b)
$$1 = \int_{-\infty}^{+\infty} \psi^2_{(x)} \, dx = C^2 \int_{-\infty}^{+\infty} x^2 e^{-2\alpha x^2} dx$$

$$= 2C^2 \int_0^{\infty} x^2 e^{-2\alpha x^2} dx$$

From pb. # 33

$$\int_0^{\infty} x^2 e^{-ax^2} dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}} \qquad a > 0$$

In our case $a = 2\alpha$

$$\Rightarrow 1 = 2C^2 \frac{1}{8\alpha}\sqrt{\frac{\pi}{2\alpha}} \Rightarrow C^2 = \sqrt{\frac{32\alpha^3}{\pi}}$$

$$\Rightarrow \boxed{C = \left(\frac{32\alpha^3}{\pi}\right)^{1/4}} \text{ with } \alpha = \frac{m\omega}{2\hbar}$$

b # 31.

For an infinite square well $\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$

$$\langle x \rangle = \int_0^L x \, \psi^2_{(x)} \, dx = \frac{2}{L} \int_0^L x \sin^2\left(\frac{n\pi}{L} x\right) dx$$

Change variables $\theta = \frac{\pi}{L} x$ $\quad d\theta = \frac{\pi}{L} dx \Rightarrow dx = \frac{L}{\pi} d\theta$

$x = 0 \rightarrow \theta = 0$ $\qquad$ and $x = \frac{L}{\pi} \theta$

$x = L \rightarrow \theta = \pi$

$$\Rightarrow \langle x \rangle = \frac{2}{L} \frac{L^2}{\pi^2} \int_0^{\pi} \theta \sin^2 n\theta \, d\theta$$

but $2\sin^2\alpha = 1 - \cos 2\alpha$