Solutions to 1^{st} Major 111

Q1.

Consider a cube of iron of mass 8.0 kg and side 4.0 inches. What is its density in kg/m^3? (1 inch = 2.54 cm)

A) 7.6×10^3
B) 6.9×10^3
C) 9.8×10^3
D) 4.3×10^3
E) 10×10^3

$$l = 4.0\ inch \times \left(\frac{2.54\ cm}{1\ inch}\right) \times \left(\frac{1\ m}{100\ cm}\right) = 1.02\times10^{-3}\ m$$

$$V = l^3 = \left(1.02\times10^{-3}\ m\right)^3 = 1.05\times10^{-3}\ m^3$$

$$\rho = \frac{m}{V} = \frac{8.0}{1.05\times10^{-3}} = 7.6\times10^3\ kg/m^3$$

Q2.

Consider the following physical relation: M = C $\rho^a r^b$, where M is mass, ρ is density, r is distance and a and b are exponents. C is a dimensionless constant. What are the values of a and b so that the equation is dimensionally correct?

A) a = 1 and b = 3
B) a = 1 and b = 2
C) a = 2 and b = 2
D) a = 2 and b = 1
E) a = 3 and b = 1

$$\mathrm{M} = \mathrm{C}\,\rho^{\mathrm{a}}\mathrm{r}^{\mathrm{b}}$$

$$[\mathrm{M}] = [\mathrm{C}]\,[\rho]^{\mathrm{a}}[\mathrm{r}]^{\mathrm{b}} = [\rho]^{\mathrm{a}}[\mathrm{r}]^{\mathrm{b}} \qquad [\mathrm{C}] = 1$$

$$[\mathrm{M}] = [\rho]^{\mathrm{a}}[\mathrm{r}]^{\mathrm{b}} = \left[\frac{M}{L^3}\right]^{\mathrm{a}}[\mathrm{L}]^{\mathrm{b}}$$

$$M = M^a / L^{3a} \bullet L^b \rightarrow ML^{-b} = M^a L^{-3a}$$

$$a = 1 \;\&\; b = 3a = 3$$

Q3.

A hot air balloon carrying a 10.0 kg block is descending vertically at a constant speed of 10.0 m/s. When the balloon is 100 m above the ground, the block is released. How long does it take the block to reach the ground? (Neglect air resistance)

A) 3.61 s
B) 2.53 s
C) 1.64 s
D) 5.43 s
E) 9.12 s

$$v^2 = v^2_0 + 2ay$$

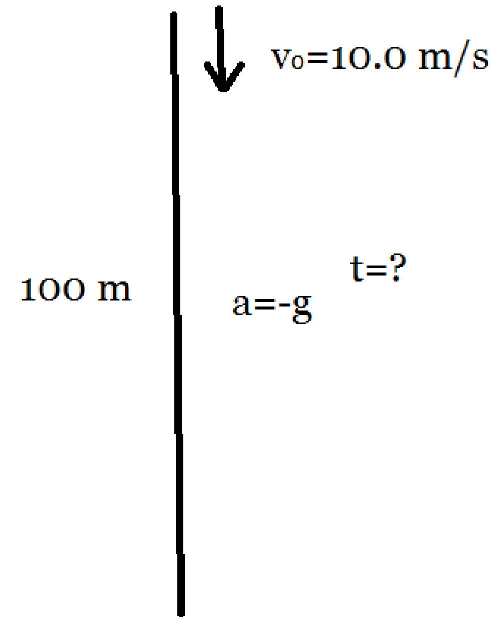
$$v^2 = (-10.0)^2 - 2\times 9.80\times(-100)$$

$$v = -45.4\ m/s$$

$$v = v + at$$

$$-45.4 = -10.0 - 9.80t$$

$$t = 3.61\ s$$



Q4.

Figure 1 shows the velocity-time graph of a particle moving along the x-axis. What is the average acceleration of the particle during the time interval t = 1.0 s to t = 8.0 s?

A) $-2.1\ m/s^2$
B) $-3.8\ m/s^2$
C) $+3.8\ m/s^2$
D) $+2.1\ m/s^2$
E) Zero

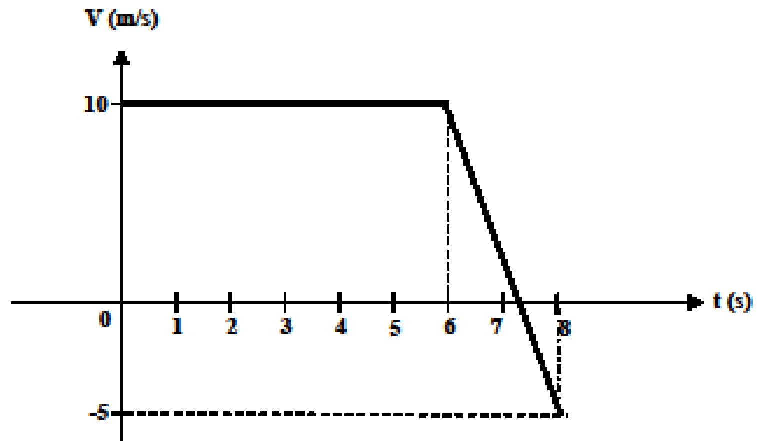


Figure 1

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{-5-10}{8-1} = -2.1\, m/s^2$$

Q5.
Which of the graphs shown in Figure 2 represents an object moving with a **negative constant velocity?**

A) (3)
B) (2)
C) (1)
D) (4)
E) (5)

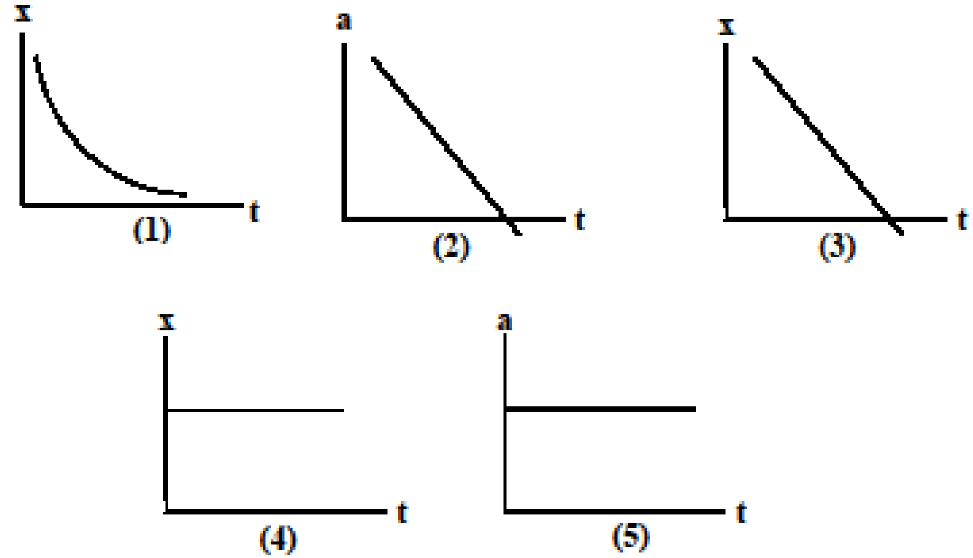


Figure 2

Q6.
A car moving along the positive x-axis with constant acceleration covered the distance between two points 60 m apart in 6.0 s. Its velocity as it passes the second point was 15 m/s. What was its velocity at the first point?

A) 5.0 m/s
B) 10 m/s
C) 2.0 m/s
D) 4.0 m/s
E) 15 m/s

$$a \text{ is constant. }, v = 15\ m/s$$

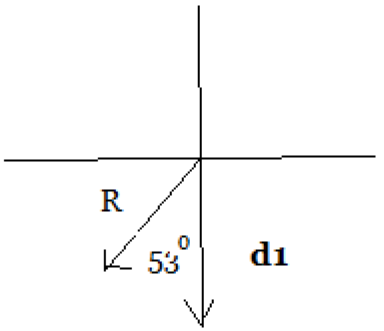
$$\Delta x = 60\ m, t = 6.0\ s, v_0 = ?$$

$$\Delta x = \frac{v + v_0}{2} t \rightarrow v_0 = \frac{2\Delta x}{t} - v \rightarrow \text{ find } v_0$$

Q7.

A car travels 30 km due south and then D km in an unknown direction. The magnitude of the resultant displacement is 50 km and its direction is 53° west of south. Find the magnitude and direction of the unknown displacement D.

A) 40 km due west
B) 40 km due east
C) 45 km due west
D) 45 km due east
E) 54 km due west



$$\vec{d}_1 = -30j, \vec{R} = -50\sin 53i - 50\cos 53j = -40i - 30j$$

$$\vec{R} = \vec{d}_1 + \vec{d}_2 \rightarrow \vec{d}_2 = \vec{R} - \vec{d}_1 = -40i - 30j + 30j = -40i$$

Q8.

A vector $\vec{A}$ is defined by $\vec{A} = 1.50\,\hat{i} + 1.50\,\hat{j}$. Find a vector $\vec{B}$ that makes an angle of 60.0° with $\vec{A}$ in the counterclockwise direction and has magnitude of 4 units.

A) $-1.04\,\hat{i} + 3.86\,\hat{j}$

B) $-1.04\,\hat{i} - 3.86\,\hat{j}$

C) $+1.04\,\hat{i} + 3.86\,\hat{j}$

D) $-3.86\,\hat{i} - 1.04\,\hat{j}$

E) $+3.86\,\hat{i} + 1.04\,\hat{j}$

$$\vec{A}\ makes\ 45^o\ with + ve\ the\ x - axis$$

$$\therefore \vec{B}\ makes\ 45^o + 60^o = 105^o\ with + ve\ the\ x - axis$$

$$\vec{B} = 4\cos 105^o\, i + 4\sin 105^o\, j = -1.04i + 3.86\,j$$

Q9.
Which one of the following statements concerning vectors and scalars is FALSE?

A) A vector that has zero magnitude may have components other that zero.
B) A vector that has a negative component, has a positive magnitude.
C) A scalar component may be either positive or negative.
D) Two vectors are equal only if they have the same magnitude and same direction.
E) In calculations, the vector components of a vector may be used in place of the vector itself.

Q10.

An airplane makes a gradual 90.0° turn while flying at a constant speed of 200 m/s. The process takes 20.0 seconds to complete. For this turn the **magnitude of the average acceleration of the plane is:**

A) 14.1 m/s²
B) zero
C) 40.4 m/s²
D) 20.8 m/s²
E) 10.3 m/s²

$$assume\ v_i = 200i\ and\ v_f = 200j$$

$$\vec{a}_{avg} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{200j - 200i}{20.0} = -10.0i + 10.0j$$

$$\left|\vec{a}_{avg}\right| = \sqrt{10.0^2 + 10.0^2} = 10.0\sqrt{2} = 14.1\,m/s^2$$

Q11.
Identical guns fire identical bullets horizontally at the same speed from the same height above level planes, one on the Earth and one on the Moon. Which of the following three statements is/are **TRUE? (gmoon = 1/6 gearth)**

I. The horizontal distance traveled by the bullet is greater on the Moon.
II. The flight time is less for the bullet on the Earth.
III. The velocities of the bullets at impact are the same.

A) I and II only
B) I only
C) I and III only
D) II and III only
E) I, II and III

Q12.

A ball is thrown horizontally from the top of a 20-m high hill as shown in Figure 3. It strikes the ground at an angle of 45°. With what speed was it thrown?

A) 20 m/s
B) 14 m/s
C) 28 m/s
D) 32 m/s
E) 40 m/s

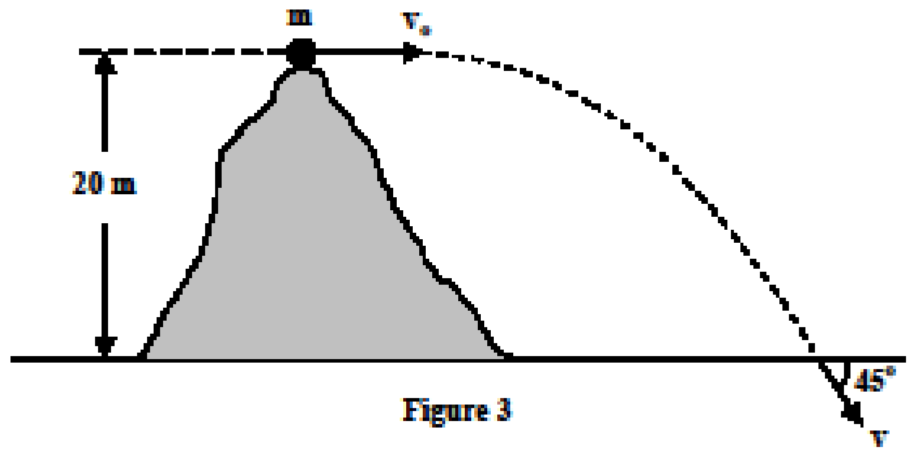


Figure 3

$$on\ the\ y-axis:$$

$$v_{yo}=0,\ y=-20\ m,\ a=-g \rightarrow v_y^{\ 2}=2ay=2\times 9.8\times 20=400$$

$$v_y=20\ m/s$$

$$on\ the\ ground\ (45^o\ angle)\ v_x=v_y=20\ m/s \rightarrow v_o=v_x=20\ m/s$$

Q13.
A toy racing car moves with constant speed around the circle as shown in Figure 4. When it is at point A its coordinates are x = 0, y = 3.0 m and its velocity is $(6.0\ \text{m/s})\hat{i}$. When it is at point B its velocity and acceleration are:

A) $+(6.0\ \text{m/s})\hat{j}$ and $+(12\ \text{m/s}^2)\hat{i}$, respectively
B) $-(6.0\ \text{m/s})\hat{j}$ and $+(12\ \text{m/s}^2)\hat{j}$, respectively
C) $-(6.0\ \text{m/s})\hat{j}$ and $-(12\ \text{m/s}^2)\hat{i}$, respectively
D) $+(6.0\ \text{m/s})\hat{j}$ and $+(12\ \text{m/s}^2)\hat{j}$, respectively
E) $+(6.0\ \text{m/s})\hat{j}$ and $+(2.0\ \text{m/s}^2)\hat{i}$, respectively

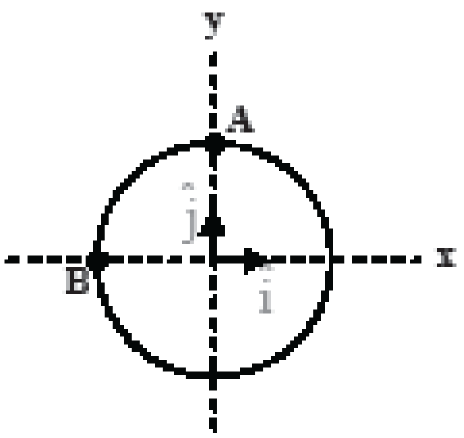
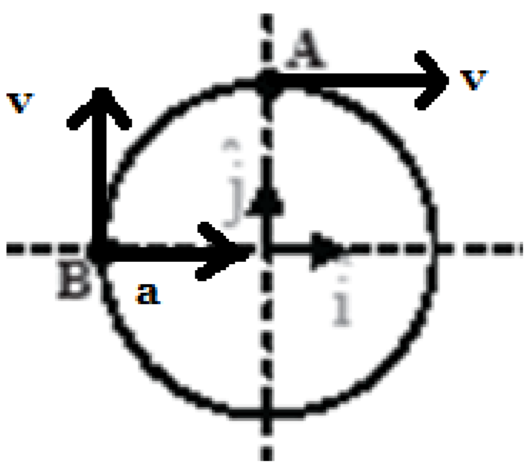


Figure 4



Q14.

A boy wishes to swim across a river to a point directly opposite as shown in Figure 5. He can swim at 2.0 m/s in still water. The river is flowing at 1.0 m/s toward the west. At what angle θ with respect to the line joining the starting and finishing points should he swim?

A) 30° east of north
B) 30° west of north
C) 60° west of north
D) 60° east of north
E) 0°

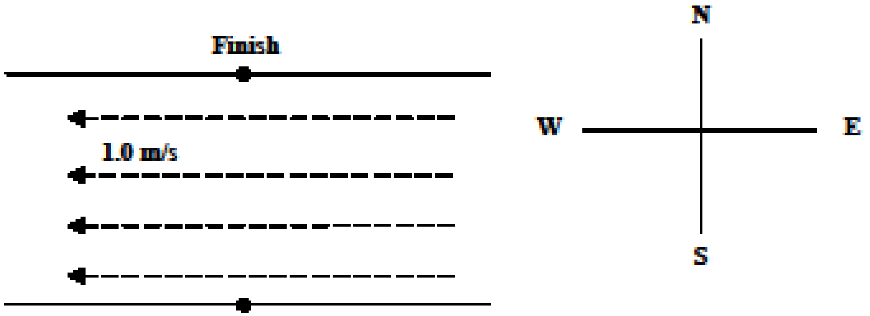
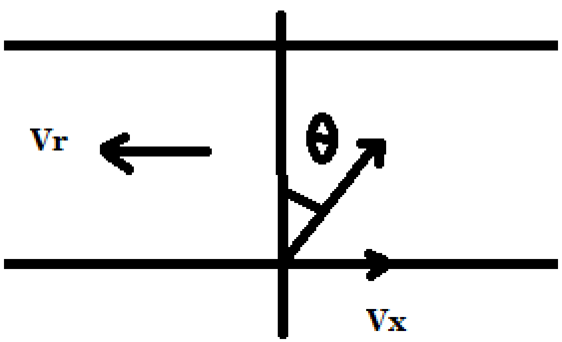


Figure 5



$$his\ v_x = |V_r|\ speed\ of\ river\ (to\ v\ in\ the\ x\ direction)$$

$$\therefore 2.0 \sin\theta = 1.0 \rightarrow \sin\theta = 0.5 \rightarrow \theta = 30^o\ east\ of\ north$$

Q15.

Two forces act on a 2 kg object to produce an acceleration $\vec{a} = (2\ \mathrm{m/s^2})\hat{i} + (-1\ \mathrm{m/s^2})\hat{j} + 0\,\hat{k}$.

One of the forces is $\vec{F}_1 = (2\ \mathrm{N})\hat{i} + (3\ \mathrm{N})\hat{j} + (-1\ \mathrm{N})\hat{k}$.

What is the other force?

A) $\vec{F}_2 = (2\ \mathrm{N})\hat{i} + (-5\ \mathrm{N})\hat{j} + (1\ \mathrm{N})\hat{k}$

B) $\vec{F}_2 = (-2\ \mathrm{N})\hat{i} + (5\ \mathrm{N})\hat{j} + (1\ \mathrm{N})\hat{k}$

C) $\vec{F}_2 = (2\ \mathrm{N})\hat{i} + (-5\ \mathrm{N})\hat{j} + (-1\ \mathrm{N})\hat{k}$

D) $\vec{F}_2 = (-2\ \mathrm{N})\hat{i} + (5\ \mathrm{N})\hat{j} + (-1\ \mathrm{N})\hat{k}$

E) None of the others

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F}_1 + \vec{F}_2 = 2\vec{a}$$

$$(2i + 3j - k) + \vec{F}_2 = 2(2i - j)$$

$$\begin{aligned} \vec{F}_2 &= 2(2i - j) - (2i + 3j - k) \\ &= 4i - 2j - 2i - 3j + k \\ &= 2i - 5j + k \end{aligned}$$

Q16.
Figure 6 shows the force versus time graph of a force F_x acting on a 5.0 kg object moving in the x-direction along a frictionless one-dimensional track. At t = 0 the object is moving in the negative direction of the x-axis with a speed of 2.0 m/s. What are the speed and direction of the object at t = 7.0 s?

A) 7.0 m/s in the positive x-direction
B) 7.0 m/s in the negative x-direction
C) 43 m/s in the positive x-direction
D) 43 m/s in the negative x-direction
E) zero

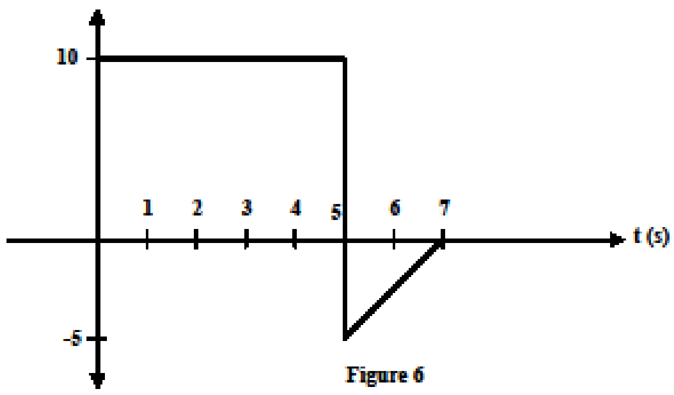


Figure 6

$$a = \frac{F}{m} \to a(0) = \frac{10}{5.0} = 2.0 m/s^2 \ \& \ a(5) = \frac{-5}{5.0} = 1\, m/s^2$$

$$\Delta v = \int a dt = area\ under\ the\ acc.\ curve = +2.0 \times 5 - \tfrac{1}{2}(7-5)\times 1 = 10 - 1 = 9$$

$$9 = \Delta v = v_f - v_i = v_f + 2 \to v_f = 9 - 2 = 7\ m/s$$

Q17.

Figure 7 shows two connected blocks that are pulled to the right on a rough table by a force T_2=120 N . If m_1=10.0 kg , m_2= 20.0 kg , and μ_k = 0.2, calculate the tensions T_1 .

A) 40.0 N
B) 20.0 N
C) 30.0 N
D) 50.0 N
E) 10.0 N

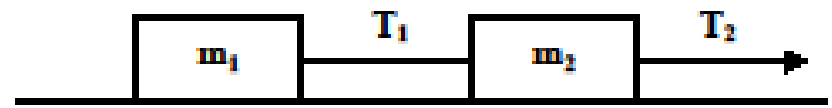
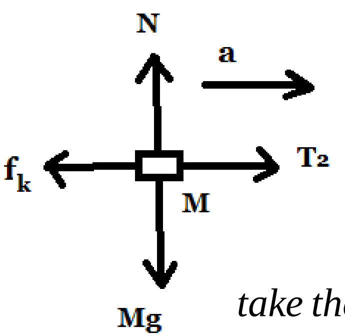


Figure 7

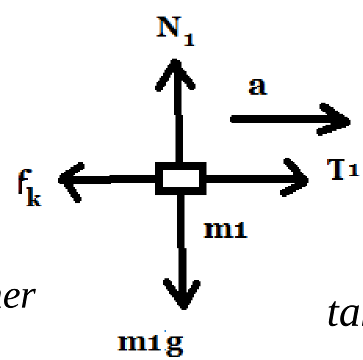


take the system to the two blocks together

$$M = m_1 + m_2 = 30.0kg$$

$$N = Mg, \quad f_k = \mu_k N = \mu_k Mg$$

$$T_2 - f_k = Ma \rightarrow \quad find \quad a$$



take the system to block one only

$$N = m_1 g, \quad f_k = \mu_k N_1 = \mu_k m_1 g$$

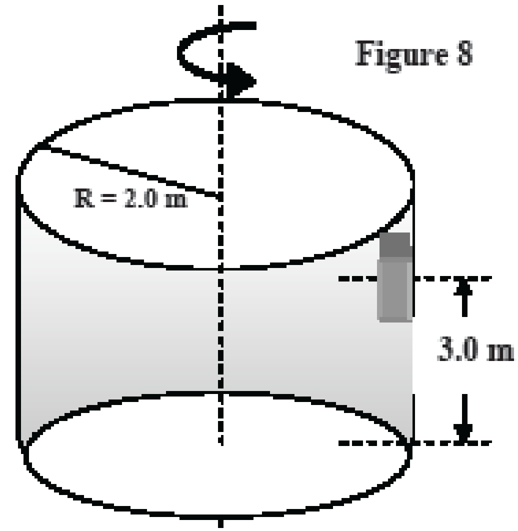
$$T_1 - f_k = m_1 a \rightarrow \quad find \quad T_1$$

Q18.

A block is 3.0 m up above the ground and is in contact with the inner side of a rotating cylinder of 2.0 m radius as shown in Figure 8. If the coefficient of static friction between the block and the cylinder is 0.50, what is the minimum speed the cylinder must have in order for the block not to fall down?

A) 6.3 m/s
B) 2.4 m/s
C) 3.4 m/s
D) 4.0 m/s
E) None of the others

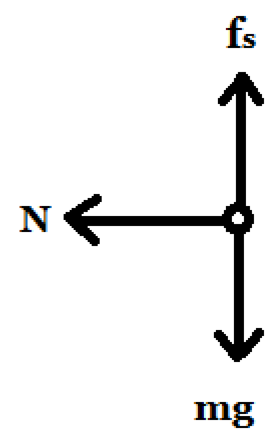
Figure 8



$$f_s = mg, \quad N = \frac{mv^2}{r}$$

$$slipping \;\; f_s = f_{s,\max} = \mu_s N = mg \rightarrow \mu_s\left(\frac{mv^2}{r}\right) = mg$$

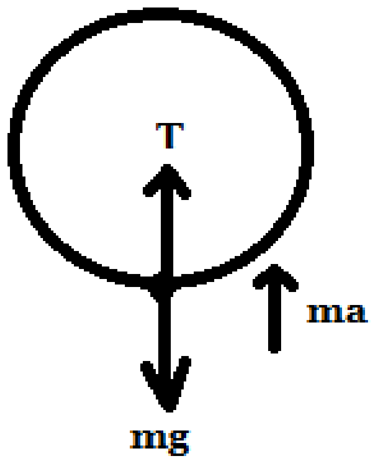
$$v^2 = \frac{gr}{\mu_s} \rightarrow v = \sqrt{\frac{gr}{\mu_s}} \qquad (r = 2.0m,\ \mu_s = 0.50)$$



Q19.

A 0.10 kg stone is tied to the end of a 1.0-m long rope. The stone is moved in a circle in the vertical plane. What is the tension in the rope when the stone is at its lowest position and has a speed of 5.0 m/s?

A) 3.5 N
B) 0.98 N
C) 0 N
D) 0.49 N
E) 1.5 N



$$a = \frac{v^2}{r} = \frac{5.0^2}{1.0} = 25\, m/s^2$$

$$F_{net} = T - mg = ma$$

$$T = m(g + a) = 0.10(9.8 + 25) = 3.5\, N$$

Q20.

An object is being accelerated in the absence of friction by a 100-N force. A second force of 100-N is then applied to the object in a direction opposite to the direction of motion. The object with these two forces acting on it will

A) Move at a constant velocity
B) Slow down
C) Move in a circle
D) Stop rapidly
E) Move backward