



For the harmonic oscillator  $U(x) = \frac{1}{2} m \omega^2 x^2$

Schrodinger equation is

$$\frac{d^2 \psi(x)}{dx^2} = -\frac{2m}{\hbar^2} \left( \frac{1}{2} m \omega^2 x^2 - E \right) \psi(x)$$

The ground state wavefunction is

$$\psi_0(x) = \left( \frac{m\omega}{\pi\hbar} \right)^{1/4} e^{-\frac{m\omega}{2\hbar} x^2} = C_0 e^{-\alpha x^2}$$

The ground state energy is

$$E_0 = \frac{1}{2} \hbar \omega$$

The first excited state wavefunction is

$$\psi_1(x) = C_0 x e^{-\alpha x^2}$$

and

$$E_1 = \frac{3}{2} \hbar \omega$$