Solution of equation (3) are the "associated Legendre polynomials". See Table 7.2 in your textbook. They are called $P_\ell^{m_\ell}(\cos\theta)$

The product $\Theta(\theta)\,\Phi(\phi)$ specify the full angular dependence of the central force wavefunction and are know as "spherical harmonics", denoted $Y_\ell^{m_\ell}(\theta,\phi)$. See Table 7.3 in your text book.

In our case $|\vec{L}|$, $L_z$ and $E$ are sharp observables

$$|\vec{L}| = \sqrt{\ell(\ell+1)}\,\hbar \qquad \ell = 0, 1, 2, \ldots$$

$$L_z = m_\ell \hbar \qquad -\ell \le m_\ell \le \ell$$

$\ell$: "orbital quantum number"

$m_\ell$: "magnetic quantum number"

"Angular momentum and its z-component are QUANTIZED"

So: $\psi_{n\ell m_\ell}(\vec{r}) = R_{n\ell}(r)\, Y_\ell^{m_\ell}(\theta,\phi)$

Total wavefunction — radial wave — angular wave