Homework Solution
Chapter 7

Pb # 3.

A particle of mass m in a three dimensional box of side L.

The energy of the particle can be calculated from

$$E = \frac{\hbar^2 \pi^2}{2mL^2}(n_1^2 + n_2^2 + n_3^2) = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

The ground state correspond to $n_1 = n_2 = n_3 = 1 \Rightarrow n^2 = 3$

The first excited state correspond to $n_1 = 1, n_2 = 1, n_3 = 2$

$\Rightarrow n^2 = 6$

The second excited state correspond to $n_1 = 2, n_2 = 1, n_3 = 2$

$\Rightarrow n^2 = 9$

The third excited state correspond to $n_1 = 3, n_2 = 1, n_3 = 1$

$\Rightarrow \boxed{n^2 = 11}$

$$E = \frac{11\,\pi^2\hbar^2}{2mL^2}$$

Combinations are $n_1 = 3, n_2 = 1, n_3 = 1$

$n_1 = 1, n_2 = 3, n_3 = 1$

$n_1 = 1, n_2 = 1, n_3 = 3$

The state is 3-fold degenerate.

c) $\psi_{311} = \left(\frac{2}{L}\right)^{3/2} \sin\left(\frac{3\pi}{L}x\right) \sin\left(\frac{\pi}{L}y\right) \sin\left(\frac{\pi}{L}z\right)$

$\psi_{131} = \left(\frac{2}{L}\right)^{3/2} \sin\left(\frac{\pi}{L}x\right) \sin\left(\frac{3\pi}{L}y\right) \sin\left(\frac{\pi}{L}z\right)$

$\psi_{113} = \left(\frac{2}{L}\right)^{3/2} \sin\left(\frac{\pi}{L}x\right) \sin\left(\frac{\pi}{L}y\right) \sin\left(\frac{3\pi}{L}z\right)$