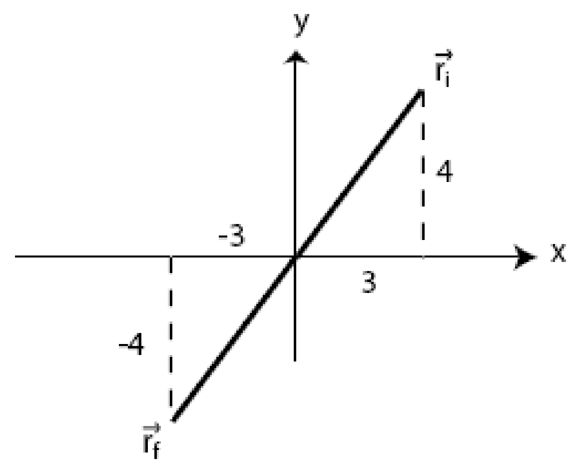
Selected Problems From Chapter 4

Q1 The position of a particle is initially ri = (3.0 m)i + (4.0 m)j, and 10 s later it is rf = -(3.0 m)i - (4.0 m)j (see Fig). What is its average velocity during this time interval ?

A1 (-0.6i - 0.8j) m/s
A2 (0.6i + 0.8j) m/s
A3 0 m/s
A4 10 m/s, at angle 45 degree
A5 10 m/s, at angle -45 degree



$$\Delta\vec{v} = \frac{\Delta\vec{r}}{\Delta t} = \frac{r_2 - r_1}{\Delta t}$$

$$= \frac{[(-3i - 4j) - (3i + 4j)]}{10}$$

$$= \frac{-6i - 8j}{10} = -0.6i - 0.8j$$

Q2 At t=0, a particle leaves the origin with a ve
of vo = (4i + 2j) m/s. After 20.0 s its veloci
v = (20i - 4j) m/s. Find its acceleration
(assumed constant).

A1 (0.8i - 0.3j) m/s**2
A2 (0.5i + 0.4j) m/s**2
A3 (0.3i - 0.7j) m/s**2
A4 (0.7i + 0.7j) m/s**2
A5 0 m/s**2

$$a = a_{avg} \quad (a \text{ constant})$$

$$= \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t}$$

$$= \frac{(20i - 4j) - (4i + 2j)}{20.0}$$

$$= \frac{16i - 6j}{20.0} = 0.8i - 0.3j$$

Q3 A stone is thrown from the ground into the air with an initial velocity V = (5.0i + 9.0j) m/s. To what maximum height will the stone rise?

A1 4.1 m
A2 1.3 m
A3 9.0 m
A4 5.0 m
A5 7.0 m

$$on\ y-axis:$$

$$v_i = 9.0 \text{ m/s}, v_f = 0, a = -g, y = ?$$

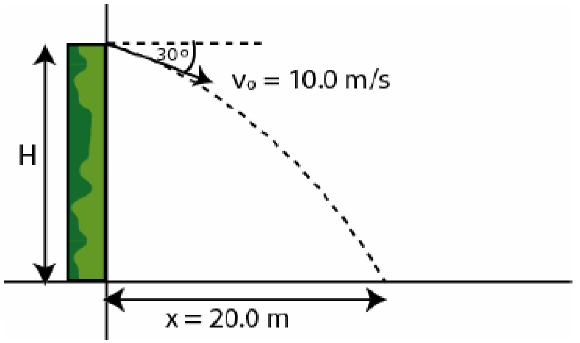
$$v^2{}_f = v^2{}_i + 2ay$$

$$0 = 9.0^2 - 2 \times 9.8y$$

$$y = \frac{81}{19.6} = 4.1 \text{ m}$$

Q 4 A projectile is thrown from a height H with a speed of 10.0 m/s at an angle of 30 degrees below horizontal as shown in Fig. Find H, if the horizontal distance x = 20.0 m.

A1 37.7 m
A2 98.0 m
A3 49.0 m
A4 20.0 m
A5 67.8 m



$$on\ the\ x-axis:$$

$$v = 10.0\cos(30) = 8.66\ \ m/s$$

$$t = \frac{\Delta x}{v} = \frac{20.0}{8.66} = 2.31\,s$$

$$on\ the\ y-axis:$$

$$t = 2.31\,s, v_0 = -10.0\sin(30) = -5.00\ m/s, a = -g, y = -H$$

$$y = v_0 t - \tfrac{1}{2}gt^2 \Rightarrow -H = -5.00\times 2.31 - \tfrac{1}{2}\times 9.80\times(2.31)^2$$

$$H = 11.55 + 26.14 = 37.7\ \text{m}$$

Q5 A stone is tied to the end of a string and is rotated with constant speed around a horizontal circle of radius 1.0 m. If the magnitude of its acceleration is 225 m/s**2, What is the period (T) of the motion?

A1 0.42 s
A2 1.0 s
A3 0.028 s
A4 5.0 s
A5 2.0 s

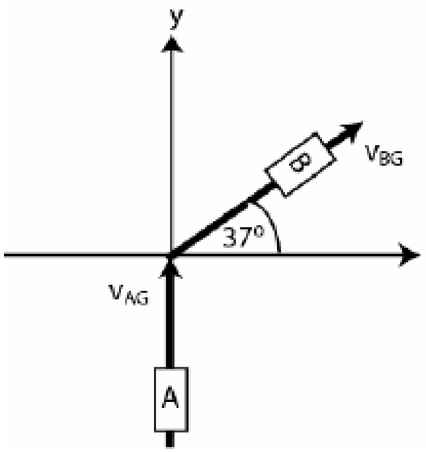
$$R = 1.0 \text{ m}, \ a = 225 \text{ m/s}^2$$

$$a = \frac{v^2}{R} \Rightarrow v = \sqrt{aR} = \sqrt{225 \times 1.0^2} = \sqrt{225} = 15 \text{ m/s}$$

$$\mathrm{T} = \frac{2\pi \mathrm{R}}{\mathrm{v}} = \frac{2\pi(1.0)}{15} = 0.42\ s$$

Q6 Car A travels with velocity (30 j) m/s (relative to the ground) and car B travels with speed of 50 m/s in a direction making an angle of 37 degrees with +x axis (relative to the ground) (see Fig). What is the velocity of car A relative to car B ?

A1 (-40i) m/s
A2 (40i+30j) m/s
A3 (-40i-60j) m/s
A4 (40i) m/s
A5 (-40i-30j) m/s



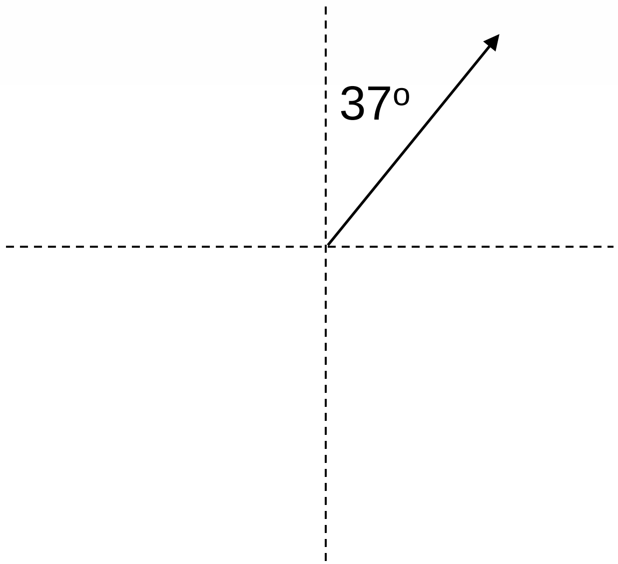
$$\vec{v}_{AG} = 30j$$

$$\vec{v}_{BG} = 50\cos(37)i + 50\sin(37)j$$
$$= 40i + 30j$$

$$\vec{v}_{AB} = \vec{v}_{AG} + \vec{v}_{GB}$$
$$= \vec{v}_{AG} - \vec{v}_{BG}$$
$$= 30j - (40i + 30j)$$
$$= -40i \ m/s$$

Q.7 A boat is sailing due North at a speed of 4.0 m/s with respect to the water of a river. If the water is moving due East at a speed of 3.0 m/s relative to the ground, what is the velocity of the boat relative to the ground?

A1 5.0 m/s making an angle 37 degrees east of north
A2 5.0 m/s making an angle 53 degrees east of north
A3 5.0 m/s east of north
A4 1.0 m/s west of south
A5 1.0 m/s west



$$\vec{v}_{BW} = 4.0\,j$$

$$\vec{v}_{WG} = 3.0i$$

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG} = 4.0\,j + 3.0i$$

$$v_{BG} = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5.0 \text{ m/s}$$

$$\tan^{-1}\frac{4}{3} = \tan^{-1}1.33 = 53^o \; north\ of\ east = 37^o \; east\ of\ north$$

••8 A plane flies 483 km east from city A to city B in 45.0 min and then 966 km south from city B to city C in 1.50 h. For the total trip, what are the (a) magnitude and (b) direction of the plane's displacement, the (c) magnitude and (d) direction of its average velocity, and (e) its average speed?

8. Our coordinate system has $\hat{i}$ pointed east and $\hat{j}$ pointed north. All distances are in kilometers, times in hours, and speeds in km/h. The first displacement is $\vec{r}_{AB} = 483\hat{i}$ and the second is $\vec{r}_{BC} = -966\hat{j}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483 \text{ km})\hat{i} - (966 \text{ km})\hat{j}$$

which yields $|\vec{r}_{AC}| = \sqrt{(483)^2 + (-966)^2} = 1.08 \times 10^3$ km.

(b) The angle is given by

$$\tan^{-1}\left(\frac{-966}{483}\right) = -63.4°.$$

We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(c) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives

$$\vec{v}_{avg} = \frac{483\hat{i} - 966\hat{j}}{2.25} = 215\hat{i} - 429\hat{j}.$$

with a magnitude $|\vec{v}_{avg}| = \sqrt{(215)^2 + (-429)^2} = 480$ km/h.

(d) The direction of $\vec{v}_{avg}$ is 26.6° east of south, same as in part (b). In magnitude-angle notation, we would have $\vec{v}_{avg} = (480 \angle -63.4°)$.

(e) Assuming the AB trip was a straight one, and similarly for the BC trip, then $|\vec{r}_{AB}|$ is the distance traveled during the AB trip, and $|\vec{r}_{BC}|$ is the distance traveled during the BC trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483 + 966}{2.25} = 644 \text{ km/ h}.$$

••15 A particle leaves the origin with an initial velocity $\vec{v} = (3.00\hat{i})$ m/s and a constant acceleration $\vec{a} = (-1.00\hat{i} - 0.500\hat{j})$ m/s^2. When it reaches its maximum x coordinate, what are its (a) velocity and (b) position vector? SSM ILW

15. Constant acceleration in both directions (x and y) allows us to use Table 2-1 for the motion along each direction. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. The x component is $v_x = v_{0x} + a_x t = 3.00 - 1.00t$, and the y component is $v_y = v_{0y} + a_y t = -0.500t$ since $v_{0y} = 0$. When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.00 - 1.00t_m = 0$ or $t_m = 3.00$ s. The y component of the velocity at this time is

$$v_y = 0 - 0.500(3.00) = -1.50 \text{ m/s};$$

this is the only nonzero component of $\vec{v}$ at t_m.

(b) Since it started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. At $t = t_m$ this becomes

$$\vec{r} = \left(3.00\hat{\mathrm{i}}\right)(3.00) + \frac{1}{2}\left(-1.00\,\hat{\mathrm{i}} - 0.50\,\hat{\mathrm{j}}\right)(3.00)^2 = (4.50\,\hat{\mathrm{i}} - 2.25\,\hat{\mathrm{j}}) \text{ m}.$$

•20 A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. (a) How long is the ball in the air? (b) What is its speed at the instant it leaves the table?

20. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and $y = -1.20$ m indicates the level at which the ball hits the floor, then

$$t = \sqrt{\frac{2(-1.20)}{-9.80}} = 0.495\,\mathrm{s}.$$

(b) The initial (horizontal) velocity of the ball is $\vec{v} = v_0\,\hat{\mathrm{i}}$. Since x = 1.52 m is the horizontal position of its impact point with the floor, we have $x = v_0 t$. Thus,

$$v_0 = \frac{x}{t} = \frac{152}{0.495} = 3.07\ \mathrm{m/s}.$$

•23 A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0°$ below the horizontal when the pilot releases a radar decoy (Fig. 4-34). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700$ m. (a) How long is the decoy in the air? (b) How high was the release point?

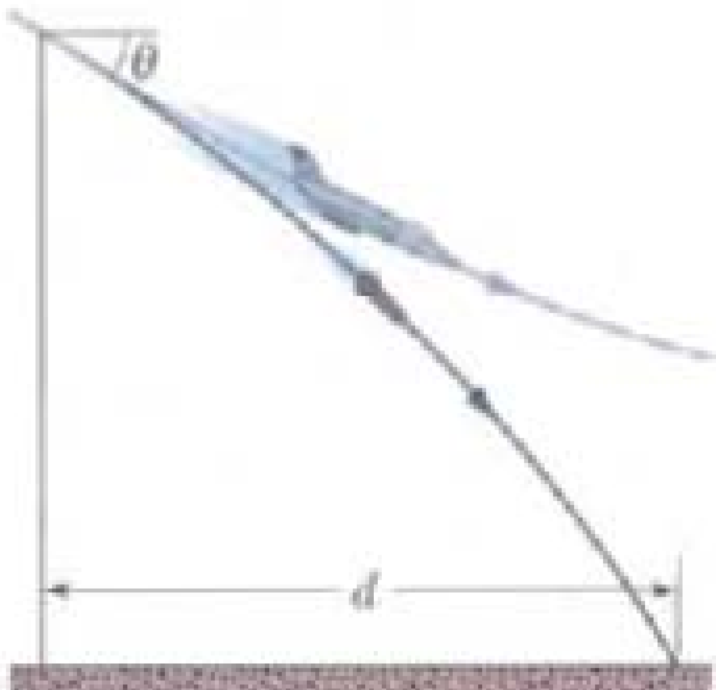


Fig. 4-34 Problem 23.

23. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -30.0°$ since the angle shown in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the plane's speed at the moment of release: $v_0 = 290$ km/h, which we convert to SI units: (290)(1000/3600) = 80.6 m/s.

(a) We use Eq. 4-12 to solve for the time:

$$\Delta x = (v_0 \cos\theta_0)\, t \quad \Rightarrow \quad t = \frac{700}{(80.6)\cos(-30.0°)} = 10.0 \text{ s}.$$

(b) And we use Eq. 4-22 to solve for the initial height y_0:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad 0 - y_0 = (-40.3)(10.0) - \frac{1}{2}(9.80)(10.0)^2$$

which yields $y_0 = 897$ m.

••31 A rifle that shoots bullets at 460 m/s is to be aimed at a target 45.7 m away. If the center of the target is level with the rifle, how high above the target must the rifle barrel be pointed so that the bullet hits dead center?

31. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$. The projectile motion equations lead to $d = v_0 t \cos\theta_0$ and $0 = v_0 t \sin\theta_0 - \frac{1}{2} g t^2$. Eliminating t leads to $2v_0^2 \sin\theta_0 \cos\theta_0 - gd = 0$. Using $\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin\ (2\theta_0) = gd \Rightarrow \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80)(45.7)}{(460)^2}$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$ and consequently $\theta_0 = 0.0606°$. If the gun is aimed at a point a distance ℓ above the target, then $\tan\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = 45.7\tan(0.0606°) = 0.0484\text{ m} = 4.84\text{ cm}.$$

•46 A centripetal-acceleration addict rides in uniform circular motion with period $T = 2.0\ \text{s}$ and radius $r = 3.00\ \text{m}$. At t_1 his acceleration is $\vec{a} = (6.00\ \text{m/s}^2)\hat{\text{i}} + (-4.00\ \text{m/s}^2)\hat{\text{j}}$. At that instant, what are the values of (a) $\vec{v} \cdot \vec{a}$ and (b) $\vec{r} \times \vec{a}$?

46. (a) During constant-speed circular motion, the velocity vector is perpendicular to the acceleration vector at every instant. Thus, $\vec{v} \cdot \vec{a} = 0$.

(b) The acceleration in this vector, at every instant, points towards the center of the circle, whereas the position vector points from the center of the circle to the object in motion. Thus, the angle between $\vec{r}$ and $\vec{a}$ is 180º at every instant, so $\vec{r} \times \vec{a} = 0$.

••62 After flying for 15 min in a wind blowing 42 km/h at an angle of 20° south of east, an airplane pilot is over a town that is 55 km due north of the starting point. What is the speed of the airplane relative to the air?

62. Velocities are taken to be constant; thus, the velocity of the plane relative to the ground is $\vec{v}_{PG} = (55\ \text{km})/(1/4\ \text{hour})\ \hat{\text{j}} = (220\ \text{km/h})\hat{\text{j}}$. In addition,

$$\vec{v}_{AG} = 42(\cos 20°\hat{\text{i}} - \sin 20°\hat{\text{j}}) = (39\ \text{km/h})\hat{\text{i}} - (14\ \text{km/h})\hat{\text{j}}.$$

Using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, we have

$$\vec{v}_{PA} = \vec{v}_{PG} - \vec{v}_{AG} = -(39\ \text{km/h})\hat{\text{i}} + (234\ \text{km/h})\hat{\text{j}}.$$

which implies $|\vec{v}_{PA}| = 237\ \text{km/h}$, or 240 km/h (to two significant figures.)