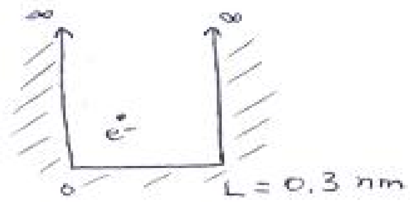
Pb # 16.



a) In general $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$

In the ground state $n=1$

$$\psi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L} x\right)$$

$$P = \int_0^{\frac{L}{3}} \psi_1^2(x)\, dx = \frac{2}{L} \int_0^{L/3} \sin^2\left(\frac{\pi}{L}x\right) dx$$

but $2\sin^2\theta = 1 - \cos 2\theta$

$$\Rightarrow P = \frac{2}{2L} \int_0^{L/3} \left[1 - \cos\left(\frac{2\pi}{L}x\right)\right] dx = \frac{1}{L}\left[x\Big|_0^{\frac{L}{3}} - \frac{L}{2\pi} \sin\left(\frac{2\pi}{L}x\right)\Big|_0^{\frac{L}{3}} \right]$$

$$P = \frac{1}{L}\left[\frac{L}{3} - \frac{L}{2\pi}\sin\left(\frac{2\pi}{3}\right)\right] = \frac{1}{3} - 0.138 = \boxed{19.5\%}$$

b) $n = 100$ $\quad \psi_{100}(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{100\pi}{L}x\right)$

$$P = \int_0^{L/3} \psi_{100}^2(x)\, dx = \frac{2}{L}\int_0^{L/3} \sin^2\left(\frac{100\pi}{L}x\right) dx$$

$$= \frac{1}{L}\int_0^{L/3}\left[1 - \cos\left(\frac{200\pi}{L}x\right)\right] dx$$

$$= \frac{1}{L}\left[x\Big|_0^{L/3} - \frac{L}{200\pi}\sin\left(\frac{200\pi}{L}x\right)\Big|_0^{L/3}\right]$$

$$= \frac{1}{L}\left[\frac{L}{3} - \frac{L}{200\pi}\sin\left(\frac{200\pi}{3}\right)\right] = \frac{1}{3} - \underbrace{\frac{1}{200\pi}\sin\left(\frac{200\pi}{3}\right)}_{\to 0}$$

$$\boxed{P = \frac{1}{3} \simeq 33\%}$$

This is also the classical probability! consistent with Bohr correspondence principle.