

   Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

   (b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

   (c) Consequently, Eq. 11-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad/s}^2 \ .$$

   (d) And Eq. 11-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(4 - 6t + 3t^2\right) = -6 + 6t \ .$$

   Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s^2.

   (e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

14. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha = 2.00\ \text{rad/s}^2$. Between t_1 and t_2 it turns through $\Delta\theta = 90.0$ rad, where $t_2 - t_1 = \Delta t = 3.00$ s.

(a) We use Eq. 11-13 (with a slight change in notation) to describe the motion for $t_1 \le t \le t_2$:

$$\Delta\theta = \omega_1 \Delta t + \frac{1}{2}\alpha\,(\Delta t)^2 \quad \Longrightarrow \quad \omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha \Delta t}{2}$$

which we plug into Eq. 11-12, set up to describe the motion during $0 \le t \le t_1$:

$$\begin{aligned} \omega_1 &= \omega_0 + \alpha t_1 \\ \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} &= \alpha t_1 \\ \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} &= (2.00)t_1 \end{aligned}$$

yielding $t_1 = 13.5$ s.

(b) Plugging into our expression for ω_1 (in previous part) we obtain

$$\omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = 27.0\ \text{rad/s}\ .$$

24. (a) Converting from hours to seconds, we find the angular velocity (assuming it is positive) from Eq. 11-18:

$$\omega = \frac{v}{r} = \frac{\left(2.90 \times 10^4 \, \text{km/h}\right)\left(\frac{1.00\,\text{h}}{3600\,\text{s}}\right)}{3.22 \times 10^3 \, \text{km}} = 2.50 \times 10^{-3} \, \text{rad/s} \; .$$

(b) The radial (or centripetal) acceleration is computed according to Eq. 11-23:

$$a_r = \omega^2 r = \left(2.50 \times 10^{-3} \, \text{rad/s}\right)^2 \left(3.22 \times 10^6 \, \text{m}\right) = 20.2 \, \text{m/s}^2 \; .$$

(c) Assuming the angular velocity is constant, then the angular acceleration and the tangential acceleration vanish, since

$$\alpha = \frac{d\omega}{dt} = 0 \quad \text{and} \quad a_t = r\alpha = 0 \; .$$

37. The particles are treated "point-like" in the sense that Eq. 11-26 yields their rotational inertia, and the rotational inertia for the rods is figured using Table 11-2(e) and the parallel-axis theorem (Eq. 11-29).

(a) With subscript 1 standing for the rod nearest the axis and 4 for the particle farthest from it, we have

$$\begin{aligned} I &= I_1 + I_2 + I_3 + I_4 \\ &= \left(\frac{1}{12}Md^2 + M\left(\frac{1}{2}d\right)^2\right) + md^2 + \left(\frac{1}{12}Md^2 + M\left(\frac{3}{2}d\right)^2\right) + m(2d)^2 \\ &= \frac{8}{3}Md^2 + 5md^2 \ . \end{aligned}$$

(b) Using Eq. 11-27, we have

$$K = \frac{1}{2}I\omega^2 = \left(\frac{4}{3}Md^2 + \frac{5}{2}md^2\right)\omega^2 \ .$$

48. The net torque is

$$\begin{aligned} \tau &= \tau_A + \tau_B + \tau_C \\ &= F_A r_A \sin\phi_A - F_B r_B \sin\phi_B + F_C r_C \sin\phi_C \\ &= (10)(8.0)\sin 135^\circ - (16)(4.0)\sin 90^\circ + (19)(3.0)\sin 160^\circ \\ &= 12\ \text{N}\cdot\text{m}\ . \end{aligned}$$

66. From Table 11-2, the rotational inertia of the spherical shell is $2MR^2/3$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega_{\text{sphere}}^2 + \frac{1}{2}I\omega_{\text{pulley}}^2 + \frac{1}{2}mv^2 \ .$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$