Pb # 6.

a) $$U = -\frac{k\alpha e^2}{r} + \frac{B}{r^m}$$

but $$B = \frac{\alpha k e^2}{m} r_0^{m-1}$$

$$\Rightarrow U = -\frac{k\alpha e^2}{r} + \frac{\alpha k e^2 r_0^{m-1}}{m\, r^m}$$

$$F = -\frac{dU}{dr} = -\frac{k\alpha e^2}{r^2} + \alpha \frac{k e^2 r_0^{m-1}}{r^{m+1}}$$

$$\boxed{F = -\frac{k\alpha e^2}{r^2}\left(1 - \left(\frac{r_0}{r}\right)^{m-1}\right)}$$

b) replace r by $r_0 + x$ where x is small

$$F = -\frac{k\alpha e^2}{(r_0+x)^2}\left[1 - \left(\frac{r_0}{r_0+x}\right)^{m-1}\right] \qquad \text{note that } \frac{x}{r_0} \ll 1$$

$$= -\frac{k\alpha e^2}{r_0^2\left(1+\frac{x}{r_0}\right)^2}\left[1 - \frac{r_0^{m-1}}{r_0^{m-1}\left(1+\frac{x}{r_0}\right)^{m-1}}\right]$$

$$= -\frac{k\alpha e^2}{r_0^2\left(1+\frac{x}{r_0}\right)^2}\left[1 - \frac{1}{\left(1+\frac{x}{r_0}\right)^{m-1}}\right]$$

$\frac{x}{r_0} = \delta \ll 1$ $\qquad (1+\delta)^2 = 1 - 2\delta$

$$(1+\delta)^{m-1} = 1 + (m-1)\,(\cdot$$