

$$\langle x \rangle = \frac{L}{\pi^2} \left[\underbrace{\int_0^\pi \theta d\theta}_{=\frac{\pi^2}{2}} - \underbrace{\int_0^\pi \theta \cos 2n\theta d\theta}_{\substack{\downarrow \\ \text{2nd term integrate by parts} \\ \downarrow}}$$

$$\left. \begin{array}{l} u = \theta \\ dv = \cos 2n\theta d\theta \end{array} \right\} \int u dv = uv - \int v du$$

$$\Rightarrow \text{2nd term} = \frac{\theta}{2n} \sin 2n\theta \Big|_0^\pi - \frac{1}{n} \int_0^\pi \sin 2n\theta d\theta$$

$$\Rightarrow \text{2nd term} = 0 \quad \underbrace{\frac{1}{2n^2} \cos 2n\theta \Big|_0^\pi = 0}$$

$$\Rightarrow \langle x \rangle = \frac{L}{\pi^2} \times \frac{\pi^2}{2} = \frac{L}{2} \Rightarrow \boxed{\langle x \rangle = \frac{L}{2}}$$

$$\langle x^2 \rangle = \frac{L^2}{\pi^3} \left(\underbrace{\int_0^\pi \theta^2 d\theta}_{=\frac{\pi^3}{3}} - \underbrace{\int_0^\pi \theta^2 \cos 2n\theta d\theta}_{\downarrow}$$

$$\int_0^\pi \theta^2 \cos 2n\theta d\theta = \frac{\theta^2}{2n} \sin 2n\theta \Big|_0^\pi - \frac{1}{n} \int_0^\pi \theta \sin 2n\theta d\theta$$

$$- \frac{1}{n} \left[\frac{\theta}{2n} \cos 2n\theta \Big|_0^\pi + \frac{1}{2n} \int_0^\pi \cos 2n\theta d\theta \right]$$

$$= -\frac{1}{n} \times \left(-\frac{\pi}{2n} \right) = \frac{\pi}{2n^2}$$

$$\Rightarrow \langle x^2 \rangle = \frac{L^2}{\pi^3} \left[\frac{\pi^3}{3} - \frac{\pi}{2n^2} \right] = \boxed{\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}}$$