3. Consider a proton in a three dimensional box of lengths $L_1 = 0.1$ Å, $L_2 = 0.2$ Å, and $L_3 = 0.4$ Å. Find
   (a) The energy of the particle in the ground state.
   (b) The energy of the particle in the first excited state.
   (c) The normalized wave function of the particle in the first excited state. Is the level degenerate? Explain.

a) $$E = \frac{\pi^2 \hbar^2}{2m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right)$$

Ground state correspond to $n_1 = n_2 = n_3 = 1$

$$E = \frac{\pi^2 \hbar^2}{2m}\left(\frac{1}{L_1^2} + \frac{1}{L_2^2} + \frac{1}{L_3^2}\right)$$

$$= \frac{\pi^2 \times (1.05\times10^{-34})^2}{2 \times 1.67\times10^{-27}}\left(\frac{1}{(0.1)^2} + \frac{1}{(0.2)^2} + \frac{1}{(0.4)^2}\right)\times10^{20}$$

$$= 4.28\times10^{-19}\ \mathrm{J} = \boxed{2.67\ \mathrm{eV}}$$

b) The first excited state correspond to $n_1 = 1$, $n_2 = 1$, and $n_3 = 2$

$$E = \frac{\pi^2 \hbar^2}{2m}\left(\frac{1}{L_1^2} + \frac{1}{L_2^2} + \frac{4}{L_3^2}\right)$$

$$= \frac{\pi^2 (1.05\times10^{-34})^2}{2 \times 1.67\times10^{-27}}\left(\frac{1}{(0.1)^2} + \frac{1}{(0.2)^2} + \frac{4}{(0.4)^2}\right)\times10^{20}$$

$$= 4.89\times10^{-19}\ \mathrm{J} = \boxed{3.05\ \mathrm{eV}}$$

c) $$\psi_{112} = \sqrt{\frac{8}{0.8\times10^{-33}}}\ \sin\left(\frac{\pi\times10^{10}}{0.1}x\right)\ \sin\left(\frac{\pi\times10^{10}}{0.2}y\right)\ \sin\left(\frac{2\pi\times10^{10}}{0.4}z\right)$$

$$\boxed{\psi_{112} = 3.16\times10^{16}\ \sin\left(3.14\times10^{11}x\right)\ \sin\left(1.57\times10^{11}y\right)\ \sin\left(1.57\times10^{11}z\right)}$$