King Fahd University of Petroleum and Minerals – Physics Department
PHYS 305 – Electricity and Magnetism I
Term 071 (Fall 2008) – Instructor: Dr. A. Mekki
Quiz # 6

Name Key ID #

A dielectric sphere of radius R and dielectric constant ε_r contains a uniform density of free charge ρ_f. Find
(a) The electric displacement inside and outside the sphere.
(b) The electric field inside and outside the sphere.
(c) The electric potential at the center of the sphere.

a) $r < R$ $\quad \oint \vec{D} \cdot d\vec{a} = Q_{f\,encl}$

$$D(4\pi r^2) = \rho_f \frac{4}{3}\pi r^3$$

$$\vec{D} = \frac{\rho_f}{3}\vec{r} = \frac{\rho_f}{3} r\,\hat{r}$$

$r > R$ $\quad \oint \vec{D} \cdot d\vec{a} = Q_{f\,encl}$

$$D(4\pi r^2) = \rho_f \frac{4}{3}\pi R^3$$

$$\vec{D} = \rho_f \frac{R^3}{3r^2}\hat{r}$$

b) $r < R$ $\quad \vec{D} = \varepsilon \vec{E} \Rightarrow \vec{E} = \frac{\rho_f}{3\varepsilon}\vec{r} = \frac{\rho_f}{3\varepsilon} r\,\hat{r}$

$r > R$ $\quad \vec{D} = \varepsilon_0 \vec{E} \Rightarrow \vec{E} = \frac{\rho_f R^3 \hat{r}}{3\varepsilon_0 r^2} = \frac{\rho_f R^3}{3\varepsilon_0 r^2}\hat{r}$

c) $$V(0) = -\int_\infty^0 \vec{E}\cdot d\vec{r} = -\int_\infty^R \vec{E}\cdot d\vec{r} - \int_R^0 \vec{E}\cdot d\vec{r}$$

$$= -\frac{\rho_f R^3}{3\varepsilon_0}\int_\infty^R \frac{1}{r^2}dr - \frac{\rho_f}{3\varepsilon}\int_R^0 r\,dr$$

$$= \frac{\rho_f R^3}{3\varepsilon_0}\left.\frac{1}{r}\right|_\infty^R - \frac{\rho_f}{3\varepsilon}\left.\frac{r^2}{2}\right|_R^0 = \boxed{\frac{\rho_f R^2}{3\varepsilon_0}\left(1 + \frac{1}{2\varepsilon_r}\right)}$$