In the case of central forces acting on a particle we use spherical spherical coordinates for the Laplacian.

Schrodinger equation is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial\psi}{\partial\theta}$$

$$+\frac{2\mu}{\hbar^2}\left[E-U\right]\psi=0 \quad —(1)$$

The wave function $\psi(r,\theta,\phi) = R(r)\,\Theta(\theta)\,\Phi(\phi)$

This leads to three differential equations

$$\frac{d^2\Phi}{d\phi^2}+m_\ell^2\,\Phi=0 \quad —(2)$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\left[\ell(\ell+1)-\frac{m_\ell^2}{\sin^2\theta}\right]\Theta=0 \quad —(3)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2\mu}{\hbar^2}\left[E-U-\frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}\right]R=0 \quad —(4)$$

Solving the Schrodinger equation (1) means solving the above three equations.

Solution of equation (2) is $\Phi(\phi) = e^{i m_\ell \phi}$