5. The electron in a hydrogen atom is in a state described by the wave function $\psi_{322}(\vec{r})$.

(a) What is the magnitude of the orbital angular momentum of the electron?

(b) What is the angle between the angular momentum vector and the z-axis?

(c) Calculate $\langle r \rangle$ for the electron in this state.

$$R_{32} = \left(\frac{1}{3a_0}\right)^{3/2} \frac{2\sqrt{2}}{27\sqrt{5}} \left(\frac{r}{a_0}\right)^2 e^{-\frac{r}{3a_0}}$$

and $a_0 = 0.53$ Å is the Bohr radius.

Given: $\int_0^\infty x^n e^{-x} dx = n!$

(Hint: change variable to $z = \frac{2r}{3a_0}$ in (c))

(a) $\ell = 2 \Rightarrow |\vec{L}| = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{6}\,\hbar = \boxed{2.57\times10^{-34}\ \text{J.s}}$

$m_\ell = 2$

(b) $\cos\theta = \frac{L_z}{|\vec{L}|} = \frac{m_\ell \hbar}{\sqrt{\ell(\ell+1)}\,\hbar} = \frac{2}{\sqrt{6}} = 0.82$

$\Rightarrow \boxed{\theta = 35.3^\circ}$

(c) $\langle r \rangle = \int_0^\infty r\, P_{32}(r)\, dr$

$P_{32}(r) = r^2 |R_{32}(r)|^2 = r^2 \left(\frac{1}{3a_0}\right)^3 \frac{8}{27^2\times5} \left(\frac{r}{a_0}\right)^4 e^{-\frac{2r}{3a_0}}$

$= \frac{8}{27^3\times5} \frac{1}{a_0^7} r^6 e^{-\frac{2r}{3a_0}}$

$\Rightarrow \langle r \rangle = \frac{8}{27^3\times5} \frac{1}{a_0^7} \int_0^\infty r^7 e^{-\frac{2r}{3a_0}} dr$

let $z = \frac{2r}{3a_0} \Rightarrow dr = \frac{3a_0}{2} dz$

$\Rightarrow \langle r \rangle = \frac{8}{27^3\times5} \frac{1}{a_0^7} \left(\frac{3a_0}{2}\right)^8 \underbrace{\int_0^\infty z^7 e^{-z} dz}_{7!} = 10.5\, a_0 = \boxed{5.6\ \text{Å}}$