King Fahd University of Petroluem and Minerals
Department of Physics

Particle Physics (PHYS 441)
(041)

Solutions of HW # 2

Assignment #2

Solution

Q.1: "Problem # 4.11 in Griffiths"

In the decay

$$\Delta^{++} \longrightarrow P + \pi^{+},$$

the initial total angular momentum is $J^P = \frac{3}{2}^+$.

Since the total angular momentum is conserved, $J_F = \frac{3}{2}$

the final angular momentum $= \frac{3}{2}$

$$\left.\begin{array}{l} \text{The spin of } \pi^+ \text{ is } 0 \\ \text{ " " " } P \text{ is } \frac{1}{2} \end{array}\right\} \text{total spin} = \frac{1}{2}$$

Now

$$\vec{J} = \vec{L} + \vec{S}$$

possible values of $\vec{L}$ are 1 and 2

Since $1 + \frac{1}{2} = \frac{3}{2}$ & $\frac{1}{2}$

$2 + \frac{1}{2} = \frac{3}{2}$ & $\frac{5}{2}$

Continue Q.1:

Parity

The parity of Δ^{++} is $+1$

since $\Delta^{++} \longrightarrow p + \pi^{+}$ proceeds by strong interaction, parity is conserved

for $p + \pi^{+}$, the parity is $\eta_p^P \eta_{\pi^+}^P (-1)^{\ell}$

$$\Rightarrow \quad 1 = (+1)(-1)(-1)^{\ell}$$

$$\Rightarrow \quad 1 = (-1)^{\ell+1}$$

$\Rightarrow$ ℓ should be odd

$$\Rightarrow \boxed{\ell = 1}$$

Q.2: "Problem # 4.37 in Griffiths"

a) The decay of η to 2π is forbidden for both strong and electromagnetic interactions

$$\eta \longrightarrow 2\pi$$

For η: $J^{PC} = 0^{-+}$

For 2π: if $\ell = 0$, $J^{PC} = 0^{++}$

Recall $\eta^P_\pi = -1 \rightarrow \eta^P_{\pi\pi} = +1$

Thus we see in the above decay parity is violated

Thus strong and electromagnetic reactions are forbidden.

b) Consider $\eta \longrightarrow 3\pi$

G-parity of $\eta = (-1)^I \times \eta^c_\eta = (-1)^0 \times 1 = +1$

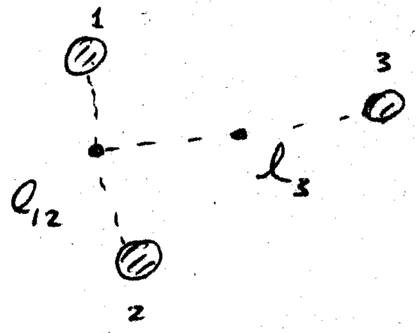
" of $3\pi = (-1)^3 = -1$

$\Rightarrow$ G-parity is violated

$\Rightarrow$ $\eta \rightarrow 3\pi$ cannot go by strong interaction but can go by electromagnetic. "Recall G is not conserved in electromagnetic"

Q.3 :

The three-pion case is a three-body problem. One should consider the orbital momentum of two particles with respect to their mutual center of mass and then the orbital angular momentum of the third particle about the centre of mass of the pair



For kaons decaying into three pions, for example, the total spin is zero. Therefore the total angular momentum

$$\vec{L} = \vec{l}_{12} + \vec{l}_3 = 0$$

$$\Rightarrow \quad l_{12} = l_3 = 0$$

Now the parity $\eta^P_{3\pi}$ of the three pions is given by

$$\eta^P_{3\pi} = \eta^P_{\pi^+} \eta^P_{\pi^-} \eta^P_{\pi^0} (-1)^{l_{12}} (-1)^{l_3} = -1 \quad (\text{recall } \eta^P_\pi = -1) \text{; } L = 0$$

Now

$$\eta^C_{3\pi} = \eta^C_{\pi^0} \eta^C_{\pi^+\pi^-}$$

we have shown in class using Pauli exclusion principle that

$$\left.\begin{array}{l} \eta^C_{\pi^+\pi^-} = (-1)^{l_{12}} \\ \text{and } \eta^C_{\pi^0} = +1 \end{array}\right\} \Rightarrow \eta^C_{3\pi} = (-1)^{l_{12}}$$

$$\Rightarrow \eta^{CP}_{3\pi} = (-1)(-1)^{l_{12}} = (-1)^{l_{12}+1} = -1 \quad \text{for } l_{12} = 0$$

Q.4:

Consider a state which is described by $\psi(\phi)$. Suppose that this state is rotated around the z-axis by an angle $\delta\phi$. Then the transformed state ψ' is

$$\psi'(\phi) = \psi(\phi+\delta\phi) = \psi(\phi) + \delta\phi \frac{\partial\psi}{\partial\phi}$$

but

$$L_z = i\frac{\partial}{\partial\phi} \qquad \text{" } \hbar = 1 \text{ "}$$

$$\Rightarrow \psi'(\phi) = \psi(\phi) - i L_z \psi(\phi)\, \delta\phi$$

$$= (1 - i\,\delta\phi\, L_z)\, \psi(\phi)$$

For finite rotation, the state is rotated n times

$$\Rightarrow \psi'(\phi) = e^{-i L_z \Delta\phi}\, \psi(\phi)$$

If the system is invariant under rotation

$$\Rightarrow [H, e^{-i L_z \Delta\phi}] = 0$$

$$\Rightarrow [H, L_z] = 0$$

$\Rightarrow$ L_z is conserved

" The above argument can be generalized for an angle around $\hat{n}$ "

Q.5

This problem is exactly similar to the example that we have solved in class (section 4.5, p 119 in Griffiths). Notice that $\Sigma^{0,\pm}$ & $K^{0,+}$ have the following isospin states:

$$K^0 \equiv |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \qquad ; \qquad |K^+\rangle = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

while

$$\Sigma^+ = |1\,1\rangle \quad ; \quad \Sigma^0 \equiv |1\,0\rangle \quad ; \quad \Sigma^- \equiv |1\,-1\rangle$$

Then

$$\Sigma^0 K^0 = |1\,0\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \quad \text{is similar to } \pi^0 n$$

$$\Sigma^- K^+ = |1\,-1\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle \quad '' \quad '' \quad \pi^- p$$

$$\Sigma^+ K^+ = |1\,1\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle \quad '' \quad '' \quad \pi^+ p$$

Then the ratio of the total cross-section of $\pi^- p$ to $\pi^+ p$ is the same as in the example.

Q.6 :

The isospin of π is $I=1$

For two π $\quad I = 2, 1, 0$

$I=2$:

Let us denote
$$\pi^+ = |I=1;\ I_3=+1\rangle$$
$$\pi^0 \equiv |I=1;\ I_3=0\rangle$$
$$\pi^- \equiv |I=1;\ I_3=-1\rangle$$

possible $I=2$ states:

$$|2\,2\rangle = \pi^+\pi^+$$
$$|2\,1\rangle = \frac{1}{\sqrt{2}}\left(\pi^+\pi^0 + \pi^0\pi^+\right)$$
$$|2\,0\rangle = \frac{1}{\sqrt{6}}\left(\pi^+\pi^- + 2\,\pi^0\pi^0 + \pi^-\pi^+\right)$$
$$|2\,{-1}\rangle = \frac{1}{\sqrt{2}}\left(\pi^0\pi^- + \pi^-\pi^0\right)$$
$$|2\,{-2}\rangle = \pi^-\pi^-$$

$I=1$:

$$|1\,1\rangle = \frac{1}{\sqrt{2}}\left(\pi^+\pi^0 - \pi^0\pi^+\right)$$
$$|1\,0\rangle = \frac{1}{\sqrt{2}}\left(\pi^+\pi^- - \pi^-\pi^+\right)$$
$$|1\,{-1}\rangle = \frac{1}{\sqrt{2}}\left(\pi^0\pi^- - \pi^-\pi^0\right)$$

$I=0$

$$|0\,0\rangle = \frac{1}{\sqrt{3}}\left(\pi^+\pi^- - \pi^0\pi^0 + \pi^-\pi^+\right)$$

Q.7 :

Consider a meson X that decays strongly to $\pi^0\pi^0$

$$X \longrightarrow \pi^0\pi^0$$

Since the decay goes by strong interaction, I is conserved. For $\pi^0\pi^0$ "the final state",

$$\pi^0\pi^0 = |10\rangle|10\rangle = \sqrt{\tfrac{2}{3}}\,|20\rangle - \tfrac{1}{\sqrt{3}}|00\rangle$$

i.e $I = 0, 2$

But X is a meson $\Rightarrow$ I should be 0

"no meson has $I=2$"

G-parity :

$$X \longrightarrow \pi^0\pi^0$$

For X : $G = (-1)^I \eta^c_X = (-1)^0 \eta^c_X = \eta^c_X$

For $\pi^0\pi^0$ $G = (-1)^2 = +1$

since G is conserved $\Rightarrow \eta^c_X = +1$

C-Parity : "charge conjugation"

For X ; $\eta^c_X = +1$

For $\pi^0\pi^0$ $\eta^c_{\pi^0\pi^0} = (+1)\times(+1) = +1$

$\Rightarrow \eta^c = +1$

Continue Q.7

Parity

$$X \longrightarrow \pi^0 \pi^0$$

Parity is conserved $= \eta^P_{\pi^0}\,\eta^P_{\pi^0}\,(-1)^l = (-1)^l$

where l is the angular momentum of $\pi^0\pi^0$.

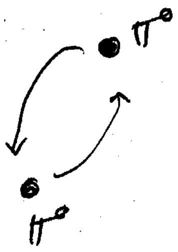
but $\vec{J} = \vec{L} + \vec{S}$ and $\vec{S} = 0$ for $\pi^0\pi^0$

$\Rightarrow l = J$

$\Rightarrow$ parity $= (-1)^J$

But under space inversion of $\pi^0\pi^0$, nothing is changed $\Rightarrow \eta^P = +1$

$\Rightarrow J$ is even



Thus the allowed values are

$$0^{++},\ 2^{++},\ 4^{++},\ \ldots$$

Q.8 :

consider the decay $\pi^+ \to \ell^+ + \nu_\ell$ in the rest frame of π^+.

a) As explained in class, the charged lepton has same helicity as neutrino in this reaction. the ν_ℓ is left-handed, thus ℓ^+ is left-handed

b) Massless leptons (or relativistic leptons) have left-handed helicity, while massless anti-leptons (or relativistic anti-leptons) have right-handed helicity. However non-relativistic leptons can have both helicities.

We would expect relativistic anti-leptons to be right-handed. left-handed anti-leptons would be suppressed by a factor of the order $1 - \frac{v}{c}$ or ($1-v$ for $c=1$)

"Recall $E^2 = \frac{m^2c^4}{1-\frac{v^2}{c^2}} \Rightarrow 1 - \frac{v}{c} \approx \frac{m^2c^4}{2E^2}$."

~~the~~ Positrons are light and in this reaction they recoil relativistically while anti-muons recoil non-relativistically implying that positrons are suppressed by a factor of

$$\frac{m^2}{2E^2} \sim \left(\frac{m_e}{m_\pi}\right)^2 \sim 1 \times 10^{-5} \Rightarrow \frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} \sim 10^{-5}$$