Selected Problems from Chapter 12

1) The diagrams in Fig.   show forces applied to a wheel of weight W=20 N. Which diagram is the wheel in equilibrium?

A1 diagram (3)
A2 diagram (2)
A3 diagram (1)
A4 diagram (4)
A5 none of them

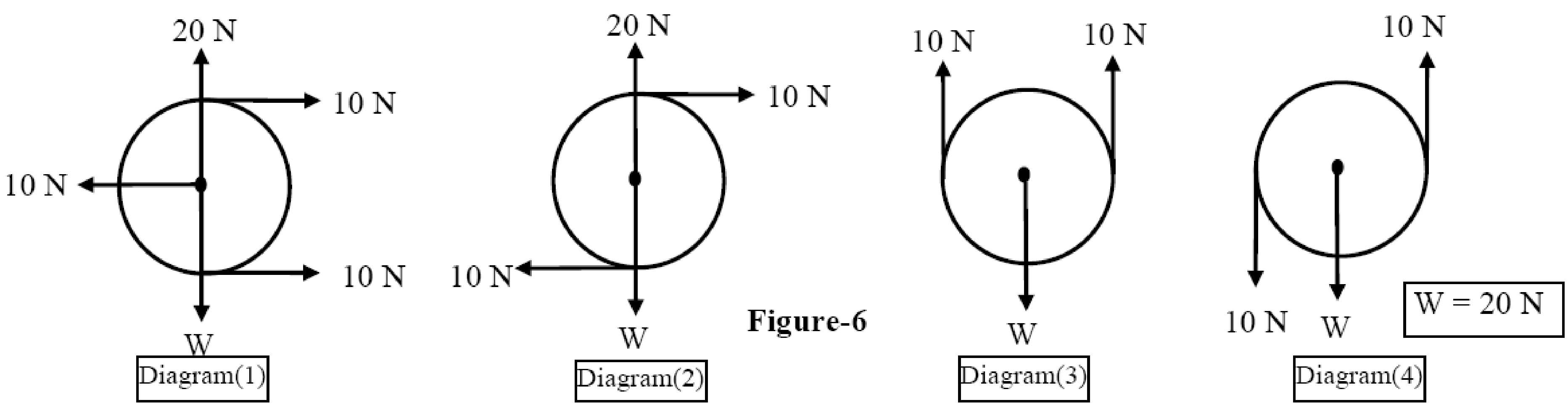
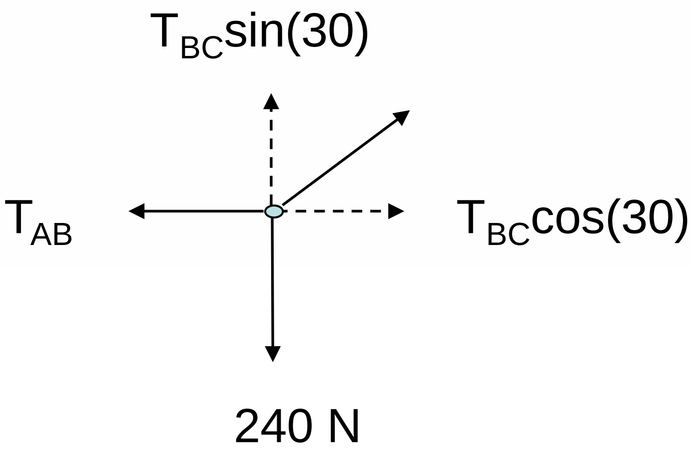


Figure-6

2) A 240 N weight is hung from two ropes AB and BC as shown in Fig . The tension in the horizontal rope AB is:

A1 416 N
A2 0 N
A3 656 N
A4 480 N
A5 176 N



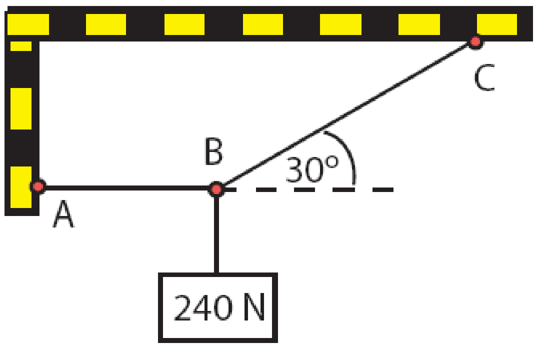


Figure 3

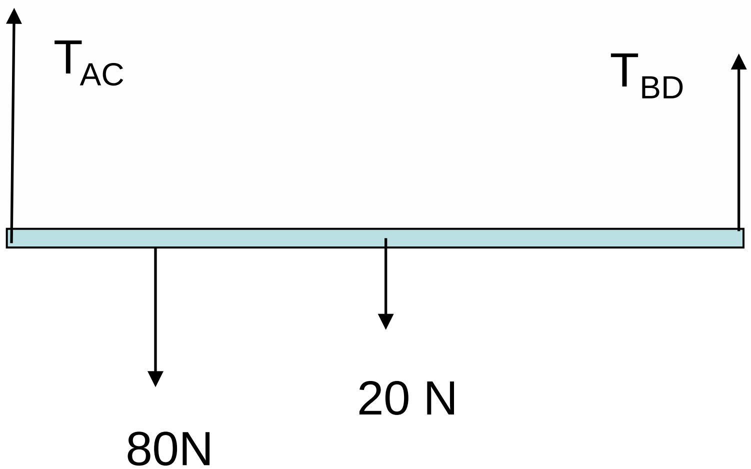
$$T_{BC}\sin(30) = W$$

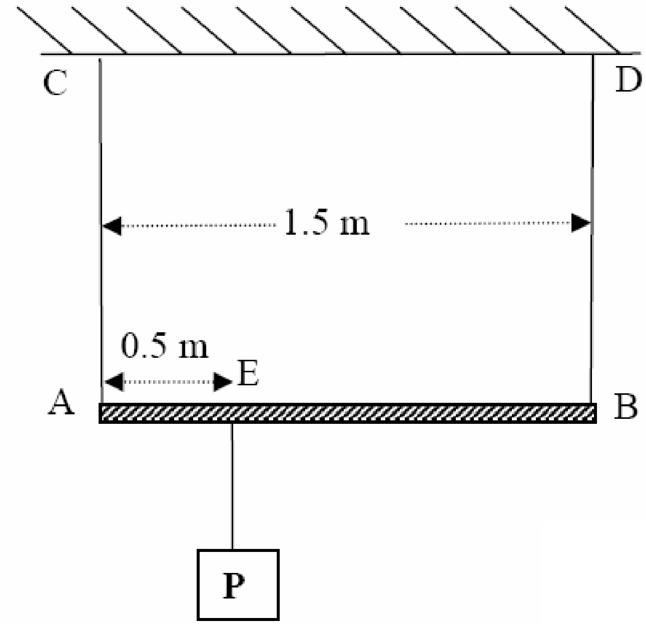
$$T_{BC} = \frac{240}{0.5} = 480\ N$$

$$T_{AB} = T_{BC}\cos(30) = 480 \times \frac{\sqrt{3}}{2} = 416\ N$$

3) A uniform rod AB is 1.5 m long and weighs 20 N. It is suspended by wires AC and BD as shown in Fig. 1. A block P weighing 80 N is attached at E, 0.50 m from A. The tension in the wire BD is:

A1 37 N
A2 63 N
A3 100 N
A4 28 N
A5 72 N





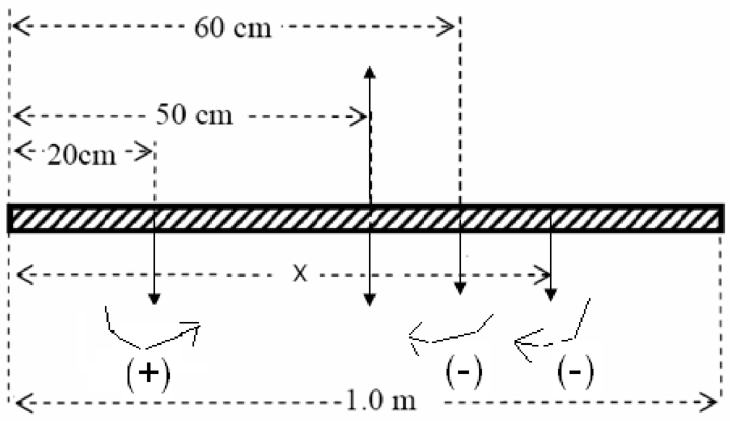
Take the torque around A:

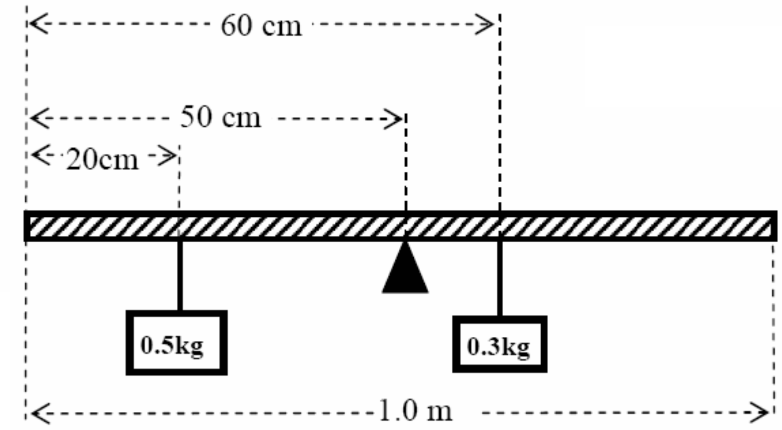
$$T_{BD} \times 1.5 = 80 \times 0.5 + 20 \times 0.75$$

Find T_{BD}.

4) A horizontal uniform meter stick is supported at the 50-cm mark. A mass of 0.50 kg is hanging from it at the 20-cm mark and a 0.30 kg mass is hanging from it at the 60-cm mark (see Fig.). Determine the position on the meter stick at which one would hang a third mass of 0.60 kg to keep the meter stick balanced.

A1 70 cm
A2 74 cm
A3 65 cm
A4 86 cm
A5 62 cm





Take the torque around the middle point.

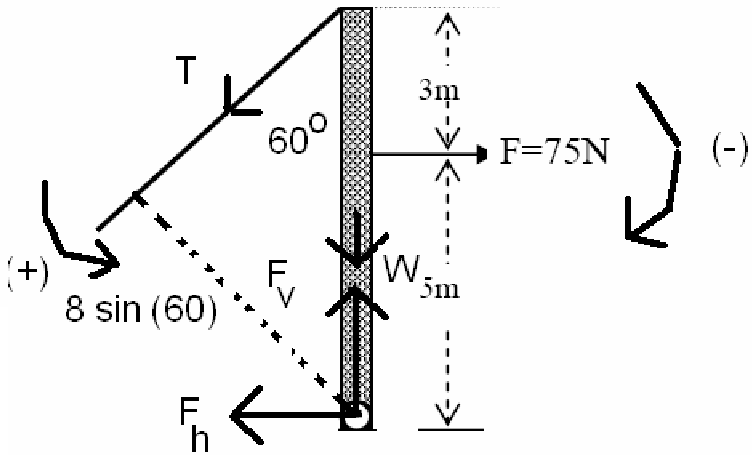
Total positive torque = total negative torque

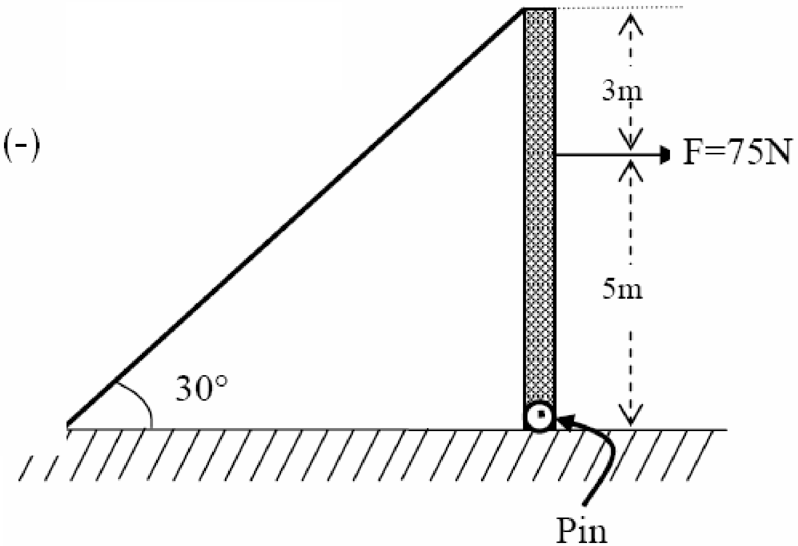
$$0.5\times 9.8\times 30 = 0.3\times 9.8\times 10 + 0.6\times 9.8\times (x - 50)$$

Find the value of x

5) A uniform 50-kg beam is held in a vertical position by a pin at its lower end and a cable at its upper end. A horizontal force F = 75 N acts as shown in the figure. What is the tension in the cable?

A1 54 N
A2 69 N
A3 47 N
A4 61 N
A5 75 N





Take the torque around the pin point

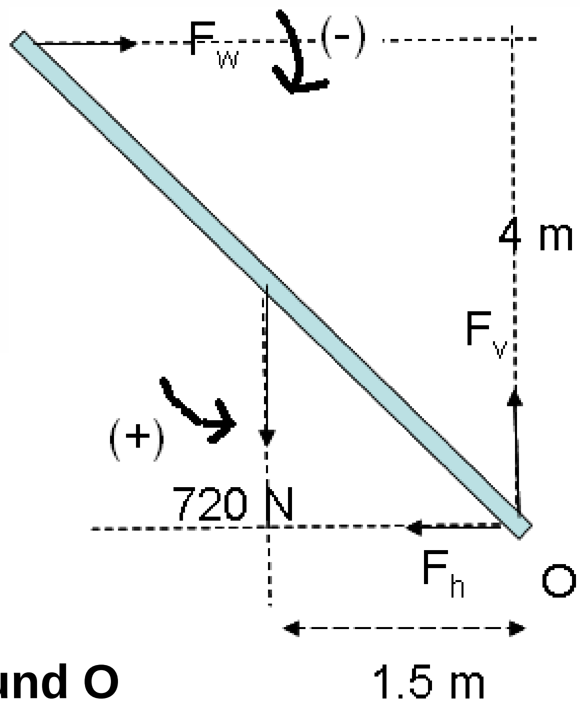
Total positive torque = total negative torque

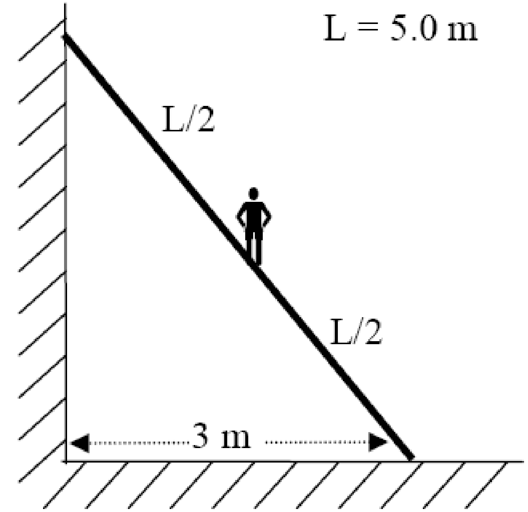
$$T \times 8 \sin(60) = 75 \times 5$$

$$find\ T$$

6) A man weighing 720 N stands halfway up a 5.0 m ladder of negligible weight. The base of the ladder is 3.0 m from the wall as shown in Fig. 2. Assume that the wall-ladder contact is frictionless. With what force does the wall push against the ladder?

A1	270 N
A2	350 N
A3	500 N
A4	600 N
A5	720 N





Take the torques around O

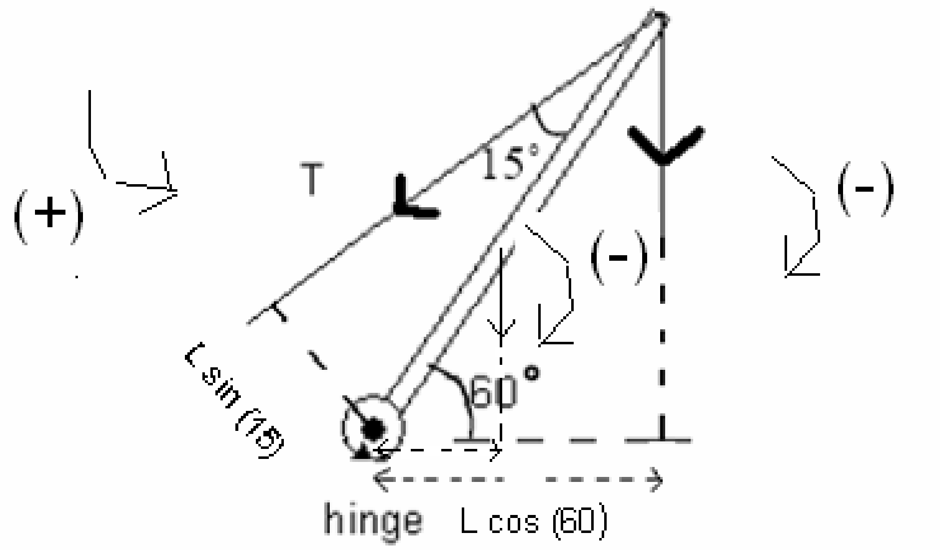
Total positive torque = total negative torque

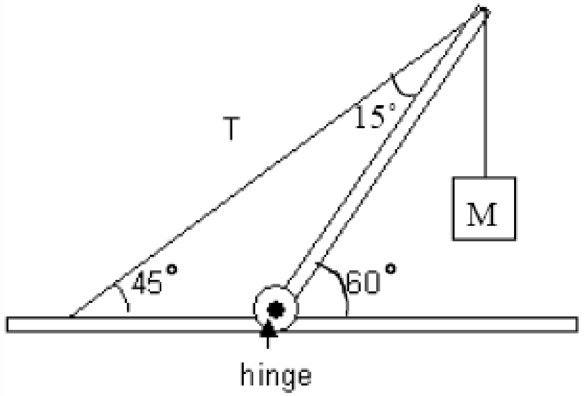
$$720 \times 1.5 = F_w \times 4$$

Find F_w

7) The system in Fig   is in equilibrium. A mass (M) of 5.0 kg hangs from the end of the uniform beam of mass = 10.0 kg. The tension in the cable is:

A1	190	N
A2	210	N
A3	69	N
A4	51	N
A5	98	N





Total positive torque = total negative torque

Take the torques around the hinge point

T x L sin (15) = 10.0 x g x L/2 cos (60) + 5.0 x g x L cos (60)

Find T.

8) A 500 kg mass is hung from the ceiling with a steel wire. The wire has a length = 45.0 cm, radius = 4.00 mm and has negligible mass. Calculate the change in the length of the wire. (Youngs modulus of steel E = 2.00* 10**11 N/m**2)

A1	0.22 mm
A2	0.15 mm
A3	0.75 mm
A4	0.50 mm
A5	0.05 mm

$$E = \frac{F / A}{\Delta L / L}$$

$$2.00 \times 10^{11} = \frac{500 \times 9.8 / \pi (4.00 \times 10^{-3} m)^2}{\Delta L / 4.5\, mm}$$

$$Find\ \Delta L \quad in\ mm.$$

9) A certain wire stretches 1.0 cm when a force F is applied to it. The same force is applied to a second wire of the same material but with twice the diameter and twice the length. The second wire stretches:

A1 0.50 cm
A2 0.25 cm
A3 1.0 cm
A4 2.0 cm
A5 4.0 cm

$$E = \frac{F/A_1}{\Delta L_1/L_1} = \frac{F/A_2}{\Delta L_2/L_2}$$

$$\Delta L_2 = \left(\frac{A_1}{A_2}\right)\times\left(\frac{L_2}{L_1}\right)\times \Delta L_1 = \left(\frac{D_1}{D_2}\right)^2 \times \left(\frac{L_2}{L_1}\right) = (1/2)^2 \times (2) \times 1.0 = 0.50\, cm$$

•6 A scaffold of mass 60 kg and length 5.0 m is supported in a horizontal position by a vertical cable at each end. A window washer of mass 80 kg stands at a point 1.5 m from one end. What is the tension in (a) the nearer cable and (b) the farther cable?

6. Let $\ell_1 = 1.5\,\text{m}$ and $\ell_2 = (5.0 - 1.5)\ \text{m} = 3.5\ \text{m}$. We denote tension in the cable closer to the window as F_1 and that in the other cable as F_2. The force of gravity on the scaffold itself (of magnitude $m_s g$) is at its midpoint, $\ell_3 = 2.5\,\text{m}$ from either end.

(a) Taking torques about the end of the plank farthest from the window washer, we find

$$F_1 = \frac{m_w g \ell_2 + m_s g \ell_3}{\ell_1 + \ell_2} = \frac{(80\,\text{kg})(9.8\,\text{m/s}^2)(3.5\,\text{m}) + (60\,\text{kg})(9.8\,\text{m/s}^2)(2.5\,\text{m})}{5.0\,\text{m}}$$
$$= 8.4 \times 10^2\ \text{N}.$$

(b) Equilibrium of forces leads to

$$F_1 + F_2 = m_s g + m_w g = (60\,\text{kg} + 80\,\text{kg})(9.8\,\text{m/s}^2) = 1.4 \times 10^3\ \text{N}$$

which (using our result from part (a)) yields $F_2 = 5.3 \times 10^2\,\text{N}$.

•10 In Fig. 12-26, a man is trying to get his car out of mud on the shoulder of a road. He ties one end of a rope tightly around the front bumper and the other end tightly around a utility pole 18 m away. He then pushes sideways on the rope at its midpoint with a force of 550 N, displacing the center of the rope 0.30 m from its previous position, and the car barely moves. What is the magnitude of the force on the car from the rope? (The rope stretches somewhat.)

Fig. 12-26 Problem 10.

10. The angle of each half of the rope, measured from the dashed line, is

$$\theta = \tan^{-1}\left(\frac{0.30\,\text{m}}{9.0\,\text{m}}\right) = 1.9°.$$

Analyzing forces at the "kink" (where $\vec{F}$ is exerted) we find

$$T = \frac{F}{2\sin\theta} = \frac{550\,\text{N}}{2\sin 1.9°} = 8.3\times10^{3}\ \text{N}.$$

••21 In Fig. 12-35, one end of a uniform beam of weight 222 N is hinged to a wall; the other end is supported by a wire that makes angles $\theta = 30.0°$ with both wall and beam. Find (a) the tension in the wire and the (b) horizontal and (c) vertical components of the force of the hinge on the beam.

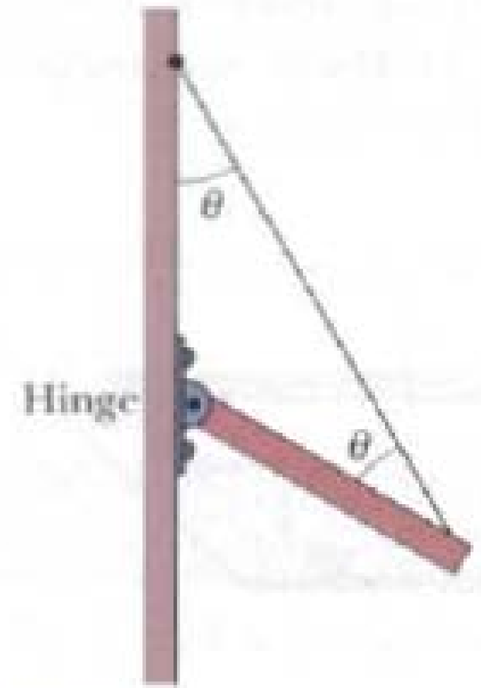


Fig. 12-35 Problem 21.

21. The beam is in equilibrium: the sum of the forces and the sum of the torques acting on it each vanish. As we see in the figure, the beam makes an angle of 60º with the vertical and the wire makes an angle of 30º with the vertical.

(a) We calculate the torques around the hinge. Their sum is $TL \sin 30º - W(L/2) \sin 60º = 0$. Here W is the force of gravity acting at the center of the beam, and T is the tension force of the wire. We solve for the tension:

$$T = \frac{W \sin 60^\circ}{2 \sin 30^\circ} = \frac{(222\,\text{N}) \sin 60^\circ}{2 \sin 30^\circ} = 192\,\text{N}.$$

(b) Let F_h be the horizontal component of the force exerted by the hinge and take it to be positive if the force is outward from the wall. Then, the vanishing of the horizontal component of the net force on the beam yields $F_h - T \sin 30º = 0$ or

$$F_h = T \sin 30^\circ = (192.3\,\text{N}) \sin 30^\circ = 96.1\,\text{N}.$$

(c) Let F_v be the vertical component of the force exerted by the hinge and take it to be positive if it is upward. Then, the vanishing of the vertical component of the net force on the beam yields $F_v + T \cos 30º - W = 0$ or

$$F_v = W - T \cos 30^\circ = 222\,\text{N} - (192.3\,\text{N}) \cos 30^\circ = 55.5\,\text{N}.$$

••27 In Fig. 12-40, suppose the length L of the uniform bar is 3.00 m and its weight is 200 N. Also, let the block's weight W = 300 N and the angle θ = 30.0°. The wire can withstand a maximum tension of 500 N. (a) What is the maximum possible distance x before the wire breaks? With the block placed at this maximum x, what are the (b) horizontal and (c) vertical components of the force on the bar from the hinge at A?

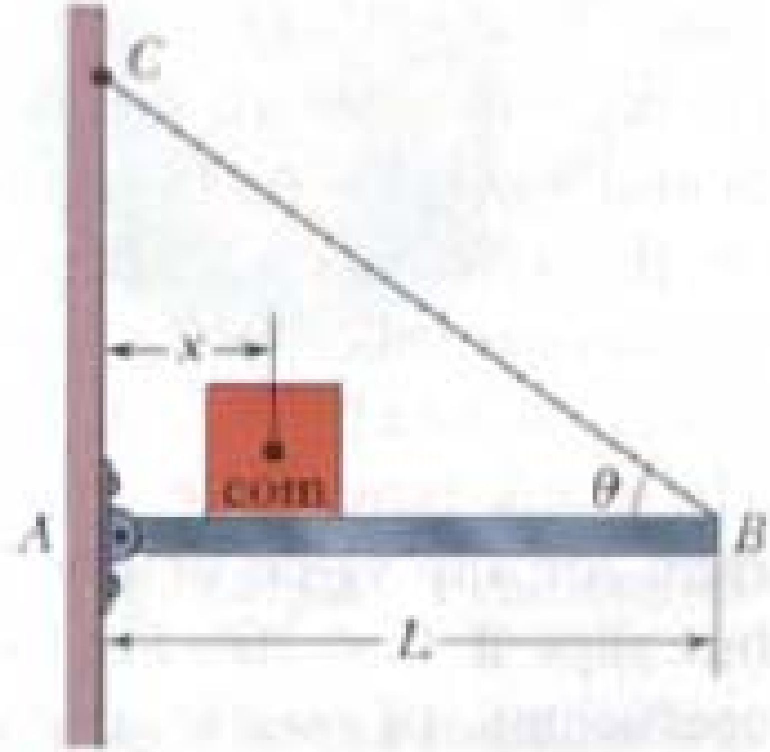


Fig. 12-40 Problems 27 and 28.

27. (a) Computing torques about point *A*, we find

$$T_{\max} L \sin\theta = W x_{\max} + W_b \left(\frac{L}{2} \right).$$

We solve for the maximum distance:

$$x_{\max} = \left(\frac{T_{\max} \sin\theta - W_b / 2}{W} \right) L = \left(\frac{500 \sin 30.0^\circ - 200/2}{300} \right) (3.00) = 1.50\,\text{m}.$$

(b) Equilibrium of horizontal forces gives $F_x = T_{\max} \cos\theta = 433\,\text{N}$.

(c) And equilibrium of vertical forces gives $F_y = W + W_b - T_{\max} \sin\theta = 250\,\text{N}$.

••31 In Fig. 12-42, a uniform plank, with a length L of 6.10 m and a weight of 445 N, rests on the ground and against a frictionless roller at the top of a wall of height $h = 3.05$ m. The plank remains in equilibrium for any value of $\theta \geq 70°$ but slips if $\theta < 70°$. Find the coefficient of static friction between the plank and the ground.

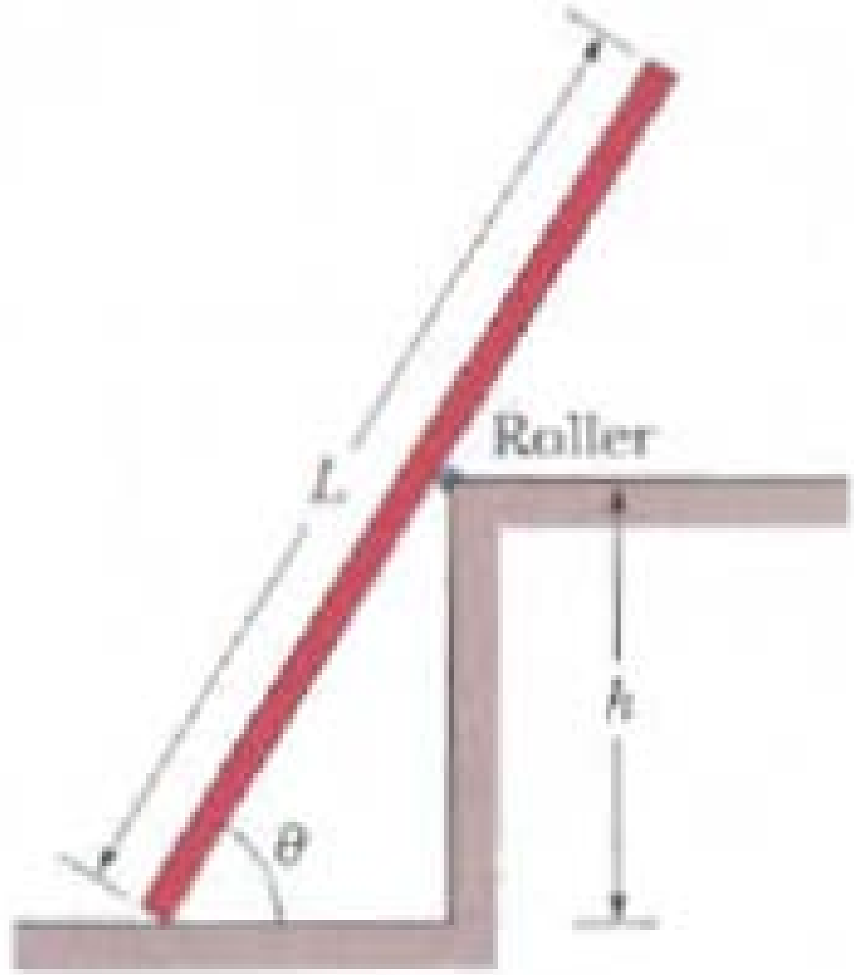
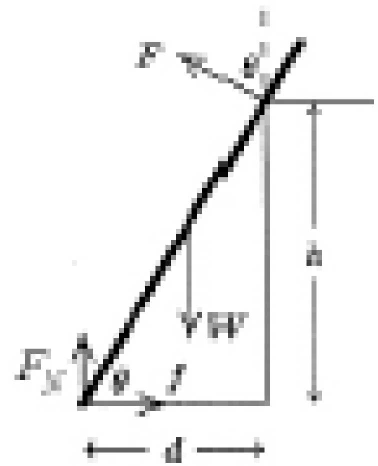


Fig. 12-42 Problem 31.

31. The diagram below shows the forces acting on the plank. Since the roller is frictionless the force it exerts is normal to the plank and makes the angle θ with the vertical. Its magnitude is designated F. W is the force of gravity; this force acts at the center of the plank, a distance $L/2$ from the point where the plank touches the floor. F_N is the normal force of the floor and f is the force of friction. The distance from the foot of the plank to the wall is denoted by d. This quantity is not given directly but it can be computed using $d = h/\tan\theta$.



The equations of equilibrium are:

horizontal force components	$F\sin\theta - f = 0$
vertical force components	$F\cos\theta - W + F_N = 0$
torques	$F_N d - fh - W\left(d - \frac{L}{2}\cos\theta\right) = 0.$

The point of contact between the plank and the roller was used as the origin for writing the torque equation.

When $\theta = 70°$ the plank just begins to slip and $f = \mu_s F_N$, where μ_s is the coefficient of static friction. We want to use the equations of equilibrium to compute F_N and f for $\theta = 70°$, then use $\mu_s = f/F_N$ to compute the coefficient of friction.

The second equation gives $F = (W - F_N)/\cos\theta$ and this is substituted into the first to obtain

$$f = (W - F_N)\sin\theta/\cos\theta = (W - F_N)\tan\theta.$$

This is substituted into the third equation and the result is solved for F_N:

$$F_N = \frac{d - (L/2)\cos\theta + h\tan\theta}{d + h\tan\theta}W = \frac{h(1+\tan^2\theta) - (L/2)\sin\theta}{h(1+\tan^2\theta)}W,$$

where we have use $d = h/\tan\theta$ and multiplied both numerator and denominator by $\tan\theta$. We use the trigonometric identity $1+\tan^2\theta = 1/\cos^2\theta$ and multiply both numerator and denominator by $\cos^2\theta$ to obtain

$$F_N = W\left(1 - \frac{L}{2h}\cos^2\theta\sin\theta\right).$$

Now we use this expression for F_N in $f = (W - F_N)\tan\theta$ to find the friction:

$$f = \frac{WL}{2h}\sin^2\theta\cos\theta.$$

We substitute these expressions for f and F_N into $\mu_s = f/F_N$ and obtain

$$\mu_s = \frac{L\sin^2\theta\cos\theta}{2h - L\sin\theta\cos^2\theta}.$$

Evaluating this expression for $\theta = 70°$, we obtain

$$\mu_s = \frac{(6.1\,\text{m})\sin^2 70°\cos 70°}{2(3.05\,\text{m}) - (6.1\,\text{m})\sin 70°\cos^2 70°} = 0.34.$$

••39 In Fig. 12-48, a 103 kg uniform log hangs by two steel wires, A and B, both of radius 1.20 mm. Initially, wire A was 2.50 m long and 2.00 mm shorter than wire B. The log is now horizontal. What are the magnitudes of the forces on it from (a) wire A and (b) wire B? (c) What is the ratio d_A/d_B?

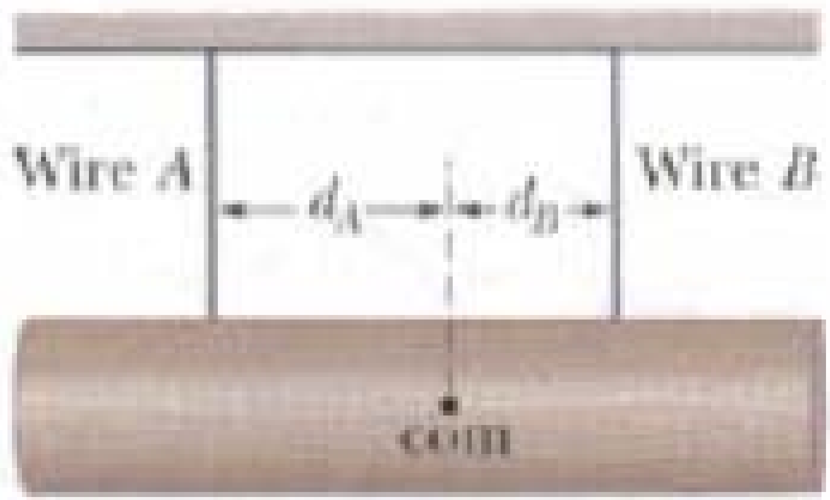


Fig. 12-48 Problem 39.

39. (a) Let F_A and F_B be the forces exerted by the wires on the log and let m be the mass of the log. Since the log is in equilibrium $F_A + F_B - mg = 0$. Information given about the stretching of the wires allows us to find a relationship between F_A and F_B. If wire A originally had a length L_A and stretches by ΔL_A, then $\Delta L_A = F_A L_A / AE$, where A is the cross–sectional area of the wire and E is Young's modulus for steel (200×10^9 N/m^2). Similarly, $\Delta L_B = F_B L_B / AE$. If ℓ is the amount by which B was originally longer than A then, since they have the same length after the log is attached, $\Delta L_A = \Delta L_B + \ell$. This means

$$\frac{F_A L_A}{AE} = \frac{F_B L_B}{AE} + \ell .$$

We solve for F_B:

$$F_B = \frac{F_A L_A}{L_B} - \frac{AE\ell}{L_B}.$$

We substitute into $F_A + F_B - mg = 0$ and obtain

$$F_A = \frac{mgL_B + AE\ell}{L_A + L_B}.$$

The cross–sectional area of a wire is $A = \pi r^2 = \pi\left(1.20\times 10^{-3}\,\text{m}\right)^2 = 4.52\times10^{-6}\,\text{m}^2$. Both L_A and L_B may be taken to be 2.50 m without loss of significance. Thus

$$F_A = \frac{(103\,\text{kg})(9.8\,\text{m/s}^2)(2.50\,\text{m})+(4.52\times10^{-6}\,\text{m}^2)(200\times10^9\,\text{N/m}^2)(2.0\times10^{-3}\,\text{m})}{2.50\,\text{m}+2.50\,\text{m}}$$

$$= 866\,\text{N}.$$

(b) From the condition $F_A + F_B - mg = 0$, we obtain

$$F_B = mg - F_A = (103\,\text{kg})(9.8\,\text{m/s}^2) - 866\,\text{N} = 143\,\text{N}.$$

(c) The net torque must also vanish. We place the origin on the surface of the log at a point directly above the center of mass. The force of gravity does not exert a torque about this point. Then, the torque equation becomes $F_A d_A - F_B d_B = 0$, which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{143\,\text{N}}{866\,\text{N}} = 0.165.$$