Pb# 25.

The wavefunction $\psi(x) = C x e^{-\alpha x^2}$ describe the first excited state of the quantum oscillator.

1) Schrodinger equation in this case is

$$\frac{\partial^2 \psi(x)}{\partial x^2} = \frac{2m}{\hbar^2}\left(\frac{1}{2} m\omega^2 x^2 - E\right)\psi(x) \quad \text{———} \quad (1)$$

$$\frac{\partial \psi}{\partial x} = C e^{-\alpha x^2} - 2C x^2 \alpha e^{-\alpha x^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = -2\alpha x C e^{-\alpha x^2} - 4C\alpha x e^{-\alpha x^2} + 4C x^3 \alpha^2 e^{-\alpha x^2}$$

$$= -6\alpha\left(C x e^{-\alpha x^2}\right) + 4\alpha^2 x^2\left(C x e^{-\alpha x^2}\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} = \left(-6\alpha + 4\alpha^2 x^2\right)\psi(x) \quad \text{———} \quad (2)$$

Compare eq. (1) and eq. (2)

$$\Rightarrow \quad \frac{2m}{\hbar^2} E = 6\alpha \quad \text{———} \quad (3)$$

$$\frac{m^2\omega^2}{\hbar^2} = 4\alpha^2 \quad \text{———} \quad (4)$$

$$\Rightarrow \quad \boxed{\alpha = \frac{m\omega}{2\hbar}}$$

and $\frac{2\cancel{m}E}{\hbar^2} = 6\alpha = \frac{6\cancel{m}\omega}{2\hbar}$

$$\Rightarrow \quad \boxed{E = \frac{3}{2}\hbar\omega}$$