King Fahd University of Petroluem and Minerals
Department of Physics

Particle Physics (PHYS 441)
(041)

Solutions of HW # 1

Q.1:

We have shown in class that from Uncertainty principle one can show that

$$R \sim \frac{\hbar}{mc} = \frac{\hbar c}{mc^2}$$

for 1fm (the size of the nucleus) $\Rightarrow$ $M \sim 200$ MeV

Q.2:

$$1\,gm = 10^{-3}\,kg \times \frac{c^2}{c^2} = 10^{-3} \times (3\times10^8)^2 \,\frac{m^2}{s^2}\cdot\frac{kg}{c^2}$$

$$= 9\times10^{13}\,\frac{J}{c^2} = 9\times10^{13}\,\frac{J}{c^2} \times \frac{1\,eV}{1.6\times10^{-19}\,J}$$

$$= 5.625\times10^{32}\,\frac{eV}{c^2} = 5.625\times10^{23}\,\frac{GeV}{c^2}$$

In natural units, $[M]$ = Energy ; $c=1$

$$\Rightarrow \quad 1\,gm = 5.625\times10^{23}\ GeV$$

a) $1\,N = 1\,kg\,\frac{m}{s^2} = \frac{1\,J}{m}$

But $1\,eV = 1.6\times10^{-19}\,J$

and in natural units $1\,fm = 10^{-15}\,m = 5\,GeV^{-1}$

$$\Rightarrow \quad 1\,m = \frac{5\,GeV^{-1}}{10^{-15}}$$

Thus

$$1\,N = \frac{eV}{1.6\times10^{-19}} \times \frac{10^{-15}}{5\,GeV^{-1}} = \frac{10^{-9}\,GeV}{1.6\times10^{-4}} \times \frac{GeV}{5}$$

$$= 1.25\times10^{-6}\ GeV^2$$

b) $G = 6.67 \times 10^{-11} \ \frac{N \cdot m^2}{kg^2}$

In natural units, $1\,N = 1.25 \times 10^{-6}\ GeV^2$

$1\,m = 5 \times 10^{15}\ GeV^{-1}$

$1\,kg = 5.625 \times 10^{26}\ GeV$

$$\Rightarrow G = \frac{6.67 \times 10^{-11} \times 1.25 \times 10^{-6}\ GeV^2 \times (5 \times 10^{15}\ GeV^{-1})^2}{(5.625 \times 10^{26}\ GeV)^2}$$

$$\Rightarrow G = 6.6 \times 10^{-39}\ GeV^{-2}$$

Q3:

According to the uncertainty principle, if energy is violated, this violation will last for a short time which is given by

$$\Delta t \, \Delta E \sim \hbar$$

But $\Delta E \sim mc^2$; $\Delta t \simeq \frac{R}{c}$ "assuming the particle moves with $v \sim c$"

Then the range of the force R

$$R = \frac{\hbar}{mc}$$

m here is the mass of the mediator

a) For electromagnetic force, $m_\gamma = 0$

$\Rightarrow R_{em} = \infty$ as expected

b) For weak force $m \sim 80$ GeV

$$\Rightarrow R_W = \frac{\hbar c}{mc^2} = \frac{0.197 \text{ GeV. fm}}{80 \text{ GeV}}$$

$R_W \simeq 2.5 \times 10^{-3}$ fm. "very short range"

Q.4:

All forces can be estimated by

$$F_f \sim \frac{\alpha_f}{r^2}$$

where α_f is the dimensionless coupling constant of the force

Now

$$F_g = \frac{\alpha_g}{r^2} = \frac{G_N m_p^2}{r^2} = \frac{6.6 \times 10^{-39} \times 1 \text{ GeV}^{-2} \times \text{GeV}^2}{(5 \text{ GeV}^{-1})^2}$$

$$= 2.64 \times 10^{-40} \text{ GeV}^2$$

$$F_{em} = \frac{\alpha}{r^2} = \frac{1}{137} \times \frac{1}{25 \text{ GeV}^{-2}} = 2.92 \times 10^{-4} \text{ GeV}^2$$

$$F_W = \frac{\alpha_W}{r^2} \simeq \frac{G_F m_p^2}{r^2} = \frac{10^{-5}\,GeV^{-2} \times 1\,GeV^2}{25\,GeV^{-2}}$$

$$= 4 \times 10^{-7}\ GeV^2$$

$$F_s = \frac{\alpha_s}{r^2} \simeq \frac{1}{25\,GeV^{-2}} = 4 \times 10^{-2}\,GeV^2$$

Thus

$$F_s : F_{em} : F_W : F_g$$

$$4\times10^{-2} : 3\times10^{-4} : 4\times10^{-7} : 3\times10^{-40}$$

$$\simeq 1 : 10^{-2} : 10^{-5} : 10^{-38}$$

Q.5:

The weak interaction becomes significant when

$\alpha_W \sim 1$, But

$$\alpha_W \sim G_F M_x^2$$

So at energy scale $M_x = \frac{1}{\sqrt{G_F}} \Rightarrow \alpha_W \sim 1$

i.e

$$M_x = \frac{1}{\sqrt{10^{-5}\,GeV^{-2}}} = 300\ GeV$$

Q.6:

Recall that

$$\tau = \frac{1}{\Gamma_{tot}}$$

Also

$$B = \frac{\Gamma_i}{\Gamma_{tot}}$$; Γ_i is the decay rate for a specific reaction 'decay'.

$$\Rightarrow \quad \tau = \frac{B}{\Gamma_i}$$

Thus

$$\frac{\tau_\tau}{\tau_\mu} = \frac{B(\tau^- \longrightarrow e^- \bar{\nu}_e \nu_\tau)}{\Gamma(\tau^- \longrightarrow e^- \bar{\nu}_e \nu_\tau)} \times \frac{\Gamma(\mu^- \longrightarrow e^- \bar{\nu}_e \nu_\mu)}{B(\mu^- \longrightarrow e^- \bar{\nu}_e \nu_\mu)}$$

But

$$\Gamma(\ell \rightarrow e^- \bar{\nu}_e \nu_\ell) \simeq K G_F^2 m_\ell^5$$; K is some constant

$$\Rightarrow \quad \frac{\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \simeq \left(\frac{m_\mu}{m_\tau}\right)^5$$

Thus

$$\frac{\tau_\tau}{\tau_\mu} = \frac{0.177}{1} \left(\frac{105 \text{ MeV}}{1784 \text{ MeV}}\right)^5 \simeq 1.25 \times 10^{-7}$$

From particle data book: $\tau_{\tau^-} = 2.9 \times 10^{-13}$ s

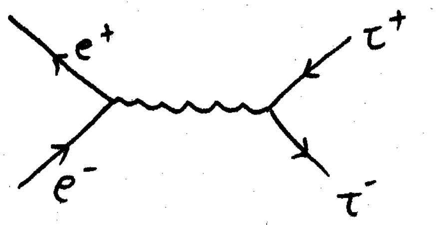
$\tau_{\mu^-} = 2.2 \times 10^{-6}$ s

$$\Rightarrow \quad \frac{\tau_\tau}{\tau_\mu} = 1.32 \times 10^{-7}$$ in good agreement with our prediction.

Q.7:

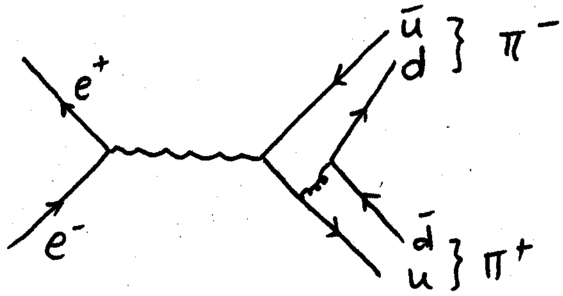
$e^- + e^+ \to \tau^- + \tau^+$

a) Possible. The leading interaction is electromagnetic



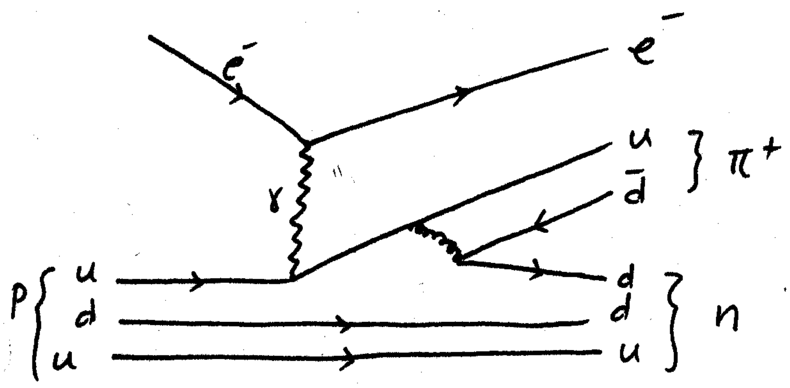
$e^- + e^+ \longrightarrow \pi^- + \pi^+$

b) Possible. Electromagnetic



$e^- + p \to e^- + n + \pi^+$

c) Possible. Electromagnetic

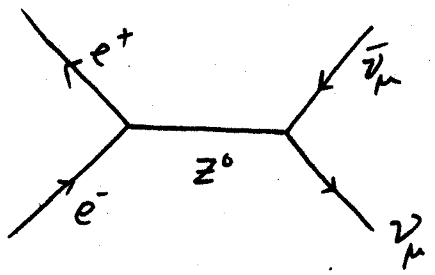


d) $\bar{\nu}_e + e^- \longrightarrow \nu_e + e^+$

Impossible. It violates conservation of charge.

e) $e^+ + e^- \longrightarrow \bar{\nu}_\mu + \nu_\mu$

possible. Weak interaction



f) $p + \bar{p} \longrightarrow \Sigma^+ + \Sigma^-$

Forbidden by baryon number conservation.
Recall that $\Sigma^+(uus)$ and $\Sigma^-(dds)$ are not antiparticles of each other.

g) $\Omega^- \longrightarrow \Xi^0 + K^-$

Forbidden by conservation of energy. The mass of Ω^-

$m_{\Omega^-} = 1672\,\text{MeV}$, while $m_{\Xi^0} = 1315\,\text{MeV}$, $m_{K^-} = 493\,\text{MeV}$

Thus

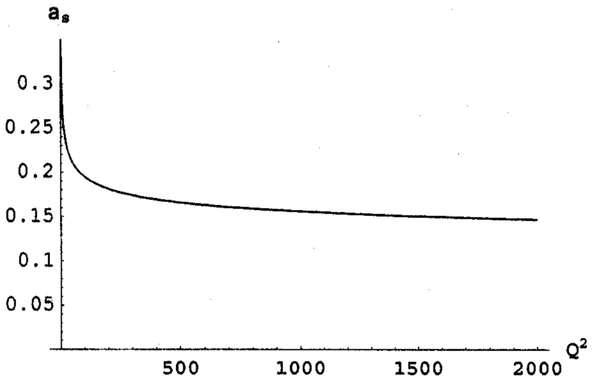
$$m_{\Omega^-} < m_{\Xi^0} + m_{K^-}$$

$\Rightarrow$ Decay cannot proceed this way.

$N_c = 3;\ N_f = 6;$

$\alpha_s[Qsqr\_,\ \Lambda\_] := \dfrac{12\,\pi}{(11\,N_c - 2\,N_f)\ \mathrm{Log}[\frac{Qsqr}{\Lambda^2}]}$

```
Plot[αs[Qsqr, 0.1], {Qsqr, 1, 2000}, PlotRange → {0, 0.35}, AxesLabel → {"Q²", "αs"}]
```



- Graphics -

```
αs[1, 0.1]
```

0.389822

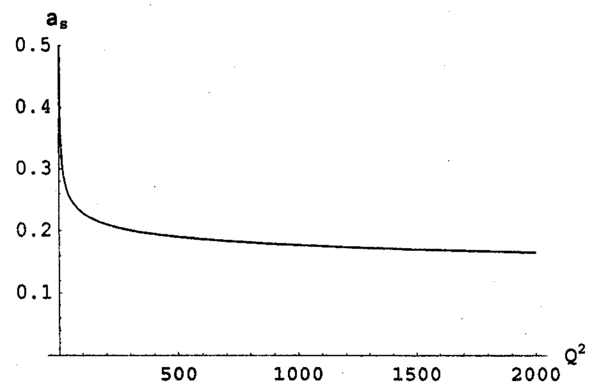
```
αs[10, 0.1]
```

0.259881

```
αs[100, 0.1]
```

0.194911

```
Plot[αs[Qsqr, 0.2], {Qsqr, 1, 2000}, PlotRange → {0, 0.5}, AxesLabel → {"Q²", "αs"}]
```



- Graphics -

```
αs[1, 0.2]
```

0.557709

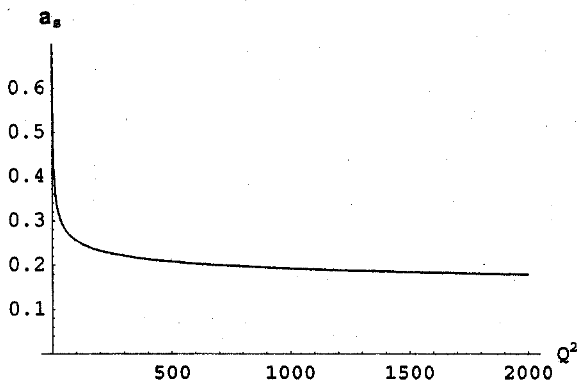
```
αs[10, 0.2]
```

0.325131

```
αs[100, 0.2]
```

0.229446

```
Plot[αs[Qsqr, 0.3], {Qsqr, 1, 2000}, PlotRange → {0, 0.7}, AxesLabel → {"Q²", "αs"}]
```



- Graphics -

```
αs[1, 0.3]
```

0.74553

```
αs[10, 0.3]
```

0.381103

```
αs[100, 0.4]
```

0.278854

QCD behaves like QED if: $2\,N_f - 11\,N_c > 0$.