$$\pi \int_0^\infty \frac{1}{\pi} \frac{1}{a_0^3} e^{-\frac{2r}{a_0}} r^2 \, dr = \frac{4}{a_0^3} \int_0^\infty r^2 e^{-\frac{2r}{a_0}} dr$$

change of variables $z = \frac{2r}{a_0} \Rightarrow dr = \frac{a_0}{2} dz \; ; \; r^2 = \left(\frac{a_0}{2}\right)^2 z^2$

$$\Rightarrow \frac{4}{a_0^3} \left(\frac{a_0}{2}\right)^3 \underbrace{\int_0^\infty z^2 e^{-z} dz}_{(z^2+2z+2)e^{-z}\big|_0^\infty = 2} \Rightarrow \frac{8}{a_0^3} \times \frac{a_0^3}{8} = 1$$

$\Rightarrow$ The wavefunction is normalized !

b # 19.

6g state

a) $n = 6$

b) $E = -\frac{13.6}{n^2} (eV) = -\frac{13.6}{64} = \boxed{-0.378\,eV}$

c) $\ell = 4 \Rightarrow L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{20}\,\hbar = \boxed{4.72 \times 10^{-34}\ J.s}$

d) $-\ell \le m_\ell \le \ell \Rightarrow m_\ell = 4, 3, 2, 1, 0, -1, -2, -3, -4.$

$L_z = m_\ell \hbar$

$m_\ell = 4$ $\quad L_z = 4\hbar = 4.22 \times 10^{-34}$ $\quad \cos\theta = \frac{L_z}{|\vec{L}|} = \frac{m_\ell}{\sqrt{\ell(\ell+1)}}$

$\cos\theta = \frac{4}{\sqrt{20}} = 0.89 \Rightarrow \boxed{\theta = 26.6°}$

$m_\ell = 3$ $\quad L_z = 3\hbar = 3.165 \times 10^{-34}$ $\quad \cos\theta = \frac{3}{\sqrt{20}} = 0.671$

$\Rightarrow \boxed{\theta = 47.9°}$

etc...