Pb # 22

$$\psi(r,\theta,\phi) = R_{n\ell}(r)\, Y_\ell^{m_\ell}(\theta,\phi)$$

$\psi_{2s} \Rightarrow n=2 \quad \ell=0 \quad m_\ell=0$

$$\psi_{2s} = R_{20}(r)\, Y_0^0(\theta,\phi) = \frac{1}{2\sqrt{\pi}}\left(\frac{1}{2a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}$$

Note that ψ_{2s} does not depend on θ and ϕ!

at $r = a_0$

$$\psi_{2s} = \frac{1}{2\sqrt{\pi}}\left(\frac{1}{2a_0}\right)^{3/2}\left(2-\frac{a_0}{a_0}\right)e^{-\frac{a_0}{2a_0}}$$

$a_0 = 0.529$

$$\psi_{2s} = \frac{1}{2\sqrt{\pi}}\left(\frac{1}{2\times 0.529\times 10^{-10}}\right)^{3/2} e^{-\frac{1}{2}} = 1.57\times 10^{+14}\,(m^{-3/2})$$

↑ unit

b) $|\psi_{2s}(a_0)|^2 = 2.47\times 10^{28}\ m^{-3}$

c) $P_{2s}(a_0) = |\psi_{2s}(a)|^2\, 4\pi a_0^2$

$= 8.7\times 10^{8}\ m^{-1}$

Pb # 25.

$P_{1s} = \frac{4}{a_0^3} r^2 e^{-\frac{2r}{a_0}}$ for hydrogen atom in the ground state.

$$\langle U\rangle = \int_0^\infty U\, P_{1s}\, dr$$

$$U = -\frac{ke^2}{r} \Rightarrow \langle U\rangle = -ke^2\frac{4}{a_0^3}\int_0^\infty r\, e^{-\frac{2r}{a_0}}\, dr$$

Let $z = \frac{2r}{a_0} \Rightarrow dr = \frac{a_0}{2}\, dz$ and $r = \frac{a_0}{2} z$